Inequality: from identity politics to policy polarization

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Abstract

This paper studies the relationship between economic inequality and political polarization in an electoral context where voters (poor or rich and cosmopolitan or nationalist) have preferences over a redistributive and a migration policy. Building on Besley and Persson (2021), I propose a different version of their theoretical model where the two parties that compete to win the election do not have symmetric strategies and loyal voters of traditional left and right wing movements place different salience on migration. I then study how an increase in economic inequality can affect the electoral competition: inequality leads both parties to please more nationalist voters, however the polarization between the two increases. The results reflect the outcomes of recent elections in western democracies.

Keywords: Inequality, Identity Politics, Redistribution, Polarization
JEL Classification: D31 D63 D72

Introduction

The relationship between inequality and support for redistribution is one of the main topics of recent political economy. If we think about how much western politics changed compared to the XX century, we cannot help being surprised. The classical division between right wing and left wing parties along an economic dimension is really difficult to find. Even in democracies where a system with two parties had the strongest tradition, we witnessed the entry of new populist actors and the political division between two forces became less evident, specially from an economic point of view.

Considering the case of the United States: the Democratic Party and the Republican Party are still the two main political actors in the country. However, the first election of Donald Trump showed that a significant number of low income people coming from a disadvantage background voted the Republican Party, traditionally located at the right of the political spectrum. How is it possible that a working class voter supported a New York billionaire that does not have a history of supporting the rights of blue collar workers? Same phenomenon if we consider the case of the United Kingdom where the far right rhetoric of Nigel Farage and his pro Brexit movement or Boris Johnson's conservatives captured the attention of many voters from the *red wall* of the North.

In other democracies with a multi-party system circumstances are a bit different, but it is still possible to find a similar pattern: in Germany, the far right movement Alternative für Deutschland is very popular in the east of the country, probably the most disadvantaged area. In France, Marine Le Pen's party is strong in rural areas and the same happened in Italy with Matteo Salvini's League and Giorgia Meloni's Brothers of Italy.

The aspect that seems to bring together all these different political experiences is a common pattern of low income people supporting movements that emphasise an identity rhetoric that create a sort of division between *"us and*

them". This tool is particularly useful because it can be applied to several issues, from civil rights to immigration, from the green economy to globalization. There is always a way to spread this sentiment of identity. As a consequence, it seems that redistribution is not a central theme anymore and the support for it decreased or, in general, became less important compared to other topics and society as a whole looks more polarized.

In the literature, this phenomenon became known as *identity politics.* The goal of this paper is to contribute to this literature and linking it with inequality and political polarization. I propose a different version of the popular model proposed by Besley and Persson (2021) to account for some aspects that I believe are important in today's politics. They could explain why right wing parties became popular in past decades even among poorer people and why left wing parties could not keep pace. In this way, this paper is also related to the issue of political polarization, because surely voters express their preferences, but these preferences are accepted by political parties that engage in a tougher fight that sometimes lead to a delegitimisation of the opponent.

The logic that inspired our modelling choice is connected to the recent European and American experience: it seems that in past years, rhetoric around certain cultural themes has became more polarized and, as a consequence, political parties had to choose strategies that needed to balance between extremists and moderates. In particular, these issues are often unrelated to the economy, but they concern the moral values of a person, such as civil rights, national identities, religion and migration. However, it might be the case that inequality and disadvantaged economic backgrounds can boost these themes in the public debate and raise their importance also for political parties. Based on the composition of loyal and swing voters of a party, this phenomenon could lead to a higher degree of polarization in the society.

I first conduct a preliminary empirical analysis just to underline the fact that, in major European countries, immigration is a topic that can complicate the traditional political division of society based on the social class. This step

gives strength to the assumptions of the model and to the interpretation of the results. Specifically, it shows that class identification alone is not sufficient anymore to explain the preferences of voters.

I will then introduce the model and present the main results of the analysis. When a cultural theme is introduced in the political debate, identity starts to play a role and citizens will vote not only based on their income, but also based on their identification and this is in line with Besley and Persson (2021) and with Bonomi et al. (2021). Furthermore, it shows that an increase in inequality leads to a stricter cultural policy and a higher degree of polarization between the two parties and, as a consequence, within society as a whole. Finally, I conduct a small empirical analysis just to verify if the conclusions of the model can be found in a sample of western democracies.

The paper is organized as follow: Section II briefly discusses the literature on identity politics and polarization. In Section III, I present the results of a preliminary empirical analysis. Section IV contains the main model. Section V has the second empirical analysis in order to verify the main results. All the proofs are in the mathematical appendix at the end of the paper.

1 Literature

The literature on identity politics is vast and it can be easily related to the literature on political polarization if we consider that themes where identity plays a role are also those which can polarize an electorate because they involve moral values. In this brief review, I will try to link the two literature since in this paper I use the theoretical tools of identity politics in order to draw some conclusion about political polarization in a world where inequality is increasing.

Shayo (2009) is one of the first papers that shows how a different type of identification (class vs nation) leads to different equilibrium tax rates. The models of Besley and Persson (2021) and Bonomi et al. (2021) complicate this frame-

work by introducing the interactions between voters and political parties. Their contribution is crucial to understand how politicians can exploit identification to gain consensus and winning the elections. In the model proposed by Besley and Persson (2021), since voters are divided between cosmopolitans and nationalists, the salience of nationalism is the main feature that boosts identification and leads the two political parties to a stricter immigration policy. In Bonomi et al. (2021), voters can identify with the social class or with the cultural class, resulting in a combination of poor and rich citizens that affects the equilibrium tax rate. This model is complicated in Gennaioli and Tabellini (2023) where political leaders can exploit the identity of voters to spread propaganda and mobilise their core audience. The common aspect of these papers is that identity, when it is not related to the social class, dampens the redistributive conflict and can explain why, despite raising inequality, people do not support more redistribution. Empirically, the role of identity politics is discussed in Matakos and Xefteris (2020), Gethin et al. (2022) and Kuziemko and Washington (2018): all these papers show that when issues connected to cultural identity are important they cause a switch in the traditional borders of social classes and redistributive preferences.

How can we link this literature with political polarization? First of all, if we think about the recent electoral campaigns in the US or in France, the cultural issues are the focus of attention and it is possible to see politicians that adopt some extreme positions related to immigration or, for instance, abortion. The work by Glaeser et al. (2005) already underlined the role of *strategic extremism* adopted by political parties. According to their model, the key characteristic that induces leaders to choose an extreme platform is heterogeneity in the level of information between core and swing voters. If the sensitivity of loyal voters is higher, politicians will target them specifically with policies that are closer to them. The consequence is a departure from the median voter theorem. Interestingly, they suggested that in this way left wing parties have more chances to propose radical redistributive policies. On the contrary, despite a rise in in-

equality, left wing parties engaged a battle with right wing parties on different topics, such as the ones previously mentioned. The recent elections in Europe are a good indicators: traditional left wing parties focused more on civil rights and immigration.

This might have something to do with the salience of these topics compared to redistribution. Empirically, Danieli et al. (2022) underlined that far right parties exploited the fact that people changed their political priorities: they might still support redistribution, but if there are cultural issues at stake, they place a higher value on them. Related to the American context, Gunderson (2021) found no relationship between inequality and polarization on economic matters when these factors are not particularly salient in the elections, suggesting that voters might choose their representatives based on other issues. Expanding the sample to European countries, Gunderson (2022) found, instead, a positive relationship between raising inequality and support for parties at the end of the political spectrum, especially far left movements. More recently, Barilari (2024) showed that mass shooting events in the US, a shock that impacts on a divisive topic, can increase polarization in the Congress not only on guns control, but also on a wide range of different topics, suggesting that polarization can be contagious across different domains. In addition, polarization from such shocks decreases the probability of passing new laws, impacting the quality of the legislative process.

However, if we consider the recent electoral campaigns, the events are more complex. For instance, in France, it is true that *La France insoumise*, the populist left wing movement, gained a lot of consensus in recent elections by supporting redistributive policies, but it seems a peculiarity. In England, Italy and Germany far right parties are those who have benefited from the increase in inequality. In the US the situation is even more complex: the Democratic party is stuck between members that would like a more liberal platform and others that would like to keep it moderate. It is in this sense that we can think of a connection between the identity politics and polarization. Members of the same

political parties do not place the same value on cultural issues. It might be the case that immigration has a higher salience for poorer voters while progressive citizens have more disutility when their party implements policies that are far from their ideal point. This is why I believe that modifying the model proposed by Besley and Persson (2021) by taking into account this difference between voters, helps us understand how income inequality, identity politics and political polarization are related.

2   Motivation: immigration and income

In the last decade, immigration has become a prominent theme of all electoral campaigns in major western democracies. Interestingly, the focus, supported mainly by right wing parties, is not on the economic effects of immigration, but on the cultural ones. For instance, the approach could have been on the impact that immigrants have on the local economy: do they lower wages? Are they welfare consumer? On the contrary, the public debate concentrates on values and how the traditions and the way of life of natives can be threatened by migrants.

This phenomenon poses an identity issue that can easily shape people's political preferences if ethnic dynamics became more important than social class dynamics. The recent events in the United Kingdom where the government had to face riots against immigrants and the police after the spread of a fake news or the focus on the rights of the LGBTQ community during the Olympics suggest that cultural identification is already having political consequences, as underlined also by the aforementioned literature. As it will be explained later in detail, we set up a model, which builds on Besley and Persson (2021), that divides the society into two main groups: rich and poor. Among those we can find both nationalist and cosmopolitans citizens. But before going into the details of the model, I conduct a brief empirical analysis on how identity can shape class dynamics. The only aim of this passage is to provide support to the main

assumptions of the model.

The European Social Survey (ESS) offers the possibility to build a repeated cross-section dataset that takes into account all these different dimensions. The ESS started in 2002 and since then it is a very rich source to study attitudes and political preferences in Europe. In 2002 they started to interview a random sample from all European countries asking questions about their political preferences, their family structure, their social background and other useful information. Since then, every two years they interview a different sample of citizens asking the same core questions. In addition, it is possible, then, to match the people interviewed with the country of origin. In this way we can account for the state dimension in the analysis and see if there is a relevant difference between countries. I include in the analysis Italy, France, Spain, Germany and the United Kingdom.

Table 1 reports the summary statistics for ESS variables that I will use in the estimations for the period 2002-2020. *Migration* captures the feeling of the respondent towards immigration from non European countries where 1 indicates that the individual would like to close the borders and 4 stands for allowing more people to come in. *Anti_Migrants* is a dummy variable that takes value 1 if the respondent is in favour of closing the borders to migrants or allowing just a small number to come in. *Redistribution* measures the support for the role of government in reducing income differences on a scale from 1 to 5. *Happy* tells us how happy the individual feels in the moment of the survey, *Gender* is the gender, while *Income expectations* is the feeling of the respondent about household income where 1 is the most negative sentiment and 4 is the most positive. Finally, *Left-wing* and *Right-wing* are dummy variables that capture how the respondent places herself in the political spectrum while *Income* is the income quartile of the individual. I add a dummy variable *poor* that takes value 1 if the respondent belongs to the lowest part of the income distribution and two dummies (*Left* and *Right*) that take value 1 if the respondent voted for a mainstream left-wing or right-wing party in the last elections. As we can

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Table 1: Summary Statistics

Statistic	N	Mean	St. Dev.	Min	Max
Happy	103,693	7.322	1.881	0	10
Gender	103,665	1.521	0.500	1	2
Income expectations	99,199	1.871	0.781	1	4
Income	80,527	2.617	1.302	1	5
Poor	80,527	0.264	0.441	0	1
Left-wing	104,072	0.342	0.474	0	1
Right-wing	104,072	0.241	0.428	0	1
Redistribution	102,294	3.850	1.021	1	5
Migration	101,048	2.591	0.891	1	4
Anti_migrants	104,072	0.547	0.498	0	1
Left	104,072	0.212	0.409	0	1
Right	104,072	0.186	0.389	0	1

see, there is a slightly higher share of left-wing people in the sample and the average support for redistribution and immigration is higher than we might have expected.

2.1 Methodology

On these data, we run a simple OLS regression where y_{it} is the main dependent variable for individual i in year t. As dependent variables I use the support for redistribution, the preferred migration policy, the position on the political spectrum and the support for mainstream parties.

$$y_{it} = \alpha + \beta X_{it} + \gamma_c + \delta_t + \eta_{it} \tag{1}$$

X_{it} is a matrix of covariates for individual i in time t, while γ_c and δ_t are respectively the country fixed effects and the time fixed effects. Finally, η_{it} is the error term.

Table 2 reports the results of two regressions where the main independent variable is a dummy that takes value 1 if the individual is in the lowest part of the income distribution and the dependent variables are the preferred migration policy and the level of redistribution. Interestingly, lower income people are less in favour of migration. One possible explanation is that they see migrants from outside Europe as rivals for low skilled jobs or government subsidies or they might be more traditional compared to richer individuals. Not surprisingly, they support more redistribution. This is in line with the assumptions of the model where both rich and poor nationalists are swing voters: if the salience of migration is high, lower income nationalist voters can support a political party that implements a stricter immigration policy, even though the redistributive policy is not the optimal one for them.

Table 3 confirms the results of Table 2 with respect to redistribution, but

Table 2: Lower class, preferences for redistribution and migration

	Migration	Redistribution
	Dependent variable:	
	(1)	(2)
Poor	−0.094***	0.082***
	(0.021)	(0.014)
Controls	Yes	Yes
Country Fixed Effects	Yes	Yes
Year Fixed Effects	Yes	Yes
Observations	76,997	77,857
R^2	0.059	0.070

Notes: Column (1) is an OLS regression of form: $Migration_i = \alpha + \beta Poor_i + \gamma X_i + \delta_c + \epsilon_t + \eta$. Column (2) is an OLS regression of form: $Redistribution_i = \alpha + \beta Poor_i + \gamma X_i + \delta_c + \epsilon_t + \eta$. Controls included are: Income expectations, gender,level of happiness, year and country fixed effects. Standard Errors in parenthesis are clustered at country level. *, ** and *** indicate significance at the 10%, 5% and 1% levels, respectively.

using as the main independent variable the income quartile of the individual: if the voter's income increases, the preferred level of redistribution decreases. This fact simply confirms that rich and poor people have different optimal taxation rates and, as in the model, it is reasonable to assume that there are two political parties that represent the interests of different social classes, traditionally left wing for the lower class and right wing for the upper class.

Table 4 reports the results of 4 OLS regressions where the main dependent variables are: the position in the political spectrum of the respondent (left wing

Table 3: Income quartiles and preferences for redistribution

	Dependent variable: Preferences for redistribution	
	(1)	(2)
Income	−0.118***	−0.095***
	(0.010)	(0.008)
Controls	Yes	Yes
Country Fixed Effects	No	Yes
Year Fixed Effects	No	Yes
Observations	77,857	77,857
R^2	0.048	0.080

Notes: Column (1) is an OLS regression of form: $Redistribution_i = \alpha + \beta Income_i + \gamma X_i + \eta$. Column (2) is an OLS regression of form: $Redistribution_i = \alpha + \beta Income_i + \gamma X_i + \delta_c + \rho_t + \eta$. Controls included are: Income expectations, gender, level of happiness, year and country fixed effects. Standard Errors in parenthesis are clustered at country level. *, ** and *** indicate significance at the 10%, 5% and 1% levels, respectively.

Table 4: Low income, position in the political spectrum and support for mainstream right-wing/left-wing parties

	Dependent variable:			
	right_value	left_value	right	left
	(1)	(2)	(3)	(4)
Poor	−0.077***	0.033***	−0.061***	0.039***
	(0.014)	(0.011)	(0.007)	(0.014)
Poor * Anti_migrants	0.055***	−0.059***	0.038***	−0.047***
	(0.011)	(0.010)	(0.006)	(0.014)
Controls	Yes	Yes	Yes	Yes
Country Fixed Effects	Yes	Yes	Yes	Yes
Year Fixed Effects	Yes	Yes	Yes	Yes
Observations	78,666	78,666	78,666	78,666
R^2	0.035	0.045	0.042	0.023

Notes: Column (1) is an OLS regression of form: $Rightvalue_i = \alpha + \beta Poor_i + \gamma X_i + \delta_c + \rho_t + \eta$. Column (2) is an OLS regression of form: $Leftvalue_i = \alpha + \beta Poor_i + \gamma X_i + \delta_c + \rho_t + \eta$. Column 3 is of form: $RightParty_i = \alpha + \beta Poor_i + \gamma X_i + \delta_c + \rho_t + \eta$. Finally column 4 is an OLS regression of form: $LeftParty_i = \alpha + \beta Poor_i + \gamma X_i + \delta_c + \rho_t + \eta$. Controls included are: Income expectations, gender, level of happiness, year and country fixed effects. Standard Errors in parenthesis are clustered at country level. *, ** and *** indicate significance at the 10%, 5% and 1% levels, respectively.

or right wing) and if the respondent voted for a mainstream party in the last elections (left wing or right wing). As we can see, low income people tend to have leftist values and support progressive parties. However, once we take into

account the interaction with migration, we see that the signs of the coefficients are the opposite. This should suggest that migration is a topic that can switch political preferences of individuals. The results of table 4 are in line with the set up of the model because we allow poor nationalists to be swing voters and to support a party that does not have the optimal fiscal policy of low income people, if it chooses a stricter migration policy.

The main takeaways of these regressions, in line with the set up of the model, is that voters can change their preferences if they consider not only redistribution, but other divisive issues. In this sense, immigration is a topic that can complicate the classical division of citizens based on the social class. This could lead hypothetically to an electoral "alliance" between the conservative upper class and the left-wing lower class. Another important aspect to consider is that the salience of immigration is not the same for the different social classes and, as a consequence, for political parties. This fact limits the strategies that politicians can implement to attract swing voters or please the core base of the party. All these aspects can be found in the model that I present in the next section.

3 Baseline Model

3.1 Voters and Parties

In this model we follow the base structure proposed by Besley and Persson (2021): the members of a society are divided into two groups, rich (R) and poor (P), and for simplicity we assume that every citizen belongs to one of these: $j \in \{P, R\}$ and the shares of the two groups in the population are equal. The income of the rich is always higher than the income of the poor, so $y^P < y^R$. People are assumed to be divided also by an issue that is exogenous to their income level: in our case, following the work of Besley and Persson (2021) national identity and immigration. As a consequence, electors can be nationalist (N) or cosmopolitans (C). The share of nationalists in each social class is the same

and we label it as μ so this means that there are rich and poor voters against immigration.

Citizens have both economic and identity preferences that involve a tax rate t^J where $t^P > t^R$ and a migration policy $x^J \in [0, 1]$ where $x = 0$ means the highest nationalist society and $x = 1$ is the highest cosmopolitan society. Equation (2), (3) and (4) capture the preferences, respectively, of poor cosmopolitans, rich cosmopolitans and both rich and poor nationalist. As in Besley and Persson (2021), U is a symmetric, decreasing convex functions.

$$V^{CP}(t, x) = \begin{cases} U(t - t^P) + \bar{\gamma}(-(1 - x)) \\ \qquad\qquad \text{If P wins} \\ \\ U(t - t^P) + (-(1 - x)) \\ \qquad\qquad \text{If R wins} \end{cases} \tag{2}$$

$$V^{CR}(t, x) = \begin{cases} U(t - t^R) + \underline{\gamma}(-(1 - x)) \\ \qquad\qquad \text{If R wins} \\ U(t - t^R) + (-(1 - x)) \\ \qquad\qquad \text{If P wins} \end{cases} \tag{3}$$

$$V^{NJ}(t, x) = U(t - t^J) + \theta(-x) \tag{4}$$

γ is a parameter that captures the importance of identity for cosmopolitans. I assume that that $\bar{\gamma} > \underline{\gamma}$. This is due to the fact that poor cosmopolitan voters might have a sense of class consciousness towards immigrants given the fact that they usually come from a difficult economic background. θ on the

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contrary, represents the salience of immigration for nationalist voters and it is assumed to be the same for poor and rich. I assume that $\theta > 0$. The use of γ is the main difference with the model of Besley and Persson (2021)[1] because, as I will show later, it introduces a different cost of strategies that political parties can implement. As we can see from equations (2) and (3), they are step functions: if poor cosmopolitans (V^{CP}) see the migration policy realised by Party P, their loss function is the one with $\bar{\gamma}$. This happens because they have a different level of disutility if a migration policy that does not match their ideal one is implemented by their reference party or by the opponent. The same logic applies to rich cosmopolitans (V^{CR}), but, as stated earlier, I assume that $\bar{\gamma} > \underline{\gamma}$.

Polarization can happen over the economy and the difference in utility between the two optimal tax rates is captured by parameter z, where $z = U(0) - U(t^P - t^R)$. $U(t - t^J)$ is symmetric loss function around 0 and it decreases in the distance from the bliss point. As it can be seen, z increases with inequality. When $z = 0$ in the society and $t^R = t^P$, it means that R and P agree over the optimal tax rate.

There are two political parties {P,R} that represent the interest of the two social classes. Their division is mainly along the economic spectrum, but they have to choose a fiscal policy t_j and the level of opening of the country to immigrants x_j. The objective functions of parties coincide with the preferences of cosmopolitan voters, in line with Besley and Persson (2021). Therefore, parties need to maximise their expected payoffs by taking into account the probability of winning. This introduces a relevant difference with the model of Besley and Persson (2021) because it implies that the weights assigned to the objective functions of the two parties are not equal[2]. As a consequence, the equilibrium

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[1] In Besley and Perssson (2021) γ is assumed to be equal to 1 and identical for both parties.

[2] In the Appendix, Besley and Persson (2021) relax the symmetry assumption and model a society where low income people are the majority of the population and, among them, there are more nationalists than cosmopolitans. This gives a natural advantage to the poor party and raises the incentive for the rich party to choose a stricter immigration policy. However,

strategies are not symmetric. Both parties have only one optimal redistributive policy $t_J = t^J$ because they represent the economic interest of the poor or the rich and, as in Besley and Persson (2021), I assume that they cannot commit to a policy that is different from the optimal one for poor and rich. However, given the fact that they want to win the elections and, as explained in more detail later, nationalists are swing voters, they can commit to an immigration policy x_j that does not coincide with the optimal one of cosmopolitans ($x_J^C = 1$).

Cosmopolitans are loyal voters in the sense that when they belong to social class P, they will never vote for the Party R and vice versa. Mathematically, this is a consequence of the assumption that $z > \overline{\gamma}$ because otherwise they might have the incentive for certain values of $\overline{\gamma}$ to switch party. On the other hand, nationalists are swing voters and they vote for the party that offers them the higher utility and they are subject to random shocks. We can think of loyal voters as activists: they engage in political activities, they support the party materially and they are those who can also campaign outside the traditional spaces of political communication. In this sense, it is true that a party wants to maximise the probability of winning, but if it loses the core voters, this might turn into a higher cost in the future.

Equation (5) tells us the utility that Party J can offer to nationalist of group K where $K \in \{P, R\}$.

$$v_J^K(t_J, x_J) = U(t^J - t^K) - \theta x_J \tag{5}$$

As in Besley and Persson (2021) the model is subject to random shocks as in standard probability voting theory. As a consequence, a poor nationalist will vote for P if $v_P^P + \epsilon + \delta \geq v_R^P$ where ϵ and δ are the idiosyncratic and

our modelling choice is different: we introduce asymmetry by assuming that cosmopolitans supporter of poor party are more demanding when it comes to cultural values compared to rich cosmopolitans.

aggregate random shocks with c.d.f. $H(\epsilon)$ and c.d.f. $G(\delta)$. We assume that ϵ is uniformly distributed with mean equal to zero and sufficiently large support, and δ is uniformly distributed between $-\frac{1}{\psi}$ and $\frac{1}{\psi}$. A similar logic applies to rich nationalists.

Party P will win the elections if its vote share is larger than Party R The winning probabilities, taking into account the aggregate shock from c.d.f. $G(\delta) = \frac{x + \frac{1}{\psi}}{\frac{1}{\psi} + \frac{1}{\psi}}$, are:

$$P^P = \frac{\theta(-x_P + x_R) + \frac{1}{\psi}}{\frac{1}{\psi} + \frac{1}{\psi}}$$

$$P^R = (1 - P^P) = [1 - \frac{\theta(-x_P + x_R) + \frac{1}{\psi}}{\frac{1}{\psi} + \frac{1}{\psi}}]$$

where $\psi > 0$ and sufficiently small. Note that, as we have not changed any assumption about the nationalists compared with Besley and Persson (2021),[3] those winning probabilities behave in the same way.

Since nationalists are swing voters and are assumed to be identical between poor and rich, the difference with Besley and Persson (2021) can be found in the gains from winning for Party P and R. This takes into account the different weights that cosmopolitan voters place on the migration policy: since $\overline{\gamma} > \underline{\gamma}$, Party P will gain less from choosing the same migration policy of Party R.

The gains from winning for each parties, i.e. the difference in policy payoffs between winning and loosing, are:

$$Z^P = [z + \overline{\gamma}(-(1 - x_P)) + (1 - x_R)] \tag{6}$$

$$Z^R = [z + \underline{\gamma}(-(1 - x_R)) + (1 - x_P)] \tag{7}$$

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[3]We assigned closed forms to the distribution function of the shocks and to the loss function of the immigration policies, in order to get closed form solutions

Our solution concept is pure strategy Nash Equilibrium (henceforth, equilibrium).

3.2   Strategies and solutions

Both political parties want to maximise the probability of winning and the gains. Choosing a migration policy that is far from the core voters' ideal point represents a cost that might turn into a higher loss even after winning the elections. Therefore, they choose their equilibrium migration policy as follows:

For Party P:

$$x_P \in argmax\{[z + \overline{\gamma}(-(1 - x_P)) + (1 - x_R)]\frac{\theta(-x_P + x_R) + \frac{1}{\psi}}{\frac{1}{\psi} + \frac{1}{\psi}}\}$$

For Party R:

$$x_R \in argmax\{[z + \underline{\gamma}(-(1 - x_R)) + (1 - x_P)][1 - \frac{\theta(-x_P + x_R) + \frac{1}{\psi}}{\frac{1}{\psi} + \frac{1}{\psi}}]\}$$

The FOC for both parties and their best responses (BR^J) to other player's strategy are:

for party P:

$$\overline{\gamma}[\frac{1}{2} + \frac{\psi\theta}{2}(- x_P + x_R)] - \frac{\psi\theta}{2}[z + \overline{\gamma}(-(1 - x_P)) + (1 - x_R)] = 0$$

and for Party R:

$$\underline{\gamma}[\frac{1}{2} - \frac{\psi\theta}{2}(- x_P + x_R)] - \frac{\psi\theta}{2}[z + \underline{\gamma}(-(1 - x_R)) + (1 - x_P)] = 0$$

Before finding the optimal migration policies for both parties, we check that the second order conditions are satisfied.

Indeed, for Party P:

$$-\frac{\overline{\gamma}\psi\theta}{2} - \frac{\overline{\gamma}\psi\theta}{2} < 0$$

Which is always the case because the left hand side is always negative since the parameters of the model are assumed to be positive. For Party R:

$$-\frac{\underline{\gamma}\psi\theta}{2} - \frac{\gamma\psi\theta}{2} < 0$$

It is useful to stop for a moment on the Best Responses and write them after solving for x_P and x_R to try to give some interpretation to players' strategies before finding the equilibrium.

For Party P:
$$x_P = \frac{1}{2\psi\theta} + \frac{x_R}{2} - \frac{z}{2\overline{\gamma}} + \frac{1}{2} - \frac{1}{2\overline{\gamma}} + \frac{x_R}{2\overline{\gamma}} \tag{8}$$

For Party R:
$$x_R = \frac{1}{2\psi\theta} + \frac{x_P}{2} - \frac{z}{2\underline{\gamma}} + \frac{1}{2} - \frac{1}{2\underline{\gamma}} + \frac{x_P}{2\underline{\gamma}} \tag{9}$$

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As we can see, the optimal choice of x_P depends on a set of parameters and on x_R. A stricter immigration policy from Party R increases the incentive for P to follow the same strategy even though the fact that $\overline{\gamma} > \underline{\gamma}$ makes P less responsive than the opponent to changes in the other party's migration policy. As expected, x_P is decreasing in θ because a higher salience of immigration for nationalists raises the importance of choosing a low x in terms of victory probabilities since it appeals more to swing voters. Interestingly, x_P is decreasing in economic polarization (z) and increasing in $\overline{\gamma}$, which again makes sense because it raises the cost for Party P to choose a policy that is further from the loyal voters' ideal point.

Solving the system of equations of the Best Responses we get the optimal choice for both parties x_P^* and x_R^*:

$$x_P^* = \frac{1}{2\psi\theta} - \frac{z}{2\overline{\gamma}} + \frac{1}{2} - \frac{1}{2\overline{\gamma}} + \left(\frac{1}{2} + \frac{1}{2\overline{\gamma}}\right)\left(\frac{-3\overline{\gamma}\underline{\gamma} + \underline{\gamma}\psi\theta z - 3\overline{\gamma}\underline{\gamma}\psi\theta + \underline{\gamma}\psi\theta + 2\overline{\gamma}\psi\theta z + \overline{\gamma}\psi\theta - \overline{\gamma} + \psi\theta z + \psi\theta}{-3\overline{\gamma}\underline{\gamma}\psi\theta + \underline{\gamma}\psi\theta + \overline{\gamma}\psi\theta + \psi\theta}\right)(10)$$

$$x_R^* = \left(\frac{-3\overline{\gamma}\underline{\gamma} + \underline{\gamma}\psi\theta z - 3\overline{\gamma}\underline{\gamma}\psi\theta + \underline{\gamma}\psi\theta + 2\overline{\gamma}\psi\theta z + \overline{\gamma}\psi\theta - \overline{\gamma} + \psi\theta z + \psi\theta}{-3\overline{\gamma}\underline{\gamma}\psi\theta + \underline{\gamma}\psi\theta + \overline{\gamma}\psi\theta + \psi\theta}\right)(11)$$

Proposition 1 *In the game where Party P and Party R have to choose simultaneously an immigration policy x_J, an equilibrium where both x_P and x_R are strictly between 0 and 1 exists for a nonempty set of parameters with*

$$z \in \left(\frac{6\overline{\gamma}^2\underline{\gamma} + 2\overline{\gamma}\underline{\gamma} + \psi\theta(6\overline{\gamma}^2\underline{\gamma} + 4\overline{\gamma}\underline{\gamma} - 2\overline{\gamma}^2 - 4\overline{\gamma} - 2\underline{\gamma} - 2)}{\psi\theta(4\overline{\gamma}\underline{\gamma} + 2\overline{\gamma} + 2\overline{\gamma}^2)}, \frac{\overline{\gamma} + 3\overline{\gamma}\underline{\gamma}}{\psi\theta(\underline{\gamma} + 2\overline{\gamma} + 1)}\right)$$

and

$$z \in \left(\frac{6\overline{\gamma}^2\underline{\gamma} + 2\overline{\gamma}\underline{\gamma} + \psi\theta(6\overline{\gamma}^2\underline{\gamma} + 4\overline{\gamma}\underline{\gamma} - 2\overline{\gamma}^2 - 4\overline{\gamma} - 2\underline{\gamma} - 2)}{\psi\theta(4\overline{\gamma}\underline{\gamma} + 2\overline{\gamma} + 2\overline{\gamma}^2)}, \frac{3\overline{\gamma}^2\underline{\gamma} + \overline{\gamma}\underline{\gamma} + \psi\theta(3\overline{\gamma}\underline{\gamma} - \overline{\gamma} - \underline{\gamma} - 1)}{\psi\theta(2\overline{\gamma}\underline{\gamma} + \overline{\gamma} + \overline{\gamma}^2)}\right)$$

Proposition 1 identifies a set of values where $x_J \in (0, 1)$ meaning that the parties do not choose a migration policy equal to 0 or 1. All the proofs are in appendix. However, it is important to mention that corner equilibria may exist and mixed equilibria as well, but in this paper I focus on the internal solution because it is also the one which seems to give us an interpretation of the European political context. Giving the specific nature of borders and the system of asylum, it is almost impossible that western democracies can implement a zero immigration policy. Even during the famous experience of the Italian government ruled by the two populist parties Five Stars Movement and Lega when they claimed that *"ports are closed"*, immigrants and refugees were not completely stopped.

Proposition 2 *In equilibrium, where $x_J \in (0, 1)$ $\forall J$, x_P will always be larger than x_R if $z > \frac{1}{\psi\theta}$.*

We assume that this condition is verified throughout the paper. The intuition behind Proposition 2 is pretty simple. Poor cosmopolitan voters are more sensitive towards migration and a stricter border policy implies a higher loss for them. Therefore, to balance the probability of winning with the gains from winning, Party P will have to choose a higher x compared to Party R (recall that $x = 1$ is the optimal migration policy for cosmopolitans).

3.3 Comparative Statics

I can now explore how a change in inequality will affect the equilibrium of the model. I believe that this is the most interesting parameter of the setting if we consider the rise in inequality of the last decades along with a more pronounced nationalist rhetoric. However, nationalism is not the only feature of the recent political debate, but it is important to include polarization as well, as underlined by the electoral campaigns in France, United States, England and Germany in 2024.

Proposition 3 *An increase of parameter z leads P and R to choose a lower* x_J.

An increase in inequality leads both parties to choose a stricter immigration policy. The intuition behind this result is that an increase in economic polarization raises incentive for both parties to win the elections and, in order to increase the probability of winning, they have to choose a policy that will appeal more to nationalist swing voters. This is in line with what we witnessed in the past years when even traditional left wing parties adopted a rigid position towards immigration.

What about cultural polarization? Does an increase in inequality reduce the distance between the two parties when they have to choose x_J or, on the contrary, inequality will increase polarization also on the cultural issue?

Proposition 4 *The distance between* x_P *and* x_R *is increasing in z.*

In the model proposed by Besley and Persson (2021), in the case of symmetry of parties' strategies, this does not happen because the cosmopolitan voters are assumed to be identical. On the contrary, for Party P it is more costly to adopt a migration policy closer to the optimal point of nationalist swing voters because this will lead to a higher disutility for its loyal base even with an increase of economic polarization. Therefore, both parties will respond to an increase in inequality with a stricter immigration policy, but party P will react less than party R. As a consequence, the distance between x_P^* and x_R^* will increase. Again, this is what we witnessed in European politics in the last years where both left and right wing parties reduce their support for immigration, but progressive voters could not accept tougher policies such as the ones proposed by nationalist/conservatives.

Figure 1 shows the logic of the comparative statics: in the first equilibrium, as stated in Proposition 2, $x_P^* > x_R^*$. However in the second equilibrium, an
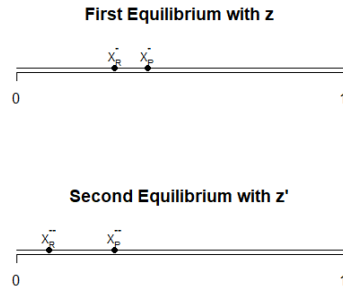
First Equilibrium with z

Second Equilibrium with z'

Figure 1: Equilibrium cultural policy of Party P and Party R with z and $z' > z$

increase of z, where $z' > z$, leads to a stricter migration policy for both parties, but polarization will increase. In the second equilibrium, in fact, we can see that $x_P^* - x_R^* < x_P^{**} - x_R^{**}$.

4 Empirical Analysis

The following analysis is conducted in order to verify if it is possible to find some empirical evidence consisted with the model presented so far. I use the Varieties of Democracy (V-Dem) dataset, an attempt to measure democracy and several aspects of the political life of countries. The data for some countries go back to 1789, but I decided to use only a sample of western countries, including new democracies, for the years 1995-2020. The reason behind this choice is connected to the fact that Europe was split in two during cold war and it would be conceptually misleading to compare Italy and Poland, for instance, in 1980. The sample of countries includes: Italy, United Kingdom, United States of America, France, Spain, Portugal, Germany, Belgium, Austria, Denmark, Sweden, Ireland, Greece, Canada, Finland, Australia, New Zealand, Netherlands, Czechia, Estonia, Hungary, Latvia, Lithuania, Poland, Slovakia and Slovenia.

Figure 2 reports the Gini index from the World Inequality Database for the main western democracies. I included a smaller sample of countries to

avoid the figure to be confusing. As we can notice, there is a certain degree of heterogeneity in the trend of Gini Index of the past decades with the United States that recorded the highest increase.
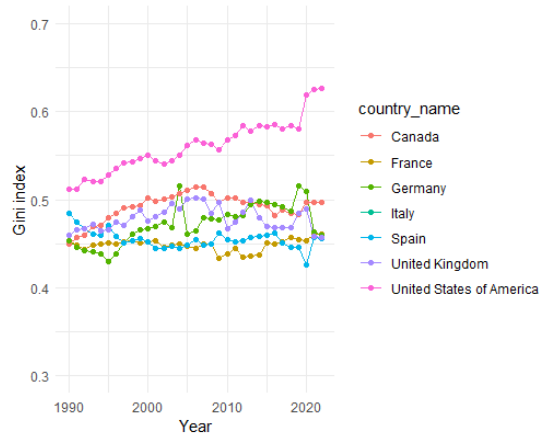

Figure 2: Gini index, 1990-2020

Figure 3, on the contrary, reports the level of polarization according to V-Dem dataset. A higher value indicates that supporters of different political parties avoid to have friendly interactions. The set of countries is the same. As we can see, more or less around the years of the economic crisis in the United States and Europe (2008-2011) there is a sharp increase in political polarization in almost every country. It is in this sense that we can think of a context where identity started playing a role. Visibly, Brexit, the election of Donald Trump, the migration issue polarized our society and with the recent events in the United Kingdom, Germany and France, this trend does not seem to stop.

4.1   Methodology and Results

To understand the correlation between inequality and political polarization, I will use data from different sources. Table 5 reports the summary statistics for
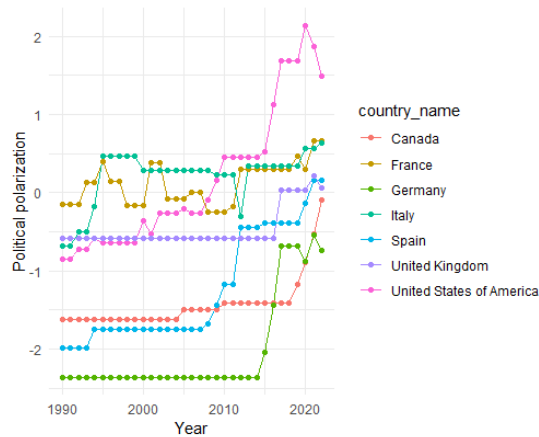
Figure 3: Political polarization, 1990-2020

the sample. Observations are different countries for different years.

Table 5: Summary Statistics

Statistic	N	Mean	St. Dev.	Min	Max
Polarization in society	777	−1.245	1.237	−3.806	3.046
Gini	687	0.455	0.051	0.307	0.616
Gdp	777	1,205,448.000	2,905,833.000	9,809.954	20,136,688.000
Unemployment	752	8.714	4.260	1.100	27.686
Population	777	31,259.500	57,208.620	1,314.545	328,330.000

The variable *political polarization* comes from the V-Dem dataset and it summarises the level of polarization within a society, as in Figure 3. The Gini index is taken from the World Inequality Database, while data on GDP, Unemployment Rate and Population are from the World Development Indicators.

The following results are obtained with a simple OLS regression where the main dependent variable is the level of political polarization in the society and the main independent one is the Gini index at the country level.

$$y_{ct} = \alpha + \beta X_{ct} + \delta_t + \gamma_c + \eta_{ct} \tag{12}$$

y_{ct} is the level of political polarization for country c in year t. X_{ct} represents a set of independent variables where I include the Gini coefficient, the GDP, the Unemployment rate and the population. δ and γ are, respectively, year and country fixed effects. Finally, η is the error term. Since I am dealing with a panel data, I can add both country and time fixed effects to account for unobserved factors related to specific countries or years where some particular events might affect the relationship.

Table 6: Inequality and polarization

	Dependent variable: Polarization in society			
	(1)	(2)	(3)	(4)
Gini	1.733**	2.943***	2.469**	3.836***
	(0.866)	(1.071)	(1.036)	(0.957)
Country Fixed Effects	No	Yes	Yes	Yes
Year Fixed Effects	No	No	Yes	Yes
Controls	No	No	No	Yes
Observations	687	687	687	665
R^2	0.005	0.818	0.843	0.863

Notes: The controls included are the GDP, the population, and the unemployment rate. All the standard errors are clustered at the country level and reported in parentheses. *, ** and *** indicate significance at the 10%, 5% and 1% levels, respectively.

As it is possible to see from Table 6, the correlation between inequality and polarization in society is positive and significant even after controlling for country fixed effects, year fixed effects, gdp, population and unemployment. A one standard deviation increase in the Gini coefficient is associated with an increase in political polarization by approximately 0.2340 standard deviation.

This is in line with the model and the interpretation of the comparative statics where an increase in the level of inequality leads to a more polarized society in the sense that the distance between political parties will increase when they have to choose a cultural policy. In general, the variable *political polarization* captures the fact that citizens from different side of the political spectrum avoid to engage in friendly interactions. Considering western democracies, this logic can be easily applied to political parties in the last years when politicians from opposite sides started to radicalize their policies and delegitimise the opponents.

Conclusion

This paper proposes a theoretical model that contributes to the literature on identity politics, inequality and political polarization. These issues became more pressing in western countries in recent years and political polarization, in particular, is something that we witness nowadays in every electoral campaign. The goal of the model is to explain why people can decide to vote for the political party that do not represent their economic interest and how the resulting equilibrium might lead to a more polarized society.

Following the work of Besley and Persson (2021), I analyse a model where citizens are divided along an economic line and along a cultural line, in our case nationalists and cosmopolitans. Two political parties have to propose an immigration policy that can attract nationalist swing voters and, therefore, increase the probability of winning, or please the loyal cosmopolitan voters and increase their utility once they are in power. It is a standard trade-off between wanting a higher chance of ruling the country or a larger utility from policies closer to the bliss points of loyal voters. The change to the standard model that I introduce is a different salience of immigration for the cosmopolitans of the two groups. This is due to the fact that preferences are heterogeneous and it is possible to have voters that place a different salience on each topic.

This difference modifies the original model by introducing asymmetry in

the two parties' strategies. As a consequence, the resulting equilibrium will be a situation where the party whose loyal voters place more importance on the cultural dimension will have to choose a less strict immigration policy, lowering the probability of winning. Interestingly, an increase in economic inequality, will result in a increase in polarization with respect to the cultural issue. This is in line with what we witnessed in recent years when both left and right wing parties opposed immigration, but with a different salience put on the topic. In general, it seems that progressive parties suffer this dynamic more than conservative ones and this could explain why in several countries, right wing parties are stronger nowadays. A tougher identity policy it is more costly for progressive loyal voters. In addition, this passage from economic inequality to political polarization on another domain could also explain why the electoral campaigns seem more focused on a moral clash rather than addressing poverty with redistributive policies.

Finally, I briefly tested the model empirically, using the V-Dem dataset. With a sample composed of the main democratic countries for the period 1995-2020, the results suggest a positive correlation between inequality and political polarization, as in the model results. One possible extension of the model is to consider the case where immigrants are needed for economic growth, a topic that is starting to become more central with the common trend of ageing population of the West or what would happen if voters place a different weight on inequality.

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A - Proofs

Proof of Proposition 1. An interior equilibrium exist when the following inequalities are simultaneously verified:

$x_P^* > 0$ or:

$$z > \frac{6\overline{\gamma}^2\underline{\gamma} + 2\overline{\gamma}\underline{\gamma} + \psi\theta(6\overline{\gamma}^2\underline{\gamma} + 4\overline{\gamma}\underline{\gamma} - 2\overline{\gamma}^2 - 4\overline{\gamma} - 2\underline{\gamma} - 2)}{\psi\theta(4\overline{\gamma}\underline{\gamma} + 2\overline{\gamma} + 2\overline{\gamma}^2)} \tag{13}$$

$x_P^* < 1$ or:

$$z < \frac{3\overline{\gamma}^2\underline{\gamma} + \overline{\gamma}\underline{\gamma} + \psi\theta(3\overline{\gamma}\underline{\gamma} - \overline{\gamma} - \underline{\gamma} - 1)}{\psi\theta(2\overline{\gamma}\underline{\gamma} + \overline{\gamma} + \overline{\gamma}^2)} \tag{14}$$

$x_R^* > 0$ or:

$$z > \frac{3\overline{\gamma}\underline{\gamma} + \overline{\gamma} + \psi\theta(-3\overline{\gamma}\underline{\gamma} - \overline{\gamma} - \underline{\gamma} - 1)}{\psi\theta(\underline{\gamma} + 2\overline{\gamma} + 1)} \tag{15}$$

$x_R^* < 1$ or:

$$z < \frac{\overline{\gamma} + 3\overline{\gamma}\underline{\gamma}}{\psi\theta(\underline{\gamma} + 2\overline{\gamma} + 1)} \tag{16}$$

Graphically, this conditions are satisfied in Fig.4:

The shaded area of Figure 4 represents the area where (13), (14), (15) and (16) are simultaneously true. I also impose the condition that $\overline{\gamma} < z$ to satisfy the assumption of the model that state that cosmopolitans are loyal voters and $z > \frac{1}{\psi\theta}$ in line with the proof of Proposition 2. The choice of the other parameters are made according to the set up of the model.
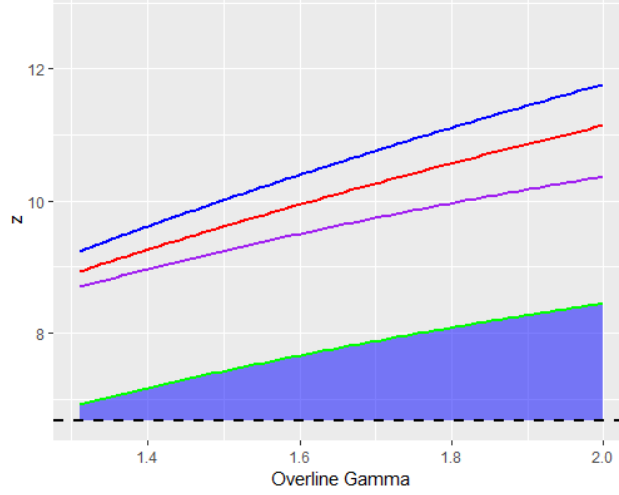
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Figure 4: Interior solutions where $z > (\overline{\gamma}, \frac{1}{\psi\theta})$

Since (13) is larger than (15), to prove that the conditions are met we just need $z >$ (13) and $z <$ (16) or, combined, (16) $-$ (13) > 0 and $z >$ (13) and $z <$ (14) or, combined, (14) $-$ (13) > 0.

(16) $-$ (13) > 0 is:

$$\frac{\overline{\gamma} + 3\overline{\gamma}\underline{\gamma}}{\psi\theta(\underline{\gamma} + 2\overline{\gamma} + 1)} - \frac{6\overline{\gamma}^2\underline{\gamma} + 2\overline{\gamma}\underline{\gamma} + \psi\theta(6\overline{\gamma}^2\underline{\gamma} + 4\overline{\gamma}\underline{\gamma} - 2\overline{\gamma}^2 - 4\overline{\gamma} - 2\underline{\gamma} - 2)}{\psi\theta(4\overline{\gamma}\underline{\gamma} + 2\overline{\gamma} + 2\overline{\gamma}^2)} > 0$$

Which, after some algebra, can be simplified to:

$$\psi\theta > \frac{\overline{\gamma}(\underline{\gamma} + \underline{\gamma}^2 + 3\overline{\gamma}^2\underline{\gamma} - 3\overline{\gamma}\underline{\gamma}^2 - \overline{\gamma}^2 - \overline{\gamma})}{(1 + 2\underline{\gamma} + \underline{\gamma}^2 + \overline{\gamma}(4 + 5\overline{\gamma} + 2\overline{\gamma}^2 + 2\underline{\gamma} - 2\underline{\gamma}^2 - 4\overline{\gamma}\underline{\gamma} - 6\overline{\gamma}^2\underline{\gamma} - 3\overline{\gamma}\underline{\gamma}^2))} \quad (17)$$

(17) is verified for certain values of $\overline{\gamma}$ and $\underline{\gamma}$ that make the numerator negative and, since ψ and θ are positive, the right hand side is bigger than the left hand side.

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$(14) - (13) > 0$ is:

$$\frac{3\overline{\gamma}^2\underline{\gamma}+\overline{\gamma}\underline{\gamma}+\psi\theta(3\overline{\gamma}\underline{\gamma}-\overline{\gamma}-\underline{\gamma}-1)}{\psi\theta(2\overline{\gamma}\underline{\gamma}+\overline{\gamma}+\overline{\gamma}^2)} - \frac{6\overline{\gamma}^2\underline{\gamma}+2\overline{\gamma}\underline{\gamma}+\psi\theta(6\overline{\gamma}^2\underline{\gamma}+4\overline{\gamma}\underline{\gamma}-2\overline{\gamma}^2-4\overline{\gamma}-2\underline{\gamma}-2)}{\psi\theta(4\overline{\gamma}\underline{\gamma}+2\overline{\gamma}+2\overline{\gamma}^2)} > 0$$

Which, after some algebra, can be simplified to:

$$\psi\theta > \frac{\overline{\gamma}(3\overline{\gamma}\underline{\gamma}+\underline{\gamma})}{-\overline{\gamma}\underline{\gamma}+3\overline{\gamma}+\underline{\gamma}+1-6\overline{\gamma}^2\underline{\gamma}+2\overline{\gamma}^2} \tag{18}$$

Again, (18) is verified for certain values of $\overline{\gamma}$ and $\underline{\gamma}$ which make the denominator negative.

Proof of Proposition 2

Note that $x_P^* > x_R^*$ implies that $x_P^* - x_R^* > 0$.

Recall that:

$$x_P^* = \frac{1}{2\psi\theta} - \frac{z}{2\overline{\gamma}} + \frac{1}{2} - \frac{1}{2\overline{\gamma}} + \left(\frac{1}{2}+\frac{1}{2\overline{\gamma}}\right)\left(\frac{-3\overline{\gamma}\underline{\gamma}+\underline{\gamma}\psi\theta z-3\overline{\gamma}\underline{\gamma}\psi\theta+\underline{\gamma}\psi\theta+2\overline{\gamma}\psi\theta z+\overline{\gamma}\psi\theta-\overline{\gamma}+\psi\theta z+\psi\theta}{-3\overline{\gamma}\underline{\gamma}\psi\theta+\underline{\gamma}\psi\theta+\overline{\gamma}\psi\theta+\psi\theta}\right)$$

$$x_R^* = \left(\frac{-3\overline{\gamma}\underline{\gamma}+\underline{\gamma}\psi\theta z-3\overline{\gamma}\underline{\gamma}\psi\theta+\underline{\gamma}\psi\theta+2\overline{\gamma}\psi\theta z+\overline{\gamma}\psi\theta-\overline{\gamma}+\psi\theta z+\psi\theta}{-3\overline{\gamma}\underline{\gamma}\psi\theta+\underline{\gamma}\psi\theta+\overline{\gamma}\psi\theta+\psi\theta}\right)$$

Define $D = \frac{3\overline{\gamma}\underline{\gamma}+\overline{\gamma}-Q-\psi\theta z(\underline{\gamma}+2\overline{\gamma}+1)}{Q}$ and $Q = \psi\theta(-3\overline{\gamma}\underline{\gamma}+\underline{\gamma}+\overline{\gamma}+1)$, where Q is always negative because $\underline{\gamma}+\overline{\gamma}+1 < 3\overline{\gamma}\underline{\gamma}$ and $D \in (0,1)$ in every interior equilibrium as we showed in the proof of Proposition 1.

then $x_P^* - x_R^* > 0$ becomes:

$$\frac{1}{2\psi\theta} - \frac{z+1}{2\overline{\gamma}} + \frac{1}{2} + (\frac{1}{2} + \frac{1}{2\overline{\gamma}})D - D > 0 \tag{19}$$

After some algebra, it is possible to simplifies (19) to:

$\frac{\overline{\gamma}}{\psi\theta} + (\overline{\gamma} - 1)(1 - D) > z$

If we expand D and manipulate the expressions we have:

$\frac{\overline{\gamma}\psi\theta(3\overline{\gamma}\underline{\gamma} - \underline{\gamma}\overline{\gamma} - 1)}{\psi\theta} + \psi\theta z 2\overline{\gamma}(\overline{\gamma} - \underline{\gamma}) > (\overline{\gamma} - 1)(3\overline{\gamma}\underline{\gamma} + \overline{\gamma})$

Rearrange and expand everything:

$\overline{\gamma}\underline{\gamma} + 2\overline{\gamma^2} + \overline{\gamma} - z\psi\theta\overline{\gamma}\underline{\gamma} - 2z\psi\theta\overline{\gamma^2} - \overline{\gamma}z\psi\theta > z\psi\theta(-3\overline{\gamma} + \underline{\gamma} + \overline{\gamma} + 1) + 3\overline{\gamma}\underline{\gamma} +$
$\overline{\gamma} - z\psi\theta\underline{\gamma} - 2z\psi\theta\overline{\gamma} - z\psi\theta$

Which simplifies to:

$-2\overline{\gamma}\underline{\gamma}z\psi\theta - 2z\psi\theta\overline{\gamma^2} > 2\overline{\gamma}\underline{\gamma} - 2\overline{\gamma^2}$

Rearrange to get $\psi\theta z > 1$ or $z > \frac{1}{\psi\theta}$ which we assume to be true to avoid particular equilibria.

Proof of Proposition 3

$\frac{\mathrm{d}x_P^*}{\mathrm{d}z} < 0$ implies that $-\frac{1}{2\overline{\gamma}} + (\frac{1}{2} + \frac{1}{2\overline{\gamma}})(\frac{\underline{\gamma} + 2\overline{\gamma} + 1}{-3\overline{\gamma}\underline{\gamma} + \underline{\gamma} + \overline{\gamma} + 1}) < 0$ which is always negative.

While $\frac{\mathrm{d}x_R^*}{\mathrm{d}z} < 0$ implies that $\frac{\underline{\gamma} + 2\overline{\gamma} + 1}{-3\overline{\gamma}\underline{\gamma} + \underline{\gamma} + \overline{\gamma} + 1} < 0$ which is negative.

Proof of Proposition 4

Consider the case when $\dfrac{\mathrm{d}(x_P^* - x_R^*)}{\mathrm{d}z} > 0$. This implies that:

$$-\frac{1}{2\overline{\gamma}} + \left(\frac{1}{2} + \frac{1}{2\overline{\gamma}}\right)\frac{\mathrm{d}D}{\mathrm{d}z} - \frac{\mathrm{d}D}{\mathrm{d}z} > 0$$

If we expand D:

$$-\frac{1}{2\overline{\gamma}} + \left(\frac{1}{2} + \frac{1}{2\overline{\gamma}}\right)\left(\frac{1}{2} + \frac{1}{2\overline{\gamma}}\right)\left(\frac{\gamma + 2\overline{\gamma} + 1}{-3\overline{\gamma}\underline{\gamma} + \gamma + \overline{\gamma} + 1}\right) - \left(\frac{\gamma + 2\overline{\gamma} + 1}{-3\overline{\gamma}\underline{\gamma} + \gamma + \overline{\gamma} + 1}\right) > 0$$

After some algebra, we get:

$$\frac{(\overline{\gamma} - 1)(1 + 2\overline{\gamma} + \gamma)}{3\overline{\gamma}\underline{\gamma} - \gamma - \overline{\gamma} - 1} > 1$$

Get rid of the denominator (which is always positive) and simplify the whole expression:

$$2\overline{\gamma^2} > 2\overline{\gamma}\underline{\gamma}$$

And finally:

$\overline{\gamma} > \underline{\gamma}$, which is always true under the assumptions of the model.