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**Dynamic Incentives:  
Temporary Vs. Permanent Disability**

Pierpaolo Giannoccolo  
Silvia Platoni

Quaderno n. 130/marzo 2018

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## ABSTRACT

This article extends Diamond and Mirrlees' (1978) disability model in a different and more detailed framework that contemplates both temporary and permanent disability. By introducing different degrees of disability into this seminal framework, the paper contributes to the recent debate among empirical scholars on the growth of disability insurance programmes in several OECD countries (e.g., the US, Norway, Sweden). This approach allows us to analyse and consequently compare able, temporarily disabled, and permanently disabled workers. Furthermore, in a numerical simulation, the analysis demonstrates that the system of dynamic incentives should adapt the disability benefits to the different disability statuses

**Keywords:** disability insurance; dynamic incentives.

**JEL classification:** H53; H31.



# 1. INTRODUCTION

Disability can be defined in a variety of different ways according to different contexts. In an economic framework, disability implies that individuals have a lower ability to perform work and sustain a sufficient income. This disability can be partial or full, temporary or permanent.<sup>1</sup>

In the second half of the last century, public disability insurance programmes were introduced to mitigate the failure of private savings and private disability insurance to eliminate, or at least limit, the risks associated with a permanent loss of earnings capacity (e.g., Bound and Burkhauser, 1999; Barr and Diamond, 2009; Braun et al., 2017).

Therefore, disability insurance programmes have become a growing part of modern welfare systems. For example, in 2012, the North American disability insurance programme paid cash benefits three times higher than those paid by unemployment insurance. Moreover, between 1985 and 2012, the proportion of disability insurance beneficiaries has more than doubled (Low and Pistaferri, 2015). Similar dynamics can be found in all developed countries (e.g., Bratberg, 1999; Brinch, 2009). This rapid growth of disability insurance programmes has generated a stimulating debate among scholars and policymakers (e.g., Evans, 2002; Goudswaard and Caminada, 2015).<sup>2</sup>

On the one hand, this debate focuses on three main concerns: (i) the welfare benefits and financial impact of these programmes (e.g., Stephens, 2001; Meyer and Mok, 2013; Ball and Low 2014; Low and Pistaferri, 2015; Braun et al., 2017)<sup>3</sup> and the suitability of reforming these programmes (e.g., Bound et al., 2004 and 2010); (ii) the inefficient distortion of these programmes on the labour supply (e.g.,

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<sup>1</sup> There are several cases of disability insurance systems that allow for more than two degrees of disability (e.g., Germany and Sweden recognize several degrees of disability). See Sim (1999) for further details.

<sup>2</sup> Evans (2002) analyses European social security programmes, and Goudswaard and Caminada (2015) focus on OECD countries.

<sup>3</sup> In particular, Low and Pistaferri (2015) compare the insurance and disincentive effects of disability benefits and study how policy reforms impact welfare and agents' behaviour, and Braun et al. (2017) show that important determinants of poverty can be found in the lifetime earnings risk, longevity, sickness/disability, and marital status risk.

Hoynes and Moffitt 1997; Brinch, 2009; Evans, 2002; Maestas et al. 2013; French and Song, 2014; Kostol and Mogstad, 2014; Goudswaard and Caminada, 2015; Blundell et al., 2016; Meyer and Mok, 2016; Koning and van Sonsbeek, 2017); and (iii) the impact of these programmes on private savings.<sup>4</sup> On the other hand, several theoretical and empirical studies show that a disability insurance programme can incur two types of errors: (i) type-I errors, which arise when some disabled workers are judged to be able, and (ii) type-II errors, which arise when some able workers are judged to be disabled.<sup>5</sup>

This article aims to contribute to this debate on disability insurance by focusing on the distinction between permanent and temporary disability and by investigating how this distinction can influence policymakers in implementing an optimal system of dynamic incentives.

Although disability can be permanent or temporary, a large body of literature considers disability as a persistent and, indeed, permanent skill shock. In their seminal paper, Diamond and Mirrlees (1978) assume that disability is a permanent state.<sup>6</sup> This paper has been

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<sup>4</sup> First, in the presence of liquidity constraints and in the case of persistent or permanent shocks to labour earnings, private savings become completely ineffective (e.g., Deaton 1991). Second, private disability insurance alone is not a perfect substitute of private savings. On the one hand, similar to all insurance policies, disability insurance programmes are affected by the moral hazard problem (e.g., Diamond and Mirrlees, 1978). On the other hand, these programmes are also affected by the adverse selection problem (e.g., Whinston, 1983; Lantto, 1989). In particular, it is possible to identify two types of moral hazard: *ex ante* moral hazard characterised by different probabilities of becoming disabled and *ex post* moral hazard involving healthcare expenditures in the case of disability (Boadway et al. 2003 and 2006).

<sup>5</sup> Among the empirical articles, Benitez-Silva et al. (2004) analyse the US Social Security Administration and evaluate 20 per cent type-I errors and 60 per cent type-II ones. Among theoretical papers, Diamond and Sheshinski (1995) and Parsons (1996) analyse optimal disability and welfare benefits in a two-type model with an imperfect disability evaluation (i.e., they recognize the presence of both type-I and type-II errors). In particular, Diamond and Sheshinski (1995) show that the levels of both disability and welfare benefits affect the labour supply, and Parsons (1996) shows that it is optimal to provide able individuals with work incentives to avoid them claiming to be disabled.

<sup>6</sup> They define a model of public insurance where individuals can be (randomly) able or disabled and where an optimal disability insurance can be implemented, with a moral hazard constraint, by the government. As result, the more generous the social insurance system, the higher the risk of moral hazard. Furthermore, in this influential article, the authors aim to explore the interactions between public and private

extended in several directions: Whinston (1983) introduces the presence of adverse selection caused by multiple unobservable types associated with different probabilities of illness; Anderberg and Andersson (2000) study an economy in which disability risk is observed but endogenous (i.e., workers can influence their probability of disability through occupational choice); Thomas and Worrall (2007) propose an infinite horizon version of Diamond and Mirrlees's (1978) model where there are no moral hazard problems (i.e., ability can be observed) and where the private insurance scheme is voluntary (i.e., individuals participate if they expect long-term benefits from the scheme); and Golosov and Tsyvinski (2006) reformulate Diamond and Mirrlees (1978) to find a tax system that implements the optimal allocation.<sup>7</sup> Finally, Platoni (2017) proposes a simplified version of Diamond and Sheshinski (1995) in which she assumes that disability is temporary.<sup>8</sup>

This article introduces a further extension of Diamond and Mirrlees (1978) by following the direction suggested by Platoni (2017). In particular, we introduce the assumption that disability can be temporary or permanent.

Recently, the distinction among disability types has generated an interesting debate among empirical scholars. For example, Meyer and Mok (2013) estimate that a person reaching age 50 has a 36 per cent chance of having been at least temporarily disabled once during his working years and a 9 per cent chance of having suffered a chronic

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insurance (i.e., the possible crowding-out effects and the optimality of a mixture of public and private insurance). The authors find that under plausible conditions, an optimally designed public insurance programme implies that, at the optimum, able individuals are indifferent about whether to work, and thus they decide to work.

<sup>7</sup> Diamond and Mirrlees (1978) design a system based on a linear tax (equal to the intertemporal wedge in the optimal allocation) that does not implement the optimum. Differently, Golosov and Tsyvinski (2006) propose an asset-tested disability programme (i.e., agents receive a disability transfer only if their assets are below a specified threshold) and introduce an intertemporal provision of dynamic incentives (i.e., the social planner rewards an agent for working by increasing the continuation utility when the agent becomes disabled). They show that this effect encourages increased consumption for agents who become disabled later in life.

<sup>8</sup> In particular, Platoni (2017), in a numerical simulation, compares the results of the dynamic incentives model with those of a private savings model characterised by a stationary tax-transfer policy.

and severe disability.<sup>9</sup> Ward et al. (2017, pp. 707-8), analysing the Current Population Survey (2008-2015) sponsored by the US Department of Health and Human Services, find that

[...] the temporarily disabled is also important. A person does not need to be permanently disabled to experience the adverse consequences of a disability. Experiencing a disability for a few months a year could affect employment, domestic responsibilities, and community participation. Policies targeting temporary disability may focus on employment accommodations or health promotion.

Temporary disability is also studied by Fevang et al. (2017), who analyse the Norwegian temporary disability insurance (TDI) programme. Those authors explore the impacts of financial incentives on the duration and outcomes of disability insurance spells. They find that there is an efficiency loss associated with the relatively generous TDI benefit level in Norway (i.e., too low transition rates to regular employment). In particular, they show that the 2002 reform, which increased the TDI benefit scheme by approximately 14 per cent, explains the large increase in Norwegian TDI applicants.

In contrast, to the best of our knowledge, no published theoretical paper analyses both temporary and permanent disability in a unified framework. The only exception is Rehn (2007), who extends Parsons (1996) by introducing three types of disability (i.e., able, disabled, and partially disabled) and explores the presence of imperfect tagging in disability insurance. Rehn (2007, pp. 30) finds that considering three types of disability implies essentially the same conclusions as considering only able and disabled agents with imperfect tagging:

[...] it is optimal to reward individuals working in line with their ability and that this leaves room for improved replacement rates for the targeted groups.

Differently, in our model, able individuals are induced to work; therefore, the problem of imperfect tagging is neutralised and the

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<sup>9</sup> Meyer and Mok (2013) use the Panel Study of Income Dynamics (PSDI), a longitudinal dataset for the period 1968-2009 with an initial sample of approximately 4,800 US households and 18,000 individuals. They move from Charles (2003) by dividing the disabled into three persistence groups: (i) the one-time disabled (who report a disability once); (ii) the temporarily disabled (who have another one or two positive limitation reports within the ten years after the initial disability onset); (iii) and the chronically disabled (who have other three or more positive limitation reports during the ten years after the initial disability onset).

consequent screening process is not necessary. Furthermore, we explore the impact of temporary and permanent disability on the design of dynamic incentives.

In this article, we propose a model that extends Diamond and Mirrlees (1978) in a more realistic framework in which different ranges of disability coexist. We extend Platoni (2017) by assuming that disability is either temporary or permanent. This assumption helps us compare able, temporarily disabled, and permanently disabled workers. Furthermore, we provide a numerical simulation that shows how the system of dynamic incentives changes according different disability statuses. In particular, we aim to answer the following questions: i) Under what conditions are able workers induced to work in the presence of publicly provided disability insurance? ii) Are the work disincentives different in the presence of temporary or permanent disability? iii) How does the consumption path depend on a person being disabled? How does the consumption path depend on his being temporarily or permanently disabled?

The remainder of the paper proceeds as follows. Section 2 describes the setup of the model; that is, it outlines Diamond and Mirrlees' (1978) model. Section 3 incorporates a system of dynamic incentives into this framework. Section 4 analyses the consumption paths and the consumption gaps between able and disabled individuals, both temporarily and permanently. Finally, Section 5 draws some conclusions.

## 2. THE DISABILITY MODEL: DIAMOND AND MIRRLEES (1978) REVISED

In this economy, (i) there are three types of individuals, the able,  $A$ ; the temporarily disabled,  $T$ ; and the permanently disabled,  $P$ , and (ii) time is discrete and finite. Hence, the life of an individual can be represented through time as  $t = 0, \dots, n + 1$ , where  $t = 0$  identifies the pre-working period,  $t = 1, \dots, n$  the working periods, and  $t = n + 1$  the retirement period.

Let  $a_t \in \{A, T, P\}$  be the individual's ability realisation at time  $t$ . In the pre-working period, all individuals are supposed to be able, and therefore,  $a_0 = A$ ; because no individual works during retirement, the ability realisation at time  $t = n + 1$  is  $a_{n+1} = P$ .

If it is assumed that the probability that an individual will be in a particular ability state during a given time period  $t = 1, \dots, n$  depends only on his ability state during the previous time period, then the stochastic ability process follows a Markov chain:

$$\pi = \begin{bmatrix} \pi(A|A) & \pi(T|A) & 0 \\ \pi(A|T) & 0 & \pi(P|T) \\ 0 & 0 & \pi(P|P) \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

where  $\pi(a_t|a_{t-1})$  is the (one-step) probability of moving from ability state  $a_{t-1}$  to ability state  $a_t$ . For each individual the transition probabilities are assumed to be the same. Note that the temporary disability state is characterised by  $\pi(T|T) = 0$ , i.e., an individual can be temporarily disabled for only one period, and the permanent disability state is characterised by  $\pi(P|P) = 1$ , i.e., the permanent disability state is an absorbing state (as in Golosov and Tsyvinski, 2006). Moreover, an individual becomes permanently disabled only after (s)he has been temporarily disabled, i.e.,  $\pi(P|A) = 0$ .

A simple disability model in the spirit of Fair (1971), Mirrlees (1971), Akerlof (1978), Diamond and Mirrlees (1978), Diamond and Sheshinski (1995), and Parsons (1996) will be analysed throughout this paper. A critical distinction between the ability state and the disability state (both temporary and permanent) is the ability to work; the disabled cannot work. The able have only one decision to make: whether to work. If they work, they produce a positive quantity of output  $y > 0$ .

To simplify the analysis, (i) the utility function over consumption and labour of the able workers is assumed to be quasilinear in labour such that in this world with a binary work decision, utility differs by a constant across work states for a given consumption level, and (ii) a similar functional form exists for the utility characterising the disabled states. Therefore, the consumption utility is state independent:

$$(1) \quad U^a = u(c^a) - d \cdot L^a, a \in \{A, D\} \text{ and } D \in \{T, P\},$$

where  $d > 0$  is the per-unit disutility from working and  $u$  satisfies the Inada conditions, i.e.,  $u'(c) > 0$  and  $u''(c) < 0$  with  $u'(c) \rightarrow \infty$  as  $c \rightarrow 0$  and  $u'(c) \rightarrow 0$  as  $c \rightarrow \infty$ . In the following, it is assumed that all able workers are induced to work; hence,  $L^A = 1$  and  $L^D = 0$ .<sup>10</sup>

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<sup>10</sup> This framework would allow us to analyse partial disability (PD) as well by considering  $0 < L^{PD} < 1$ .

The social insurance programme is defined by only two consumption levels, one for able workers and another for (temporarily or permanently) disabled workers. The problem is to maximise ex ante the expected utility using the consumption levels as policy variables subject to a budget constraint (*BC*) and an incentive constraint (*IC*):

$$(2) \quad \begin{aligned} E(U) &\equiv \max_{c^a} \sum_{a=A,D} \pi^a \cdot [u(c^a) - d \cdot L^a] \\ \text{s.to} \quad &\sum_{a=A,D} \pi^a \cdot (y \cdot L^a - c^a) \geq 0 \quad (\lambda) \\ &u(c^A) - d \geq u(c^D) \quad (\mu), \end{aligned}$$

with  $\pi^a = \pi(a_t | a_{t-1})$ , and where  $D = T$  if  $a_{t-1} = A$  and  $D = P$  if  $a_{t-1} = T$ . The indifference curve is tangent to the *BC* on the 45° line. Given  $d > 0$ , the *IC* is flatter than the 45° line, and the optimum coincides with the intersection of the *BC* and the *IC* on a lower indifference curve. Therefore, the entire set of incentive-compatible allocations  $\{(c^A, c^D) | u(c^A) - d \geq u(c^D)\}$  is below the 45° line; thus,  $c^A \geq c^D$ .

Rather than maximising the expected utility, it is possible to minimise resource use (*R*), which is equivalent to revenue maximisation, subject to a level of expected utility  $E(U) = \bar{U}$  promised to agent  $a$ , i.e., subject to a promised utility-keeping constraint (*PK*). Furthermore, because the consumption required to give agent  $a \in \{A, D\}$  utility  $u^a$  is  $c(u^a) = u^{-1}(u^a)$ , the analysis can be significantly simplified if the utilities, rather than the consumption levels, are used as choice variables. Therefore, the minimisation problem can be written as follows:

$$(3) \quad \begin{aligned} C(\bar{U}) &\equiv \min_{u^a} \sum_{a=A,D} \pi^a \cdot [c(u^a) - y \cdot L^a] \\ \text{s.to} \quad &\sum_{a=A,D} \pi^a \cdot (u^a - d \cdot L^a) \geq \bar{U} \quad (\lambda) \\ &u^A - d \geq u^D \quad (\mu). \end{aligned}$$

As observed for the maximisation problem (2), given  $d > 0$ , the *IC* is flatter than the 45° line. The optimum coincides with the intersection of the *PK* and the *IC*, and the entire set of incentive-compatible allocations  $\{(u^A, u^D) | u^A - d \geq u^D\}$  is below the 45° line.

### 3. THE DYNAMIC INCENTIVES MODEL

The ability model proposed by Diamond and Mirrlees (1978), and abridged in Section 2, is reconsidered to analyse a system of dynamic incentives.

The history is defined as a sequence of ability realisations  $h^t = (a_1, \dots, a_t)$  for  $t = 1, \dots, n$  and with  $h^t \in H^t$ . The number of histories in each working period  $\mathcal{N}(t)$  can be computed on the basis of the Fibonacci sequence  $\mathcal{F}$  as<sup>11</sup>

$$\mathcal{N}(t) = \mathcal{F}(t+1) + \mathcal{F}(t) + \sum_{\tau=1}^t \mathcal{F}(\tau-1) = \mathcal{N}(t-2) + \mathcal{N}(t-1) + 1,$$

where  $\mathcal{F}(t+1)$  identifies the number of able states in the working period  $t$ ,  $\mathcal{F}(t)$  is the number of temporarily disabled states, and  $\sum_{\tau=1}^{t-1} \mathcal{F}(\tau-1)$  is the number of permanently disabled states.

Because  $\pi(a_{t+1}|a_t)$  is the transition probability characterising the Markov process, the probability of the history  $h^t$  being realised given the initial ability realisation  $a_0 = A$  is

$$\pi(h^t) = \pi(a_t|a_{t-1}) \dots \pi(a_1|a_0).$$

Each retired individual can be considered permanently disabled  $a_{n+1} = P$ . Hence, the history regarding the ability realisation at  $n+1$  is  $h^{n+1} = (a_1, \dots, a_t, \dots, a_n, P)$ , and then with  $\pi(P|a_n) = 1$ , it is possible to write  $\pi(h^{n+1}) = \pi(h^n)$ .

With  $r$  as the real interest rate, as in Golosov and Tsyvinski (2006), it is assumed that  $\beta = 1/(1+r)$  and that the utility maximisation problem, with  $c(h^t)$  as choice variables, is

$$(4) \quad \max_{c(h^t)} \sum_{t=1}^{n+1} \sum_{h^t \in H^t} \beta^{t-1} \cdot \pi(h^t) \cdot [u(c(h^t)) - d \cdot L(h^t)]$$

$$(5) \quad \text{s. to} \quad \sum_{t=1}^{n+1} \sum_{h^t \in H^t} \beta^{t-1} \cdot \pi(h^t) \cdot [y \cdot L(h^t) - c(h^t)] \geq 0 \quad (\lambda_t)$$

$$(6) \quad \sum_{s=t}^{n+1} \sum_{h^s | a_t = A} \beta^{s-1} \cdot \pi(h^s) \cdot [u(c(h^s)) - d \cdot L(h^s)] \geq \\ \sum_{s=t}^{n+1} \sum_{h^s | a_t = D} \beta^{s-1} \cdot \pi(h^s) \cdot [u(c(h^s)) - d \cdot L(h^s)] \quad (\mu_t),$$

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<sup>11</sup> The Fibonacci sequence is defined by the recurrence relation  $\mathcal{F}(t) = \mathcal{F}(t-2) + \mathcal{F}(t-1)$ . Hence, with the seed values  $\mathcal{F}(0) = 0$  and  $\mathcal{F}(1) = 1$ , the sequence is  $\mathcal{F}(2) = 0 + 1 = 1$ ,  $\mathcal{F}(3) = 1 + 1 = 2$ ,  $\mathcal{F}(4) = 1 + 2 = 3$ ,  $\mathcal{F}(5) = 2 + 3 = 5$ ,  $\mathcal{F}(6) = 3 + 5 = 8$ ,  $\mathcal{F}(7) = 5 + 8 = 13$ , and so on.

with  $a_t \in \{A, T\}$  if  $a_{t-1} = A$  and  $a_t \in \{A, P\}$  if  $a_{t-1} = T$ , and where the *IC* (6) states that able workers are induced to work; that is, if a worker is able in  $t$  ( $a_t = A$ ), then he is guaranteed greater utility not only in the current period but also in future periods ( $s = t, \dots, n + 1$ ).

As previously suggested in Section 2, the problem can be analysed in terms of cost minimisation, where the choice variables are the utilities  $u(h^t)$  instead of consumption levels  $c(h^t)$ . Because the consumption required to give agent  $a_t \in \{A, D\}$ —with  $a_t \in \{A, T\}$  if  $a_{t-1} = A$  and  $a_t \in \{A, P\}$  if  $a_{t-1} = T$ —utility  $u(h^t)$  is  $c(u(h^t)) = u^{-1}(u(h^t))$ , with  $c''(u) > 0$ , it is possible to write

$$\begin{aligned}
 (7) \quad & \min_{u(h^t)} \sum_{t=1}^{n+1} \sum_{h^t \in H^t} \beta^{t-1} \cdot \pi(h^t) \cdot [c(u(h^t)) - y \cdot L(h^t)] \\
 (8) \quad & \text{s. to } \sum_{t=1}^{n+1} \sum_{h^t \in H^t} \beta^{t-1} \cdot \pi(h^t) \cdot [u(h^t) - d \cdot L(h^t)] \geq U(\lambda_t) \\
 (9) \quad & \sum_{s=t}^{n+1} \sum_{h^s | a_t = A} \beta^{s-1} \cdot \pi(h^s) \cdot [u(h^s) - d \cdot L(h^s)] \geq \\
 & \sum_{s=t}^{n+1} \sum_{h^s | a_t = D} \beta^{s-1} \cdot \pi(h^s) \cdot [u(h^s) - d \cdot L(h^s)] (\mu_t),
 \end{aligned}$$

where in the *PK* (8),  $U$  is the utility that, in the optimal scheme, implies the *BC* (5) in the original maximisation problem (4) holds with equality.

### 3.1 The Recursive Formulation

Next, a recursive formulation of the problem previously analysed is proposed.

Individuals may work in period  $n$ , i.e., the last working period of their working life, and are retired in period  $n + 1$ . Then, the lifetime utility function is given by the following:

$$(10) \quad E(U_n^a) = [u(c_n^a) - d \cdot L^a] + \beta \cdot u(c_{n+1}^a), a \in \{A, D\},$$

where the choice variables are  $c_t$  (for  $t = n, n + 1$ ) with  $u'(c) > 0$  and  $u''(c) < 0$ .

The government can choose the agent's consumption in both the working period  $n$  and retirement period  $n + 1$ . Moreover, it can make consumption in both periods conditional on the labour supply in period  $n$ .

Staying with the dual approach, suppose that the government guarantees the agent expected utility  $U_n$  over the two periods. Therefore, the government's objective is to minimise the (discounted) resource use required to provide the agent with the guaranteed expected utility  $U_n$ . Because the consumption required to give agent  $a \in \{A, D\}$  utility  $u_t^a$  (for  $t = n, n + 1$ ) is  $c(u_t^a) = u^{-1}(u_t^a)$ , with  $c''(u) > 0$ , the utilities  $u_t^a$  in these two periods (current utility and promised future utility), rather than the consumption levels, can be used as controls. Thus, the government's problem can be written as follows:

$$(11) \quad \begin{aligned} C_n^{a_{n-1}}(U_n) &\equiv \min_{u_n^{a_n|a_{n-1}}, u_{n+1}^{a_n|a_{n-1}}} \sum_{a_n=A,D} \pi(a_n|a_{n-1}) \cdot \left\{ c(u_n^{a_n|a_{n-1}}) - y \cdot L^{a_n} \right\} + \\ &\quad \beta \cdot c(u_{n+1}^{a_n|a_{n-1}}) \\ \text{s.to } \sum_{a_n=A,D} \pi(a_n|a_{n-1}) \cdot \left[ (u_n^{a_n|a_{n-1}} - d \cdot L^{a_n}) + \beta \cdot u_{n+1}^{a_n|a_{n-1}} \right] &\geq U_n \quad (\lambda_n^{a_{n-1}}) \\ (u_n^{A|a_{n-1}} - d \cdot L^A) + \beta \cdot u_{n+1}^{A|a_{n-1}} &\geq (u_n^{D|a_{n-1}} - d \cdot L^D) + \beta \cdot u_{n+1}^{D|a_{n-1}} \quad (\mu_n^{a_{n-1}}), \end{aligned}$$

where  $a_{n-1} \in \{A, T\}$ ,  $a_n \in \{A, T\}$  if  $a_{n-1} = A$ , and  $a_n \in \{A, P\}$  if  $a_{n-1} = T$  (in other words,  $D = T$  if  $a_{n-1} = A$  and  $D = P$  if  $a_{n-1} = T$ ). Because the solution is binding at both the *PK* and *IC* (see Appendix A), the *FOCs* related to the able, temporarily disabled, and permanently disabled workers are, respectively:

$$(12) \quad \begin{aligned} c'(u_n^{A|a_{n-1}}) &= c'(u_{n+1}^{A|a_{n-1}}) = \frac{\lambda_n^{a_{n-1}} \cdot \pi(A|a_{n-1}) + \mu_n^{a_{n-1}}}{\pi(A|a_{n-1})} \text{ with } a_{n-1} \in \{A, T\}, \\ c'(u_n^{T|A}) &= c'(u_{n+1}^{T|A}) = \frac{\lambda_n^A \cdot \pi(T|A) - \mu_n^A}{\pi(T|A)}, \\ c'(u_n^{P|T}) &= c'(u_{n+1}^{P|T}) = \frac{\lambda_n^T \cdot \pi(P|T) - \mu_n^T}{\pi(P|T)}. \end{aligned}$$

Thus, the solution entails

$$(13) \quad \begin{aligned} u_n^{A|a_{n-1}} &= u_{n+1}^{A|a_{n-1}} = u^{A|a_{n-1}} \text{ with } a_{n-1} \in \{A, T\}, \\ u_n^{T|A} &= u_{n+1}^{T|A} = u^{T|A}, \\ u_n^{P|T} &= u_{n+1}^{P|T} = u^{P|T}. \end{aligned}$$

Because the multipliers  $\mu_n^A, \mu_n^T > 0$ , the disequalities  $c'(u^{A|A}) > c'(u^{T|A})$  and  $c'(u^{A|T}) > c'(u^{P|T})$  hold; hence,  $u^{A|A} > u^{T|A}$  and  $u^{A|T} > u^{P|T}$ . The government provides incentives to work in period  $n$  by offering individuals who work a higher utility (and therefore consumption) in the retirement period  $n + 1$  as well.

In the previous working periods ( $t = 1, \dots, n-1$ ), the consumption level required to give agent  $a_t$  utility  $u_t^{a_t|a_{t-1}}$  in the current working period  $t$  is  $c(u_t^{a_t|a_{t-1}}) = u^{-1}(u_t^{a_t|a_{t-1}})$  with  $c''(u) > 0$ . With reference to future working periods (i) if  $a_{t-1} \in \{A, T\}$  the cost level required to give able agent  $a_t = A$  utility  $U_{t+1}^{A|a_{t-1}}$  in the future working periods and in the retirement period is  $C_{t+1}^A(U_{t+1}^{A|a_{t-1}}) = U_{t+1}^{-1}(U_{t+1}^{A|a_{t-1}})$ , with  $C_{t+1}^{A''}(U_{t+1}) > 0$ ; (ii) if  $a_{t-1} = A$  the cost level required to give temporarily disabled agent  $a_t = T$  utility  $U_{t+1}^{T|A}$  in the future working periods and in the retirement period is  $C_{t+1}^T(U_{t+1}^{T|A}) = U_{t+1}^{-1}(U_{t+1}^{T|A})$ , with  $C_{t+1}^{T''}(U_{t+1}) > 0$ ; and (iii) if  $a_{t-1} = T$  the cost level required to give permanently disabled agent  $a_t = P$  utility

$$\sum_{\tau=t+1}^{n+1} \beta^{\tau-(t+1)} \cdot (u_{t+1}^{P|T} - d \cdot L^D) = (1 - \beta^{(n+1)-t}) / (1 - \beta).$$

$(u_{t+1}^{P|T} - d \cdot L^D)$  in the future working periods and in the retirement period is

$$\sum_{\tau=t+1}^{n+1} \beta^{\tau-(t+1)} \cdot [c(u_{t+1}^{P|T}) - y \cdot L^D] = (1 - \beta^{(n+1)-t}) / (1 - \beta).$$

$$[c(u_{t+1}^{P|T}) - y \cdot L^D].^{12}$$

As for period  $n$ , the utilities in these periods (rather than the consumption levels) can be used as controls. Therefore, if an individual was able in the previous period  $a_{t-1} = A$ , the government's problem can be written as the following Bellman equation (see Bellman, 1957):

$$(14) \quad C_t^A(U_t) \equiv \min_{u_t^{a_t|A}, U_{t+1}^{a_t|A}} \sum_{a_t=A,T} \pi(a_t|A) \cdot \{ [c(u_t^{a_t|A}) - y \cdot L^{a_t}] + \beta \cdot C_{t+1}^{a_t}(U_{t+1}^{a_t|A}) \}$$

$$\text{s.to} \quad \sum_{a_t=A,T} \pi(a_t|A) \cdot [(u_t^{a_t|A} - d \cdot L^{a_t}) + \beta \cdot U_{t+1}^{a_t|A}] \geq U_t \quad (\lambda_t^A)$$

$$(u_t^{A|A} - d \cdot L^A) + \beta \cdot U_{t+1}^{A|A} \geq (u_t^{T|A} - d \cdot L^D) + \beta \cdot U_{t+1}^{T|A} \quad (\mu_t^A),$$

In addition, if an individual was temporarily disabled in the previous period  $a_{t-1} = T$ , the government's problem can be written as the following Bellman equation (see Bellman, 1957):

---

<sup>12</sup> Note that  $u_{t+1}^{P|T}$  takes the same value for  $\tau = t+1, \dots, n+1$ .

$$\begin{aligned}
c_t^T(U_t) &\equiv \min_{u_t^{A|T}, u_{t+1}^{A|T}, u_t^{P|T}, u_{t+1}^{P|T}} \sum_{a_t=A,P} \pi(a_t|T) \cdot [c(u_t^{a_t|T}) - y \cdot L^{a_t}] + \\
&\quad \pi(A|T) \cdot \beta \cdot C_{t+1}^A(U_{t+1}^{A|T}) + \pi(P|T) \cdot \beta \cdot \sum_{\tau=t+1}^{n+1} \beta^{\tau-(t+1)} \cdot [c(u_{t+1}^{P|T}) - y \cdot L^D] \\
\text{s. to } &\sum_{a_t=A,P} \pi(a_t|T) \cdot (u_t^{a_t|T} - d \cdot L^{a_t}) + \pi(A|T) \cdot \beta \cdot U_{t+1}^{A|T} + \\
(15) \quad &\pi(P|T) \cdot \beta \cdot \sum_{\tau=t+1}^{n+1} \beta^{\tau-(t+1)} \cdot (u_{t+1}^{P|T} - d \cdot L^D) \geq U_t(\lambda_t^T) \\
&(u_t^{A|T} - d \cdot L^A) + \beta \cdot U_{t+1}^{A|T} \geq (u_t^{P|T} - d \cdot L^D) + \\
&\beta \cdot \sum_{\tau=t+1}^{n+1} \beta^{\tau-(t+1)} \cdot (u_{t+1}^{P|T} - d \cdot L^D) (\mu_t^T),
\end{aligned}$$

where  $\sum_{\tau=t+1}^{n+1} \beta^{\tau-(t+1)} = (1 - \beta^{(n+1)-t}) / (1 - \beta)$ . As in the minimisation problem (11), in the minimisation problems (14) and (15), the solutions are binding at both the *PK* and *IC* (see Appendix A); hence, the *FOCs* related to the able, temporarily disabled, and permanently disabled workers are, respectively, as follows:

$$\begin{aligned}
c'(u_t^{A|a_{t-1}}) &= C_{t+1}^{A'}(U_{t+1}^{A|a_{t-1}}) = \frac{\lambda_t^{a_{t-1}} \cdot \pi(A|a_{t-1}) + \mu_t^{a_{t-1}}}{\pi(A|a_{t-1})} \text{ with } a_{t-1} \in \{A, T\}, \\
(16) \quad c'(u_t^{T|A}) &= C_{t+1}^{T'}(U_{t+1}^{T|A}) = \frac{\lambda_t^A \cdot \pi(T|A) - \mu_t^A}{\pi(T|A)}, \\
c'(u_t^{P|T}) &= c'(u_{t+1}^{P|T}) = \frac{\lambda_t^T \cdot \pi(P|T) - \mu_t^T}{\pi(P|T)} \Rightarrow u_t^{P|T} = u_{t+1}^{P|T} = u^{P|T}.
\end{aligned}$$

Because the multipliers  $\mu_t^A, \mu_t^T > 0$ , the disequalities  $c'(u_t^{A|A}) > c'(u_t^{T|A})$  and  $c'(u_t^{A|T}) > c'(u^{P|T})$  hold; hence,  $u_t^{A|A} > u_t^{T|A}$  and  $u_t^{A|T} > u^{P|T}$ . Moreover, if  $u_t^{A|A} - d \cdot L^A < u_t^{T|A} - d \cdot L^D$  then  $U_{t+1}^{A|A} > U_{t+1}^{T|A}$  and if  $u_t^{A|T} - d \cdot L^A < u^{P|T} - d \cdot L^D$ , then  $U_{t+1}^{A|T} > (1 - \beta^{(n+1)-t}) / (1 - \beta) \cdot (u^{P|T} - d \cdot L^D)$ . Thus, the government provides incentives to work in period  $t$  by offering individuals who work a higher utility level also in future periods.

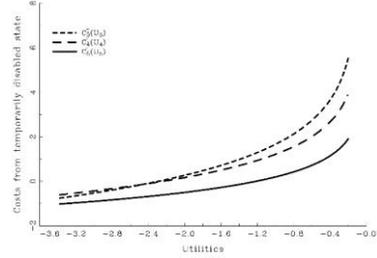
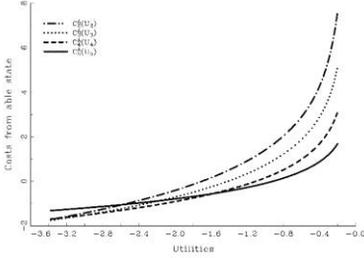
The model is investigated and clarified by means of numerical simulations (see Appendix B), in which individuals are assumed to have *CARA* preferences (see, e.g., Shimer and Werning, 2007; Platoni, 2017). The cost functions derived on the basis of this simulation in the cases of  $n = 5$  and  $n = 10$  are represented in Figure 1.

FIGURE 1

Cost Functions at  $\alpha = 2$ ,  $d = 0.05$ ,  $r = 0.03$ ,  $\pi^{AlA} = 0.8$ , and  $\pi^{AlT} = 0.5$

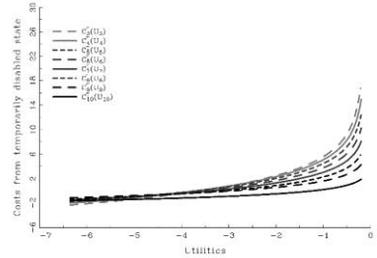
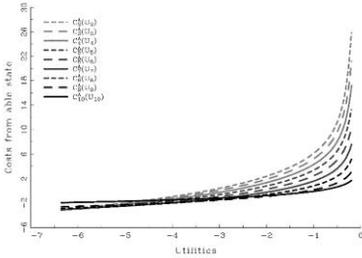
(a)  $n = 5$  – from able state

(b)  $n = 5$  – from temporarily disabled state



(c)  $n = 10$  – from able state

(d)  $n = 10$  – from temporarily disabled state



Due to the concavity of the utility function, the cost functions are convex; the curves are lower when the retirement period is closer; that is, the cost of guaranteeing a certain level of utility decreases when the retirement period is closer. Moreover, if  $n = 5$ , then  $C_t^A(\bullet) < C_t^T(\bullet)$  with  $C_t^A(\bullet) > C_t^{T'}(\bullet)$ , and if  $n = 10$ , then  $C_t^A(\bullet) > C_t^T(\bullet)$  for  $t = 3 \dots 6$  and  $C_t^A(\bullet) < C_t^T(\bullet)$  for  $t = 7 \dots 10$  with  $C_t^A(\bullet) > C_t^{T'}(\bullet)$ . Given the cost functions as represented in Figure 1, the multi-period minimisation problem with  $n$  working periods can be solved: from  $t = 1, \dots, n - 1$ , the government's problems are (14) and (15), and in the last working period  $n$ , the government's problem is (11).

### 3.2 The Properties of the Optimal Allocation

The aim is to explore the properties of the optimal dynamic allocation. Specifically, the analysis considers how an individual's utility allocation depends on his being disabled and the difference between temporary and permanent disability.

Tables 1 and 2 report the results of the simulation with five working periods ( $n = 5$ ). Table 1 displays how the government saves

in the working periods  $t = 1, \dots, 4$  to finance consumption in the last working period  $n = 5$  and in the retirement period  $n + 1 = 6$ .

TABLE 1

Government Balance ( $n = 5$ )							
	$t = 1$	$t = 2$	$t = 3$	$t = 4$	$t = 5$	$t = 6$	
$Y$	0.8000	0.7184	0.6334	0.5597	0.4944	0.0000	3.2060
$C$	0.5751	0.5581	0.5416	0.5257	0.5102	0.4953	3.2060
$Y - C$	0.2249	0.1603	0.0918	0.0340	-0.0157	-0.4953	0.0000

Table 2 displays the discounted promised future utility  $\beta^t \cdot U_{t+1}^a$  and the sum of the discounted utilities  $\beta^{t-1} \cdot U_t^a$  (i.e., the sum of the discounted current utility  $\beta^{t-1} \cdot (u_t^a - d \cdot L^a)$  and discounted promised future utility  $\beta^t \cdot U_{t+1}^a$ ) for each possible history (with  $n = 5$ , the number of histories is  $\mathcal{N}(5) = \mathcal{F}(6) + \mathcal{F}(5) + \sum_{\tau=1}^5 \mathcal{F}(\tau - 1) = 8 + 5 + 7 = 20$ , where  $\mathcal{F}(6) = 8$  is the number of able states,  $\mathcal{F}(5) = 5$  the number of temporarily disabled states, and  $\sum_{\tau=1}^5 \mathcal{F}(\tau - 1) = 7$  the number of permanently disabled states).

*Theorem 1 Binding IC:* The IC (9) will bind, i.e., will be satisfied with equality, for the optimal solution.

*Proof.* A proof by contradiction is used to show that the IC (9) will bind. If  $\sum_{s=t}^{n+1} \sum_{h^s | a_t=A} \beta^{s-1} \cdot \pi(h^s) \cdot [u(h^s) - d \cdot L(h^s)] > \sum_{s=t}^{n+1} \sum_{h^s | a_t=D} \beta^{s-1} \cdot \pi(h^s) \cdot [u(h^s) - d \cdot L(h^s)]$ , then the gap between the sum of current and future utilities of a worker able in  $t$  and the sum of current and future utilities of a worker disabled in  $t$  can be lowered by an amount  $\delta$ :

$$\sum_{s=t}^{n+1} \sum_{h^s | a_t=A} \beta^{s-1} \cdot \pi(h^s) \cdot [u(h^s) - d \cdot L(h^s)] > \sum_{s=t}^{n+1} \sum_{h^s | a_t=D} \beta^{s-1} \cdot \pi(h^s) \cdot [u(h^s) - d \cdot L(h^s)] + \delta,$$

and this makes it easier to satisfy the PK constraint:

$$\sum_{t=1}^{n+1} \sum_{h^t \in H^t} \beta^{t-1} \cdot \pi(h^t) \cdot [u(h^t) - d \cdot L(h^t)] + \delta \geq U.$$

Therefore, the utilities can be lowered by an amount  $\varepsilon$ :

$$\sum_{t=1}^{n+1} \sum_{h^t \in H^t} \beta^{t-1} \cdot \pi(h^t) \cdot [u(h^t) - d \cdot L(h^t) - \varepsilon] + \delta \geq U,$$

but then, the original solution was not resource use minimising. ■

TABLE 2  
*Utilities and consumption with  $n = 5$  at  $\alpha = 2$ ,  $d = 0.05$ ,  $r = 0.03$ ,  $\pi^A/A = 0.8$ , and  $\pi^A/I = 0.5$*

	A		T		A		T	
	3.2060		0.0292		0.0292		0.0290	
$U_t$								
$Y = C$								
$u_t^c$	-0.313	-0.316	-0.313	-0.316	-0.313	-0.316	-0.313	-0.316
$u_t^d - d \cdot I_t^d$	-0.363	-0.326	-0.342	-0.346	-0.342	-0.346	-0.342	-0.346
$U_t^d$	-1.615	-1.320	-0.900	-1.008	-0.900	-1.008	-0.900	-1.008
$\beta \cdot U_t^d$	-1.568	-1.249	-0.868	-0.923	-0.868	-0.923	-0.868	-0.923
$U_t^e$	-1.951	-1.568	-1.249	-1.249	-1.249	-1.249	-1.249	-1.249
$C_t^e$	0.5809	0.5560	0.5809	0.5560	0.5809	0.5560	0.5809	0.5560
$\Delta c_t(I^e) = \Delta c_t(I^e)$								
$u_t^e$	-0.309	-0.326	-0.309	-0.326	-0.309	-0.326	-0.309	-0.326
$\beta \cdot (u_t^e - d \cdot I_t^e)$	-0.349	-0.331	-0.342	-0.346	-0.342	-0.346	-0.342	-0.346
$U_t^e$	-1.204	-0.977	-0.654	-0.674	-0.654	-0.674	-0.654	-0.674
$\beta^2 \cdot U_t^e$	-1.219	-0.912	-0.589	-0.599	-0.589	-0.599	-0.589	-0.599
$\beta \cdot U_t^e$	-1.588	-1.219	-0.884	-0.912	-0.884	-0.912	-0.884	-0.912
$C_t^e$	0.5870	0.5604	0.5870	0.5604	0.5870	0.5604	0.5870	0.5604
$\Delta c_t(I^e)$								
$u_t^e$	-0.305	-0.326	-0.305	-0.326	-0.305	-0.326	-0.305	-0.326
$\beta^2 \cdot (u_t^e - d \cdot I_t^e)$	-0.335	-0.331	-0.329	-0.331	-0.329	-0.331	-0.329	-0.331
$U_t^e$	-0.967	-0.707	-0.577	-0.603	-0.577	-0.603	-0.577	-0.603
$\beta^2 \cdot U_t^e$	-0.884	-0.674	-0.496	-0.506	-0.496	-0.506	-0.496	-0.506
$\beta \cdot U_t^e$	-1.219	-0.912	-0.606	-0.606	-0.606	-0.606	-0.606	-0.606
$C_t^e$	0.5954	0.5649	0.5954	0.5649	0.5954	0.5649	0.5954	0.5649
$\Delta c_t(I^e)$								
$u_t^e$	-0.301	-0.323	-0.301	-0.323	-0.301	-0.323	-0.301	-0.323
$\beta^2 \cdot (u_t^e - d \cdot I_t^e)$	-0.321	-0.296	-0.313	-0.313	-0.313	-0.313	-0.313	-0.313
$U_t^e$	-0.634	-0.663	-0.654	-0.674	-0.654	-0.674	-0.654	-0.674
$\beta^2 \cdot U_t^e$	-0.563	-0.589	-0.581	-0.599	-0.581	-0.599	-0.581	-0.599
$\beta \cdot U_t^e$	-0.884	-0.884	-0.912	-0.912	-0.912	-0.912	-0.912	-0.912
$C_t^e$	0.6003	0.5649	0.5834	0.5649	0.5834	0.5649	0.5834	0.5649
$\Delta c_t(I^e)$								
$u_t^e$	-0.296	-0.322	-0.307	-0.332	-0.304	-0.330	-0.304	-0.330
$\beta^2 \cdot (u_t^e - d \cdot I_t^e)$	-0.308	-0.286	-0.311	-0.299	-0.315	-0.293	-0.315	-0.293
$U_t^e$	-0.296	-0.322	-0.311	-0.336	-0.304	-0.330	-0.304	-0.330
$\beta^2 \cdot U_t^e$	-0.256	-0.277	-0.268	-0.290	-0.263	-0.284	-0.263	-0.284
$\beta \cdot U_t^e$	-0.563	-0.563	-0.589	-0.589	-0.577	-0.577	-0.577	-0.577
$C_t^e$	0.6082	0.5671	0.5911	0.5513	0.5947	0.5546	0.5947	0.5546
$\Delta c_t(I^e) = \Delta c_t(I^e)$								

In fact, from Table 2, it is possible to verify that the *IC* (9) binds along each working history both from the able state,

$$\begin{aligned}\beta^{t-1} \cdot u_t^{AlA} &= \beta^{t-1} \cdot u_t^{TlA}, \\ u_t^{AlA} &= u_t^{TlA}, \\ (u_t^{AlA} - d \cdot L^A) + \beta \cdot u_{t+1}^{AlA} &= u_t^{TlA} + \beta \cdot u_{t+1}^{TlA}, t = 1, \dots, n,\end{aligned}$$

and from the temporarily disabled state,

$$\begin{aligned}\beta^{t-1} \cdot u_t^{AlT} &= \beta^{t-1} \cdot \frac{1 - \beta^{(n+1)-(t-1)}}{1 - \beta} \cdot u_t^{PlT}, \\ u_t^{AlT} &= \frac{1 - \beta^{(n+1)-(t-1)}}{1 - \beta} \cdot u_t^{PlT}, \\ (u_t^{AlT} - d \cdot L^A) + \beta \cdot u_{t+1}^{AlT} &= u_t^{PlT} + \beta \cdot \frac{1 - \beta^{(n+1)-t}}{1 - \beta} \cdot u_{t+1}^{PlT}, t = 2, \dots, n,\end{aligned}$$

that is, in  $t = 1$ ,  $U_1^A = U_1^T = -1.931$ , in  $t = 2$ ,  $\beta \cdot U_2^{AA} = \beta \cdot U_2^{AT} = -1.568$  and  $\beta \cdot U_2^{TA} = \beta \cdot (1 - \beta^5)/(1 - \beta) \cdot u_2^{TP} = -1.599$ , in  $t = 3$ , following the first path  $\beta^2 \cdot U_3^{AAA} = \beta^2 \cdot U_3^{AAT} = -1.219$  and following the eighth path  $\beta^2 \cdot U_3^{ATA} = \beta^2 \cdot (1 - \beta^4)/(1 - \beta) \cdot u_3^{ATP} = -1.249$ , in  $t = 4$ , following the first path  $\beta^3 \cdot U_4^{AAAA} = \beta^3 \cdot U_4^{AAAT} = -0.884$  and following the fifth path  $\beta^3 \cdot U_5^{AATA} = \beta^3 \cdot (1 - \beta^3)/(1 - \beta) \cdot u_4^{AATP} = -0.912$ , and, finally, in  $t = 5$ , following the first path  $\beta^4 \cdot U_5^{AAAAA} = \beta^4 \cdot U_5^{AAAAAT} = -0.563$  and following the third path  $\beta^4 \cdot U_5^{AATAA} = \beta^4 \cdot (1 - \beta^2)/(1 - \beta) \cdot u_5^{AAATP} = -0.589$ .

Hence, the system of dynamic incentives implies that in each working period  $t = 1, \dots, n$  able individuals are indifferent between working and not working and consequently able individuals are induced to work.

It is interesting to examine whether, under the optimal dynamic incentives scheme, a worker is better off if his disability occurs early versus late in his lifecycle, in the case of both temporary disability and permanent disability. Because it is important to consider whether the disability spell is closer to or further from retirement, as in Shimer and Werning (2007), the analysis is phrased in terms of the number of working periods remaining before retirement, defined as  $q = n - t + 1$ .

*Result 1 Early versus late disability:* Whereas (a) late temporary disability is worse than early temporary disability (that is, the dynamic

incentives scheme increases with time), (b) early permanent disability is worse than late permanent disability (that is, the dynamic incentives scheme decreases with time).

*Proof a.* The focus is on the cases with one temporary disability spell at  $t = 3$  and precisely on the sum of the discounted utilities of a temporarily disabled individual when  $q = 4$  working periods remain (at  $t = 2$ ) before retirement  $\beta^2 \cdot U_3^{ATa_3}$  and the sum of the discounted utilities of a temporarily disabled individual when  $q = 5$  working periods remain (at  $t = 1$ )  $\beta^2 \cdot U_3^{TAa_3}$ . From Table 2, because  $\beta^2 \cdot U_3^{ATa_3}[-1.249] < \beta^2 \cdot U_3^{TAa_3}[-1.244]$ , the sum of the discounted utilities of an individual disabled when  $q = 4$  working periods remain is lower than the sum of the discounted utilities of an individual disabled when  $q = 5$  working periods remain. ■

*Proof b.* The focus is on the cases with one permanent disability spell at  $t = 4$  and precisely on the sum of the discounted utilities of a permanently disabled individual when  $q = 3$  working periods remain (at  $t = 3$ ) before retirement  $\beta^3 \cdot U_4^{ATPa_4}$  and the sum of the discounted utilities of a permanently disabled individual when  $q = 4$  working periods remain (at  $t = 2$ )  $\beta^3 \cdot U_4^{TPPa_4}$ . From Table 2, because  $\beta^3 \cdot U_4^{ATPa_4}[-0.923] > \beta^3 \cdot U_4^{TPPa_4}[-0.931]$ , the sum of the discounted utilities of an individual permanently disabled when  $q = 3$  working periods remain is higher than the sum of the discounted utilities of an individual permanently disabled when  $q = 4$  working periods remain. ■

When the time horizon is shorter, the dynamic incentives are restricted in the case of temporary disability and expanded in the case of permanent disability. Consequently, the system of dynamic incentives guarantees higher disability benefits if the temporary disability occurs in the early periods of the working life, i.e., if younger temporarily disabled individuals are better insured than older temporarily disabled individuals, and higher disability benefits if the permanent disability occurs in the late periods of the working life, i.e., if older permanently disabled individuals are better insured than younger permanently disabled individuals. Therefore, late temporary

disability is worse than early temporary disability, and late permanent disability is better than early permanent disability.

Result 1 confirms the findings of both Platoni (2017) in the case of temporary disability and Golosov and Tsyvinski (2006) in the case of permanent disability. Thus, this system of dynamic incentives implies that, if disability is a temporary state, then the intertemporal provision of dynamic incentives encourages higher consumption for agents who become disabled early in life; therefore late disability is worse than early disability. A possible interpretation is that older individuals—supposedly more skilled and more efficient workers thanks to their longer work experience—are more encouraged to work thanks to lower disability benefits (see, e.g., Platoni, 2017). In contrast, if disability is a permanent state, then the intertemporal provision of dynamic incentives encourages higher consumption for agents who become disabled later in life; therefore, late disability is better than early disability. A possible interpretation is that older individuals—supposedly less healthy and less productive workers because of their age—are less encouraged to work because they will receive higher disability benefits.

### 3.3 The Consumption Paths and Gaps

The purpose is to analyse the consumption paths and the consumption gaps between able and disabled individuals, both temporarily and permanently.

Table 2, where  $n = 5$ , not only displays the discounted promised future utilities and the sums of the discounted utilities but also the consumption gaps. A worker who is able in period  $t$ , i.e.,  $a_t = A$ , and a worker who is disabled in period  $t$ , i.e.,  $a_t = T$  if  $a_{t-1} = A$  or  $a_t = P$  if  $a_{t-1} = T$ , are considered. The able worker has a higher level of consumption, and the gap in consumption is as follows:

$$(17) \quad \begin{aligned} \Delta c(a_0 = A) &= c(a_1 = A) - c(a_1 = T), t = 1, \\ \Delta c(h^{t-1}; a_{t-1} = A) &= c(h^{t-1}; a_t = A) - c(h^{t-1}; a_t = T), t = 2, \dots, n, \\ \Delta c(h^{t-1}; a_{t-1} = T) &= c(h^{t-1}; a_t = A) - c(h^{t-1}; a_t = P), t = 2, \dots, n, \end{aligned}$$

where  $(h^{t-1}; a_t = A)$  is the period  $t$  history when the worker is able in  $t$  and  $(h^{t-1}; a_t = T)$  and  $(h^{t-1}; a_t = P)$  are the period  $t$  histories when the worker is, respectively, temporarily or permanently disabled in  $t$ . As observed from Equation (17), if  $t > 1$ , this consumption gap generally depends on the history in the periods preceding  $t$ , i.e.,  $h^{t-1}$ .

*Result 2 Temporary disability:* In the case of temporary disability, the consumption gaps ( $a$ ) not only increase when the retirement period becomes closer ( $b$ ) but also increase if the disability spell is further from retirement.

*Proof a.* From Table 2, in  $t = 5$ , the consumption gaps along the working life of a constantly able individual are  $\Delta c(A, A, A, A)[0.0411] > \Delta c(A, A, A)[0.0353] > \Delta c(A, A)[0.0330] > \Delta c(A)[0.0309]$ , those of an individual temporarily disabled in  $t = 1$  are  $\Delta c(T, A, A, A)[0.0402] > \Delta c(T, A, A)[0.0346] > \Delta c(T, A)[0.0323]$ , and, finally, those of an individual temporarily disabled in  $t = 2$  are  $\Delta c(A, T, A, A)[0.0400] > \Delta c(A, T, A)[0.0344]$ . ■

*Proof b.* From Table 2, in  $t = 3$ , the consumption gaps of temporarily disabled individuals are  $\Delta c(A, A)[0.0330] > \Delta c(T, A)[0.0323]$ , in  $t = 4$  they are  $\Delta c(A, A, A)[0.0353] > \Delta c(T, A, A)[0.0346] > \Delta c(A, T, A)[0.0344]$ , and, finally, in  $t = 5$ , they are  $\Delta c(A, A, A, A)[0.0411] > \Delta c(T, A, A, A)[0.0402] > \Delta c(A, T, A, A)[0.0400] > \Delta c(A, A, T, A)[0.0397]$ . ■

*Result 3 Permanent disability:* In the case of permanent disability, the consumption gaps ( $a$ ) not only decrease when the retirement period becomes closer ( $b$ ) but also decrease if a sporadic temporary disability spell (i.e., temporary disability not followed by permanent disability) occurs closer to retirement.<sup>13</sup>

*Proof a.* From Table 2, in  $t = 5$ , the consumption gaps of individuals able in  $t = 1$  and permanently disabled in  $t = 5$ ,  $t = 4$ , and  $t = 3$  are  $\Delta c(A, A, A, T)[0.0392] < \Delta c(A, A, T)[0.0470] < \Delta c(A, T)[0.0496]$ , and, those of a temporarily disabled individual in  $t = 1$  and permanently disabled in  $t = 5$  and  $t = 4$  are  $\Delta c(T, A, A, T)[0.0384] < \Delta c(T, A, T)[0.0461]$ . ■

*Proof b.* From Table 2, in  $t = 4$ , the consumption gaps of permanently disabled individuals are  $\Delta c(T, A, T)[0.0461] < \Delta c(A, A, T)[0.0470]$ , and, in  $t = 5$ , they are  $\Delta c(A, T, A, T)[0.0383] < \Delta c(T, A, A, T)[0.0384] < \Delta c(A, A, A, T)[0.0392]$ . ■

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<sup>13</sup> In other words, in the case of permanent disability, as in the case of temporary disability (see Result 2), the consumption gaps increase if a sporadic temporary disability spell occurs further from retirement.

In summary, the disadvantage of being temporarily disabled rises (i.e., the disability benefits decrease) with the age of the individual, and the disadvantage of being permanently disabled diminishes (i.e., the disability benefits increase) with the age of the individual; hence, Results 2(a) and 3(a) confirm Result 1. Moreover, Results 2(b) and 3(b) state that a sporadic temporary disability spell increases the disadvantage of being both temporarily and permanently disabled if this disability spell is further from retirement.

### 3.4 Longer Working Life

Let us now extend our analysis to a sufficiently long working life. Hence, it is now appropriate to present a simulation with ten working periods ( $n = 10$ ). The number of histories is  $\mathcal{N}(10) = \mathcal{F}(11) + \mathcal{F}(10) + \sum_{\tau=1}^{10} \mathcal{F}(\tau - 1) = 89 + 55 + 88 = 232$ , where  $\mathcal{F}(11) = 89$  is the number of able states,  $\mathcal{F}(10) = 55$  is the number of temporarily disabled states, and  $\sum_{\tau=1}^{10} \mathcal{F}(\tau - 1) = 88$  is the number of permanently disabled states.

Table 3 indicates that the government saves in the working periods  $t = 1, \dots, 5$  to finance consumption in the working periods  $t = 6, \dots, 9$  in the last working period  $n = 10$ , and in the retirement

TABLE 3

*Government Balance ( $n = 10$ )*

	$t = 1$	$t = 2$	$t = 3$	$t = 4$	$t = 5$	...
$Y$	0.8000	0.7184	0.6334	0.5597	0.4944	...
$C$	0.5186	0.5034	0.4886	0.4742	0.4603	...
$Y - C$	0.2814	0.2151	0.1448	0.0855	0.0341	...
	$t = 6$	$t = 7$	$t = 8$	$t = 9$	$t = 10$	$t = 11$
$Y$	0.4368	0.3858	0.3409	0.3011	0.2660	0.0000
$C$	0.4468	0.4336	0.4209	0.4086	0.3966	0.3850
$Y - C$	-0.0100	-0.0478	-0.0801	-0.1075	-0.1306	-0.3850
						0.0000

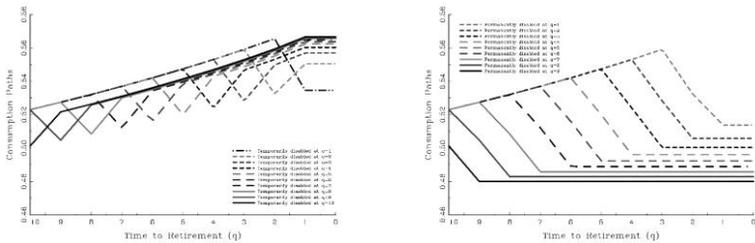
Whereas Figure 2(a) compares the consumption paths of individuals suffering from only one temporary disability spell when  $q$  working periods remain before retirement, Figure 2(b) compares the consumption paths of individuals suffering from a permanent disability spell when  $q$  working periods remain before retirement. Note that an individual who suffers from a permanent disability spell when  $q$  working periods remain before retirement suffers from a

temporary disability spell when  $q + 1$  working periods remain before retirement.<sup>14</sup>

The consumption path of a temporarily disabled individual when  $q$  working periods remain before retirement is higher than the consumption path of a temporarily disabled individual when  $q - j$  (with  $j = 1, \dots, q - 1$ ) working periods remain (see Figure 2(a)). In contrast, the consumption path of a permanently disabled individual when  $q$  working periods remain is lower than the consumption path of a permanently disabled individual when  $q - j$  (with  $j = 1, \dots, q - 1$ ) working periods remain (see Figure 2(b)).

FIGURE 2

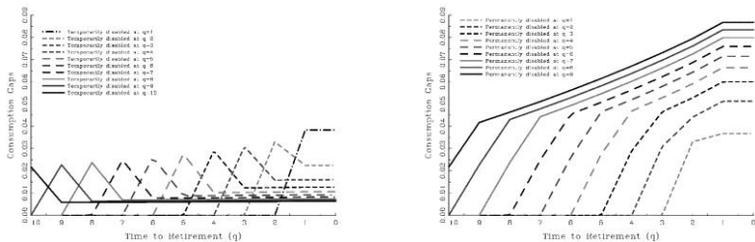
Consumptions paths with  $n = 10$  at  $\alpha = 2$ ,  $d = 0.05$ ,  $r = 0.03$ ,  $\pi^{A|A} = 0.8$ , and  $\pi^{A|T} = 0.5$   
 (a) Temporary disability (b) Permanent disability



In addition, the findings obtained analysing the consumption paths are corroborated by the findings obtained examining the consumption gaps (see Figure 3).

FIGURE 3

Consumptions gaps with  $n = 10$  at  $\alpha = 2$ ,  $d = 0.05$ ,  $r = 0.03$ ,  $\pi^{A|A} = 0.8$ , and  $\pi^{A|T} = 0.5$   
 (a) Temporary disability (b) Permanent disability



<sup>14</sup> Hence, an individual who is permanently disabled with 9 working periods remaining before retirement is temporarily disabled when 10 working periods remain before retirement, an individual who is permanently disabled with 8 working periods remaining before retirement is temporarily disabled when 9 working periods remain before retirement, and so on.

Note that in the case of temporary disability, the gaps are computed as the difference between the consumption path of an individual who faces one temporary disability spell in  $q$  and the consumption path of an always able individual (see Figure 3(a)). Put differently, in the case of permanent disability, the gaps are computed as the difference between the consumption path of an individual who faces a permanent disability spell in  $q$  and the consumption path of an individual who faces one temporary disability spell in  $q + 1$  (see Figure 3(b)).

Whereas the gap between the consumption path of an individual who faces one temporary disability spell and that of an always able individual is larger when the temporary disability spell occurs closer to the retirement period (see Figure 3(a)), the gap between the consumption path of an individual who faces a permanent disability spell in  $q$  and that of an individual who faces only one temporary disability spell in  $q + 1$  is smaller when the permanent disability spell occurs closer to the retirement period (see Figure 3(b)).

Therefore, the results obtained in the simulation with ten working periods ( $n = 10$ ), as shown in Figures 2 and 3, confirm the findings obtained in the case of five working periods ( $n = 5$ ), as stated in Results 1, 2, and 3.

## 4. CONCLUSIONS

The article compares able, temporarily disabled, and permanently disabled workers. The study of the dynamic incentives model, which extends Diamond and Mirrlees (1978) and Platoni (2017), makes it possible to answer the research questions stated in the introduction.

i) Under what conditions are able workers induced to work in the presence of publicly provided disability insurance? In line with the previous literature, we find that able workers are guaranteed the highest utility to reduce their work disincentives (i.e., Theorem 1); moreover, they receive the highest utility also in future periods (i.e., also in the retirement period).

ii) Are the work disincentives different in the presence of temporary or permanent disability? To answer to this question, we must specify that there are differences between temporary and permanent disability and that these differences are related to timing,

specifically when disability occurs (further from or closer to retirement). In particular, if disability occurs in the early periods of a person's working life, then the system of dynamic incentives guarantees higher (lower) disability benefits in the case of temporary (permanent) disability; hence, late temporary disability is worse than early temporary disability, and late permanent disability is better than early permanent disability (i.e., Result 1). In other words, we find that in the presence of temporary (permanent) disability, it is better that agents become disabled early (late) in life.

iii) How does the consumption path depend on a person being disabled? How does the consumption path depend on his being temporarily or permanently disabled? In the model, we find that an able worker is guaranteed a higher level of consumption than a disabled one is, and this consumption gap generally depends on the history in previous working periods. Finally, we show that the closer the temporary disability spell is to the retirement period, the larger the gap between the consumption path of an always able individual and that of an individual facing one temporary disability spell (i.e., Result 2). Further, the closer the permanent disability status is to the retirement period, the lower the gap between the consumption paths of an individual who, from a temporary disability spell, either comes back to able status or becomes permanently disabled (i.e., Result 3).

## APPENDICES

### A. Binding Promised Utility-keeping and Incentive Constraints

*Theorem A.1 Binding PK:* The PKs in the minimisation problems (11), (14), and (15) will bind, i.e., will be satisfied with equality, for the optimal solutions.

*Proof:* A proof by contradiction is used to show that the PK in the minimisation problem (11) will bind. If the multiplier  $\lambda_n^{a_n} = 0$ , i.e., if  $\sum_{a_n=A,D} \pi(a_n|a_{n-1}) \cdot \left[ \left( u_n^{a_n|a_{n-1}} - d \cdot L^{a_n} \right) + \beta \cdot u_{n+1}^{a_n|a_{n-1}} \right] > U_n$ , then there exists an amount  $\varepsilon$  by which the current utilities of able and disabled workers can be lowered without violating the PK constraint:

$$\sum_{a_n=A,D} \pi(a_n|a_{n-1}) \cdot \left[ \left( u_n^{a_n|a_{n-1}} - d \cdot L^{a_n} - \varepsilon \right) + \beta \cdot u_{n+1}^{a_n|a_{n-1}} \right] > U_n.$$

It is straightforward that the *IC* will hold as well

$$\begin{aligned} & \left( u_n^{A|a_{n-1}} - d \cdot L^A - \varepsilon \right) + \beta \cdot u_{n+1}^{A|a_{n-1}} \geq \left( u_t^{D|a_{n-1}} - d \cdot L^D - \varepsilon \right) + \beta \cdot u_{n+1}^{D|a_{n-1}} \\ \Rightarrow & \left( u_n^{A|a_{n-1}} - d \cdot L^A \right) + \beta \cdot u_{n+1}^{A|a_{n-1}} \geq \left( u_t^{D|a_{n-1}} - d \cdot L^D \right) + \beta \cdot u_{n+1}^{D|a_{n-1}}. \end{aligned}$$

Because both the *PK* and *IC* still hold, the current utilities can be lowered by  $\varepsilon$ , which violates the assumption that the solution was resource use minimising in the first place. The proof of the binding *PK* in the minimisation problems (14) and (15) is equivalent and thus omitted. ■

*Theorem A.2 Binding IC:* The *ICs* in the minimisation problems (11), (14), and (15) will bind, i.e., will be satisfied with equality, for the optimal solutions.

*Proof.* A proof by contradiction is used to show that the *IC* in the minimisation problem (11) will bind. If the multiplier  $\mu_n^{a_{n-1}} = 0$ , i.e., if  $\left( u_n^{A|a_{n-1}} - d \cdot L^A \right) + \beta \cdot u_{n+1}^{A|a_{n-1}} > \left( u_t^{D|a_{n-1}} - d \cdot L^D \right) + \beta \cdot u_{n+1}^{D|a_{n-1}}$ , then the gap between the sum of current and future utilities of an able worker and the sum of current and future utilities of a disabled worker can be lowered by an amount  $\delta$ :

$$\left( u_n^{A|a_{n-1}} - d \cdot L^A \right) + \beta \cdot u_{n+1}^{A|a_{n-1}} > \left( u_t^{D|a_{n-1}} - d \cdot L^D \right) + \beta \cdot u_{n+1}^{D|a_{n-1}} + \delta,$$

and this makes it easier to satisfy the *PK* constraint:

$$\sum_{a_n=A,D} \pi(a_n|a_{n-1}) \cdot \left[ \left( u_n^{a_n|a_{n-1}} - d \cdot L^{a_n} \right) + \beta \cdot u_{n+1}^{a_n|a_{n-1}} \right] + \delta > U_n.$$

Therefore, the current utilities can be lowered by an amount  $\varepsilon$ :

$$\sum_{a_n=A,D} \pi(a_n|a_{n-1}) \cdot \left[ \left( u_n^{a_n|a_{n-1}} - d \cdot L^{a_n} - \varepsilon \right) + \beta \cdot u_{n+1}^{a_n|a_{n-1}} \right] + \delta \geq U_n,$$

but then, the original solution was not resource use minimising. The proof of the binding *IC* in the minimisation problems (14) and (15) is straightforward and thus omitted. ■

## B. The Simulation on the Dynamic Incentives Model

The model is analysed by means of numerical simulations.<sup>15</sup> In line with the previous literature (e.g., Shimer and Werning, 2007), consumption preferences are assumed to exhibit constant absolute risk

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<sup>15</sup> The simulations are performed using the *GAUSS* software.

aversion (*CARA*)  $u(c) = -\exp(-\alpha \cdot c)$ , where  $\alpha$  is some positive scalar.<sup>16</sup> The *CARA* preferences allow us to abstract from wealth effects, and then the individual's decision to work or not to work is independent of his wealth level and solely dependent on the system of dynamic incentives.

Because the purpose of this simulation is to investigate and clarify the model and since the outcomes do not depend on the value of the parameters, the parameterisation is primarily selected for numerical convenience. Thus, an income value  $y = 1$  and the parameter values  $\alpha = 2$ ,  $d = 0.05$ , and  $r = 0.03$  are considered. Moreover, the transition probabilities of the Markov chain are as follows:

$$\pi = \begin{bmatrix} \pi(A|A) & \pi(T|A) & 0 \\ \pi(A|T) & 0 & \pi(P|T) \\ 0 & 0 & \pi(P|P) \end{bmatrix} = \begin{bmatrix} 0.80 & 0.20 & 0 \\ 0.50 & 0 & 0.50 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}.$$

The first step is to compute the cost function  $C_n^{a_{n-1}}(U_n)$  related to the last working period minimisation problem. In the last working period  $n$ , the government's problem (11) is as follows:

$$(B.1) \quad \begin{aligned} C_n^{a_{n-1}}(U_n) &\equiv \min_{u_n^{a_n|a_{n-1}}, u_{n+1}^{a_n|a_{n-1}}} \sum_{a_n=A,D} \pi(a_n|a_{n-1}) \cdot \left\{ \left[ -\frac{\ln(-u_n^{a_n|a_{n-1}})}{\alpha} - y \cdot L^{a_n} \right] - \right. \\ &\quad \left. \beta \cdot \frac{\ln(-u_{n+1}^{a_n|a_{n-1}})}{\alpha} \right\} \\ \text{s. to } &\sum_{a_n=A,D} \pi(a_n|a_{n-1}) \cdot \left[ (u_n^{a_n|a_{n-1}} - d \cdot L^{a_n}) + \beta \cdot u_{n+1}^{a_n|a_{n-1}} \right] \geq U_n \quad (\lambda_n^{a_{n-1}}) \\ &(u_n^{A|a_{n-1}} - d) + \beta \cdot u_{n+1}^{A|a_{n-1}} \geq u_n^{D|a_{n-1}} + \beta \cdot u_{n+1}^{D|a_{n-1}} \quad (\mu_n^{a_{n-1}}), \end{aligned}$$

where  $a_{n-1} \in \{A, T\}$ ,  $a_n \in \{A, T\}$  if  $a_{n-1} = A$ , and  $a_n \in \{A, P\}$  if  $a_{n-1} = T$ , the solution to which entails (13). Then, it is possible to compute the cost functions  $C_t^A(U_t)$  and  $C_t^T(U_t)$  recursively from  $t = n - 1$  to  $t = 2$ .<sup>17</sup> In the working period  $t$ , the government's problems (14) and (15) are as follows:

<sup>16</sup> The properties of *CARA* utility functions are  $u'(c) = \alpha \cdot \exp(-\alpha \cdot c) > 0$  and  $u''(c) = -\alpha^2 \cdot \exp(-\alpha \cdot c) < 0$ . Thus, the degree of absolute risk aversion is constant  $A(c) = -u''(c)/u'(c) = \alpha$ , and as  $A' = 0$ , the absolute risk aversion is independent of wealth.

<sup>17</sup> In the working period  $t = 1$ , the consumption level required to give agent  $a_1 = A, T$  utility  $u_1^{a_1|A}$  in the current working period is  $c(u_1^{a_1|A}) = u^{-1}(u_1^{a_1|A})$  and the cost

$$(B.2) \quad \begin{aligned} C_t^A(U_t) &\equiv \min_{u_t^{a_t|A}, U_{t+1}^{a_t|A}} \sum_{a_t=A,T} \pi(a_t|A) \cdot \left\{ \left[ -\frac{\ln(-u_t^{a_t|A})}{\alpha} - y \cdot L^{a_t} \right] + \beta \cdot C_{t+1}^{a_t}(U_{t+1}^{a_t|A}) \right\} \\ \text{s. to } \sum_{a_t=A,T} \pi(a_t|A) \cdot \left[ (u_t^{a_t|A} - d \cdot L^{a_t}) + \beta \cdot U_{t+1}^{a_t|A} \right] &\geq U_t \quad (\lambda_t^A) \\ (u_t^{A|A} - d) + \beta \cdot U_{t+1}^{A|A} &\geq u_t^{T|A} + \beta \cdot U_{t+1}^{T|A} \quad (\mu_t^A), \end{aligned}$$

and

$$(B.3) \quad \begin{aligned} C_t^T(U_t) &\equiv \min_{u_t^{A|T}, U_{t+1}^{A|T}, u_t^{P|T}, U_{t+1}^{P|T}} \sum_{a_t=A,P} \pi(a_t|T) \cdot \left[ -\frac{\ln(-u_t^{a_t|T})}{\alpha} - y \cdot L^{a_t} \right] + \\ &\quad \pi(A|T) \cdot \beta \cdot C_{t+1}^A(U_{t+1}^{A|T}) + \pi(P|T) \cdot \beta \cdot \frac{1 - \beta^{(n+1)-t}}{1 - \beta} \cdot \left[ -\frac{\ln(-u_{t+1}^{P|T})}{\alpha} \right] \\ \text{s. to } \sum_{a_t=A,P} \pi(a_t|T) \cdot (u_t^{a_t|T} - d \cdot L^{a_t}) + \pi(A|T) \cdot \beta \cdot U_{t+1}^{A|T} + \\ &\quad \pi(P|T) \cdot \beta \cdot \frac{1 - \beta^{(n+1)-t}}{1 - \beta} \cdot u_{t+1}^{P|T} \geq U_t \quad (\lambda_t^T) \\ (u_t^{A|T} - d) + \beta \cdot U_{t+1}^{A|T} &\geq u_t^{P|T} + \beta \cdot \frac{1 - \beta^{(n+1)-t}}{1 - \beta} \cdot u_{t+1}^{P|T} \quad (\mu_t^T), \end{aligned}$$

the FOCs of which (16) entail the following:

$$(B.4) \quad \begin{aligned} -\frac{1}{\alpha} \cdot \frac{1}{u_t^{A|a_{t-1}}} &= C_{t+1}^{A'}(U_{t+1}^{A|a_{t-1}}) \text{ with } a_{t-1} \in \{A, T\}, \\ -\frac{1}{\alpha} \cdot \frac{1}{u_t^{T|A}} &= C_{t+1}^{T'}(U_{t+1}^{T|A}), \\ -\frac{1}{\alpha} \cdot \frac{1}{u_t^{P|T}} &= -\frac{1}{\alpha} \cdot \frac{1}{u_{t+1}^{P|T}} \Rightarrow u_t^{P|T} = u_{t+1}^{P|T} = u^{P|T}. \end{aligned}$$

Therefore, the envelope condition is as follows:

$$(B.5) \quad \begin{aligned} C_t^A(U_t) &= \frac{\partial \mathcal{L}}{\partial U_t} \Big|_{opt} = \lambda_t^A, \\ C_t^T(U_t) &= \frac{\partial \mathcal{L}}{\partial U_t} \Big|_{opt} = \lambda_t^T. \end{aligned}$$

The cost functions are represented in Figure 1. Given the cost functions, the multiperiod minimisation problem with  $n$  working periods can be solved: from  $t = 1, \dots, n-1$ , the government's problems are (B.2) and (B.3), and in the last working period  $n$ , it is (B.1).

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level required to give agent  $a_1 = A, T$  utility  $U_2^{a_1|A}$  in the future working periods and in the retirement period is  $C_2^{a_1}(U_2^{a_1|A}) = U_2^{-1}(U_2^{a_1|A})$ .

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