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in a Disability Model**

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Quaderno n. 125/aprile 2017

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ABSTRACT

This paper investigates a system of dynamic incentives developed within the framework of the classic Diamond and Mirrlees (1978) disability model, but considering disability as a temporary state and rephrasing the analysis in terms of current and promised future utilities. The model therefore assumes that if disabled individuals receive benefits to the extent that able individuals are indifferent between working and not working, then the marginal utility of consumption is lower for working individuals. A comparison, based on a numerical simulation, between the dynamic incentives (*DI*) model and a private savings (*PS*) model characterised by a stationary tax-transfer policy allows the assertion that, even if the first system converges to the second system, the total utility guaranteed by the government in the *DI* model is greater than the total value achieved by the *PS* model, and in the *DI* model, the gap in consumption between able and disabled individuals increases not only along working histories, as in the *PS* model, but also across working histories.

Keywords: temporary disability, dynamic incentives, private savings.
JEL classification: H53, H31.

1. INTRODUCTION

At the beginning of the 20th century, the private market economies of most Western industrial countries operated almost completely devoid of government regulation. In fact, these countries primarily relied on private markets to allocate resources, including labour, and the primary role of government was viewed as the enforcement of private contracts. Near the end of the century, even though in the labour markets wages and working conditions continued to be established through supply and demand interactions, negotiations for wages and working conditions were carried out by larger entities (unions and firms) through collective bargaining, and regulations, that were intended to ensure minimum working conditions for all workers, established socially determined boundaries for labour market transactions, i.e., health and safety regulations, maximum working hours, and minimum wages. Furthermore, beginning in the second half of the last century, social insurance systems were implemented with the aim of protecting workers from economic hardship related to exit from a job, i.e., unemployment insurance, old-age and survivors' insurance, sickness and accident insurance, and disability insurance (see, e.g., Bound and Burkhauser, 1999).

Disability is defined as a condition that makes individuals unable to perform or that limits their ability to perform work, and it can be permanent or temporary. The economic and political rationale for public disability insurance resides in the fact that both private savings and private disability insurance are not likely to be effective mechanisms for limiting the risks associated with a permanent loss of earning capacity (see, e.g., Bound and Burkhauser, 1999).

Deaton (1991), within a context of a simple model of optimal private savings by liquidity-constrained consumers, shows that the effectiveness of private savings at insuring individuals against shocks to labour earnings declines as the persistence of these shocks rises, and private savings become completely ineffective—individuals do not save at all—when labour earnings follow a random walk and shocks are permanent. Moreover, private disability insurance alone is not a viable alternative to private savings. In fact, as with all insurance programmes—private and public—disability insurance is affected not only by the moral hazard problem (see, e.g., Diamond and Mirrlees, 1978) but also by the adverse selection problem (see, e.g., Whinston, 1983): individuals not only can ex post decrease their ability to supply labour by adopting certain consumer behaviours, but individuals are also ex ante characterised by different probabilities of becoming disabled.

In this respect, public disability insurance is designed to reduce the risks associated with lost earnings resulting from poor or deteriorating health through mandatory actuarially fair insurance. However, public disability insurance—similar to other social insurance programmes—pursues not only the insurance goal but also a redistributive goal that is justified by equity concerns.

Therefore, disability insurance programmes have become an important part of the modern welfare state and thus an important feature of modern economies. In fact, a sizable proportion of the working age population is enrolled in public disability insurance schemes in all developed countries (see, e.g., Bratberg, 1999; Brinch, 2009).

The growth and magnitude of disability insurance programmes generate at least two concerns, and a consequent debate, involving both economists and policymakers: the first is that the programmes strain public finance; the second is that the programmes contribute to generating an inefficiently low payoff for working and thereby waste resources by reducing the labour supply below efficient levels (see, e.g., Brinch, 2009). Diamond and Sheshinski (1995) analyse optimal disability and welfare (or early retirement) benefits with an imperfect disability evaluation (i.e., some able workers are judged disabled and some disabled workers are judged able). The authors show not only that the levels of both disability and welfare benefits affect the labour supply because of the wide range of difficulties or disutilities associated with working but also that the alternative source of income for certain non-working individuals is disability benefits rather than welfare benefits, even though, in principle, disability benefits are only available to those individuals who are unable to find any remunerative employment as a consequence of their disability.

As previously mentioned, disability can be permanent or temporary. In Meyer and Mok (2013) the degree of persistence or severity of an individual's disabling condition is determined based on the frequency of positive limitation reports after disability onset. In detail, those authors divide the disabled into three persistence groups, building on Charles (2003): the one-time disabled are those who report a disability once, the temporarily disabled are those who have other one or two positive limitation reports within the ten years after the initial disability onset, and, lastly, the chronically disabled are those who have other three or more positive limitation reports during the ten years after the initial disability onset.¹

¹ In the US all workers' compensation claims are initially classified as temporary cases and temporary total benefits are paid; if the disability persists beyond the date of

A large body of literature considers disability as a persistent, in fact permanent, skill shock. Golosov and Tsyvinski (2006, p. 259) justify this assumption by arguing that ‘less than 1 per cent of those who start receiving disability benefits from the Social Security Disability Insurance (*SSDI*) system return to work’.

Diamond and Mirrlees (1978) assume that disability is a permanent state and propose a model of public insurance with a continuum of individuals in which an individual’s ability to supply labour is affected by a random variable (health) that is unobservable by the government. Hence, it is impossible to know whether an individual is truly disabled, and the government faces a moral hazard constraint: if social insurance is overly generous, workers will be tempted to claim disability when they are actually able to work. The aim of Diamond and Mirrlees (1978) is to analyse optimal private insurance and examine the interactions between public and private insurance, that is, to verify whether public insurance crowds out private insurance and whether a mixture of public and private insurance is optimal. The authors find that under plausible conditions, at the optimum individuals are indifferent whether to work or not, but they do work when they are able.

Whinston (1983) extends the Diamond and Mirrlees (1978) model to allow for adverse selection caused by multiple unobservable types that have different probabilities of illness. Anderberg and Andersson (2000) consider an economy in which disability risk is observed but endogenous, as workers can influence their probability of disability through occupational choice.

Thomas and Worrall (2007) propose an infinite horizon version of the Diamond and Mirrlees (1978) disability model in which individuals can observe the ability of others to work; hence, the advantage of a private insurance scheme is that it does not face a moral hazard problem. However, the private insurance scheme is voluntary—it cannot mandate payments—and hence, individuals participate if they expect long-term benefits from the scheme.

Golosov and Tsyvinski (2006) reconsider the analysis proposed by Diamond and Mirrlees (1978) and investigate possible tax systems that could implement the optimal allocation. Because the system designed by Diamond and Mirrlees (1978)—a linear tax equal to the intertemporal wedge in the optimal allocation—does not implement the optimum, Golosov and Tsyvinski (2006) propose an asset-tested

maximum medical improvement, the case is reclassified as a permanent disability, and thus the worker is eligible for permanent total benefits (see, e.g., Meyer, 2002).

disability programme: a person only receives a disability transfer if his assets are below a specified threshold. An important feature of this model is the intertemporal provision of dynamic incentives: the social planner rewards an agent for working by increasing the continuation utility when the agent becomes disabled. This effect encourages increased consumption for agents who become disabled later in life.

Nevertheless, as also noted by Golosov and Tsyvinski (2006, p. 259), a low number of disabled returning to work does not necessarily mean that disability is a permanent state. In fact, relatively generous benefits could have significant work disincentives not only for able individuals but also for those affected by temporary disability who return to able status (see, e.g., Bound and Burkhauser, 1999). Meyer and Mok (2013) using the Panel Study of Income Dynamics (PSDI)—a longitudinal dataset for the period 1968-2009 with an initial sample of approximately 4,800 US households and 18,000 individuals—estimate that a person reaching age 50 has a 36 per cent chance of having been at least temporarily disabled once during his working years and a 9 per cent chance of having suffered a chronic and severe disability.

Therefore, this paper implements a system of dynamic incentives developed within the framework of the seminal Diamond and Mirrlees (1978) disability model but assuming that disability is temporary. Furthermore, the analysis is considerably simplified because it is framed in terms of current and promised future utilities. Thus, by means of numerical simulations, the results of the dynamic incentives (*DI*) model are compared with those of a private savings (*PS*) model characterised by a stationary tax-transfer policy².

The remainder of the paper proceeds as follows. Section 2 describes the setup of the model, i.e., it outlines the Diamond and Mirrlees (1978) model, and Section 3 incorporates a system of dynamic incentives within this framework. Section 4 develops a *PS* model implemented through a stationary tax-benefit system. Finally, Section 5 draws conclusions.

² The comparison between the *DI* model and the *PS* model is carried on following the same approach proposed by Shimer and Werning (2007). The authors analyse how a worker behaves when confronted with a constant unemployment benefit system and consider two opposite situations: a worker with no liquidity problems (with access to borrowing and lending) who holds his consumption constant during employment spells (see, e.g., Thomas and Worrall, 1990) and a “hand-to-mouth” worker facing liquidity constraints, who is thus unable to borrow, lacks access to private insurance markets, and must consume his current income (see, e.g., Hansen and İmrohorođlu, 1992).

2. THE DISABILITY MODEL: DIAMOND AND MIRRLEES (1978)

A simple disability model in the spirit of Fair (1971), Mirrlees (1971), Akerlof (1978), Diamond and Mirrlees (1978), Diamond and Sheshinski (1995), and Parsons (1996) will be analysed throughout this paper.

There are two types of individuals in the economy, the able A and the disabled D . As aforementioned in Section I, disability is assumed to be temporary; the exogenous disability status is *i.i.d.* over time and across individuals, with $\Pr[A] = \pi^A$ and $\Pr[D] = \pi^D = 1 - \pi^A$. Moreover, if the population size is assumed to be large, the ability-disability status probabilities are also the population shares of the two types.

A critical distinction between the two states is the ability to work; the disabled cannot work. The able have only one decision to make: whether to work. If they work, they produce a positive quantity of output $y > 0$.

To simplify the analysis, (i) the utility function over consumption and labour of the able workers is assumed to be quasilinear in labour, such that in this world with a binary work decision, utility differs by a constant across work states for a given consumption level; and (ii) a similar functional form exists for the utility characterising the disabled state. Therefore, consumption utility is state independent:

$$(1) \quad U^i = u(c^i) - d \cdot L^i, i = A, D,$$

where d is the per-unit disutility from working, $u'(c) > 0$ and $u''(c) < 0$ with $u'(c) \rightarrow \infty$ as $c \rightarrow 0$ and $u'(c) \rightarrow 0$ as $c \rightarrow \infty$. In the following, it is assumed that all able workers are induced to work; hence, $L^A = 1$ and $L^D = 0$.

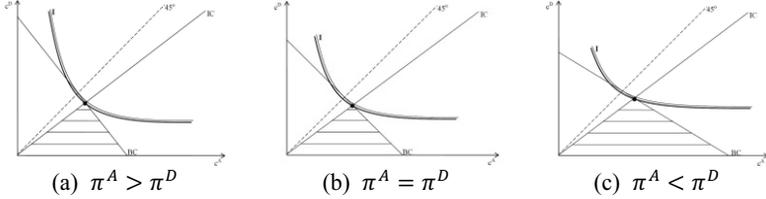
The social insurance programme is defined by only two consumption levels, one for able workers and another for disabled workers. The problem is to maximise ex ante expected utility using the consumption levels as policy variables subject to a budget constraint (BC) and an incentive constraint (IC):

$$(2) \quad \begin{aligned} E(U) \equiv \max_{c^i} & \sum_{i=A,D} \pi^i \cdot [u(c^i) - d \cdot L^i] \\ \text{s. to} & \sum_{i=A,D} \pi^i \cdot (y \cdot L^i) \geq \sum_{i=A,D} \pi^i \cdot c^i \\ & u(c^A) - d \geq u(c^D). \end{aligned}$$

Given $d > 0$, the slope of the IC is less than 1; hence, the IC is flatter than the 45° line (see Figure 1, in which the IC is assumed to

be linear). The indifference curve (I) is tangent to the BC on the 45° line, and the optimum coincides with the intersection of the BC and the IC (on a lower indifference curve). Therefore, as Figure 1 shows, the entire set of incentive-compatible allocations $\{(c^A, c^D) | u(c^A) - d \geq u(c^D)\}$ is below the 45° line.

FIGURE 1. Maximisation Problem

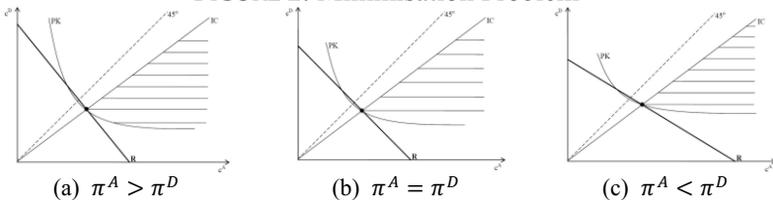


Rather than maximising expected utility, it is possible to minimise resource use (R), which is equivalent to revenue maximisation, subject to a level of expected utility $E(U) = \bar{U}$ promised to agent i , i.e., subject to a promised utility-keeping constraint (PK). Furthermore, because the consumption required to give agent $i = A, D$ utility u^i is $c(u^i) = u^{-1}(u^i)$, the analysis can be significantly simplified if the utilities, rather than the consumption levels, are used as choice variables. Therefore, the minimisation problem can be written as:

$$(3) \quad \begin{aligned} C(\bar{U}) \equiv \min_{u^i} & \sum_{i=A,D} \pi^i \cdot [c(u^i) - y \cdot L^i] \\ \text{s. to} & \sum_{i=A,D} \pi^i \cdot (u^i - d \cdot L^i) \geq \bar{U} \\ & u^A - d \geq u^D. \end{aligned}$$

As observed for the maximisation problem, given $d > 0$, the slope of the IC is less than 1; hence, the IC is flatter than the 45° line (see Figure 2, in which the IC is again assumed to be linear). The optimum coincides with the intersection of the PK and the IC , and the entire set of incentive-compatible allocations $\{(u^A, u^D) | u^A - d \geq u^D\}$ is below the 45° line.

FIGURE 2. Minimisation Problem



3. THE DYNAMIC INCENTIVES MODEL

The ability model proposed by Diamond and Mirrlees (1978) is reconsidered to analyse a system of dynamic incentives. In this economy, time is discrete and finite; hence, the life of an individual can be represented by timing as $t = 1, \dots, n + 1$, where $t = 1, \dots, n$ identifies the working periods, and $t = n + 1$ the retirement period.

Let the ability realisation at time t be $h_t \in \{A, D\}$; because no individual works during retirement, the ability realisation at time $t = n + 1$ is $h_{n+1} = D$. If the history is defined as a sequence of ability realisations $h^t = (h_1, \dots, h_t)$, for $t = 1, \dots, n + 1$ and with $h^t \in H^t$, then it is also possible to write $\pi(h^t) = \pi(h_t) \dots \pi(h_1)$, where $\pi(h_t) \in \{\pi^A, \pi^D\}$ in the working periods $t = 1, \dots, n$ and $\pi(h_{n+1}) = 1$ in the retirement period $n + 1$.

With r as the real interest rate, as in Golosov and Tsyvinski (2006), it is assumed that $\beta = 1/(1 + r)$, and the utility maximisation problem, with $c(h^t)$ as choice variables, is:

$$\begin{aligned}
 (4) \quad & \max_{c(h^t)} \sum_{t=1}^{n+1} \sum_{h^t \in H^t} \beta^{t-1} \cdot \pi(h^t) \cdot [u(c(h^t)) - d \cdot L(h^t)] \\
 (5) \quad & \text{s. to } \sum_{t=1}^{n+1} \sum_{h^t \in H^t} \beta^{t-1} \cdot \pi(h^t) \cdot [y \cdot L(h^t) - c(h^t)] \geq 0 \\
 (6) \quad & \sum_{s=t}^{n+1} \sum_{h^s | h_t = A} \beta^{s-1} \cdot \pi(h^s) \cdot [u(c(h^s)) - d \cdot L(h^s)] \geq \\
 & \sum_{s=t}^{n+1} \sum_{h^s | h_t = D} \beta^{s-1} \cdot \pi(h^s) \cdot [u(c(h^s)) - d \cdot L(h^s)],
 \end{aligned}$$

where the IC (6) states that able workers are induced to work, i.e., if a worker is able in t ($h_t = A$), then he is guaranteed greater utility not only in the current period but also in future periods ($s = t, \dots, n + 1$).

As previously suggested, the problem can be analysed in terms of cost minimisation, where the choice variables are the utilities $u(h^t)$ instead of consumption levels $c(h^t)$. Because the consumption required to give agent $i = A, D$ utility $u(h^t)$ is $c(u(h^t)) = u^{-1}(u(h^t))$, with $c''(u) > 0$, it is possible to write:

$$\begin{aligned}
 (7) \quad & \min_{u(h^t)} \sum_{t=1}^{n+1} \sum_{h^t \in H^t} \beta^{t-1} \cdot \pi(h^t) \cdot [c(u(h^t)) - y \cdot L(h^t)] \\
 (8) \quad & \text{s. to } \sum_{t=1}^{n+1} \sum_{h^t \in H^t} \beta^{t-1} \cdot \pi(h^t) \cdot [u(h^t) - d \cdot L(h^t)] \geq U
 \end{aligned}$$

$$(9) \quad \sum_{s=t}^{n+1} \sum_{h^s | h_t = A} \beta^{s-1} \cdot \pi(h^s) \cdot [u(h^s) - d \cdot L(h^s)] \geq \\ \sum_{s=t}^{n+1} \sum_{h^s | h_t = D} \beta^{s-1} \cdot \pi(h^s) \cdot [u(h^s) - d \cdot L(h^s)],$$

where in the *PK* (8), U is the utility that, in the optimal scheme, implies the *BC* (5) in the original maximisation problem (4) holds with equality.

3.1 THE RECURSIVE FORMULATION

Next, a recursive formulation of the problem previously analysed is proposed.

Individuals may work in period n , i.e., the last working period of their working life, and are retired in period $n + 1$. Then, the lifetime utility function is given by:

$$(10) \quad E(U_n^i) = [u(c_n^i) - d \cdot L^i] + \beta \cdot u(c_{n+1}^i), i = A, D,$$

where the choice variables are c_t (for $t = n, n + 1$) with $u'(c) > 0$ and $u''(c) < 0$.

The government can choose the agent's consumption in both the working period n and retirement period $n + 1$. Moreover, it can make consumption in both periods conditional on the labour supply in period n .

Staying with the dual approach, suppose that the government guarantees the agent expected utility U_n over the two periods. Therefore, the government's objective is to minimise the (discounted) resource use required to provide the agent with the guaranteed expected utility U_n . Because the consumption required to give agent $i = A, D$ utility u_t^i (for $t = n, n + 1$) is $c(u_t^i) = u^{-1}(u_t^i)$, with $c''(u) > 0$, the utilities u_t^i in these two periods (current utility and promised future utility), rather than the consumption levels, can be used as controls. Thus, the government's problem can be written as:

$$(11) \quad C_n(U_n) \equiv \min_{u_n^i, u_{n+1}^i} \sum_{i=A,D} \pi^i \cdot \{[c(u_n^i) - y \cdot L^i] + \beta \cdot c(u_{n+1}^i)\} \\ \text{s. to} \quad \sum_{i=A,D} \pi^i \cdot [(u_n^i - d \cdot L^i) + \beta \cdot u_{n+1}^i] \geq U_n \quad (\lambda_n) \\ (u_n^A - d \cdot L^A) + \beta \cdot u_{n+1}^A \geq \\ (u_n^D - d \cdot L^D) + \beta \cdot u_{n+1}^D \quad (\mu_n),$$

where the superscript $i = A, D$ in the retirement period $n + 1$ refers

to the ability status in the last working period n . Because the solution is binding at both the PK and IC (see Appendix A), the $FOCs$ related to the able and disabled workers are, respectively,

$$(12) \quad \begin{aligned} c'(u_n^A) &= c'(u_{n+1}^A) = \frac{\lambda_n \cdot \pi^A + \mu_n}{\pi^A} \\ c'(u_n^D) &= c'(u_{n+1}^D) = \frac{\lambda_n \cdot \pi^D - \mu_n}{\pi^D}, \end{aligned}$$

where λ_n is the multiplier on the PK and μ_n is the multiplier on the IC in (11). Thus, the solution entails:

$$(13) \quad \begin{aligned} u_n^A &= u_{n+1}^A = u^A, \\ u_n^D &= u_{n+1}^D = u^D. \end{aligned}$$

Because the multiplier $\mu_n > 0$, the disequality $c'(u^A) > c'(u^D)$ holds; hence, $u^A > u^D$. The government provides incentives to work in period n by offering individuals who work a higher utility (and therefore consumption) also in the retirement period $n + 1$.

In the previous working periods ($t = 1, \dots, n - 1$), the consumption level required to give agent $i = A, D$ utility u_t^i in the current working period t is $c(u_t^i) = u^{-1}(u_t^i)$, and the cost level required to give agent $i = A, D$ utility U_{t+1}^i in the future working periods and in the retirement period is $C_{t+1}(U_{t+1}^i) = U_{t+1}^{-1}(U_{t+1}^i)$, with $c''(u) > 0$ and $C_{t+1}''(U_{t+1}) > 0$. As for period n , the utilities in these periods (rather than the consumption levels) can be used as controls. Therefore, the government's problem can be written as the Bellman equation (see Bellman, 1957):

$$(14) \quad \begin{aligned} C_t(U_t) &\equiv \min_{u_t^i, U_{t+1}^i} \sum_{i=A,D} \pi^i \cdot \{[c(u_t^i) - y \cdot L^i] + \beta \cdot C_{t+1}(U_{t+1}^i)\} \\ \text{s. to} &\quad \sum_{i=A,D} \pi^i \cdot [(u_t^i - d \cdot L^i) + \beta \cdot U_{t+1}^i] \geq U_t \quad (\lambda_t) \\ &\quad (u_t^A - d \cdot L^A) + \beta \cdot U_{t+1}^A \geq \\ &\quad (u_t^D - d \cdot L^D) + \beta \cdot U_{t+1}^D \quad (\mu_t). \end{aligned}$$

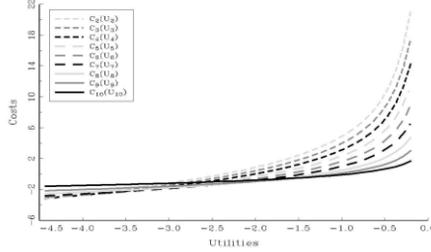
As in the minimisation problem (11), also in the minimisation problem (14) the solution is binding at both the PK and IC (see Appendix A); hence, the $FOCs$ related to the able and disabled workers are, respectively,

$$(15) \quad \begin{aligned} c'(u_t^A) &= C'_{t+1}(U_{t+1}^A) = \frac{\lambda_t \cdot \pi^A + \mu_t}{\pi^A} \\ c'(u_t^D) &= C'_{t+1}(U_{t+1}^D) = \frac{\lambda_t \cdot \pi^D - \mu_t}{\pi^D}, \end{aligned}$$

where λ_t is the multiplier on the *PK* and μ_t is the multiplier on the *IC* in (14). Because the multiplier $\mu_t > 0$, the disequalities $c'(u_t^A) > c'(u_t^D)$ and $C'_{t+1}(U_{t+1}^A) > C'_{t+1}(U_{t+1}^D)$ hold; hence, $u_t^A > u_t^D$ and $U_{t+1}^A > U_{t+1}^D$. Thus the government provides incentives to work in period t by offering individuals who work a higher utility level also in future periods.

The *DI* model is investigated and clarified by means of numerical simulations (see Appendix B) in which individuals are assumed to have *CARA* preferences (see, e.g., Shimer and Werning, 2007). The cost functions derived on the basis of this simulation are represented in Figure 3.

FIGURE 3. $C_t(U_t)$ at $\alpha = 2$, $d = 0.05$, $\pi^A = 0.8$, and $r = 0.03$



Due to the concavity of the utility function, the cost functions are convex. Moreover, the curves are lower when the retirement period is closer, i.e., the cost of guaranteeing a certain level of utility decreases when the retirement period is closer. Given the cost functions as represented in Figure 3, the multiperiod minimisation problem with n working periods can be solved: from $t = 1, \dots, n - 1$, the government's problem is (14), and in the last working period n , it is (11).

3.2 THE PROPERTIES OF THE OPTIMAL ALLOCATION

The aim is to explore the properties of the optimal dynamic allocation. Specifically, the analysis considers how an individual's utility allocation depends on his being disabled and the consecutiveness of the disability spells.

Tables 1 and 2 report the results of the simulation with four working periods ($n = 4$).

TABLE 1. Government Balance ($n = 4$)

	$t = 1$	$t = 2$	$t = 3$	$t = 4$	$t = 5$	
Y	0.8000	0.7767	0.7541	0.7321	0.0000	3.0629
C	0.6496	0.6306	0.6121	0.5940	0.5766	3.0629
$Y - C$	0.1504	0.1461	0.1420	0.1381	-0.5766	0.0000

Table 1 displays how the government saves in all working periods $t = 1, \dots, n$ (with $n = 4$) to finance consumption in the retirement period $n + 1 = 5$.

Table 2 displays the discounted promised future utility $\beta^t \cdot U_{t+1}^i$ and the sum of the discounted utilities $\beta^{t-1} \cdot U_t^i$ (i.e., the sum of discounted current utility $\beta^{t-1} \cdot (u_t^i - d \cdot L^i)$ and discounted promised future utility $\beta^t \cdot U_{t+1}^i$) for each possible history (with $n = 4$, the number of histories is $2^4 = 16$).

Theorem 1 Binding IC: The IC (9) will bind, i.e., will be satisfied with equality, for the optimal solution.

Proof. A proof by contradiction is used to show that the IC (9) will bind. If

$$\begin{aligned} & \sum_{s=t}^{n+1} \sum_{h^s | h_t = A} \beta^{s-1} \cdot \pi(h^s) \cdot [u(h^s) - d \cdot L(h^s)] > \\ & \sum_{s=t}^{n+1} \sum_{h^s | h_t = D} \beta^{s-1} \cdot \pi(h^s) \cdot [u(h^s) - d \cdot L(h^s)], \end{aligned}$$

then the gap between the sum of current and future utilities of a worker able in t and the sum of current and future utilities of a worker disabled in t can be lowered by an amount δ :

$$\begin{aligned} & \sum_{s=t}^{n+1} \sum_{h^s | h_t = A} \beta^{s-1} \cdot \pi(h^s) \cdot [u(h^s) - d \cdot L(h^s)] > \\ & \sum_{s=t}^{n+1} \sum_{h^s | h_t = D} \beta^{s-1} \cdot \pi(h^s) \cdot [u(h^s) - d \cdot L(h^s)] + \beta^{t-1} \cdot \delta, \end{aligned}$$

and this makes it easier to satisfy the PK constraint:

$$\sum_{t=1}^{n+1} \sum_{h^t \in H^t} \beta^{t-1} \cdot \{\pi(h^t) \cdot [u(h^t) - d \cdot L(h^t)] + \delta\} \geq U.$$

So the utilities can be lowered by an amount ε :

$$\sum_{t=1}^{n+1} \sum_{h^t \in H^t} \beta^{t-1} \cdot \{\pi(h^t) \cdot [u(h^t) - d \cdot L(h^t) - \varepsilon] + \delta\} \geq U,$$

but then the original solution was not resource use minimising. ■

In fact, from Table 2, it is possible to verify that the IC (9) binds along each working history:

$$\begin{aligned} \beta^{t-1} \cdot U_t^A &= \beta^{t-1} \cdot U_t^D, \\ U_t^A &= U_t^D, \\ (u_t^A - d) + \beta \cdot U_{t+1}^A &= u_t^D + \beta \cdot U_{t+1}^D, \quad t = 1, \dots, n, \end{aligned}$$

that is, in $t = 1$, $U_1^A = U_1^D = -1.4414$, in $t = 2$, $\beta \cdot U_2^{AA} = \beta \cdot U_2^{AD} = -1.1207$ and $\beta \cdot U_2^{DA} = \beta \cdot U_2^{DD} = -1.1601$, in $t = 3$, following the first path $\beta^2 \cdot U_3^{AAA} = \beta^2 \cdot U_3^{AAD} = -0.8118$ and following the last path $\beta^2 \cdot U_3^{DDA} = \beta^2 \cdot U_3^{DDD} = -0.8768$, and, finally, in $t = 4$, following the first path $\beta^3 \cdot U_4^{AAAA} = \beta^3 \cdot U_4^{AAAD} = -0.5150$ and following the last path $\beta^3 \cdot U_4^{DDDA} = \beta^3 \cdot U_4^{DDDD} = -0.5886$.

Hence, the system of dynamic incentives implies that in each working period $t = 1, \dots, n$ able individuals are indifferent between working and not working, and consequently able individuals are induced to work.

It is interesting to examine whether, under the optimal dynamic incentives scheme, a worker is better off (i) if his disability occurs early versus late in his lifecycle, and (ii) if his disability spells are consecutive or non-consecutive. Because it is important to consider whether the disability spell is closer to or further from retirement, as in Shimer and Werning (2007), the analysis is phrased in terms of the number of working periods remaining before retirement, defined as $q = n - t + 1$.

Result 1 Early versus late disability: Late disability is worse than early disability.

Proof. The focus is on the cases with one disability spell at $t = 3$ and precisely on the sum of the discounted utilities of a disabled individual when $q = 3$ working periods remain (at $t = 2$) before retirement $\beta^2 \cdot U_3^{ADh_3}$ and the sum of the discounted utilities of a disabled individual when $q = 4$ working periods remain (at $t = 1$) $\beta^2 \cdot U_3^{DAh_3}$. From Table 2, because $\beta^2 \cdot U_3^{ADh_3} [-0.8477] < \beta^2 \cdot U_3^{DAh_3} [-0.8409]$, the sum of the discounted utilities of an individual disabled when $q = 3$ working periods remain is lower than the sum of the discounted utilities of an individual disabled when $q = 4$ working periods remain. ■

When the time horizon is shorter, the dynamic incentives are restricted, and, consequently, the system of dynamic incentives guarantees higher disability benefits if the temporary disability occurs in the early periods of the working life, i.e., younger disabled individuals are better insured than older disabled individuals.

Therefore, late disability is worse than early disability³. Thus, this system of dynamic incentives implies that older individuals—supposedly more skilled and more efficient workers—are more encouraged to work (see, e.g., Golosov and Tsyvinski, 2006). Moreover, if temporary disability occurs very close to retirement, the alternative source of income could be early retirement benefits rather than disability benefits (see, e.g., Diamond and Sheshinski, 1995).

Result 2 Consecutive versus non-consecutive disability spells: The sum of the discounted utilities of an individual suffering from two consecutive disability spells is lower than the sum of the discounted utilities of an individual suffering from two non-consecutive disability spells if the long disability spell occurs closer to the retirement period.

Proof. The focus is on cases with two consecutive disability spells and two non-consecutive disability spells at $t = 4$. Specifically, the focus is on (i) the sum of the discounted utilities of a disabled individual when $q = 4$ working periods remain before retirement (at $t = 1$) and $q = 3$ working periods remain (at $t = 2$) $\beta^3 \cdot U_4^{DDAh_4}$; (ii) the sum of the discounted utilities of a disabled individual when $q = 3$ working periods remain (at $t = 2$) and $q = 2$ working periods remain (at $t = 3$) $\beta^3 \cdot U_4^{ADh_4}$; and (iii) the sum of the discounted utilities of a disabled individual when $q = 4$ working periods remain (at $t = 1$) and $q = 2$ working period remain (at $t = 3$) $\beta^3 \cdot U_4^{DADh_4}$. From Table 2, because $\beta^3 \cdot U_4^{ADh_4}[-0.570] < \beta^3 \cdot U_4^{DADh_4}[-0.565] < \beta^3 \cdot U_4^{DDAh_4}[-0.558]$, the sum of the discounted utilities of an individual suffering from two consecutive disability spells is lower than the sum of the discounted utilities of an individual suffering from two non-consecutive disability spells if the long disability spell occurs closer to the retirement period. ■

Because a short time horizon limits the effectiveness of dynamic

³ Result 1 diverges from the findings of Golosov and Tsyvinski (2006). In the model proposed by those authors—in which disability is a permanent state—the intertemporal provision of dynamic incentives encourages higher consumption for agents who become disabled later in life, and therefore early disability is worse than late disability. Golosov and Tsyvinski (2006) interpret their system as indicating that individuals who become disabled early in life receive larger transfers, whereas those who become disabled later are assumed to supplement their lower disability transfers with savings accumulated while able.

incentives, when analysing how the dynamic incentives scheme treats consecutive periods of disability, it is important to also consider whether the long disability spell is closer to or further from retirement (see Result 1).

3.3 THE CONSUMPTION GAPS

The purpose is to analyse, first, the consumption paths and the consumption gaps between able and disabled individuals, and second, the average current consumption gap and the variance in the current consumption gaps.

Table 2, where $n = 4$, not only displays the discounted promised future utilities and the sums of the discounted utilities but also the consumption gaps. A worker who is able in period t , i.e., $h_t = A$, and a worker who is disabled in period t , i.e., $h_t = D$, are considered. The able worker has a higher level of consumption, and the gap in consumption is:

$$(16) \quad \begin{aligned} \Delta c &= c(h_t = A) - c(h_t = D), & t = 1, \\ \Delta c(h^{t-1}) &= c(h^{t-1}; h_t = A) - c(h^{t-1}; h_t = D), & t = 2, \dots, n, \end{aligned}$$

where $(h^{t-1}; h_t = A)$ is the period t history when the worker is able in t , and $(h^{t-1}; h_t = D)$ is the period t history when the worker is disabled in t . As observed from equation (16), if $t > 1$, this consumption gap generally depends on the history in the periods preceding t , i.e., h^{t-1} .

Result 3 Consumption gaps in the DI model: The consumption gaps (a) not only increase when the retirement period becomes closer (b) but also if an individual has been “more able” during his working life.

Proof a. From Table 2, in $t = 4$, the consumption gaps along the working life of a constantly able individual are $\Delta c(A, A, A)[0.0465] > \Delta c(A, A)[0.0314] > \Delta c(A)[0.0238]$, those of a disabled individual in $t = 2$ and $t = 3$ are $\Delta c(A, D, D)[0.0419] > \Delta c(A, D)[0.0300] > \Delta c(A)[0.0238]$, those of a disabled individual in $t = 1$ are $\Delta c(D, A, A)[0.0448] > \Delta c(D, A)[0.0302] > \Delta c(D)[0.0229]$, and, finally, those of a constantly disabled individual are $\Delta c(D, D, D)[0.0405] > \Delta c(D, D)[0.0289] > \Delta c(D)[0.0229]$. ■

Proof b. From Table 2, in $t = 2$, the consumption gaps are $\Delta c(A)[0.0238] > \Delta c(D)[0.0229]$ and in $t = 3$, they are $\Delta c(A, A)[0.0314] > \Delta c(A, D)[0.0300]$ and $\Delta c(D, A)[0.0302] >$

$\Delta c(D, D)[0.0289]$. Finally, in $t = 4$, following the first path ($h_1 = h_2 = A$), the consumption gaps are $\Delta c(A, A, A)[0.0465] > \Delta c(A, A, D)[0.0438]$ and, following the last path ($h_1 = h_2 = D$), they are $\Delta c(D, D, A)[0.0428] > \Delta c(D, D, D)[0.0405]$. ■

Thus, the disadvantage of being disabled rises (i.e., the disability benefits decrease) with the age of the individual (see also Result 1) and diminishes (i.e., the disability benefits increase) with the number of disability spells. Hence, “more disabled” individuals are less penalised than “more able” individuals.

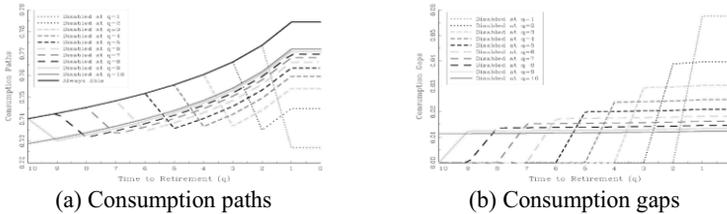
To complete the analysis on the dynamic incentives scheme a sufficiently long working life should be considered. Hence, it is now appropriate to present a simulation with ten working periods ($n = 10$) and then focus on the cases in which the individual is always able and when he suffers from only one disability spell. Table 3 indicates that the government saves in all working periods $t = 1, \dots, n$ (with $n = 10$) to finance consumption in the retirement period $n + 1 = 11$.

TABLE 3. Government Balance ($n = 10$)

	$t = 1$	$t = 2$	$t = 3$	$t = 4$	$t = 5$	$t = 6$	$t = 7$	$t = 8$	$t = 9$	$t = 10$	$t = 11$	
Y	0.8000	0.7767	0.7541	0.7321	0.7108	0.6901	0.6700	0.6505	0.6315	0.6131	0.0000	7.0289
C	0.7379	0.7163	0.6955	0.6752	0.6555	0.6363	0.6177	0.5997	0.5822	0.5646	0.5480	7.0289
$Y - C$	0.0621	0.0604	0.0586	0.0569	0.0553	0.0538	0.0523	0.0508	0.0493	0.0485	-0.5480	0.0000

Therefore, while Figure 4(a) compares the consumption paths of an always able individual and those of individuals suffering from only one disability spell when q working periods remain before retirement, Figure 4(b) compares the current consumption gaps of individuals who become disabled when q working periods remain before retirement.

FIGURE 4. $DI(n = 10) - c(h^t)$ and $\Delta c(h^{t-1})$ with $\#(h^t = D) = 1$



The consumption path of a disabled individual when q working periods remain before retirement is higher than the consumption path of a disabled individual when $q - j$ (with $j = 1, \dots, q - 1$) working periods remain before retirement (see Figure 4(a)). Thus, the gap between the consumption path of an individual who faces one

disability spell and the consumption path of an always able individual is larger when the disability spell occurs nearer to the retirement period (see Figure 4(b)). Therefore, this result confirms the findings obtained in the case of four working periods ($n = 4$) as stated in Result 1.

Considering the consumption gaps from equation (16), the average current consumption gap is defined by taking the expectation over h^{t-1} . Thus, the average current consumption gap is:

$$(17) \quad \Delta c_t = \sum_{h^{t-1} \in H^{t-1}} \pi(h^{t-1}) \cdot \Delta c(h^{t-1}).$$

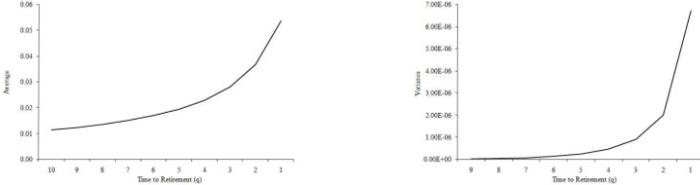
Furthermore, the variance of the current consumption gaps can be computed:

$$(18) \quad \sigma^2 = \sum_{h^{t-1} \in H^{t-1}} \pi(h^{t-1}) \cdot (\Delta c(h^{t-1}) - \Delta c_t)^2.$$

Result 4 Convergence: The system of optimal dynamic incentives effectively converges to a *PS* model with fixed taxes and benefits.

Proof. When the number of working periods remaining before retirement increases, the curve representing the average current consumption gap becomes flatter (see Figure 5(a)), and the variance of the current consumption gaps decreases (see Figure 5(b)).

FIGURE 5. *DI* ($n = 10$) - Δc_t and σ^2



(a) Average current consumption gap (b) Variance of current consumption gaps

Because moving further from retirement implies that (i) the average current consumption gap converges, and (ii) the variance of the current consumption gaps decreases, it is possible to assert that the system of optimal dynamic incentives actually converges to a system where the consumption gaps in t do not depend on the history preceding t , i.e., in each period t the consumption gap between able and disabled states is the same for any possible history preceding t . Hence, the *DI* model effectively converges to a *PS* model with fixed taxes and benefits. ■

4. THE PRIVATE SAVINGS MODEL

In the previously analysed system of dynamic incentives, the government can perfectly control individual utility (and therefore consumption) and the allocation over time. Thus, the implicit assumption is that the government can control individual savings. To evaluate and then appreciate the welfare gain associated with this system, the opposite extreme can be considered, i.e., an economy in which the government implements a stationary tax-benefit system, and workers can take advantage of a perfectly functioning capital market (in other words, workers use private savings to smooth consumption across time and states).

Workers make savings decisions knowing their ability-disability status. Able workers pay a tax T , and disabled workers receive a benefit b . Thus, for any policy (T, b) , whether an able worker would choose to work depends on his level of assets: if he has accumulated a sufficient amount of assets, he might choose not to work.

In the retirement period $n + 1$, the consumption level of all types of individuals is:

$$(19) \quad c_{n+1} = b + R \cdot s_{n+1},$$

where R is the rate of return, and $R \cdot s_{n+1}$ is the capital stock accumulated from the past (capital accumulated until the last working period n). In the previous working periods $t = 1, \dots, n$, the consumption level of working individuals—only able individuals—is:

$$(20) \quad c_t^{L=1} = y - T + R \cdot s_t - s_{t+1}^{L=1},$$

and the consumption level of not working individuals—the disabled individuals who cannot work and the able individuals who decide not to work—is:

$$(21) \quad c_t^{L=0} = b + R \cdot s_t - s_{t+1}^{L=0},$$

where $s_1 = 0$ at $t = 1$, i.e., individuals do not inherit. Therefore, the utility maximisation problem, with $s_{t+1}^L(h^t)$ as choice variables and given the assets—the savings accumulated in the past— $s_t(h^{t-1})$, is:

$$(22) \quad \max_{s_{t+1}^L(h^t)} \sum_{t=1}^{n+1} \sum_{h^t \in H^t} \beta^{t-1} \cdot \pi(h^t) \cdot [u(c(s_{t+1}^L(h^t); s_t(h^{t-1}))) - d \cdot L(h^t)]$$

$$(23) \quad \text{s. to} \quad \sum_{t=1}^{n+1} \sum_{h^t \in H^t} \beta^{t-1} \cdot \pi(h^t) \cdot [y \cdot L(h^t) - c(s_{t+1}^L(h^t); s_t(h^{t-1}))] \geq 0,$$

with $s_{n+2}^L(h^{n+1}) = 0$, i.e., individuals do not bequeath.

If no one ever accumulates sufficient assets to decide not to work when they are able, then the expected workers' discounted net tax revenue is:

$$(24) \quad \mathcal{R} = \sum_{t=1}^n \frac{1}{R^{t-1}} \cdot (\pi^A \cdot T - \pi^D \cdot b) + \frac{1}{R^n} \cdot b,$$

which equals zero if the tax-transfer policy is budget balanced. However, the aim of the government is to choose the tax-transfer policy (T, b) that maximises welfare even if the result of such a policy means that, at the optimum/equilibrium, some able workers choose not to work.

4.1 THE RECURSIVE FORMULATION

In the last working period n , individuals may work in period n and are retired in period $n + 1$. There are two value functions: $V_n^A(s_n)$ for the able state and $V_n^D(s_n)$ for the disabled state. Therefore, the value function is:

$$(25) \quad V_n(s_n) = \pi^A \cdot V_n^A(s_n) + \pi^D \cdot V_n^D(s_n).$$

Moreover, as in the *DI* model, in the following we will assume that $\beta = 1/R$ (see, e.g., Golosov and Tsyvinski, 2006).

First, the value function $V_n^A(s_n)$ is considered, i.e., the worker is assumed to be able in the last working period n and to have assets s_n . This worker type must decide (i) whether to work and (ii) how much to save (or borrow). Therefore, his value function can be written as:

$$(26) \quad V_n^A(s_n) = \max(\mathcal{V}_n^{L=1}(s_n), \mathcal{V}_n^{L=0}(s_n)).$$

Note that $\mathcal{V}_n^{L=1}(s_n)$ is the value associated with working in the last working period n , and $\mathcal{V}_n^{L=0}(s_n)$ is the value associated with not working. Because an able worker can choose, his final value $V_n^A(s_n)$ is the greater of the two options. The value associated with working $\mathcal{V}_n^{L=1}(s_n)$ is:

$$(27) \quad \mathcal{V}_n^{L=1}(s_n) \equiv \max_{s_{n+1}^{L=1}} [u(y - T + R \cdot s_n - s_{n+1}^{L=1}) - d] \\ + \beta \cdot u(b + R \cdot s_{n+1}^{L=1}),$$

the solution to which entails:

$$(28) \quad s_{n+1}^{L=1} = \frac{R \cdot s_n + (y - T - b)}{1 + R},$$

and the value associated with not working $\mathcal{V}_n^{L=0}(s_n)$ is:

$$(29) \quad \mathcal{V}_n^{L=0}(s_n) \equiv \max_{s_{n+1}^{L=0}} u(b + R \cdot s_n - s_{n+1}^{L=0}) + \beta \cdot u(b + R \cdot s_{n+1}^{L=0}),$$

the solution to which entails:

$$(30) \quad s_{n+1}^{L=0} = \frac{R \cdot s_n}{1 + R}.$$

Second, the case of a worker who becomes disabled in the last working period n and has assets s_n , i.e., $V_n^D(s_n)$, is considered. Because this worker cannot choose to work, his value function is the value associated with not working $\mathcal{V}_n^{L=0}(s_n)$, and then:

$$(31) \quad V_n^D(s_n) = \mathcal{V}_n^{L=0}(s_n),$$

the solution to which entails (30).

As in the last working period, in the previous working periods $t = 1, \dots, n - 1$, there are two value functions: $V_t^A(s_t)$ for the able state and $V_t^D(s_t)$ for the disabled state. Therefore, the value function is:

$$(32) \quad V_t(s_t) = \pi^A \cdot V_t^A(s_t) + \pi^D \cdot V_t^D(s_t).$$

The value function of an able worker $V_t^A(s_t)$ is:

$$(33) \quad V_t^A(s_t) \equiv \max(\mathcal{V}_t^{L=1}(s_t), \mathcal{V}_t^{L=0}(s_t)).$$

Because the able worker can choose, his final value $V_t^A(s_t)$ is the greater of the two options $\mathcal{V}_t^{L=1}(s_t)$ —the value associated with working in period t —and $\mathcal{V}_t^{L=0}(s_t)$ —the value associated with not working. In (33), the value associated with working $\mathcal{V}_t^{L=1}(s_t)$ is the Bellman equation (see Bellman, 1957):

$$(34) \quad \mathcal{V}_t^{L=1}(s_t) \equiv \max_{s_{t+1}^{L=1}} [u(y - T + R \cdot s_t - s_{t+1}^{L=1}) - d] + \beta \cdot V_{t+1}(s_{t+1}^{L=1}),$$

the *FOC* of which is:

$$(35) \quad u'(y - T + R \cdot s_t - s_{t+1}^{L=1}) = \beta \cdot \frac{\partial V_{t+1}(s_{t+1}^{L=1})}{\partial s_{t+1}^{L=1}},$$

and the value associated with not working $\mathcal{V}_t^{L=0}(s_t)$ is the Bellman equation (see Bellman, 1957):

$$(36) \quad \mathcal{V}_t^{L=0}(s_t) \equiv \max_{s_{t+1}^{L=0}} u(b + R \cdot s_t - s_{t+1}^{L=0}) + \beta \cdot V_{t+1}(s_{t+1}^{L=0}),$$

the *FOC* of which is:

$$(37) \quad u'(b + R \cdot s_t - s_{t+1}^{L=0}) = \beta \cdot \frac{\partial V_{t+1}(s_{t+1}^{L=0})}{\partial s_{t+1}^{L=0}}.$$

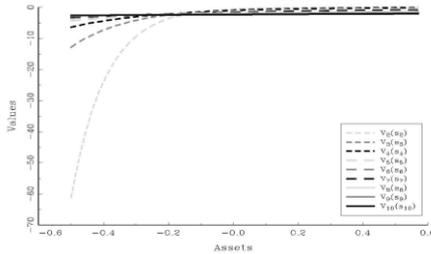
Conversely, if a worker with assets s_t is disabled in working period t , his value function $V_t^D(s_t)$ is the value associated with not working $\mathcal{V}_t^{L=0}(s_t)$, and then:

$$(38) \quad V_t^D(s_t) = \mathcal{V}_t^{L=0}(s_t),$$

the *FOC* of which is (37).

As the *DI* model, the *PS* model is also analysed by means of a simulation (see Appendix C) based on the assumption that individuals have *CARA* preferences (see, e.g., Shimer and Werning, 2007). The value functions derived on the basis of this simulation are represented in Figure 6.

FIGURE 6. $V_t(s_t)$ at $\alpha = 2$, $d = 0.05$, $\pi^A = 0.8$, and $r = 0.03$



Given the derivatives of the value functions, the multiperiod maximisation problem with n working periods can be solved.

For the working period $t = 1, \dots, n - 1$, the value function is (32). Therefore, the maximisation problem of the able worker (33) implies the *FOC* (35) if he chooses to work, i.e., if $\mathcal{V}_t^{L=1}(s_t) \geq \mathcal{V}_t^{L=0}(s_t)$, and the *FOC* (37) otherwise. The maximisation problem of the disabled worker (38) implies the *FOC* (37). In the last working period n , the value function is (25). The maximisation problem of the able worker is (26), the solution to which entails (28) if he chooses to work, i.e., if $\mathcal{V}_n^{L=1}(s_n) \geq \mathcal{V}_n^{L=0}(s_n)$, and (30) otherwise. The maximisation problem of the disabled worker is (31), the solution to which entails (30).

4.2 DYNAMIC INCENTIVES VS. PRIVATE SAVINGS

It is then possible to compare the two proposed models by means of a numerical simulation.

The constant taxation levels T (and the related benefits b), which guarantee higher levels of $V_t(s_t)$ for $t = 1, \dots, n$ and for different working periods n , are presented in Table 4.

TABLE 4. Tax-transfer policies

	$n = 1$	$n = 2$	$n = 3$	$n = 4$	$n = 5$	$n = 6$	$n = 7$	$n = 8$	$n = 9$	$n = 10$
T	0.4339	0.4250	0.2690	0.1923	0.1915	0.1835	0.1815	0.1831	0.1947	0.1846
b	0.2963	0.5013	0.4186	0.3560	0.4001	0.4194	0.4444	0.4738	0.5270	0.5186

Table 4 shows that the tax T decreases until $n = 7$, increases from $n = 7$ to $n = 9$, and decreases again from $n = 9$ to $n = 10$; the benefits b increase from $n = 1$ to $n = 2$, decrease from $n = 2$ to $n = 4$, increase from $n = 4$ to $n = 9$, and, finally, decrease from $n = 9$ to $n = 10$. Thus, the tax T and benefits b are negatively correlated from $n = 1$ to $n = 2$ and from $n = 4$ to $n = 7$.

While the first negative correlation is obtained because when there is only one working period ($n = 1$), a high taxation level T is necessary to balance the particularly low disability benefits b , the second negative correlation is obtained because if the number of working periods increases from an extremely low number ($n = 4$), then lower taxation levels T can balance higher disability benefits b : when the number of working periods n is remarkably low, an increase of n allows lower taxes T and greater benefits b . However, if the number of working periods increases from a relatively high number ($n = 7$), then the correlation between taxes T and benefits b becomes positive.

Table 5 reports the results of a simulation with four working periods ($n = 4$) and hence with $T = 0.1923$ and $b = 0.3560$ (see Table 4). This table displays the discounted future value $\beta^t \cdot V_{t+1}^i(s_{t+1}^i)$ and the discounted current value $\beta^{t-1} \cdot V_t^i(s_t)$ —i.e., the sum of discounted current utility $\beta^{t-1} \cdot (u_t^i - d \cdot L^i)$ and discounted future value $\beta^t \cdot V_{t+1}^i(s_{t+1}^i)$ —for each possible history (with $n = 4$, the number of histories is $2^4 = 16$).

In the *DI* model, as stated by Theorem 1, the *IC* (9) always binds. In the *PS* model, the government selects its tax-transfer policy (T, b) to maximise welfare.

Suppose that the government selects an excessive taxation level T ; in this case, some able workers could decide not to work because, given the assets they accumulated in the past, $\mathcal{V}_t^{L=1}(s_t) < \mathcal{V}_t^{L=0}(s_t)$, and therefore, the tax revenue decreases. Because the tax-transfer policy must be budget balanced, the consequence is that the benefit decreases, and the government does not achieve its purpose of maximising welfare.

Result 5 Better off able than disabled: The government always chooses a tax-transfer policy (T, b) such that $\mathcal{V}_t^{L=1}(s_t) \geq \mathcal{V}_t^{L=0}(s_t)$, and this implies:

$$\begin{aligned} [u(y - T + R \cdot s_t - s_{t+1}^A) - d] &+ \beta \cdot V_{t+1}(s_{t+1}^A) \geq \\ u(b + R \cdot s_t - s_{t+1}^D) &+ \beta \cdot V_{t+1}(s_{t+1}^D), \quad t = 1, \dots, n. \end{aligned}$$

Proof. From Table 5 in $t = 1$, $V_1^A[-1.4214] > V_1^D[-1.6370]$, in $t = 2$, $\beta \cdot V_2^{AA}[-1.0642] > \beta \cdot V_2^{AD}[-1.2663]$ and $\beta \cdot V_2^{DA}[-1.2630] > \beta \cdot V_2^{DD}[-1.5179]$, in $t = 3$, following the first path $\beta^2 \cdot V_3^{AAA}[-0.7321] > \beta^2 \cdot V_3^{AAD}[-0.9207]$, and following the last path $\beta^2 \cdot V_3^{DDA}[-1.0780] > \beta^2 \cdot V_3^{DDD}[-1.3923]$, and, finally, in $t = 4$, following the first path $\beta^3 \cdot V_4^{AAAA}[-0.4272] > \beta^3 \cdot V_4^{AAAD}[-0.6033]$, and following the last path $\beta^3 \cdot V_4^{DDDA}[-0.8433] > \beta^3 \cdot V_4^{DDDD}[-1.2615]$. ■

Moreover, because both systems lead to an allocation in which able individuals always work, total labour income and total consumption to be allocated are clearly identical in the two systems: $Y = C = 3.0629$ if $n = 4$ (see Tables 2 and 5).

Result 6 Welfare gain by dynamic incentives: Even if total labour disutility Y and total consumption to be allocated C are identical in the two systems, the total utility guaranteed by the government in the *DI* model is higher than the total value achieved in the *PS* model: $U_1 > V_1$. This difference measures the welfare gain associated with the government adopting the dynamic incentives system.

Proof. From Tables 2 and 5, $U_1[-1.4414] > V_1[-1.4645]$. ■

To compare the two models, the Equivalent Variation (*EV*) can also be computed (see Table 6).

TABLE 6. EV at $\alpha = 2$, $d = 0.05$, $\pi^A = 0.8$, and $r = 0.03$

$n = 4$	DI	PS	" DP "	EV
U_1 & V_1	-1.4414	-1.4645	-1.4645	
y	1	1	0.9863	0.0137
Y	3.0629	3.0629	3.0208	
$n = 10$	DI	PS	" DP "	EV
U_1 & V_1	-2.5331	-2.5436	-2.5436	
y	1	1	0.9967	0.0033
Y	7.0289	7.0289	7.0060	

The EV is the amount of income to be subtracted in the DI model to yield the same "total value" obtained in the PS model: while with $n = 4$, income decreases from 1 to 0.9863 and the EV is 0.0137, with $n = 10$, income decreases from 1 to 0.9967 and the EV is only 0.0033.

The fact that the EV drastically decreases when n increases confirms that the system of optimal dynamic incentives converges to a stationary tax-benefit system (see Result 4).

Result 7 Consumption gaps in the PS model: In the PS model, the consumption gaps only increase when the retirement period becomes closer.

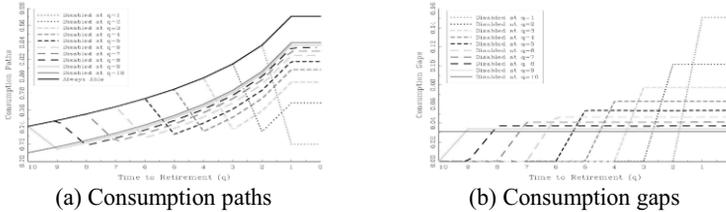
Proof. From Table 5, the consumption gaps along the working life of a constantly able individual are $\Delta c(A, A, A)[0.2292] > \Delta c(A, A)[0.1551] > \Delta c(A)[0.1180]$, those of a disabled individual in $t = 2$ and $t = 3$ are $\Delta c(A, D, D)[0.2292] > \Delta c(A, D)[0.1551] > \Delta c(A)[0.1180]$, those of a disabled individual in $t = 1$ are $\Delta c(D, A, A)[0.2292] > \Delta c(D, A)[0.1551] > \Delta c(D)[0.1180]$, and, finally, those of a constantly disabled individual are $\Delta c(D, D, D)[0.2292] > \Delta c(D, D)[0.1551] > \Delta c(D)[0.1180]$. ■

While in the DI model, the consumption gaps increase, not only when the retirement period becomes closer but also if an individual has been "more able" during his working life (see Table 2 and Result 3), in the PS model, the consumption gaps only increase when the retirement period becomes closer (see Table 5 and Result 7). Therefore, in the PS model, because the consumption gap between able and disabled states in each period t is the same regardless of individual work history, the average current consumption gap (17) equals the consumption gaps, and the variance in the current consumption gaps (18) obviously equals zero (see also Result 4).

As for the DI model, to complete the analysis on the PS model, a

sufficiently long working life should be considered. For this reason, it is opportune to consider a simulation with ten working periods ($n = 10$)—and then $T = 0.1846$ and $b = 0.5186$ (see Table 4)—and focus on cases in which the individual is always able and suffers from only one disability spell. Therefore, while Figure 7(a) compares the consumption paths of an always able individual with individuals suffering from one disability spell when q working periods remain before retirement, Figure 7(b) compares the current consumption gaps of disabled individuals when q working periods remain.

FIGURE 7. $PS(n = 10) - c(h^t)$ and $\Delta c(h^{t-1})$ with $\#(h^t = D) = 1$



As in the *DI* model (see Figure 4), in the *PS* model, (i) the consumption path of a disabled individual when q working periods remain before retirement is higher than the consumption path of a disabled individual when $q - j$ (with $j = 1, \dots, q - 1$) working periods remain before retirement (see Figure 7(a)); and (ii) the gap between the consumption path of an individual who faces one disability spell and the consumption path of an always able individual is larger when the disability spell occurs nearer to the retirement period (see Figure 7(b)), thus confirming the findings obtained with $n = 4$ and stated by Result 7.

5. CONCLUSIONS

The purpose of this paper is to examine a system of dynamic incentives—developed through the framework of the classical Diamond and Mirrlees (1978) disability model assuming disability as a temporary condition and revising the analysis in terms of current and promised future utilities—and to compare this model with a private savings model characterised by a stationary tax-benefit system. These aims are achieved by means of numerical simulations.

The paper establishes that it is preferable for an individual to be disabled in the early periods of his working life. This statement is corroborated by the fact that the gap between the consumption path of an individual facing one disability spell and the consumption path of

an always able individual is larger when the disability spell occurs nearer to the retirement period. The paper also demonstrates that consecutive disability spells harm individuals to a greater extent than non-consecutive disability spells if the long disability episode is nearer to the retirement period.

Thus, the consumption gaps along and across working histories are compared, and this comparison allows us to state that the consumption gaps become larger not only when the retirement period is closer but also if the working life is characterised by a lower number of disability spells.

Nevertheless, when the number of working periods remaining before the retirement period increases, the average current consumption gap converges, and the variance of the current consumption gaps decreases. Hence, the dynamic incentives system converges to a stationary tax-benefit system. Because both systems lead to an allocation in which able individuals always work, the total labour income and the total consumption to be allocated are identical in the two systems. However, in the *PS* model, (i) the total value is smaller than the total utility guaranteed by the government in the *DI* model (the welfare gain associated with the government adopting the dynamic incentives system), and (ii) the consumption gaps only increase along the working histories, i.e., when the retirement period becomes closer, and not across the working histories.

Hence, these findings are significant from both theoretical and policy perspectives. In fact the results are of policy relevance for at least three reasons. First, policymakers should design publicly provided disability insurance on the basis of a dynamic incentives scheme rather than a stationary tax-benefit system. Second, at least in the case of temporary disability, the dynamic incentives scheme should be time increasing (or, in other words, the continuation utility should be time decreasing), and therefore it should guarantee lower disability benefits to individuals who become disabled later in their working life, i.e., to more skilled and thus more efficient workers. Finally, the dynamic incentives scheme should penalise the individuals affected by frequent disability spells less.

In further research, the proposed *DI* model could be extended (i) to allow for adverse selection caused by multiple unobservable types that have different ability-disability status probabilities, as in Whinston (1983), and (ii) to consider individuals facing a stochastic ability-disability process that follows a Markov chain, as in Hansen and İmrohoroğlu (1992).

APPENDICES

A. BINDING PROMISED UTILITY-KEEPING AND INCENTIVE CONSTRAINTS

Theorem A.1 Binding PK: The PKs in the minimisation problems (11) and (14) will bind, i.e. will be satisfied with equality, for the optimal solutions.

Proof. A proof by contradiction is used to show that the PK in the minimisation problem (11) will bind. If the multiplier $\lambda_n = 0$, i.e., if $\sum_{i=A,D} \pi^i \cdot [(u_n^i - d \cdot L^i) + \beta \cdot u_{n+1}^i] > U_n$, then there exists an amount ε by which the current utilities of able and disabled workers can be lowered without violating the PK constraint:

$$\sum_{i=A,D} \pi^i \cdot [(u_n^i - d \cdot L^i - \varepsilon) + \beta \cdot u_{n+1}^i] > U_n.$$

Obviously, the IC will hold as well

$$\begin{aligned} (u_n^A - d \cdot L^A - \varepsilon) + \beta \cdot u_{n+1}^A &\geq (u_n^D - d \cdot L^D - \varepsilon) + \beta \cdot u_{n+1}^D \\ \Rightarrow (u_n^A - d \cdot L^A) + \beta \cdot u_{n+1}^A &\geq (u_n^D - d \cdot L^D) + \beta \cdot u_{n+1}^D. \end{aligned}$$

Because both the PK and IC still hold, the current utilities can be lowered by ε , which violates the assumption that the solution was resource use minimising in the first place. The proof of the binding PK in the minimisation problem (14) is equivalent and thus straightforward. \blacksquare

Theorem A.2 Binding IC: The ICs in the minimisation problems (11) and (14) will bind, i.e., will be satisfied with equality, for the optimal solutions.

Proof. A proof by contradiction is used to show that the IC in the minimization problem (11) will bind. If the multiplier $\mu_n = 0$, i.e., if $(u_n^A - d \cdot L^A) + \beta \cdot u_{n+1}^A > (u_n^D - d \cdot L^D) + \beta \cdot u_{n+1}^D$, then the gap between the sum of current and future utilities of an able worker and the sum of current and future utilities of a disabled worker can be lowered by an amount δ :

$$(u_n^A - d \cdot L^A) + \beta \cdot u_{n+1}^A > (u_n^D - d \cdot L^D) + \beta \cdot u_{n+1}^D + \delta,$$

and this makes it easier to satisfy the PK constraint:

$$\sum_{i=A,D} \pi^i \cdot [(u_n^i - d \cdot L^i) + \beta \cdot u_{n+1}^i] + \delta \geq U_n.$$

So the current utilities can be lowered by an amount ε :

$$\sum_{i=A,D} \pi^i \cdot [(u_n^i - d \cdot L^i - \varepsilon) + \beta \cdot u_{n+1}^i] + \delta \geq U_n,$$

but then the original solution was not resource use minimizing. The proof of the binding *IC* in the minimization problem (14) is straightforward. ■

B. THE SIMULATION ON THE DYNAMIC INCENTIVES MODEL

The *DI* model is analysed by means of numerical simulations⁴. Following the previous literature (see, e.g., Shimer and Werning, 2007), consumption preferences are assumed to exhibit constant absolute risk aversion (*CARA*) $u(c) = -\exp(-\alpha \cdot c)$, where α is some positive scalar⁵. The *CARA* preferences allow us to abstract from wealth effects, and then the individual's decision to work or not to work is independent of his wealth level, but solely dependent on the system of dynamic incentives.

Because the purpose of this simulation is to investigate and clarify the *DI* model—and ultimately to compare this model with the *PS* model—and since the outcomes do not depend on the value of the parameters, the parameterisation is primarily selected for numerical convenience. Thus, the income value $y = 1$ and the parameter values $\alpha = 2$, $d = 0.05$, $\pi^A = 0.8$ (and hence, $\pi^D = 0.2$), and $r = 0.03$ are considered.

The first step is to compute the cost function $C_n(U_n)$ related to the last working period minimisation problem. In the last working period n , the government's problem (11) is:

$$(B.1) \quad C_n(U_n) \equiv \min_{u_n^i, u_{n+1}^i} \sum_{i=A,D} \pi^i \cdot \left\{ \left[-\frac{\ln(-u_n^i)}{\alpha} - y \cdot L^i \right] - \beta \cdot \frac{\ln(-u_{n+1}^i)}{\alpha} \right\}$$

$$\text{s. to } \sum_{i=A,D} \pi^i \cdot [(u_n^i - d \cdot L^i) + \beta \cdot u_{n+1}^i] \geq U_n \quad (\lambda_n)$$

$$(u_n^A - d) + \beta \cdot u_{n+1}^A \geq u_n^D + \beta \cdot u_{n+1}^D \quad (\mu_n),$$

⁴ The simulations are performed using *GAUSS* software.

⁵ The properties of *CARA* utility functions are $u'(c) = \alpha \cdot \exp(-\alpha \cdot c) > 0$ and $u''(c) = -\alpha^2 \cdot \exp(-\alpha \cdot c) < 0$. Thus, the degree of absolute risk aversion is constant $A(c) = -u''(c)/u'(c) = \alpha$, and as $A' = 0$, the absolute risk aversion is independent of wealth (see, e.g., Artige, 2004).

the solution to which entails (13). Then, it is possible to compute the cost functions $C_t(U_t)$ recursively from $t = n - 1$ to $t = 2$.⁶ In the working period t , the government's problem (14) is:

$$(B.2) \quad \begin{aligned} C_t(U_t) \equiv & \min_{u_t^i, U_{t+1}^i} \sum_{i=A,D} \pi^i \cdot \left\{ \left[-\frac{\ln(-u_t^i)}{\alpha} - y \cdot L^i \right] + \right. \\ & \left. \beta \cdot C_{t+1}(U_{t+1}^i) \right\} \\ \text{s. to} & \sum_{i=A,D} \pi^i \cdot [(u_t^i - d \cdot L^i) + \beta \cdot U_{t+1}^i] \geq U_t \quad (\lambda_t) \\ & (u_t^A - d) + \beta \cdot U_{t+1}^A \geq u_t^D + \beta \cdot U_{t+1}^D \quad (\mu_t), \end{aligned}$$

the *FOCs* of which (15) entail:

$$(B.3) \quad \begin{aligned} -\frac{1}{\alpha} \cdot \frac{1}{u_t^A} &= C'_{t+1}(U_{t+1}^A), \\ -\frac{1}{\alpha} \cdot \frac{1}{u_t^D} &= C'_{t+1}(U_{t+1}^D). \end{aligned}$$

Therefore, the envelope condition is:

$$(B.4) \quad C_t(U_t) = \left. \frac{\partial \mathcal{L}}{\partial U_t} \right|_{opt} = \lambda_t,$$

where λ_t is the multiplier on the *PK* in (B.2).

The cost functions are represented in Figure 3. Given the cost functions, the multiperiod minimisation problem with n working periods can be solved: from $t = 1, \dots, n - 1$, the government's problem is (B.2), and in the last working period n , it is (B.1).

C. THE SIMULATION ON THE PRIVATE SAVINGS MODEL

As performed for the *DI* model, a simulation of the *PS* model is developed⁷ assuming that individuals have *CARA* preferences⁸ (see, e.g., Shimer and Werning, 2007): abstracting from wealth effects, this type of preferences allows us to study the effects of different policies (T, b) on an individual's decision to work or not to work. For the aforementioned purpose of numerical convenience, the income value is set to $y = 1$, and the parameter values are set to $\alpha = 2$, $d = 0.05$,

⁶ In the working period $t = 1$, the consumption level required to give agent $i = A, D$ utility u_1^i in the current working period is $c(u_1^i) = u^{-1}(u_1^i)$ and the cost level required to give agent $i = A, D$ utility U_2^i in the future working periods and in the retirement period is $C_2(U_2^i) = U_2^{-1}(U_2^i)$.

⁷ See footnote 4.

⁸ See footnote 5.

$\pi^A = 0.8$ (and hence, $\pi^D = 0.2$), and $r = 0.03$.

The first step is to compute the value function $V_n(s_n)$ related to the last working period (25). If the worker is able, his value function $V_n^A(s_n)$ is (26), where the value associated with working $\mathcal{V}_n^{L=1}(s_n)$ is (27):

$$(C.1) \quad \mathcal{V}_n^{L=1}(s_n) \equiv \max_{s_{n+1}^{L=0}} \left[-\exp(-\alpha \cdot (y - T + R \cdot s_n - s_{n+1}^{L=1})) - d \right] + \beta \cdot \left[-\exp(-\alpha \cdot (b + R \cdot s_{n+1}^{L=1})) \right],$$

the derivative of which is:

$$(C.2) \quad \frac{\partial \mathcal{V}_n^{L=1}(s_n)}{\partial s_n} = \alpha \cdot R \cdot \exp(-\alpha \cdot (y - T + R \cdot s_n - s_{n+1}^{L=1})),$$

and where the value associated with not working $\mathcal{V}_n^{L=0}(s_n)$ is (29):

$$(C.3) \quad \mathcal{V}_n^{L=0}(s_n) \equiv \max_{s_{n+1}^{L=0}} \left[-\exp(-\alpha \cdot (b + R \cdot s_n - s_{n+1}^{L=0})) \right] + \beta \cdot \left[-\exp(-\alpha \cdot (b + R \cdot s_{n+1}^{L=0})) \right],$$

the derivative of which is:

$$(C.4) \quad \frac{\partial \mathcal{V}_n^{L=0}(s_n)}{\partial s_n} = \alpha \cdot R \cdot \exp(-\alpha \cdot (b + R \cdot s_n - s_{n+1}^{L=0})).$$

If $\mathcal{V}_n^{L=1}(s_n) \geq \mathcal{V}_n^{L=0}(s_n)$, the derivative of the value function $V_n^A(s_n)$ is (C.2); otherwise, the derivative is (C.4). Conversely, if the worker is disabled, his value function $V_n^D(s_n)$ is (31), and then (C.3), the derivative of which is (C.4). Therefore, the derivative of the value function related to the last working period $V_n(s_n)$ is:

$$(C.5) \quad \frac{\partial V_n(s_n)}{\partial s_n} = \begin{cases} \pi^A \cdot \frac{\partial \mathcal{V}_n^{L=1}(s_n)}{\partial s_n} + \pi^D \cdot \frac{\partial \mathcal{V}_n^{L=0}(s_n)}{\partial s_n} & \text{if } \mathcal{V}_n^{L=1}(s_n) \geq \mathcal{V}_n^{L=0}(s_n) \\ \frac{\partial \mathcal{V}_n^{L=0}(s_n)}{\partial s_n} & \text{otherwise.} \end{cases}$$

Given $V_n(s_n)$, it is possible to compute the value functions (32) recursively from $t = n - 1$ to $t = 2$. If the worker is able, his value function $V_t^A(s_t)$ is (33), where the value associated with working $\mathcal{V}_t^{L=1}(s_t)$ is (34):

$$(C.6) \quad \mathcal{V}_t^{L=1}(s_t) \equiv \max_{s_{t+1}^{L=1}} \left[-\exp(-\alpha \cdot (y - T + R \cdot s_t - s_{t+1}^{L=1})) - d \right] + \beta \cdot V_{t+1}(s_{t+1}^{L=1}),$$

the derivative of which is:

$$(C.7) \quad \frac{\partial \mathcal{V}_t^{L=1}(s_t)}{\partial s_t} = \alpha \cdot R \cdot \exp\left(-\alpha \cdot (y - T + R \cdot s_t - s_{t+1}^{L=1})\right),$$

and where the value associated with not working $\mathcal{V}_t^{L=0}(s_t)$ is (36):

$$(C.8) \quad \mathcal{V}_t^{L=0}(s_t) \equiv \max_{s_{t+1}^{L=0}} \left[-\exp\left(-\alpha \cdot (b + R \cdot s_t - s_{t+1}^{L=0})\right) \right] + \beta \cdot V_{t+1}(s_{t+1}^{L=0}),$$

the derivative of which is:

$$(C.9) \quad \frac{\partial \mathcal{V}_t^{L=0}(s_t)}{\partial s_t} = \alpha \cdot R \cdot \exp\left(-\alpha \cdot (b + R \cdot s_t - s_{t+1}^{L=0})\right).$$

Conversely, if the worker is disabled, his value function $V_t^D(s_t)$ is (38), and then (C.8), the derivative of which is (C.9). Therefore, the derivative of the value function related to the working period t , i.e., $V_t(s_t)$, is:

$$(C.10) \quad \frac{\partial V_t(s_t)}{\partial s_t} = \begin{cases} \pi^A \cdot \frac{\partial \mathcal{V}_t^{L=1}(s_t)}{\partial s_t} + \pi^D \cdot \frac{\partial \mathcal{V}_t^{L=0}(s_t)}{\partial s_t} & \text{if } \mathcal{V}_t^{L=1}(s_t) \geq \mathcal{V}_t^{L=0}(s_t) \\ \frac{\partial \mathcal{V}_t^{L=0}(s_t)}{\partial s_t} & \text{otherwise.} \end{cases}$$

The value functions are represented in Figure 6. Given the derivatives of the value functions, the multiperiod maximisation problem with n working periods can be solved.

While in the last working period n , the value function is (25), for the working period $t = 1, \dots, n-1$, the value function is (32). Therefore, the maximisation problem of the able worker is (33), the *FOC* (35) of which is:

$$(C.11) \quad \alpha \cdot \exp\left(-\alpha \cdot (y - T + R \cdot s_t - s_{t+1}^{L=1})\right) = \beta \cdot \frac{\partial V_{t+1}(s_{t+1}^{L=1})}{\partial s_{t+1}^{L=1}}$$

if he chooses to work, i.e., if $\mathcal{V}_t^{L=1}(s_t) \geq \mathcal{V}_t^{L=0}(s_t)$; and the *FOC* (37) of which is:

$$(C.12) \quad \alpha \cdot \exp\left(-\alpha \cdot (b + R \cdot s_t - s_{t+1}^{L=0})\right) = \beta \cdot \frac{\partial V_{t+1}(s_{t+1}^{L=0})}{\partial s_{t+1}^{L=0}}$$

otherwise. The maximisation problem of the disabled worker is (38), the *FOC* (37) of which is (C.12).

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