Uncertainty, Risk, and Rationality

Ciro De Florio[1] with Luca Zanetti[2]

[1]Università Cattolica di Milano – Faculty of Economics
[2]Politecnico di Milano – Dept. of Civil and Environmental Engineering (DICA)

March 18, 2022

Outline

1. Introduction
2. First Part: Meta-reflections about uncertainty, risk, and probability
3. Second part: Uncertainty, Probability, and Normal Risk

A plea for serious philosophy

A spectre is haunting Italy. An unhealthy idea of philosophy:

- Obscure and evocative language
- All-ology
- Snobbish refusal to get their hands dirty with empirical data
- Allergy to clarity, definitions, and logic
- ...

A plea for serious philosophy

However, there is a **better** idea of philosophy (and it has a history of 2700 years)

- Rational investigation about the structural features of reality
- Specializations
- Deep connections with empirical research
- Use of mathematical models (formal logic) to characterize the concepts at play
- ...

First Part: Meta-reflections about uncertainty, risk, and probability

Uncertainty

A standard distinction is science:

1. **aleatoric uncertainty**: due to the randomness of natural phenomena; does not decrease over time;
2. **epistemic uncertainty**: due to limited knowledge, incomplete or insufficient data, etc.; can decrease over time.

A third kind of uncertainty (recently discussed in philosophy):

3. **normative uncertainty**: concerns which decisions are rational in a given situation.

Decision Theory

The interpretation of the uncertainty, its representation, its measure are fundamental ingredients of **Decision Theory**. But this is relevant not only for (the foundations of) economics;

- Public rational choices
- Management of risk
- General idea of rationality
- ...

Uncertainty

Let us assume that the representations of the world show features of uncertainty.

Example

I do not know what colour Anna's tends are. So my representation of Anna's house is **uncertain** with respect to the color of the tends. Let us notice that I am certain that Anna's tends are coloured.

Uncertainty

Problem: is the **world indeterminate**? In other words, the uncertainty of our representation mirrors (or is grounded in) the indeterminacy of the world?

This is a 2500 years old metaphysical question; there have been at least two turns of events:

- Around XVII century, with the emergence of modern science
- At the beginning of XX century, with the quantum revolution

And it is not a case that the concept of probability played a prominent role in both the scientific revolutions.

Probability

There are many ways to catch the uncertainty of our representations. By far, the best is the concept of **probability**.

Based on the concept of probability and on its mathematical properties, there are many models of rational decision.

But what is the **conceptual space** of the relationships between uncertainty and its probabilistic rendering? (Let us notice the **meta-theoretical** flavour of this question)

I Model: Standard View

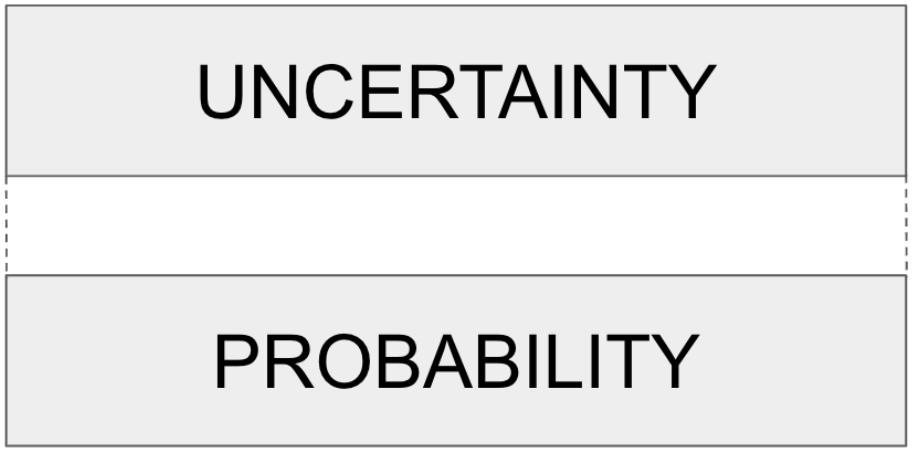


Figure: I Model

The concept of probability is able to characterize all the amount of uncertainty within (our representation of) the world. **Monistic** conception of risk: risk **is** probability.

I Model: Standard View

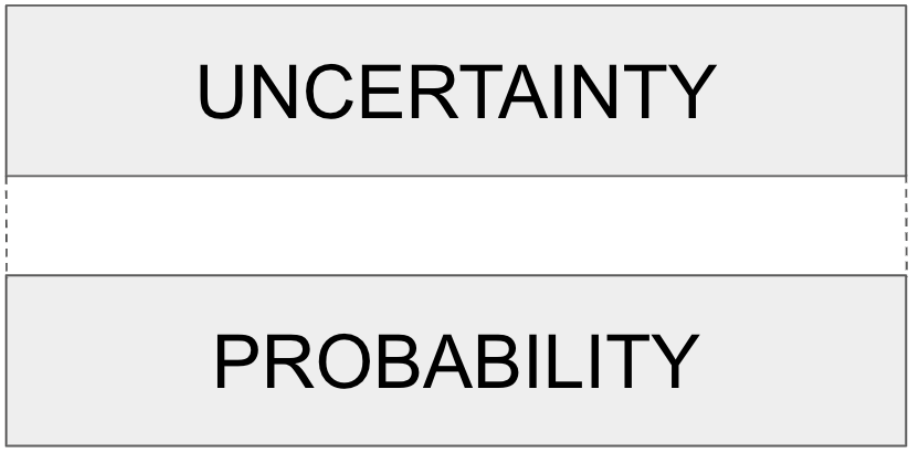


Figure: I Model

It follows that, if our judgements do not fit with the probabilistic framework, we are wrong. See for instance the classical gambler's fallacy. The uncertainty of the fact that rolling a fair dice will come out six is **fully captured** by the sentence

$$Pr(six) = 1/6$$

II Model: Incomplete Standard View

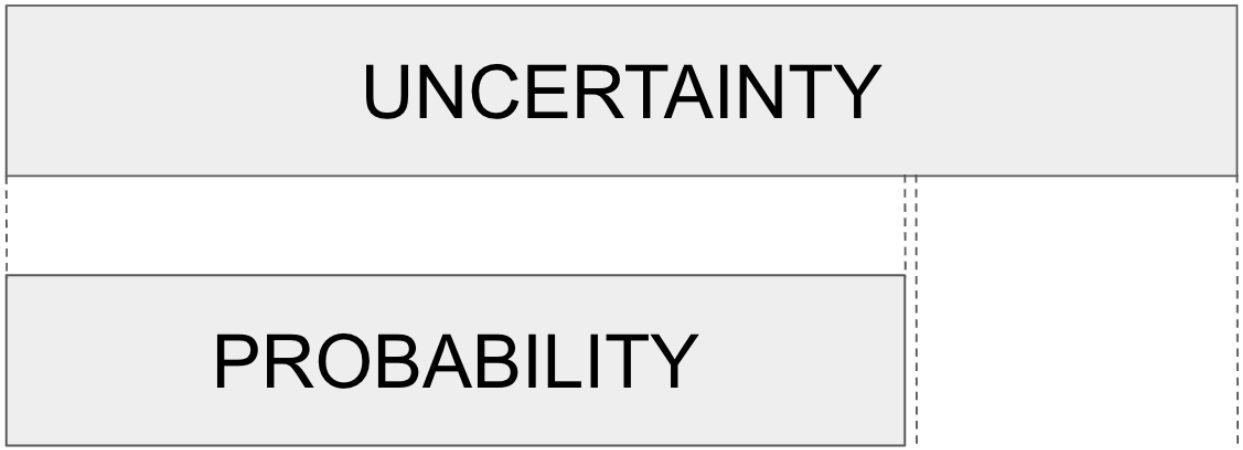


Figure: II Model

Not all the uncertainty can be captured in probabilistic terms; there are events to which is **impossible**, or it has **no sense**, to apply a probability measure.

II Model: Incomplete Standard View

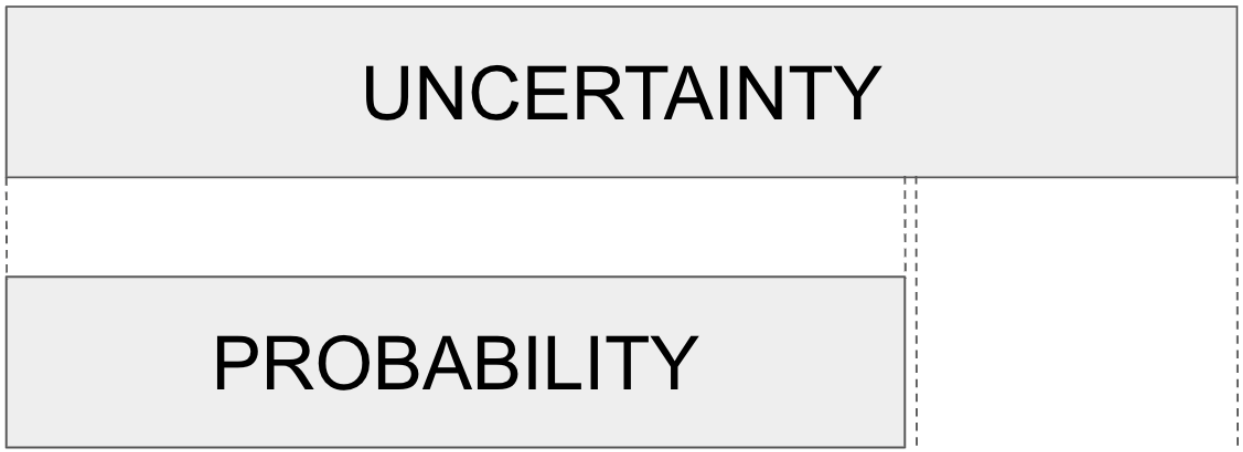


Figure: II Model

For an orthodox frequentist, this model is rather natural. It is uncertain if John will be happy by marrying Mary but it is hard to attribute an objective probability to that event.

II Model: Incomplete Standard View A-variant

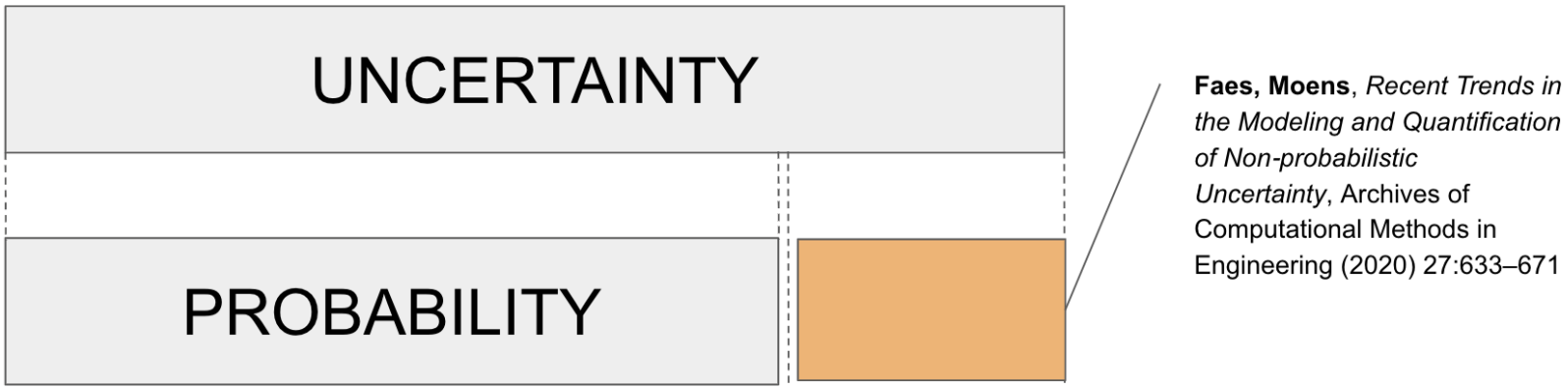


Figure: II Model (a)

There are other theoretical devices for measuring the uncertainty.

II Model: Incomplete Standard View B-variant

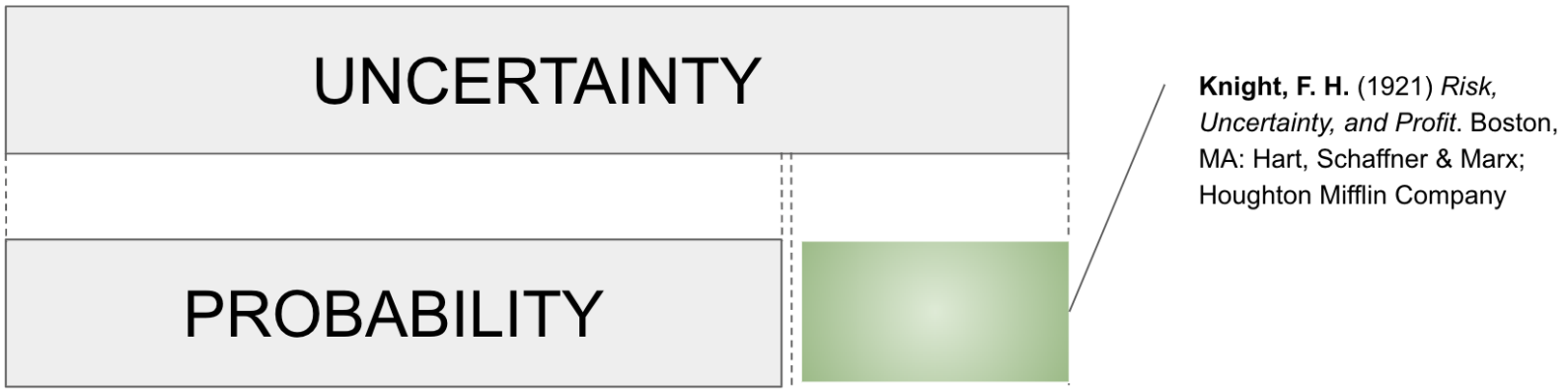


Figure: II Model (b)

There is no way to treat this uncertainty; this is the **radical** uncertainty

II Model: Incomplete Standard View B-variant

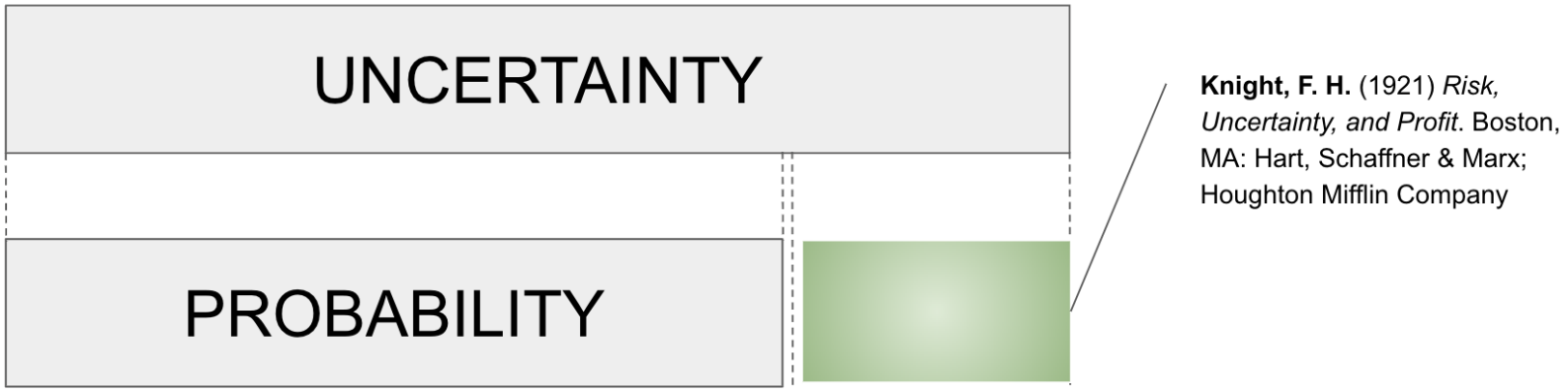


Figure: II Model (b)

Problem: how to set a decision theory framework under radical uncertainty? See, for instance, Hansson[1996]

III Model: Conflictual Overlapping

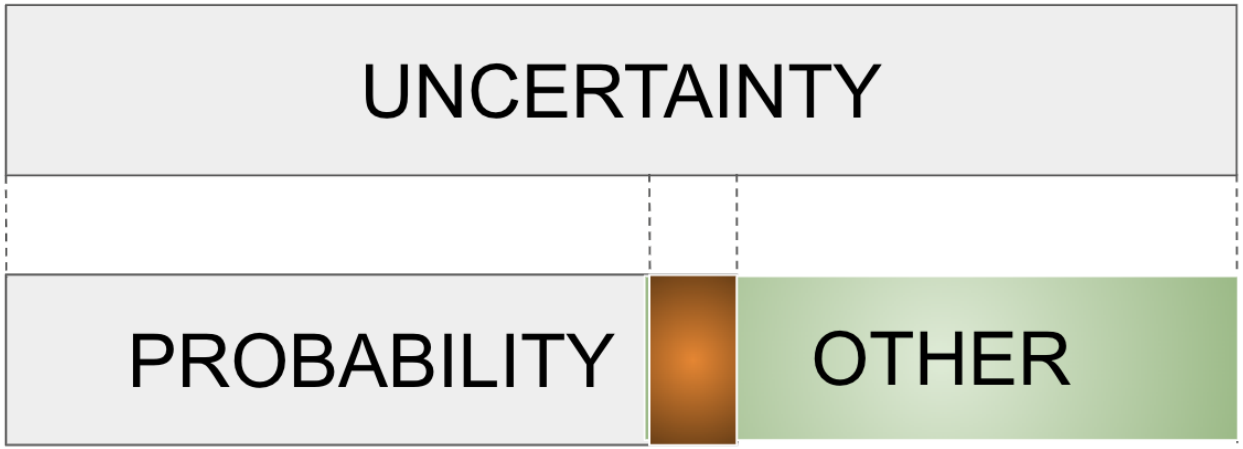
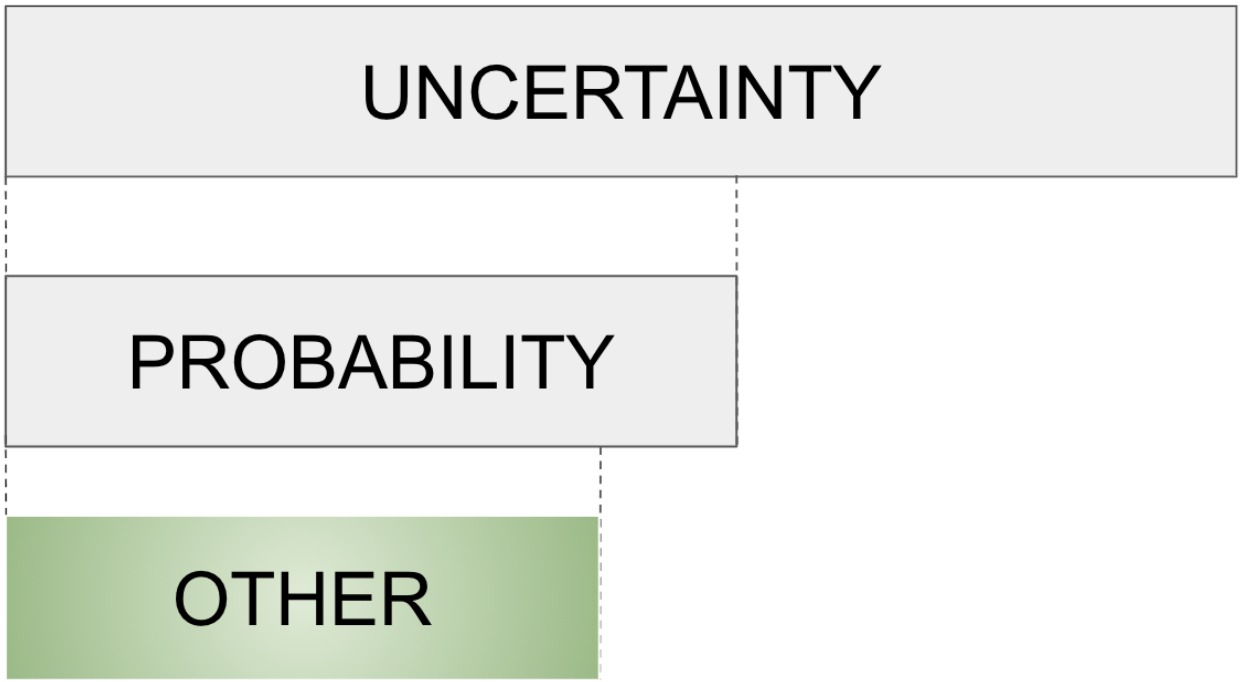


Figure: III Model

There is one (or more) approaches to the uncertainty that **conflict** with the probability approach.

IV Model: Conservative Pluralism



All (or part of) the uncertainty can be captured by the concept of probability (**conservativity** feature); however there cases in which:

- The probability ascriptions are identical
- There seem to be good reasons to not consider the two events as equally risky

A Step Back: Risk

Risk is a **many-faced** concept (Hansson[2007])

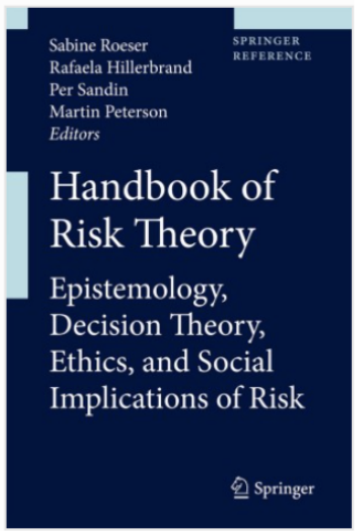
- risk_1 An unwanted event that may or may not occur;
- risk_2 The cause of an unwanted event that may occur;
- risk_3 The **probability** that the unwanted event occurs;
- risk_4 The **expected value** of the unwanted event (probability $\times$ absolute value);
- risk_5 Decisions under conditions of **risk** (known probabilities) vs. decisions under conditions of uncertainty (unknown probabilities)

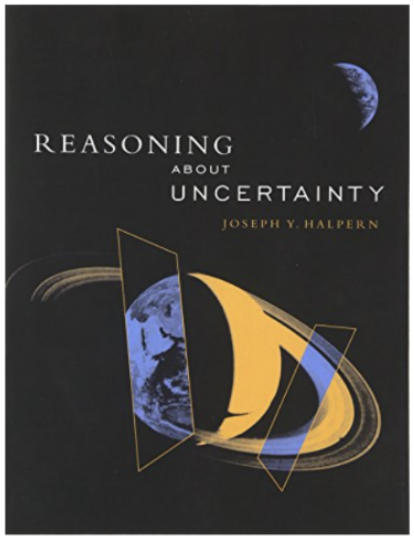
"There are **known knowns***; there are things we know we know. We also know there are* **known unknowns***; that is to say we know there are some things we do not know. But there are also* **unknown unknowns** *– the ones we don't know we don't know"*

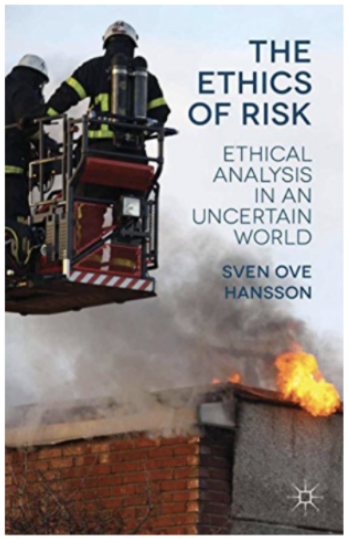
(Donald Rumsfeld, former US Secretary of Defence)

Epistemology (and Logic) of Risk

Growing and interesting literature on the epistemology of risk; many case-studies can be included in the fourth model we presented.







Epistemology (and Logic) of Risk

Typical case-study: Subjects are presented with two scenarios as the following:

Bomb1

An evil scientist has rigged up a large bomb, which he has hidden in a populated area. If the bomb explodes, many people will die. There is no way of discovering the bomb before the time it is set to detonate. The bomb will only detonate, however, if a set of six specific numbers between 1 and 49 come up on the next national lottery draw. The odds of these numbers appearing is fourteen million to one. It is not possible to interfere with this lottery draw.

Epistemology (and Logic) of Risk

Typical case-study: Subjects are presented with two scenarios as the following:

Bomb2

Same as above, however the bomb will only detonate if a series of three highly unlikely events obtains. First, the weakest horse in the field at the Grand National, Lucky Loser, must win the race by at least ten furlongs. Second, the worst team remaining in the FA Cup draw, Accrington Stanley, must beat the best team remaining, Manchester United, by at least ten goals. And third, the Queen of England must spontaneously choose to speak a complete sentence of Polish during her next public speech. The odds of this chain of events occurring are fourteen million to one.

Epistemology (and Logic) of Risk

Even if the two scenarios have, by stipulation, the same probability, there is **empirical evidence** that subjects tend to claim that Bomb 2 is **less risky**; in other words, they consider Bomb 1 more likely to happen and, since the harmfulness of the event is the same, Bomb 1 is riskier. It is important to notice that the subjects **do** know that the two events have the same probability.

$$Pr(\text{Bomb1}) = Pr(\text{Bomb2}) = n$$

nevertheless

$$Risk(\text{Bomb1}) \neq Risk(\text{Bomb2})$$

Epistemology (and Logic) of Risk

There is a family resemblance between this researches and the well-known **Ellsberg** and **Allais** paradoxes. Even in those cases, we have pairs of situations that Expected Utility Maximization Model prescribes to treat as indifferent but that subjects consider **different**.

Epistemology (and Logic) of Risk

Possible reactions:

1. It's a mistake; subjects should consider the scenarios as perfectly indifferent. We are irrational, *c'est la vie*.
2. Well, it's a mistake but it is **systematic** mistake. And as such, we must **explain** this kind of bias.
3. Well, it is not a mistake. After all, the model is conservative on the probability ascriptions. But there are **good reasons** not to consider the alternatives as indifferent.

Which are these alleged good reasons? How to characterize them in a mathematically acceptable framework?

Second part: Uncertainty, Probability, and Normal Risk

Summing up:

- We have pairs of events (scenarios, situations, as you like) ϕ, ψ such that: $Pr(\phi) = Pr(\psi)$; therefore, our *credence* should be:
- $cr(\phi) = cr(\psi)$. This follows from Lewis' Principal Principle. Constraint on rationality.
- Moreover, since $Pr(\phi) = Pr(\psi)$, we have that $Pr(\neg\phi) = Pr(\neg\psi)$, and, again, $cr(\neg\phi) = cr(\neg\psi)$.
- However, we shall investigate a kind of counter-example.

Another case-study

Another (less dramatic) case-study. Two scenarios:

Lottery

Paula bought a lottery ticket. After the draw, Paula has the only winning ticket. Yippee! (ϕ)

Forecast

Tomorrow, 15th of July, 3 feets of snow will fall in Rome. (ψ)

Another case-study

Another (less dramatic) case-study.
The two events are, obviously, rather rare. Let us **stipulate** that $Pr(\phi) = Pr(\psi)$. There could be reasons to consider ψ less risky than ϕ.

What reasons?

- **Empirical reasons** (test)
- **Linguistic evidence**. Different reactions to:
  - (Neg-Lottery) It is not true that Paula win the lottery
  - (Neg-Forecast) It is not true that tomorrow, 15 July,3 feet of snow will fall in Rome

Formal framework: Ingredients

Concept of history: a way in which the things may possibly go.

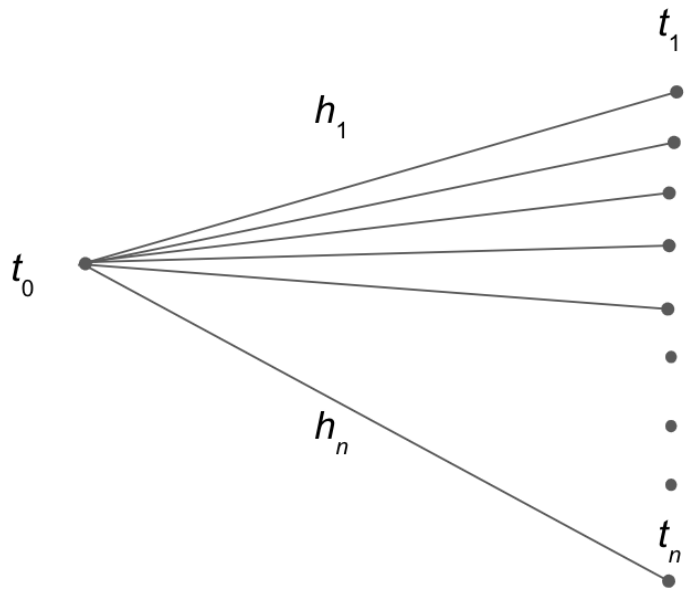


Figure: Bundle of histories

Formal framework: Ingredients

Some of these evolutions can be considered **normal** on the basis of **what we know**. On the basis of **information** we have, we can consider normal an history in which the sun rises tomorrow, the Italian prime minister is still Mario Draghi, and so on.

Let $\mathcal{I}$ be the collection of our available information:

$$\mathcal{I} = \{I_1, I_2, ..., I_n, ...\}$$

Let us define a function p which maps instants of time and information set on subsets of the available histories at t:

$$p : \mathbb{T} \times \mathcal{I} \mapsto \wp(\mathcal{H}_t)$$

The idea is that p selects the normal histories at t on the basis of the information we have.

Formal framework: Ingredients

Concept of history: a way in which the things may possibly go.

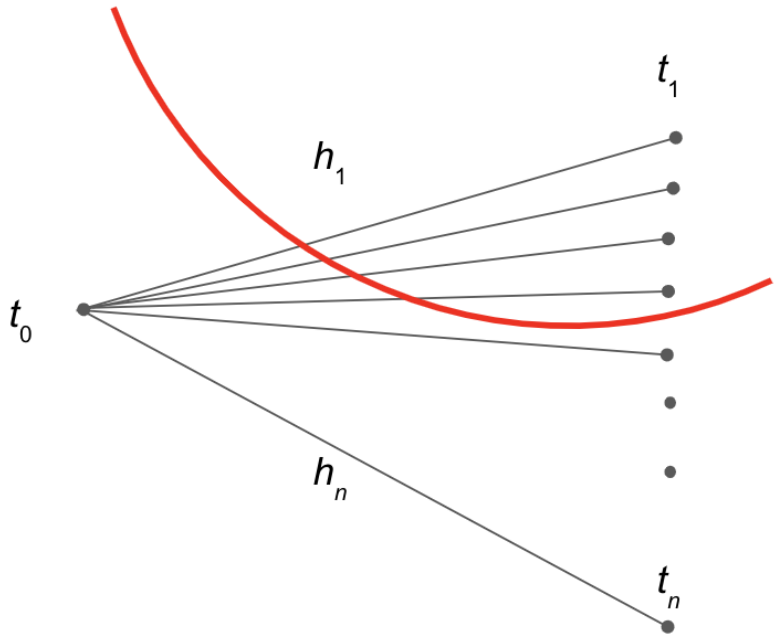


Figure: Subset of normal histories

$\mathcal{H}_t^{n,I_1}$ is the set of the normal histories at t, according to the information I_1.

Case-study: Formal Analysis

Now, let us take into account the lottery case (φ) and the forecast case (ψ). By stipulation, $Pr(\phi) = Pr(\psi)$; however, as we have seen, the risk ascriptions diverge. Why? There exists, on the basis of our information about whether forecast, a selection of the normal histories, $\mathcal{H}_t^{n,I_{\texttt{forecast}}}$ such that in every normal history it is false that there will be snow in Rome in July. But, on the other hand, there is no partition $\mathcal{H}_t^{n,I_{\texttt{lottery}}}$ such that in every history it is false that Paula will win. Instead, there is one normal history in which Paula win the lottery.

In other words $\mathcal{H}_t^{n,I_{\texttt{lottery}}} = \mathcal{H}_t$; i.e., for lotteries, any history is a normal history. Thus, the probability of ϕ is more relevant for our judgement. In the case of forecast, on the contrary, the probability of ψ is considered but it is a limit case, a remote possibility.

Case-study: Conceptual Analysis

The case-study seems to suggest that considerations about the **normality** of the development of the world has a certain weight in defining our judgements about risk and uncertainty. The intriguing feature is that, for the most part of times, we actually **don't know** the objective probability ascriptions (maybe, since they do not exist or they are meaningless). So, *a fortiori*, it follows that our guide in attributing subjective credences to uncertain events is grounded on considerations about normality. The formal model of normal histories could be an explanatory framework for many biases and it can help to track down a map of different forms of rationality.

Conclusions

- Multidimensional conception of rationality;
- The various levels are, however, grounded into the core of standard probabilistic view;
- There are possible, nevertheless, extensions of the conceptual models to handle our pre-theoretical notion of risk;
- The construction of these conceptual models and their mathematical implementation is a good agenda both for philosophers and for economists (and for the collaboration between the two!)

Thank you very much!

ciro.deflorio@unicatt.it
luca.zanetti@polimi.it