

Banach Afternoons

9 April 2021

14:45 – 15:25

Martina Maiuriello

15:35 – 16:05

Enrico Miglierina

16:20 – 17:00

Stefano Ciaci

17:10 – 17:30

Daniele Puglisi

16 April 2021

14:45 – 15:25

Paolo Leonetti

15:35 – 16:05

Elisabetta Maluta

16:15 – 16:55

Davide Ravasini

17:05 – 17:30

Simone Ferrari

JOIN ZOOM MEETING:

<https://us02web.zoom.us/j/84222158653>

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UNIVERSITÀ
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del Sacro Cuore



Schedule

9 April 2021

14:45 – 15:25 *Martina Maiuriello*

Dynamical behaviors on L^p spaces

15:35 – 16:05 *Enrico Miglierina*

Weak* fixed point property and the space of affine functions

16:20 – 17:00 *Stefano Ciaci*

A characterization of Banach spaces containing $\ell_1(\kappa)$
via ball-covering properties

17:10 – 17:30 *Daniele Puglisi*

Nonmeasurable Vitali set: Variations on theme

16 April 2021

14:45 – 15:25 *Paolo Leonetti*

Convergence of optimal paths to fixed points of correspondences

15:35 – 16:05 *Elisabetta Maluta*

Constant width and diametrically complete sets with empty interior

16:15 – 16:55 *Davide Ravasini*

Compactivorous sets in separable Banach spaces

17:05 – 17:30 *Simone Ferrari*

A topological approach to uniformly rotund renormings

Plenary talks

A characterization of Banach spaces containing $\ell_1(\kappa)$ via ball-covering properties

Stefano Ciaci (stefano.ciaci@ut.ee)

University of Tartu

In 1989, G. Godefroy proved that a Banach space contains an isomorphic copy of ℓ_1 if and only if it can be equivalently renormed to be octahedral. Recall that a normed space X is said to be octahedral if, for every finite dimensional subspace $E \subset X$ and $\varepsilon > 0$, there exists $x \in S_X$ such that for all $\lambda \in \mathbb{R}$ and $e \in E$

$$\|\lambda x + e\| \geq (1 - \varepsilon)(|\lambda| + \|e\|).$$

It is known that octahedral norms can be characterized by means of covering the unit sphere by a finite number of balls. This observation allows us to connect the theory of octahedral norms with ball-covering properties of Banach spaces introduced by L. Cheng in 2006. Following this idea, we extend G. Godefroy's result to higher cardinalities. Namely, for an infinite cardinal κ , we give an isomorphic characterization of Banach spaces containing a copy of $\ell_1(\kappa^+)$ by means of coverings of the unit sphere consisting of κ many balls.

Convergence of optimal paths to fixed points of correspondences

Paolo Leonetti (leonetti.paolo@gmail.com)

Università Bocconi

Given a correspondence Φ on a normed vector space X , a real-valued function u on X , and an ideal \mathcal{I} on the positive integers, we provide sufficient conditions to ensure that every sequence (x_n) in X which is feasible (i.e., $x_{n+1} \in \Phi(x_n)$ for all n) and maximizes the smallest \mathcal{I} -cluster point of the sequence $(u(x_n))$ has to be necessarily \mathcal{I} -convergent to an identified fixed point of Φ .

Dynamical behaviors on L^p spaces

Martina Maiuriello (martina.maiuriello@unicampania.it)

Università degli Studi della Campania "Luigi Vanvitelli"

Linear Dynamics is a relatively recent area of mathematics which lies at the intersection of Operator Theory and Dynamical Systems. We focus on important concepts in Linear Dynamics, such as hyperbolicity and shadowing, in particular for the class of the composition operators on L^p spaces. We provide an overview of basic concepts, tools, and of the main results in the literature concerning these fundamental topics. We introduce dissipative systems with bounded distortion generated by composition operators on L^p spaces, and, in this context, we present our results concerning the relations between hyperbolicity and shadowing. As it is well-known, hyperbolicity implies shadowing but the equivalence is not always true: an important bridge between them is the notion of generalized hyperbolicity. We show that the notions of generalized hyperbolicity and shadowing coincide and we provide tools for the construction of examples.

Compactivorous sets in separable Banach spaces

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Compactivorous sets were first introduced in [1] as a sufficient condition which guarantees the Haar nonnegligibility of a Borel set in a separable Banach space. A set E in a separable Banach space X is *compactivorous* if for every compact set K in X there is a nonempty, (relatively) open subset of K which can be translated into E . A few characterisations of this property are investigated.

- [1] J. Esterlé, É. Matheron, and P. Moreau, “Haar negligibility of positive cones in Banach spaces”, *St. Petersburg Mathematical Journal* **27.5**, 731–756 (2016). DOI: <https://doi.org/10.1090/spmj/1414>.

Contributions

Weak* fixed point property and the space of affine functions

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First we prove that if a separable Banach space X contains an isometric copy of an infinite-dimensional space $A(S)$ of affine continuous functions on a Choquet simplex S , then its dual X^* lacks the weak* fixed point property for nonexpansive mappings. Then, we show that the dual of a separable L_1 -predual X fails the weak* fixed point property for nonexpansive mappings if and only if X has a quotient isometric to some infinite-dimensional space $A(S)$. Moreover, we provide an example showing that “quotient” cannot be replaced by “subspace”. Finally, it is worth to be mentioned that in our characterization the space $A(S)$ cannot be substituted by any space $\mathcal{C}(K)$ of continuous functions on a compact Hausdorff K . (Joint work with Emanuele Casini and Łukasz Piasecki)

A topological approach to uniformly rotund renormings

Simone Ferrari (simone.ferrari@unisalento.it)

Università del Salento

Recall that a norm on a Banach space X is said to be uniformly rotund (UR, for short) if $\|x_n - y_n\| \rightarrow 0$, whenever $2\|x_n\|^2 + 2\|y_n\|^2 - \|x_n + y_n\|^2 \rightarrow 0$. This type of norms have been widely studied since the mid-thirties [1] and, maybe, the most famous result is the characterization of the existence of such type of renormings due to James [5], Enflo [2] and Pisier [6], where the authors prove that the existence of an UR renorming is equivalent to the fact that X is superreflexive.

Following the results obtained in some previous papers [3–4], written in collaboration with M. Raja and J. Orihuela (Univesidad de Murcia), we will show that the presence of a metric on the unit sphere generating the weak topology is equivalent to the existence of an equivalent norm satisfying a UR-type property.

- [1] J.A. Clarkson, “Uniformly convex spaces”, *Trans. Amer. Math. Soc.* **40**, 396–414 (1936).
- [2] P. Enflo, “Banach spaces which can be given an equivalent uniformly convex norm”, *Israel J. Math.* **13**, 281–288 (1972).

- [3] S. Ferrari, J. Orihuela, M. Raja, "Weakly metrizable spheres and renormings of Banach spaces", *Q. J. Math.* **67**, 15–27 (2016).
 - [4] S. Ferrari, J. Orihuela, M. Raja, "Generalized metric properties of spheres and renormings of normed spaces", *RACSAM* **113**, 2655–2663 (2019).
 - [5] R.C. James, "Uniformly nonsquare Banach spaces", *Ann. Math.* **80**, 542–550 (1964).
 - [6] G. Pisier, "Martingales with values in uniformly convex spaces" *Israel J. Math.* **20**, 326–350 (1975).
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Constant width and diametrically complete sets with empty interior

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We expect sets of constant width, i.e., sets such that the distance between any pair of parallel hyperplanes supporting the set is constant, to be "plump" sets, and this is obviously true in finite dimensional spaces. In infinite dimensional spaces this is not necessarily true and, in c_0 , a prototypical example of a constant width set with empty interior has been known for more than thirty years. Existence of such sets in some reflexive spaces has been proved only in 2019, and is guaranteed upon a suitable renorming of the space.

If we look instead for sets with empty interior in the larger class of diametrically complete sets, i.e. sets that are not strictly contained in any set with the same diameter, several results are known both in the isomorphic and in the isometric setting.

Nonmeasurable Vitali set: Variations on theme

Daniele Puglisi (dpuglisi@dmi.unict.it)

Università di Catania

We extend Vitali's procedure to get new nonmeasurable sets. Assume that G is a group acting on a Polish space. Given a subgroup H of G , by a closed H -selection we mean a closed selection on each orbit of H . We investigate measurability properties of closed H -selections with respect to G -invariant measures.