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DIPARTIMENTO DI ECONOMIA INTERNAZIONALE
DELLE ISTITUZIONI E DELLO SVILUPPO

Carlo Beretta

Reasonable rules of choice

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Abstract

Players have consonant interests if each has a strategy favourable to the pursuit of his own and the interests of the others when the latter adopt a best response. Reasonableness is to move according to such a strategy at the stage reached, reaping the gains this generates. Some overt games hide an underlying game in the choice of rules of choice in which reasonableness is substantively rational, credible and leads to a state that Pareto dominates the Nash equilibria of the original game. The paper contains an application to the finite prisoner's dilemma.

Common knowledge – rationality – backward induction - reasonableness – incomplete information – finite prisoner's dilemma

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Introduction

Whatever experiments seem to show, most of game theory assumes a world in which substantive rationality rules. The main, possibly only, defence of this kind of rationality is its instrumentality in the pursuit of one's aims. In games, it says that, given a set of strategies, a rational player must adopt one that is at least maximal,[1] i.e. not dominated. Ordering strategies is what I will refer to as level 1 reasoning.
Consider first one stage games. In a few cases, there is just one non dominated strategy. This tells how one should play the overt game in which he is involved. In these instances, the process of decision ends at this stage.[2] Out of these lucky cases, substantive rationality turns out to be indecisive and sometimes self-defeating. Level 1 reasoning is unable to decide and one must go on to discover which is the "substantively rational" strategy, i.e. one must use substantive rationality at a higher level. In fact, one builds, besides the overt game, an underlying level 2 game, usually tacit and implicit, in which reasoning, and consequently processes or rules of choice among strategies, are played against each other. Consistently, one asks that also this game be played rationally.
To give scope for the application of rationality, one has to introduce additional assumptions, for example common knowledge of the payoff matrix and of substantive rationality. Even so, in one stage games one does not go much farther. One can introduce Nash equilibrium, ask for *ex post* optimality in the choice of strategies for the overt game. However, aside from the case of existence of dominating strategies, the substantive rationality of the strategy chosen is conditional on the choices made by the other players. This leaves many relevant questions unsolved.
Nash can point out the existence of equilibria when looking just at

[1] Of course, assuming it does exist.
[2] One might take exception for the cases in which, as in the prisoner's dilemma, this leads to a Pareto inefficient state. But all the ways out suggested either change the payoff matrix or abandon substantive rationality. On the prisoner's dilemma, for example, see Binmore (1994).

pure strategies there seemed to be none.[3] But, faced with mixed strategies, one has to go back to reorder one's strategies for the overt game. Unfortunately, in these cases, substantive rationality at this point seems to make irrelevant the use substantive rationality at level 2, leading back to the level 1 situation in which substantive rationality is silent.[4]

Furthermore, when there are many Nash equilibria, substantive rationality by itself is unable to discriminate among strategies even when, looking at their implications for the overt game, these equilibria are Pareto orderable. Substantive rationality seems to fall short in its instrumental role.

To go farther it seems unavoidable to introduce explicitly the reasoning used at level 2, one's reasoning as well as that of the other players, which lead to the choices observed in the overt game.[5] When reasoning become an object of choice, the decision to use substantive rationality in playing the overt games must itself be justified by rationality at this higher level. Given the latitude of possible reasoning, this quest may sound hopeless. However, substantive rationality applied to this choice can restrict the set of possible candidates.

The reasoning must be instrumentally efficient in the pursuit of the aims of the player that adopts it and therefore cannot lead to a worse result from his point of view than using level 1 rationality. To be able to implement the strategies they suggest, they must open to one's opponents the possibility to use best responses which lead to a result they too prefer, at least weakly, to all the original Nash equilibria, and possibly better for all players, than some of them. To make best responses determinate, the reasoning in question must be unique. Finally, and this is obvious in the case of symmetric games, they must discount the fact that also one's opponents can use a reasoning of the same kind so that the strategy suggested by one's reasoning must be

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[3] As in the case of matching pennies.

[4] In matching pennies, one discovers that, faced by a rational player, it is consistent with substantive rationality to choose for sure either head or tail in playing the overt game, provided one can keep one's decision secret.

[5] This is what is done when one introduces rationalizability of strategies.

identical to the best response to the optimal strategy adopted by the other in the overt game.[6]

In one stage games, this has nice implications, though it does not lead very far. Much as one does when using Nash to solve the overt game, one can look for the existence of reasoning which best counter each other. If the overt game is sufficiently simple and one is lucky, this will leave just one candidate and thus determine the strategies to be used in the overt game. Of course, only some Nash equilibrium strategies for the overt game will satisfy the constraints rationality puts on the choice of reasoning. The advantage of putting rationality constraints is that it can restrict the set of Nash equilibria to be considered and act as a refinement criterion.[7]

For multi-stage games, the situation is quite different since there are further possibilities. Even in these games, it is usual to ask that each move of a "rational" strategy be optimal in the conditions considered,[8] i.e. to ask for substantive rationality to be used in the overt game. To act in this way is to follow a rule in the choice of moves: whatever situation one is considering, choose the rational strategy and assume the other also does so. Aumann (1995) shows that, in these conditions, common knowledge of rationality implies backward induction. This is rather discomforting since in many cases this reasoning leads both players very near to the worst state in terms of

[6] Even in symmetric games, one is not saying that since the players are in the same situation, they will necessarily make the same choices, but simply that both will follow substantive rationality when applied at the appropriate level, since they are assumed to be substantively rational.

[7] However, considering for example a battle of the sexes game, one cannot discard the possibility of many Nash equilibria in reasoning of the level 2 game. Then, substantive rationality would have to be pushed further to single out one of them and make it necessary to follow the dictates of this Nash equilibrium, and not just erode them, as it can happen in the case of mixed strategy equilibria and this could just start an infinite chain.

[8] When one uses backward induction, for example, one starts from what is rational to do at the last stage, isolating it from anything that happened before, and then reasons backward, always stage by stage.

the realization of their aims. Then one is led to ask whether it is rational to be rational.
The reasoning implied by common knowledge of rationality, the inductive reasoning, is just one of the possible reasoning, one of the possible rules of choice of a strategy. While in one stage games there is actually no other reasoning than following level 1 rationality in the overt game that satisfies the conditions of rationality above mentioned, in multistage games one knows there are rules of behaviour or reasoning that it is advantageous for a player to use instead of following the substantively rational strategy.[9]
One can now go back to the question of the rationality of being rational. Perhaps a rational person cannot choose whether to be rational or not, but it seems clear that it is possible for a rational person to decide whether to use inductive rationality and, more, whether to follow its dictates in playing the overt game. Substantive rationality in the choice of reasoning makes it doubtful whether it is substantively rational to play according to inductive rationality the overt game and can advise against it.[10]
One has already gone in this direction in the literature.[11] For example, when one introduces different types of agents, what one is in fact considering is the effect of using different rules of behaviour. The

[9] Even if they suggest the adoption of strategies in the overt game that are not part of Nash equilibria and therefore can have problems of credibility.
[10] As Aumann (1995) says: "… the inductive choice may be not only unreasonable and unwise, but quite simply irrational." On this reading, common knowledge of rationality in the choice of reasoning would be inconsistent with common knowledge of the use of rationality in deciding how to play the overt game. On this point, see also Binmore (1996a), Aumann (1996) and Binmore (1996b). Interesting experimental data can be found, for example, in Rubinstein (2004).
[11] Several alternatives have been suggested. See, for example, Reny (1992) and (1995) or the forerunner Kreps - Milgrom - Roberts - Wilson (1982), that reaches by a different way a result similar to that of Radner (1980).

rules considered were in fact “irrational”,[12] but perhaps one did not push far enough the quest.
To discard as not rational the inductively rational behaviour is not equivalent to being free to choose any strategy.[13] Rationality can put constraints even in these choices. One can rank reasoning in terms of the level of realization of the aims of the player that adopts it, on the assumption that backward induction will not be used, and discard those which are obviously dominated.
Even when there is the possibility to make, or to consider,[14] known one’s decision, there would be the problem of ensuring its credibility, at least for some stages, since it dictates a strategy that is not part of a Nash equilibrium. Furthermore, one should take into account the possibility that also one’s opponents choose a strategy on the basis of the same criteria. This requires that either there is a process leading to an agreement on this point that it is individually rational to abide to or the strategy one adopts must be “almost”[15] identical to the best response to one’s opponents using they too “rational” rules. Restrictive though they sound, all these conditions are satisfied in admittedly simple toy games like the finite prisoner’s dilemma.
Here I will concentrate on strategies which obey rules. Following rules means to single out sets of circumstances in which one will act in the same way faced with any member of the set in question. For this rea-

[12] Whatever the results obtained in experimental games and in tournaments *a la* Axelrod seemed to show. One can enforce uncertainty about one’s rationality, player A can induce B to discard the assumption that A is substantively rational simply by playing cooperative at the first stage in a finite prisoner’s dilemma and it can be “rational” to do so.
[13] As it has been said above, since it is justified by its instrumentality, the strategy chosen cannot fair worse than the inductive one. To be able to implement it, it must open to one’s opponents the possibility to use best responses that improve their payoffs with respect to the Nash equilibria of the overt game. In general, to ask uniqueness of the strategy satisfying this requirement is quite restrictive, even in two persons repeated games.
[14] In the case considered in this paper, knowledge of the payoff matrix allows educated guesses about what it would be rational to do.
[15] In a sense that will be made clear in what follows.

son, it does not necessarily dictate to adopt in every situation the best choice in those conditions.[16] However, since rules themselves are the results of reasoning and are an object of choice, rational rules must satisfy the constraints rationality puts on acceptable reasoning.

The simplest case concerns rules for just two agents called to interact repeatedly between themselves.[17] In the simple games considered in this paper, there is just one obvious candidate for the role of the instrumentally efficient reasoning and shows that, failing to distinguish the different levels at which substantive rationality must be used can unduly restrict the set of alternative reasoning one can follow at level 2.

For example, in multistage games using substantive rationality in playing the overt game leads to backward induction. Backward induction, (bi, for short), is actually a Nash equilibrium of the level 2 game in reasoning and it is necessitated by substantive rationality when applied at the overt game. But substantive rationality applied at the level of the choice in reasoning opens other possibilities. What is argued in this paper is that even sub-game perfection, while it allows to weed out non credible promises and threats, it can push the process too far and prevent trust in situations in which trusting and being trustful is substantively rational. This is what the paper aims at proving.

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[16] Of course, doing at each stage what is optimal at that point can be seen as a rule, but the behavior suggested does not necessarily follow a rule.

[17] Some rules have relevance and meaning only or mainly for the agent that adopts them, like most personal habits, and their justification lies in reducing the cost of decision or the costs of adjusting to a new environment, possibly shutting out interesting novelties. Others have a social relevance and then their main purpose is that of making the behaviour of each member somewhat predictable if not necessitated and predetermined. Abiding to the former kind of rules is supported by personal will and supported by personal reasons and give raise to the usual problems connected with weakness of the will and the possibility of temporal inconsistency. On these points, see Schelling (1984). For those of the latter kind, personal will and personal reasons are no longer sufficient. They must be known and accepted by the other members of the group if they must survive. When rules involve even not very large numbers of players, they raise immediately the problems of communication, of reaching an agreement when there more alternatives, and of enforcement.

Definition and scope of reasonableness

Reasonableness consists in looking for and exploiting all possibilities of pursuing rational, self interested cooperation.[18] A reasonable player favours the realization of the aims of the other players till this is instrumental in the pursuit of one's own. When this is no longer possible, he opts for a Nash equilibrium strategy in the overt game.
A strategy, usually conditional on the behaviour of the other players,[19] supports reasonableness, and in this limited sense is called reasonable, (for short, rea), if it induces best responses, called the accommodating strategies (ac), that lead, in the overt game, to a maximal state from the point of view of the player in question that Pareto dominates, at least weakly, any Nash equilibrium of the overt game. Existence of a reasonable strategy[20] is a precondition for reasonableness to have any role. Using best responses[21] is what rational opponents must do if they are self interested.
Applied to one shot games, reasonableness allows to bypass some of the problems of substantive rationality outlined above[22] and is justified by substantive rationality when applied at the appropriate level of reasoning. But, in multi-stage games, one can show that reasona-

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[18] In a one-off prisoner's dilemma, it is obvious that there is no such possibility, but repetition opens up to it.
[19] Or, in other words, supported by a suitable threat.
[20] The one considered in the case of the finite prisoner's dilemma is that of playing always cooperative.
[21] In the case at hand, play always cooperative till the last but one stage.
[22] As for the first point, it is easy to see that, if there is a dominant strategy, as in the one-shot prisoner's dilemma, that is also the reasonable one. When there is just one mixed strategy equilibrium, as in the case of matching pennies, reasonableness accords with any suitable randomization for the game at hand reached by reasoning at level 2, and resists the reasons for declaring indifference among all the strategies in its support. In games with many Nash equilibria, it selects the one that Pareto dominates all the others, for example, in a stag hunt game, if there is one, though, when Nash equilibria cannot be Pareto ordered, in a battle of the sexes, for example, it can lead to disaster.

bleness, besides being consistent with[23] substantive rationality when applied at the appropriate level, can justify playing the overt game at odds with what sub-game perfection dictates and can allow to reach a state that Pareto dominates that associated to the latter.

The strategy that supports reasonableness, in the relevant cases,[24] is not a best response to its best response.[25] This raises immediately the question of its consistency with rationality and credibility: in these games, at some stage, it foresees a move that everybody knows will not be taken.[26] However, reasonable strategies have just the instrumental role of showing that there is room for rational, self interested cooperation, for the use of reasonableness as a criterion of choice of reasoning. It shows that, at the stage at hand, by following the usual rationality criteria in the overt game, one is forsaking valuable occasions of cooperation which generate strictly positive gains for all the players involved, benefits independent from the behaviour kept at later stages.

By itself, this fact falsifies the belief that it is instrumentally efficient to follow the dictates of the usual rationality criteria, and therefore that it is substantively rational to use that kind of rationality.[27] Reasonableness, till it holds, shows that sub-game perfection is not an efficient criterion of choice in terms of pursuing one's aims, and therefore it is not a criterion a substantively rational player must always use.[28]

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[23] Or perhaps better, required by.

[24] i.e when it suggests to play at odds with sub-game perfection.

[25] Though, it is indistinguishable from it till almost the end of the game.

[26] Of course, this is what must happen when the reasonable strategy is not a best response to its best response, at difference from what happened in the one shot games considered above.

[27] The fact that whatever they do in the following stages will not take away the advantages they have reaped by playing cooperative, makes how they will actually play afterwards immaterial, and therefore irrelevant the credibility of the reasonable strategy in later stages, much as backward induction takes as irrelevant what has gone on before a certain stage.

[28] Not only in solving the game but also in attacking the reasonable strategy.

From this point of view, reasonableness is a criterion of choice among moves at each stage, not the choice of one strategy. It dictates and motivates the move one should make at a certain stage,[29] not the entire sequence of moves. Of course, applied consistently stage after stage it selects a strategy. Consistency and rationality, however, should not be required of the reasonable strategy but of the possibility and optimality of implementing reasonableness in the choice of moves at each stage, i.e. of reasonableness itself. It is in the interest of both players to give a possibility to reasonableness and play consistently with it, either adopting a reasonable strategy or responding optimally to it.

When each can act in a way favourable to the pursuit of the interests of the others adopting a strategy that is not dominated by any other, included that which would be chosen following backward induction, the interests of the parties involved are mutually consistent. What reasonableness aims at is ascertaining at each stage whether there is still enough consistency in the aims of the players that makes it worth, actually individually substantively rational, playing "cooperative" and delay playing out the conflict of interests. One knows that one will reach a stage in which this consistency will vanish and conflict must be faced, but exploits as far as possible the advantages offered by rational, self interested cooperation.

The real question to be faced by the players is not which strategy to employ in the overt game but whether, at the stage considered, it is rationally necessary to use sub-game perfection. In the games examined in this paper, since it is not, it proves that at the point considered it is consistent with, actually it is required by, substantive rationality to play cooperative at the stage at hand for both players, whether they choose reasonableness or accommodation.[30]

The strategy reasonableness uses in motivating the choice of

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[29] Much as backward induction does.

[30] Reasonableness is inconsistent with backward induction but playing reasonable to an accommodating opponent or playing accommodating to a reasonable opponent, is instrumentally superior to following backward induction.

move to make at a certain stage can change as the game evolves. This is known by both players, they know when and how the strategy suggested by reasonableness changes and still the adoption of reasonableness at the current stage remains advantageous and credible.

Reasonableness requires anyway to run the risk of ending up worse off than at a backward induction equilibrium. To be worth playing it, it must be the case that the other players know about reasonableness, believe that playing reasonable is credible and, more, have a strategy that, while leaving them not worse, and possibly better off, than at backward induction equilibria, makes it possible[31] and advantageous for the reasonable player to play reasonable, or, even better, that the other players themselves have the possibility to play reasonable.[32] It can be enforced on the others, only in multistage games and provided there are still enough stages to go.

The definition of a reasonable equilibrium that will be used in this paper is quite demanding and helps in outlining the conditions in which it can be relevant. A reasonable equilibrium is one in which all players have a reasonable strategy and these strategies are consistent among themselves in the sense that each incorporates a best response to each other so that acting reasonably or accommodating lead to the same choice of moves for most of the overt game.[33]

To abide by reasonableness is in the interest of each player if he can rely on the reasonableness of the others, i.e. on their sophisticated use of substantive rationality. When relevant, it asks playing cooperative but this cannot be done unconditionally; it must be supported

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[31] The best response to a reasonable strategy must be consistent with the reasonable strategy itself, must allow carrying it out.

[32] Provided their reasonable strategy is consistent with playing accommodating, as it will be argued below.

[33] In the case of the finite prisoner's dilemma her considered, the reasonable and the accommodating strategies, while differing at the very last stages, in fact dictate the same moves at all the other stages, as it will be shown, so that considering reasonableness a Nash equilibrium in the choice of reasoning does not seem inappropriate.

by a threat of punishment of a player that uses a non cooperative strategy.[34]

Given the amount of knowledge required for reasonableness to be rational, it works best when players are just two[35] that have interacted repeatedly among themselves for a long period.[36] In these cases, as it will be shown, one can simply announce one's intention to play reasonable, and support the credibility of this promise by a suitable threat of punishment that it is in the interest of the defaulted agent to carry out, at least till the very last stages of the game are reached. In this way, players can give each other the possibility to test and be tested in one's resolve to play reasonable.[37]

Finally, reasonableness is just one Nash equilibrium in the choice of reasoning. The equilibrium dictating backward induction will survive.

Two things must be noticed.

As in multistage games there is a sequence in the choice of moves in the overt game, there is a sequence of level 2 games in which players choose among rules of choice. In the case of the finite prisoner's dilemma examined in this paper, the structure of the underlying overt game changes. Though this is known by both players, this does not destroy the substantive rationality of behaving reasonable.

If one introduces a level 2 game in the choice of rules, one is led to a level 3 game in which one chooses the rules to be followed in the

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[34] Conditionality, while it makes possible to attack backward induction, which can be seen as the result of iterated deletion of donated strategies when the game is written in normal form, does not attack iterated deletion itself, for example when it stops before the last step.

[35] Or a very small group of agents.

[36] Otherwise it must become a social rule supported by suitable threat mechanisms.

[37] When the number of player increases, however, this device does not work, unless one builds a suitable social structure. Problems with groups, especially if large, concern mainly the processes that can lead to such a degree of mutual trust, the enforcement mechanisms and the kind of threats that can be used, as is common to all social rules.

choice of rules, and then a level 4, … , n, …, starting an infinite regress. In the case at hand, however, each stage reaffirms the substantive rationality of reasonableness.

A characterization of reasonable strategies

Let c and nc be the cooperative and non-cooperative strategies in the stage game and normalize payoffs as in Kreps - Milgrom - Roberts - Wilson (1982). The payoff matrix of the stage game is:

1 \ 2	nc	c
nc	0 ; 0	a ; b
c	b ; a	1 ; 1

with $a > 1$ and $b < 0$, so that at the Nash equilibrium both players get 0. Suppose also that $a + b < 2$[38] and that the rate of discount of future gains is 0. Finally, let N be the number of stages.
In a finitely repeated prisoner's dilemma, common knowledge of substantive rationality is taken to imply the use of backward induction, and then non cooperation from the start, the (bi; bi) equilibrium. However, it is easy to show that, for both players, there are strategies, which, coupled with their best response, lead to a state that both strictly prefer to the Nash equilibrium of the overt game. Till there are enough stages still to go, the maximal one asks playing always cooperative, the last stage included but, as it has been said, the strategy suggested by reasonableness changes as the number of stages remaining diminishes.
The promise to play reasonable is supported by a lenient threat,[39] that

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[38] If $a + b \geq 2$, reasonable or just rational cooperative players would use a more complicated strategy than the "always cooperate unless …" here considered, for example they could agree on 1 playing c and 2 playing nc at even stages, reversing roles at odd stages, but it is easy to adapt the reasoning to consider also this case.
[39] One must discard the trigger strategy: start with cooperation and keep cooperating till the other does; from the stage in which the other turns non co-

of playing nc for n_a stages, where n_a is the smallest integer greater than a, going back to cooperation afterwards.[40] This threat ensures that a one time deviation is worse than consistent cooperation over a span of $n_a + 1$ stages, and therefore ensures that, if credible, it will induce as a best response to the reasonable strategy that of playing always cooperative in the stages in exam. The fact the punishment period is finite allows a defaulted player to go back to cooperation having reaffirmed one's image of reasonableness. It allows then both players to test and be tested on this score. To acquire and support this image is in the best interest of both players and this makes credible that the punishment will be ministered if necessity arises.

Since most of the problems of reasonableness arise at the last stages of the game, it is expedient to start with examining the behaviour it suggests at them.

At stage $n \geq N + b$,[41] even reasonableness dictates consistent non cooperation, since there is no other strategy which coupled with its best response gives a player a payoff at least as great as that associated to consistent use of nc.[42] At the level 2 game, choosing reasonableness

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operative, play non cooperative till the end of the game. This strategy dominates any other that induces a best response with a shorter span of cooperation. However, one knows that, by itself, the announcement of such a strategy will not produce any effect unless it justifies doubts about the rationality of the player that makes it; furthermore, it is not the best of this kind and therefore is not consistent with reasonableness. Besides, it has the usual problems of credibility and needs to be supported by an infinite hierarchy of threats of punishment of anybody who has not punished … the defaulter. The advice never to use disproportionate threats here holds *a fortiori*. More lenient strategies can produce the same effects, at a much lower cost, allowing a best response which leads to Pareto superior results so that they accord better with reasonableness.

[40] It is also possible to make the length of the period of punishment conditional on the behaviour kept by the other in this span of the game, for example, shortening it if the other plays cooperative before the end of the announced round of non cooperation.

[41] And, of course, if $N < -b$.

[42] Notice that, even if – b □ 2, playing consistently c till the end of the game

or backward induction[43] give the same result, whatever each player decides to do, so the choice is irrelevant. Anyway, reasonableness does not need the use of backward induction to suggest what to do.
From stage $n > N - a$ to stage $n \leq N + b$,[44] both players have a reasonable strategy that differs from that dictated by backward induction and it requires to play always c in the overt game till the end, with the threat of turning consistently non cooperative as soon as one is defaulted by the other.[45] The reasonable strategy is at least weakly dominated both by accommodation and by immediate and consistent defection but accommodation and consistent defection cannot be ranked.
One can certainly argue that, since it is dominated, the reasonable strategy will not be used, and therefore delete also the accommodating strategy. On the other hand, to argue that immediate defection is "the" rational choice, one has to use backward induction. However, the fact that the reasonable strategy coupled with its best response lead to a state that Pareto dominates that associated to backward induction shows that the latter is not an instrumentally efficient, and therefore at least not "the" substantively rational choice.
This is a situation in which there are both a reasonable and a backward induction equilibrium in the choice of rules of choice among strategies. At the level of rules of decision, while backward induction is the best response to itself, it is not with respect to reasonableness, being beaten by accommodation in this role. In fact, one has a reasonable equilibrium that Pareto dominates that associated to backward induction. At level 2, then, one is playing a stag hunt game.

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is not a reasonable strategy since, coupled with its best response, it does not give greater gains than following backwards induction and playing consistently nc.

[43] If there is any scope for it, i.e. if $- b > 1$.

[44] Or when $a > N > - b$.

[45] Note that the threat is credible, since non cooperation by one's opponent must be justified by his use of backward induction and, in that eventuality, rationality requires to shift to backward induction and therefore to non cooperation. The best response to this strategy is to play nc only at N.

Multiplicity of equilibria destroys the certainty that it is optimal to play nc at these stages.[46]

As in all stag hunt games, going for the Pareto dominating equilibrium means to run a risk of ending up worse off than at the Pareto dominated one, if defected: it must be the case that both players believe in reasonableness and both either act reasonably or accommodating, if the decision has to pay off. Only if one has reason to give a sufficiently high probability that this will be the case it makes sense to act at variance with backward induction. So, this is an unlikely choice if one comes from a history of consistent defection in the previous stages.[47]

If one comes from a history of consistent cooperation, however, one either has to give up the assumption of rationality of the players or has to explain why a rational player that has resisted backward induction up to that point starts just then to use it. What will be argued is that it is possible, in a substantively rational world, to come from a history of consistent cooperation. In such a world, consistency would require playing c also at these stages,[48] though at the price of running some risk.[49]

The fact that reasonableness asks playing c at some stage and nc at the stage immediately following may strike as contradictory. After all, playing c is justified by the existence of a strategy, that of going on to play c till the end of the game, one knows it is irrational to carry out. In fact, however, the principle underlying reasonableness is that of exploiting all possibilities of mutually advantageous cooperation till this is feasible, till there exists a strategy that, coupled with

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[46] And just this fact can stop the use of backward induction at $n < N - a$.

[47] Or if N is very low. Actually, it should be possible to test experimentally the strength of the assumption of reasonableness with respect to that of backward induction by comparing behaviour in games of different length.

[48] Because it is dictated both by reasonableness and by accommodation, the two strategies leading to the same choice of move in the conditions considered,

[49] Even the reasonable player will be forced by reasonableness to play nc as soon as $n > N + b$.

its best response, gives the player who adopts it an advantage with respect to the strategy dictated by backward induction, while using accommodation is advantageous also for the other player. At the last stage in which reasonableness suggests the use of c, such a possibility exists; there are gains for both parties if both of them stick to it. At the stage immediately following, this possibility vanishes, and it is this the reason for which, afterwards, reasonableness as well as a straight dominance argument dictate to play consistently nc till the end.
For $n \leq N - a$, reasonableness again suggest playing c till the end and either it cannot be ranked with backward induction or is only weakly dominated by it, and accommodation strictly dominates backward induction. The reasoning given above in favour of reasonableness then applies *a fortiori*.
For $N - 2a \leq n \leq N - a$, one can support the promise to play reasonable only by the threat to shift to backward induction. However, when $n < N - 2a + b$, one can use the more lenient threat of punishing just for n_a stages.

1 \ 2	bi	rea	ac
bi	0 ; 0	a ; b	a ; b
rea	b ; a	N-n ; N-n	N-n-1+b; N-n-1+a
ac	b ; a	N-n-1+a; N-n-1+b	N-n-1; N-n-1

Provided N is sufficiently great, both reasonableness and accommodation would then dictate playing c at the first stage of the game. Furthermore, for the reasons given above, rational players will either follow reasonableness or accommodation, and play consistently c up to $n \leq N + b$ and then play nc till the end of the game, or follow backward induction and play consistently nc from the start of the game.

Taking into account that there is a stark alternative between using reasonableness or backward induction, the payoff matrix of the game at stage n, with $n < N - a$ has the structure given above.

From it, it is easy to see that the game in the choice of reasoning has two equilibria, the reasonable one and that associated to backward induction, with the former Pareto dominating the latter. If it has to prove its instrumental value, substantive rationality must select the reasonable one, but the game in the choice of reasoning has a stag hunt form. Playing reasonable always entails running the risk of being defaulted. Abiding by the rule is not costless but neither too costly with respect to the prospective gains reaped[50] if one's reasonableness is affirmed and believed. Anyway, the threat of the ministration of a just sufficiently harsh punishment does not destroy the justification for following reasonableness in the rest of the game and is credible so that, in a world in which substantive rationality of the players is common knowledge, in these stages, the risk is null.[51]

Stage after stage, the reasoning given above can be repeated.

One knows that a stage will be reached in which it is no longer pos-

[50] Though prospective gains diminish as n moves towards $N + b$.

[51] The fact that one is playing a sequence of stag hunt games gives further reasons for using lenient strategies with regard to punishment in case of deviation. In a one-shot stag hunt game, there are no ways to send credible signals to the other. But in a sequence of games of this kind, behaviour at one stage, especially if potentially costly, allows signalling on the equilibrium in the game in rationality on which coordination is sought and, at least at the beginning of a sufficiently long sequence, a round differs very little from that immediately following. When using the harsh strategy, one needlessly limits the usefulness of these signals in the event the other deviates. The behaviour kept before the deviation by the player defaulted upon will not work as a signal about future behaviour since the announcement would now require the defaulted always to play according to bi. By using a harsh strategy, one puts one's image needlessly at risk.

sible to support by punishment belief in the respect of the rule associated to reasonableness, in which, therefore, coordination on the efficient pure strategy equilibrium in the stag hunt game cannot be taken for granted. But players are not forced to solve backwards the sequence of underlying games and this gives both a reason to choose reasonableness or at least accommodation, and to believe that also the other player, especially if substantively rational in the choice of rules of choice, will do so.

The sequence of choices in the sequence of stages

At n = 0, deciding what to do at n = 1, they know that this is just the first occasion in which they have to make the choice between following either reasonableness or backward induction, the first of a finite sequence of choices of the same kind. One must consider whether the fact that one knows that one is facing a finite sequence of finite sequences all of this kind, each one step shorter as one progresses in the sequence, will undermine the rationality of reasonableness. However, when looking for the reasonable strategy, one has just faced this sequence of sequences, though in a reverse order. So, there is no problem on this point.

To the level 3 one can attach a level 4, 5, …n … game, the sequence going on to infinity. But in this sequence, the level n game has the same structure of that of level n-1, for n > 2, so that what has been said for level 2 applies. How to behave in these sequences is decided privately by each agent, and consistency requires the same choice is made, stage after stage, confirming the choice made at stage 2, which however depends on the information the agent has at the stage reached in the overt game.[52]

[52] Note that, attaching an underlying game in the choice of rationality to a one-shot prisoner's dilemma leaves the substantive rationality of defection unaffected.

The traveller game

Results obtained in a variety of experimental games seem to support the approach suggested in this paper.
In the repeated prisoners' dilemma, players can talk to each other, can announce they intend to play reasonable and, if the argument given above holds, the announcement is credible. If both players are substantively rational and have common knowledge both of rationality and of the possibility of reasonableness, the announcement is however unnecessary: common knowledge of substantive rationality must induce them to believe each will play either reasonable or accommodating for most of the game since they are instrumentally superior to backward induction. Both these alternatives lead to the same sequence of moves in the overt game, except for the very last stages, but what one will do at later stages is immaterial for deciding what to do in the preceding ones.
When the game is put in normal form, what reasonableness does is to question that substantive rationality requires necessarily sub-game perfection and the use of backward induction. But these conditions are used also to solve games which require players to make just one overt move, to take a once for all decision which cannot be revised afterwards, but that are based on the solution of level 2 games played inwardly by each agent.
A simple example is Basu's traveller game.[53] Backward induction leads both players to practically the worst possible result. In fact, at each stage of the process, they are simultaneously solving a prisoner's dilemma, and the final result is the one expected when playing a finite sequence of prisoners' dilemmas, in which, in the stage game, the strategies are: push backward induction one step farther (P), stop backward induction (S). The payoff matrix of the stage game is:

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[53] Two players are asked to choose separately a number in the interval [150; 300]. The player that chooses the smallest wins and gets a payoff equal to the number chosen plus 5, while the other receives a payoff equal to the smallest number chosen minus 5. If both choose the same number, both get a payoff equal to the number chosen.

1 \ 2	P	S
P	- 1 ; - 1	+ 4 ; - 5
S	- 5 ; + 4	0 ; 0

In each stage, pushing backward induction one step farther seems forced by the fact that substantive rationality will induce also the other too to do so. In this case, the only reasonable strategy is to give up any use of backward induction since the game of choice of rationality is not going to be repeated. The accommodating strategy is to use backward induction for just one step, trusting in the reasonableness of the other.

Rubinstein (2004) gives some very interesting statistics about the way people actually play this game. In a population made up of university students from different faculties, the percentage using what appears to be the unique equilibrium strategy is exceedingly low; by far the highest plays strategies that are far from the "equilibrium" one, mostly however concentrated around the strategies which, if played by both agents, would grant them nearly the highest gain which could be obtained, the reasonable and the accommodating ones coming out on top.

If the reasoning given above holds, it can also be that players believe that substantive rationality is very common, but is applied in the underlying game of choice of rules of choice, and therefore choose either a reasonable or an accommodating strategy.[54] From this point of view, students would be substantively rational just because they do not play the "equilibrium" strategy in the overt game, and even when

[54] Reaction times of those choosing the reasonable strategy are lower than those of students choosing the "rational one". Of course, the reasoning to do once reasonableness is chosen is shorter but one must first compare reasonableness and backward induction. May be there is some unknown mental mechanism which distinguishes and separates automatically situations in which reasonableness is better.

they choose a best response to the best reasonable strategy, they trust in the substantive rationality[55] of the other.

Closing remarks

The cases examined in this paper are very particular, but show that, sensible as it is in many decision problems, subgame perfection and its implications can push distrust too far. At least, the existence of reasonable strategies at the early stages should be given a possibility. It also suggests that rationality can require much more than discarding dominated strategies

Reasonableness seems an appealing behavioural rule. Here it has been used mostly in multistage games in which it can be "enforced" on the other player, but has nice properties also in one-off games. By introducing multiple equilibria, it generates uncertainty endogenously in a "natural" way and captures most of what one wants from imperfect knowledge about the type of the other player, or bounded or ε-rationality.[56] It generates consistent choices and it can be instrumentally superior to subgame perfection in the pursuit of one's aims.

It requires assuming the existence of an underlying game a player can add to the overt one. Control for this, and more of the characteristics of the underlying game, can be substantially difficult but seems required to read experimental results.

The behaviour required by reasonableness, though it may have alternatives, has a natural candidate in the overt games examined. In different games, what behaving reasonably means can be much more obscure. This by itself does not destroy the point one has been trying to make but there are at least two *caveat*.

The fact that the overt game can be endowed with two or more equally tenable, but inconsistent, criteria of reasonableness, can give rise to an infinite regress of decisions about the rationality it is rational to use to define what reasonableness means.

There is the possibility that what reasonableness means could be

[55] i.e. in the reasonableness.

[56] Radner (1980).

quite clear for both players, but both have reasonable strategies that are however inconsistent with each other, at least for some of the initial stages, and imply best responses inconsistent with each other.
In both cases, the game in the choice of rationality would loose the simple structure here exploited.

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