Adverse Selection and Moral Hazard in the Annuity Markets

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Abstract

Several authors have analysed the case in which individuals possess hidden information about their longevity. Davies and Kuhn [2] have considered the related case in which individuals can take hidden actions to affect their longevity. In this work I will consider the case in which the annuity market is characterized by both adverse selection and moral hazard; with private information all individuals, and in particular low-risk individuals, suffer from negative estenalities.

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1 Introduction

Several authors ([9], [11], [7], [8] and [3]) have analysed the case in which individuals possess hidden information about their longevity, that is when there is adverse selection in the annuity market and a group of individuals (low risk individuals) is affected by negative externalities. Davies and Kuhn [2] have considered the related case in which individuals can take hidden actions to affect their longevity by consuming "health-related goods", that is when there is moral hazard in the annuity market and all individuals are affected by negative esternalities.

In this work I will consider the case in which the annuity market is characterized by both adverse selection (individuals possess hidden information about their longevity) and moral hazard (individuals can take hidden actions to affect their longevity). I will analyse both the situation in which the annuity market is characterized by public information and the situation in which the annuity market is characterized by private information. In this second case annuity supplying firms offer the utility-maximizing actuarially fair annuity contracts (either separating contracts or pooling contract) subject to the constraint that individuals cannot take hidden actions and subject to the constraint that individuals cannot exploit hidden information. Then all individuals, and in particular low-risk individuals, suffer from negative estenalities.

2 Public Information and Private Information in the Annuity Market: the Basic Structure

The economy to be studied is a variant of Samuelson's pure-exchange overlapping generations model [10]. At each period t ($t \geq 1$) the population consists of old members of generation $t-1$ who all die at the end of that period and young members of generation t.

Generations are of equal size (the population growth rate is $n = 0$) and there is no altruism; thus when there is no uncertainty each individual leaves a bequest of zero.

In this economy there is a single non-storable and non-producible consumption good c. Each young agent is endowed at birth with w units of the consumption good.

All individuals live for a maximum of two periods ($t = 1, 2$). All members of a given generation are alive for certain in the first period, and survive with some probability p into the second period.

If we consider the health care as "capabilities and mechanism of defense that protect an organism from external stress" [1], an individual can increase his survival probability into the second period by investing in health care in the first period: an individual survives with probability $p(h)$ into the second period, where h represents the investment level in health care in the first period [2].

Since investment in health care in the first period decreases the probability of death at the beginning of the second period, we have $\dfrac{\delta p(h)}{\delta h} > 0$. Moreover I suppose $\dfrac{\delta^2 p(h)}{\delta h \delta h} = 0$. With $h^{\max}$ the maximum feasible investment in h, an individual survives with probability $p(h)$ into the second period with $0 \leq p(h) \leq 1$, $p(h) \longrightarrow 0$ as $h \longrightarrow 0$ and $p(h) \longrightarrow 1$ as $h \longrightarrow h^{\max}$.

The cost of investing in a level of health care h is given by the function $c(h)$, with $c(0) = 0$ and $\dfrac{\delta c(h)}{\delta h} > 0$. Moreover I suppose $\dfrac{\delta^2 c(h)}{\delta h \delta h} = 0$.

Then the representative individual's expected lifetime utility is given by

$$U = u(c_1) + p(h) \cdot u(c_2)$$

with $u_c > 0$, $u_{cc} < 0$, $u_c \longrightarrow \infty$ as $c \longrightarrow 0$ and $u_c \longrightarrow 0$ as $c \longrightarrow \infty$ and the individuals' consumption sets are bounded by

$$0 \leq h \leq h^{\max}$$

2.1 Myopia

I suppose that in this economy individuals are myopic. According to traditional definition of myopia [6] consumers can fail to appreciate their later needs, either discounting the future completely or placing a lower weight upon it than would capture their true preferences, or they may make mistakes in their planning, have lack information or simply be irrational. Although the behavioural foundations of myopia differ significantly from those of consistent utility maximization, the formulation of myopic behaviour of Feldstein [4] can be incorporated into the traditional analysis. If we define μ the index which captures the degree of myopia (with $0 \leq \mu \leq 1$), the representative individual's expected lifetime utility becomes

$$U = u(c_1) + \mu \cdot p(h) \cdot u(c_2)$$

2.2 Annuity Contract

Agents allocate consumption intertemporally by purchasing annuities which are supplied by competitive firms that specialize in holding the safe asset. An annuity bond at period t is a claim to a certain quantity of the consumption good at period $t+1$ which is payable only if the original purchaser of the annuity is alive. Normalizing the purchasing price of a period t annuity to one unit of the good at t, the annuity's rate of return represents the intertemporal terms of trade faced by its buyer.

Then we define an annuity contract as a two dimensional vector (s, R) (with s quantity and R rate of return) so that if a young agent purchase this contract his consumption vector (c_1, c_2) becomes $(w - c(h) - s, s \cdot R)$ if he lives two periods and $(w - c(h) - s, 0)$ if he lives only one period.

Firms decide the values of quantity s and rate of return R by maximizing the expected utility of young agents

$$\underset{c_1, c_2, h}{Max} u(c_1) + \mu \cdot p(h) \cdot u(c_2)$$

$$\begin{aligned} s.t. \quad & c_1 = w - c(h) - s \\ & c_2 = s \cdot R \\ & 0 \leq h \leq h^{\max} \\ & s \geq 0 \end{aligned}$$

This maximization problem may be rewritten using the indirect utility form in the simpler form

$$\underset{s,h}{Max}\nu\left(w-c\left(h\right)-s\right)+\mu\cdot p\left(h\right)\cdot\nu\left(s\cdot R\right)$$

$$\begin{aligned} s.t. \quad & 0\leq h\leq h^{\max} \\ & s\geq 0 \end{aligned}$$

The constraint $s\geq 0$ marks that the consumer cannot sell off the future income for most first period income.

2.3 Equilibrium Concepts

In the following analysis I will consider a population in which each generation is partitioned into three distinct groups according to the degree of myopia (and then according to the survival probability) of agents. We suppose that the relative size of these groups is fixed. Given this heterogeneity of the population, we can think of two kinds of annuity equilibria:

1. a separating equilibrium in which agents with different survival probabilities purchase annuities with different rate of return,
2. a pooling equilibrium in which the same annuity is purchased by members of different groups.

Since relative size of groups is fixed, there is absence of aggregate uncertainty regarding the number of deaths in each group and then absence of uncertainty regarding the profits of the annuity-supplying firms. Therefore in either a pooling or separating equilibrium real profits must be equal to zero.

Rothschild and Stiglitz [9] consider firms that accomplish a screening strategy and they define an equilibrium as a set of contracts such that when agents choose contract to maximize their expected utility

1. no contract in the equilibrium set makes negative expected profits,
2. there is no contract outside the equilibrium set that, if offered, will make a non-negative profit.

Given these basic features of the equilibrium, in a Rothschild-Stiglitz equilibrium each firm assumes that the contract its competitors offer are independent of its own actions (Nash-Cournot type equilibrium) [9], [3]. Then each contract offered in equilibrium earns zero profits: positive profits on any single contract are eliminating by undercutting among the firms and cross-subsidization among different contracts offered by any given firm can be rule out by noting that firms will withdraw contracts that persistently yield negative profits.

However a Rothschild-Stiglitz equilibrium has problems of non-existence:

1. there cannot be a Rothschild-Stiglitz pooling equilibrium,

2. for a relatively small number of high-risk agents (agents with higher survival probability), there does not exist a Rothschild-Stiglitz equilibrium.

Wilson solves the problems of non-existence of a Rothschild-Stiglitz equilibrium in a contest in which firms carry on accomplishing a screening strategy: each firm will correctly anticipate which of those policies that are offered by other firms will become unprofitable as a consequence of any change in its own policies [11], [3]. Then a firm offers a new policy only if it makes non-negative profits after all the other firms have made the expected adjustment in their policy offers[1]. Then in the following analysis I will consider a Wilson equilibrium.

2.4 Parameterized Example

I will clarify the results by presenting a computational model. I assume that preferences are given by a logarithmic function. Thus representative individual's expected lifetime utility is given by

$$U = \ln(c_1) + \mu \cdot p(h) \cdot \ln(c_2)$$

With q_h the price of a unit of investment in health care h, I will consider the cost function $c(h) = q_h \cdot h$. Moreover $h^{\max} = \dfrac{w}{q_h}$ and then $p(h) = \dfrac{h}{h^{\max}} = \dfrac{h \cdot q_h}{w}$. In the numerical examples the endowed income is $w = 1000$ and the price of a unit of investment in health care is $q_h = 1.25$. The simulations are made with GAUSS.

3 The Model

Each generation t is partitioned into three distinct groups, L, M and H, whose relative size is fixed for all t, so that for each agent of type L there

[1] Riley solves the problems of non-existence of a Rothschild-Stiglitz equilibrium in a context in which firms adopt a *signalling* strategy [8, Riley (1979b)]. In this model the signal should be the investment in health care in the first period, but low-risk individuals invest less in health care than high-risk individuals. Since high-risk individuals have incentive to declare the lower investment in health care in the first period, firms cannot adopt a signalling strategy.

are γ_M agents of type M and γ_H agents of type H, with $\gamma_M, \gamma_H > 0$. Individuals of three groups have a different degree of myopia μ^i (with $0 \leq \mu^i \leq 1$): members of group L are more myopic than members of group M and members of group M are more myopic than members of group H ($\mu^L < \mu^M < \mu^H$).

The absence of aggregate uncertainty regarding the number of deaths in each group (and then the absence of uncertainty regarding the profits of the annuity-supplying firms) implies that a stationary allocation (c_1^i, c_2^i, h^i) $(i = L, M, H)$ is feasible if it satisfies

$$\begin{aligned}&\left[\left(c_1^L + c\left(h^L\right)\right) + \gamma_M \cdot \left(c_1^M + c\left(h^M\right)\right) + \gamma_H \cdot \left(c_1^H + c\left(h^H\right)\right)\right] \\ &+ \left[p\left(h^L\right) \cdot c_2^L + \gamma_M \cdot p\left(h^M\right) \cdot c_2^M + \gamma_H \cdot p\left(h^H\right) \cdot c_2^H\right] = w \cdot (1 + \gamma_M + \gamma_H)\end{aligned}$$

A feasible stationary allocation $\left(\bar{c}_1^i, \bar{c}_2^i, \bar{h}^i\right)$ $(i = L, M, H)$ is optimal if there does not exist another feasible stationary allocation (c_1^i, c_2^i, h^i) $(i = L, M, H)$ such that

$$U^i\left(c_1^i, c_2^i, h^i\right) \geq U^i\left(\bar{c}_1^i, \bar{c}_2^i, \bar{h}^i\right)$$

with strict inequality for some i. Moreover an interior allocation (c_1^i, c_2^i, h^i) is optimal if for some $\beta^i > 0$ $(i = L, M, H)$ it solves the problem

$$Max\ \beta^L \cdot U^L\left(c_1^L, c_2^L, h^L\right) + \beta^M \cdot U^M\left(c_1^M, c_2^M, h^M\right) + \beta^H \cdot U^H\left(c_1^H, c_2^H, h^H\right)$$

$$\begin{aligned}s.t.\quad &\left[\left(c_1^L + c\left(h^L\right)\right) + \gamma_M \cdot \left(c_1^M + c\left(h^M\right)\right) + \gamma_H \cdot \left(c_1^H + c\left(h^H\right)\right)\right] + \\ &+ \left[p\left(h^L\right) \cdot c_2^L + \gamma_M \cdot p\left(h^M\right) \cdot c_2^M + \gamma_H \cdot p\left(h^H\right) \cdot c_2^H\right] = \\ &w \cdot (1 + \gamma_M + \gamma_H)\end{aligned}$$

$$\begin{gathered}c_1^L, c_1^M, c_1^H \geq 0 \\ c_2^L, c_2^M, c_2^H \geq 0 \\ 0 \leq h^L, h^M, h^H \leq h^{\max}\end{gathered}$$

Ignoring the inequality constraints, if we maximize with respect to the consumption goods in the two periods we obtain

$$\frac{u_{c_1}^i}{u_{c_2}^i} = \mu^i \tag{1}$$

Then we can write

$$\frac{u_{c_1}^L}{\mu^L \cdot u_{c_2}^L} = \frac{u_{c_1}^M}{\mu^M \cdot u_{c_2}^M} = \frac{u_{c_1}^H}{\mu^H \cdot u_{c_2}^H} = \mathbf{1} \tag{2}$$

defined by Eckestein, Eichenbaum and Peled [3] as the necessary and sufficient condition for an interior allocation to be optimal.

Ignoring the inequality constraints and maximizing also with respect to the investment in health care we obtain

$$u^i\left(c_2^i\right) - c_2^i \cdot u_{c_2}^i = \frac{\dfrac{\delta c\left(h^i\right)}{\delta h}}{\dfrac{\delta p\left(h^i\right)}{\delta h}} \cdot \frac{u_{c_1}^i}{\mu^i} = \frac{\dfrac{\delta c\left(h^i\right)}{\delta h}}{\dfrac{\delta p\left(h^i\right)}{\delta h}} \cdot u_{c_2}^i \tag{3}$$

From the last equation we can argue that

- if $h^i > 0$ and then if $p\left(h^i\right) > 0$ (with $i = L, M, H$), members of three groups choose the same level of consumption in the second period: $c_2^L = c_2^M = c_2^H > 0$;
- if $h^i = 0$ and then if $p\left(h^i\right) = 0$ (with $i = L, M, H$), the utility function becomes $U^i = u\left(c_1^i\right)$.

In the following analysis I will consider only the case $h^i > 0$ (with $i = L, M, H$).

The results of simulation are summarized in table 1.

In the first period members of group L consume more and invest less in health care than members of group M and members of group M consume more and invest less in health care than members of group H (with a consumption in the second period equal for members of three groups).

Given heterogeneity with respect to degree of myopia ($\mu^L < \mu^M < \mu^H$), optimal allocations have the property that ex ante marginal rates of substitution are not equalized across members of different groups

$$\frac{1}{p\left(h^L\right)} \cdot \frac{u_{c_1}^L}{\mu^L \cdot u_{c_2}^L} \neq \frac{1}{p\left(h^M\right)} \cdot \frac{u_{c_1}^M}{\mu^M \cdot u_{c_2}^M} \neq \frac{1}{p\left(h^H\right)} \cdot \frac{u_{c_1}^H}{\mu^H \cdot u_{c_2}^H} \tag{4}$$

In a competitive equilibrium agents equate their expected intertemporal marginal rate of substitution to the rate of return on saving that they face:

$$\frac{1}{p\left(h^i\right)} \cdot \frac{u_{c_1}^i}{\mu^i \cdot u_{c_2}^i} = R^i \quad \text{for } i = L, M, H$$

	$\mu^L = 0.4$	$\mu^M = 0.7$	$\mu^H = 1.0$
c_1^i	565.44	323.11	226.18
c_2^i	**226.18**	**226.18**	**226.18**
h^i	283.52	441.63	504.87
p^i	35.44%	55.20%	63.11%
U^i	7.1061	7.8729	8.8426
	$\mu^L = 0.5$	$\mu^M = 0.7$	$\mu^H = 0.9$
c_1^i	452.35	323.11	251.31
c_2^i	**226.18**	**226.18**	**226.18**
h^i	357.30	441.63	488.47
p^i	44.66%	55.20%	61.06%
U^i	7.3251	7.8729	8.5059
	$\mu^L = 0.6$	$\mu^M = 0.7$	$\mu^H = 0.8$
c_1^i	376.96	323.11	282.72
c_2^i	**226.18**	**226.18**	**226.18**
h^i	406.49	441.63	467.98
p^i	50.81%	55.20%	58.50%
U^i	7.5849	7.8729	8.1815

Table 1: Optimal Allocations

Consequently, the competitive equilibrium will be full information Pareto optimal if and only if all agents face actuarially fair rates of return $\frac{1}{p\left(h^i\right)}$ for agents of type i (with $i = L, M, H$). When

$$R^i = \frac{1}{p\left(h^i\right)} \quad \text{for } i = L, M, H$$

we obtain

$$\frac{u_{c_1}^i}{\mu^i \cdot u_{c_2}^i} = 1 \quad \text{for } i = L, M, H$$

and the necessary and sufficient condition for an interior allocation to be optimal is satisfied.

4 Public Information

In this section the degree of myopia (and then the investment in health care in the first period h^i and the survival probability $p\left(h^i\right)$) of any given agent is assumed to be public information.

Given the postulated heterogeneity of the population with respect to the degree of myopia ($\mu^L < \mu^M < \mu^H$), we can think of two kinds of annuity equilibria: a separating equilibrium in which agents with different survival probabilities purchase annuities with different rate of return and a pooling equilibrium in which the same annuity is purchased by members of three groups.

The absence of aggregate uncertainty in this economy (if we assume that each generation is large, the survival probabilities are also the proportions of each type that live for a second period) implies a similar absence of uncertainty regarding the profits of the annuity supplying firms. Therefore, in either a separating or a pooling equilibrium real profits must be equal to zero.

4.1 Separating Equilibrium

In the case of separating equilibrium, firms choose quantity and rate of return of separating contracts by maximizing the expected utility of the representative young agent of group i, with $i = L, M, H$.

$$\underset{c_1^i, c_2^i, h^i}{Max}\, u^i\left(c_1^i\right) + \mu^i \cdot p\left(h^i\right) \cdot u^i\left(c_2^i\right)$$

$$\begin{aligned} s.t. \quad & c_1^i = w - c\left(h\right) - s^i \\ & c_2^i = s^i \cdot R\left(p\left(h^i\right)\right) \\ & 0 \leq h^i \leq h^{\max} \\ & s^i \geq 0 \end{aligned}$$

This maximization problem may be rewritten using the indirect utility form in the simpler form

$$\underset{s^i, h^i}{Max} \nu^i\left(w - c\left(h\right) - s^i\right) + \mu^i \cdot p\left(h^i\right) \cdot \nu^i\left(s^i \cdot R\left(p\left(h^i\right)\right)\right) \tag{5}$$

$$\begin{aligned} s.t. \quad & 0 \leq h^i \leq h^{\max} \\ & s^i \geq 0 \end{aligned}$$

Ignoring the inequality constraints, if firms offer the actuarially fair rates of return $R\left(p\left(h^i\right)\right) = \dfrac{1}{p\left(h^i\right)}$ we obtain

$$\frac{\nu_{c_1}^i}{\nu_{c_2}^i} = \mu^i \tag{6}$$

$$\nu^i\left(s^i \cdot \frac{1}{p\left(h^i\right)}\right) - \frac{s^i}{p\left(h^i\right)} \cdot \nu_{c_2}^i = \frac{\dfrac{\delta c\left(h^i\right)}{\delta h}}{\dfrac{\delta p\left(h^i\right)}{\delta h}} \cdot \frac{\nu_{c_1}^i}{\mu^i} = \frac{\dfrac{\delta c\left(h^i\right)}{\delta h}}{\dfrac{\delta p\left(h^i\right)}{\delta h}} \cdot \nu_{c_2}^i \tag{7}$$

	$\mu^L = 0.4$	$\mu^M = 0.7$	$\mu^H = 1.0$
s^i	80.16	124.86	142.74
h^i	283.52	441.63	504.87
p^i	35.44%	55.20%	63.11%
R^i	2.82	1.81	1.58
$s^i \cdot R^i$	**226.18**	**226.18**	**226.18**
V^i	7.1061	7.8729	8.8426
	$\mu^L = 0.5$	$\mu^M = 0.7$	$\mu^H = 0.9$
s^i	101.02	124.86	138.10
h^i	357.30	441.63	488.47
p^i	44.66%	55.20%	61.06%
R^i	2.24	1.81	1.64
$s^i \cdot R^i$	**226.18**	**226.18**	**226.18**
V^i	7.3251	7.8729	8.5059
	$\mu^L = 0.6$	$\mu^M = 0.7$	$\mu^H = 0.8$
s^i	114.92	124.86	132.31
h^i	406.49	441.63	467.97
p^i	50.81%	55.20%	58.50%
R^i	1.97	1.81	1.71
$s^i \cdot R^i$	**226.18**	**226.18**	**226.18**
V^i	7.5849	7.8729	8.1815

Table 2: Public Information

and the necessary and sufficient condition for an interior allocation to be optimal is satisfied. In the previous section we argued that if $h > 0$ (and then if $p(h) > 0$) members of three groups choose the same level of $c_2 > 0$; in this section we can conclude that $s^L \cdot R\left(p\left(h^L\right)\right) = s^M \cdot R\left(p\left(h^M\right)\right) = s^H \cdot R\left(p\left(h^H\right)\right)$. The results of simulation are summarized in table 2.

With $\mu^L < \mu^M < \mu^H$ we have $h^H > h^M > h^L$ and $p\left(h^H\right) > p\left(h^M\right) > p\left(h^L\right)$: individuals with a larger degree of myopia have a higher survival probability. Then contracts offered by firms are characterized by $s^H > s^M > s^L$ and $R\left(p\left(h^H\right)\right) < R\left(p\left(h^M\right)\right) < R\left(p\left(h^L\right)\right)$: for annuity-supplying firms members of group H are high-risk agents, members of group M are medium-risk agents and members of group L are low-risk agents.

The results of public information problem and those ones of optimal allocation are equal. Then a competitive equilibrium is Pareto optimal with full information if and only if all agents face actuarially fair rates of return $\dfrac{1}{p\left(h^{i}\right)}$ for agents of type $i = L, M, H$.

4.2 Pooling Equilibrium

In the case of pooling equilibrium $R^{L}(t) = R^{M}(t) = R^{H}(t) = \bar{R}(t)$ and the zero profits condition is given by

$$\begin{aligned} & s_{t+1}^{L} + \gamma_{M} \cdot s_{t+1}^{M} + \gamma_{H} \cdot s_{t+1}^{H} + \\ & -\bar{R}_{t} \cdot \left[p_{t}\left(h^{L}\right) \cdot s_{t}^{L} + \gamma_{M} \cdot p_{t}\left(h^{M}\right) \cdot s_{t}^{H} + \gamma_{H} \cdot p_{t}\left(h^{H}\right) \cdot s_{t}^{H}\right] = 0 \end{aligned}$$

In a stationary equilibrium $p_{t}\left(h^{L}\right) = p\left(h^{L}\right)$, $p_{t}\left(h^{M}\right) = p\left(h^{M}\right)$, $p_{t}\left(h^{H}\right) = p\left(h^{H}\right)$, $\bar{R}_{t} = \bar{R}$, $s_{t+1}^{L} = s^{L}$, $s_{t+1}^{M} = s^{M}$ and $s_{t+1}^{H} = s^{H}$ for all t; then we obtain

$$\bar{R} = \frac{s^{L} + \gamma_{M} \cdot s^{M} + \gamma_{H} \cdot s^{H}}{p\left(h^{L}\right) \cdot s^{L} + \gamma_{M} \cdot p\left(h^{M}\right) \cdot s^{H} + \gamma_{H} \cdot p\left(h^{H}\right) \cdot s^{H}} \tag{8}$$

Since $\bar{R} \leq R\left(p\left(h^{L}\right)\right)$, the pooling contract can never be an equilibrium contract because at this rate firms can obtain positive profits by restricting the sales of such annuities to only one of the groups (group L).

5 Private Information

In this section I will consider the case of private information regarding the degree of myopia (and then the investment in health care in the first period h and the survival probability $p(h)$).

5.1 Moral Hazard

In the case of private information regarding the degree of myopia (and then the investment in health care in the first period h and the survival probability $p(h)$) individuals can take hidden actions to affect their longevity (in annuity markets there is moral hazard): individuals choose the optimal level of h in response to contract $\left(s^{i}, R\left(p\left(h^{i}\right)\right)\right)$ offered by annuity-supplying firms [2]: for individuals the rate of return $R\left(p\left(h^{i}\right)\right)$ is given. The problem of a young agent of group i is

$$\underset{\breve{h}^{i}}{Max} \nu^{i}\left(w - c\left(\breve{h}^{i}\right) - s^{i}\right) + \mu^{i} \cdot p\left(\breve{h}^{i}\right) \cdot \nu^{i}\left(s^{i} \cdot R\left(p\left(h^{i}\right)\right)\right) \tag{9}$$

$$s.t. \;\; 0 \leq \breve{h}^{i} \leq h^{\max}$$

Ignoring the inequality constraints concerning the investment in health care, if firms don't consider the moral hazard problem and if they offer

	$\mu^L = 0.4$	$\breve{\mu}^L = 0.4$	$\mu^M = 0.7$	$\breve{\mu}^M = 0.7$	$\mu^H = 1.0$	$\breve{\mu}^H = 1.0$
s^i	**80.16**	**80.16**	**124.86**	**124.86**	**142.73**	**142.73**
h^i	283.52	366.96	441.63	489.31	504.87	538.24
p^i	35.44%	45.87%	55.20%	61.16%	63.11%	67.28%
R^i	**2.82**	**2.82**	**1.81**	**1.81**	**1.58**	**1.58**
V^i	7.1061	7.1284	7.8719	7.8952	8.8426	8.8649
	$\mu^L = 0.5$	$\breve{\mu}^L = 0.5$	$\mu^M = 0.7$	$\breve{\mu}^M = 0.7$	$\mu^H = 0.9$	$\breve{\mu}^H = 0.9$
s^i	**101.02**	**101.02**	**124.86**	**124.86**	**138.10**	**138.10**
h^i	357.30	366.96	441.63	489.31	488.47	525.56
p^i	44.66%	45.87%	55.20%	61.16%	61.06%	65.69%
R^i	**2.24**	**2.24**	**1.81**	**1.81**	**1.64**	**1.64**
V^i	7.3251	7.3474	7.8729	7.8952	8.5059	8.5281
	$\mu^L = 0.6$	$\breve{\mu}^L = 0.6$	$\mu^M = 0.7$	$\breve{\mu}^M = 0.7$	$\mu^H = 0.8$	$\breve{\mu}^H = 0.8$
s^i	**114.92**	**114.92**	**124.86**	**124.86**	**132.31**	**132.31**
h^i	406.49	462.12	441.63	489.31	467.98	509.70
p^i	50.81%	57.76%	55.20%	61.16%	58.50%	63.71%
R^i	**1.97**	**1.97**	**1.81**	**1.81**	**1.71**	**1.71**
V^i	7.5849	7.6072	7.8729	7.8952	8.1815	8.2038

Table 3: Moral Hazard

the actuarially fair rates of return computed in the public information case $R\left(p\left(h^i\right)\right) = \frac{1}{p\left(h^i\right)}$ we obtain

$$\nu^i\left(s^i \cdot \frac{1}{p\left(h^i\right)}\right) = \frac{\frac{\delta c\left(\breve{h}^i\right)}{\delta h}}{\frac{\delta p\left(\breve{h}^i\right)}{\delta h}} \cdot \frac{\nu^i_{c_1}}{\mu^i} \tag{10}$$

where s^i and h^i are computed from 6 and 7. The necessary and sufficient condition for an interior allocation to be optimal is not satisfied: when individuals can take hidden actions to affect their longevity the levels of investment in health care h^i are higher and the profits of firms become negative. The results of simulation are summarized in table 3.

The results show that, since $V^i\left(\breve{h}^i\right) > V^i\left(s^i, h^i\right)$, individuals choose $\breve{h}^i > h^i$ and then $p\left(\breve{h}^i_1\right) > p\left(h^i_1\right)$.

5.2 Adverse Selection

In the case of private information regarding the degree of myopia (and then the investment in health care in the first period h and the survival probability $p(h)$) no agent (individuals, firms and government) knows whether any particular individual belongs to group L, M or H.

I showed that the contract $\left(s^L, R\left(p\left(h^L\right)\right)\right)$ (low-risk contract), the contract preferred by members of group L, the contract $\left(s^M, R\left(p\left(h^M\right)\right)\right)$ (medium-risk contract), the contract preferred by members of group M, and the contract $\left(s^H, R\left(p\left(h^H\right)\right)\right)$ (high-risk contract), the contract preferred by members of group H, are characterized by $s^H > s^M > s^L$, $R\left(p\left(h^H\right)\right) < R\left(p\left(h^M\right)\right) < R\left(p\left(h^L\right)\right)$ and $s^H \cdot R\left(p\left(h^H\right)\right) = s^M \cdot R\left(p\left(h^M\right)\right) = s^L \cdot R\left(p\left(h^L\right)\right)$.

With private information both members of group M and members of group H would prefer contract $\left(s^L, R\left(p\left(h^L\right)\right)\right)$ to contracts $\left(s^M, R\left(p\left(h^M\right)\right)\right)$ and $\left(s^H, R\left(p\left(h^H\right)\right)\right)$: from the point of view of members of groups M and H contract $\left(s^L, R\left(p\left(h^L\right)\right)\right)$ dominates contracts $\left(s^M, R\left(p\left(h^M\right)\right)\right)$ and $\left(s^H, R\left(p\left(h^H\right)\right)\right)$.

Then the medium-risk individuals (members of group M) and the high-risk individuals (members of group H) have hidden information about their longevity (in annuity markets there is adverse selection). The maximization problems of medium-risk agents M and high-risk agents H become

$$\underset{\hat{h}^M}{Max} \hat{\nu}^M \left(w - c\left(\hat{h}^M\right) - s^L\right) + \mu^M \cdot p\left(\hat{h}^M\right) \cdot \hat{\nu}^M \left(s^L \cdot R\left(p\left(h^L\right)\right)\right) \quad (11)$$

$$s.t. \;\; 0 \leq \hat{h}^M \leq h^{\max}$$

and

$$\underset{\hat{h}^H}{Max} \hat{\nu}^H \left(w - c\left(\hat{h}^H\right) - s^L\right) + \mu^H \cdot p\left(\hat{h}^H\right) \cdot \hat{\nu}^H \left(s^L \cdot R\left(p\left(h^L\right)\right)\right) \quad (12)$$

$$s.t. \;\; 0 \leq \hat{h}^H \leq h^{\max}$$

Ignoring the inequality constraints concerning the investment in health care, if firms offer the actuarially fair rate of return $R\left(p\left(h^L\right)\right) = \dfrac{1}{p\left(h^L\right)}$ we obtain

$$\hat{\nu}^M \left(\mathbf{s^L} \cdot \frac{1}{p\left(\mathbf{h^L}\right)}\right) = \frac{\dfrac{\delta c\left(\hat{h}^M\right)}{\delta h}}{\dfrac{\delta p\left(\hat{h}^M\right)}{\delta h^M}} \cdot \frac{\hat{\nu}^M_{c_1}}{\mu^M} \quad (13)$$

and

$$\hat{\nu}^H\left(\mathbf{s^L}\cdot\frac{1}{p\left(\mathbf{h^L}\right)}\right)=\frac{\dfrac{\delta c\left(\hat{h}^H\right)}{\delta h}}{\dfrac{\delta p\left(\hat{h}^H\right)}{\delta h^H}}\cdot\frac{\hat{\nu}^H_{c_1}}{\mu^H} \tag{14}$$

where s^L and h^L are computed from 6 and 7. The necessary and sufficient conditions for an interior allocation to be optimal are not satisfied: when medium-risk individuals (members of group M) and high-risk individuals (members of group H) have hidden information about their longevity the levels of investment in health care h^M and h^H are higher.

Since individuals have hidden information about their longevity and since both members of group M and members of group H would prefer contract $\left(s^L, R\left(p\left(h^L\right)\right)\right)$ to contracts $\left(s^M, R\left(p\left(h^M\right)\right)\right)$ and $\left(s^H, R\left(p\left(h^H\right)\right)\right)$ the profits of firms become negative.

With $s^L\cdot R\left(p\left(h^L\right)\right)=s^M\cdot R\left(p\left(h^M\right)\right)=s^H\cdot R\left(p\left(h^H\right)\right)$, if we compare the equation concerning the moral hazard problem (equation 10) with the equations concerning the adverse selection problem (equations 13 and 14) we can argue that $\hat{h}^M$ and $\hat{h}^H$ are such that $\nu^M_{c_1}=\hat{\nu}^M_{c_1}$ and $\nu^H_{c_1}=\hat{\nu}^H_{c_1}$ (and then such that $\hat{c}^M_1=\breve{c}^M_1$ and $\hat{c}^H_1=\breve{c}^H_1$).

The results of simulation are summarized in tables 4, 5 and 6.

Since $\hat{V}^M\left(\hat{h}^M\right)>V^M\left(\breve{h}^M\right)>V^M\left(s^M,h^M\right)$ and $\hat{V}^H\left(\hat{h}^H\right)>V^H\left(\breve{h}^H\right)>V^H\left(s^H,h^H\right)$, medium-risk individuals (members of group M) and high-risk individuals (members of group H) choose $\hat{h}^M>\breve{h}^M>h^M$ and $\hat{h}^H>\breve{h}^H>h^H$.

Hence, if contracts $\left(s^L, R\left(p\left(h^L\right)\right)\right)$, $\left(s^M, R\left(p\left(h^M\right)\right)\right)$ and $\left(s^H, R\left(p\left(h^H\right)\right)\right)$ are offered, all agents will purchase contract $\left(s^L, R\left(p\left(h^L\right)\right)\right)$ and profits of firms will become negative.

6 Utility Level Curves and Annuity Contracts Curves

In this section I will study the characteristics of the annuity contracts in the case of private information in the (s,R)-plane. In the (s,R)-plane we represent the utility level curve

$$V=\nu\left(w-c\left(h\right)-s\right)+\mu\cdot p\left(h\right)\cdot\nu\left(s\cdot R\right)$$

(a higher level curve represents a higher utility level) and the annuity contracts curve

$$-\frac{\delta c\left(h\right)}{\delta h}\cdot\frac{\delta\nu\left(w-c\left(h\right)-s\right)}{\delta c_1}+\mu\cdot\frac{\delta p\left(h\right)}{\delta h}\cdot\nu\left(s\cdot\frac{1}{p\left(h\right)}\right)=0$$

	$\mu^L = 0.4$	$\breve{\mu}^L = 0.4$	
s^i	**80.16**	**80.16**	
h^i	283.52	366.96	
p^i	35.44%	45.87%	
R^i	**2.82**	**2.82**	
V^i	7.1061	7.1284	
	$\mu^M = 0.7$	$\breve{\mu}^M = 0.7$	$\hat{\mu}^M = 0.7$
s^i	124.86	124.86	**80.16**
h^i	441.63	489.31	525.07
p^i	55.20%	61.16%	65.63%
R^i	1.81	1.81	**2.82**
V^i	7.8729	7.8952	8.0648
	$\mu^H = 1.0$	$\breve{\mu}^H = 1.0$	$\hat{\mu}^H = 1.0$
s^i	142.74	142.74	**80.16**
h^i	504.87	538.24	588.31
p^i	63.11%	67.28%	73.54%
R^i	1.54	1.54	**2.82**
V^i	8.8426	8.8649	9.2042

Table 4: Adverse Selection - Case 0.4, 0.7 and 1.0

	$\mu^L = 0.5$	$\breve{\mu}^L = 0.5$	
s^i	**101.02**	**101.02**	
h^i	357.30	424.06	
p^i	44.66%	53.01%	
R^i	**2.24**	**2.24**	
V^i	7.3251	7.3474	
	$\mu^M = 0.7$	$\breve{\mu}^M = 0.7$	$\hat{\mu}^M = 0.7$
s^i	124.86	124.86	**101.02**
h^i	441.63	489.31	508.38
p^i	55.20%	61.16%	63.55%
R^i	1.81	1.81	**2.24**
V^i	7.8729	7.8952	7.9857
	$\mu^H = 0.9$	$\breve{\mu}^H = 0.9$	$\hat{\mu}^H = 0.9$
s^i	138.10	138.10	**101.02**
h^i	488.47	525.56	555.22
p^i	61.06%	65.69%	69.40%
R^i	1.64	1.64	**2.24**
V^i	8.5059	8.5281	8.7091

Table 5: Adverse Selection - Case 0.5, 0.7 and 0.9

	$\mu^L = 0.6$	$\breve{\mu}^L = 0.6$	
s^i	**114.92**	**114.92**	
h^i	406.49	462.12	
p^i	50.81%	57.76%	
R^i	**1.97**	**1.97**	
V^i	7.5849	7.6072	
	$\mu^M = 0.7$	$\breve{\mu}^M = 0.7$	$\hat{\mu}^M = 0.7$
s^i	124.86	124.86	**114.92**
h^i	441.63	489.31	497.25
p^i	55.20%	61.16%	62.16%
R^i	1.81	1.81	**1.97**
V^i	7.8729	7.8952	7.9329
	$\mu^H = 0.8$	$\breve{\mu}^H = 0.8$	$\hat{\mu}^H = 0.8$
s^i	132.31	132.31	**114.92**
h^i	467.98	509.70	523.60
p^i	58.50%	63.71%	65.45%
R^i	1.71	1.71	**1.97**
V^i	8.1815	8.2038	8.2792

Table 6: Adverse Selection - Case 0.6, 0.7 and 0.8

(for each value of s annuity supplying firms offer the rate of return $R(p(h)) = \dfrac{1}{p(h)}$ on the annuity contracts curve).

Let the slope of the utility level curve in the (s, R)-plane be denoted by

$$M(s,R,h) = -\frac{\dfrac{\delta V(s,R,h)}{\delta s}}{\dfrac{\delta V(s,R,h)}{\delta R}} = -\frac{-\dfrac{\delta \nu (w - c(h) - s)}{\delta c_1} + \mu \cdot p(h) \cdot \dfrac{\delta \nu (s \cdot R)}{\delta c_2} \cdot R}{\mu \cdot p(h) \cdot \dfrac{\delta \nu (s \cdot R)}{\delta c_2} \cdot s}$$

There exists an annuity contract (s^*, R^*) such that

$$\frac{\delta \nu (w - c(h) - s^*)}{\delta c_1} = \mu \cdot p(h) \cdot \frac{\delta \nu (s^* \cdot R^*)}{\delta c_2} \cdot R^* \longrightarrow M(s^*, R^*, h) = 0$$

With $s < s^*$ we have

$$\mu \cdot p(h) \cdot \frac{\delta \nu (s \cdot R)}{\delta c_2} \cdot R > \frac{\delta \nu (w - c(h) - s)}{\delta c_1}$$

and then

$$M(s,R,h)=\frac{\dfrac{\delta\nu(w-c(h)-s)}{\delta c_1}}{\mu\cdot p(h)\cdot\dfrac{\delta\nu(s\cdot R)}{\delta c_2}\cdot s}-\frac{R}{s}<0 \tag{15}$$

Consider two degrees of myopia such that $\mu^b<\mu^a$ and the annuity contract $\left(\bar{s},\bar{R}\right)$ with $\bar{s}<s^*$ for all individuals. Given contract $\left(\bar{s},\bar{R}\right)$ both individuals μ^b and individuals μ^a maximize their indirect utility with respect to h:

$$-\frac{\delta c\left(h^b\right)}{\delta h}\cdot\frac{\delta\nu\left(w-c\left(h^b\right)-\bar{s}\right)}{\delta c_1}+\mu^b\cdot\frac{\delta p\left(h^b\right)}{\delta h}\cdot\nu\left(\bar{s}\cdot\bar{R}\right)=0$$

and

$$-\frac{\delta c(h^a)}{\delta h}\cdot\frac{\delta\nu(w-c(h^a)-\bar{s})}{\delta c_1}+\mu^a\cdot\frac{\delta p(h^a)}{\delta h}\cdot\nu\left(\bar{s}\cdot\bar{R}\right)=0$$

Since $\dfrac{\delta^2 c(h)}{\delta h\delta h}=0$ and $\dfrac{\delta^2 p(h)}{\delta h\delta h}=0$ we have $\dfrac{\delta c\left(h^b\right)}{\delta h}=\dfrac{\delta c(h^a)}{\delta h}=\dfrac{\delta c(h)}{\delta h}$ and $\dfrac{\delta p\left(h^b\right)}{\delta h}=\dfrac{\delta p(h^a)}{\delta h}=\dfrac{\delta p(h)}{\delta h}$. Then we can write

$$\frac{\delta\nu\left(w-c\left(h^b\right)-\bar{s}\right)}{\delta c_1}=\mu^b\cdot\frac{\dfrac{\delta p(h)}{\delta h}}{\dfrac{\delta c(h)}{\delta h}}\cdot\nu\left(\bar{s}\cdot\bar{R}\right)$$

and

$$\frac{\delta\nu\left(w-c(h^a)-\bar{s}\right)}{\delta c_1}=\mu^a\cdot\frac{\dfrac{\delta p(h)}{\delta h}}{\dfrac{\delta c(h)}{\delta h}}\cdot\nu\left(\bar{s}\cdot\bar{R}\right)$$

and we can argue that $\mu^b<\mu^a$ implies $h^b<h^a$ (and then $p\left(h^b\right)<p(h^a)$). If we replace the last two equations in the $M(s,R,h)$ equation (equation 15) we obtain

$$\frac{\dfrac{\dfrac{\delta p(h)}{\delta h}}{\dfrac{\delta c(h)}{\delta h}}\cdot\nu\left(\bar{s}\cdot\bar{R}\right)}{p(h^b)\cdot\dfrac{\delta\nu\left(\bar{s}\cdot\bar{R}\right)}{\delta c_2}\cdot\bar{s}}-\frac{\bar{R}}{\bar{s}}>\frac{\dfrac{\dfrac{\delta p(h)}{\delta h}}{\dfrac{\delta c(h)}{\delta h}}\cdot\nu\left(\bar{s}\cdot\bar{R}\right)}{p(h^a)\cdot\dfrac{\delta\nu\left(\bar{s}\cdot\bar{R}\right)}{\delta c_2}\cdot s}-\frac{\bar{R}}{\bar{s}}$$

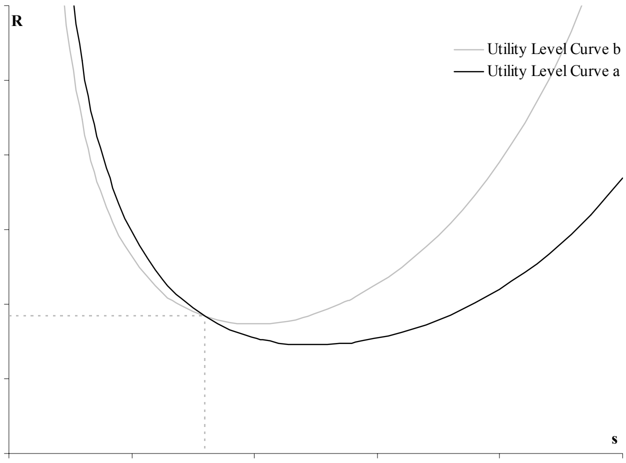

Figure 1: Utility level curves and $M(s, R, h) < 0$.

Since $M(s, R, h) < 0$ we have that the slope of μ^b is greater than the slope of μ^a (in absolute value the slope of μ^a is greater than the slope of μ^b): the slope is declining in μ. Then the utility level curve of μ^a crosses the utility level curve of μ^b from below (see figure 1).

With $s > s^*$ we have

$$\frac{\delta\nu\,(w - c\,(h) - s)}{\delta c_1} > \mu \cdot p\,(h) \cdot \frac{\delta\nu\,(s \cdot R)}{\delta c_2} \cdot R \longrightarrow M\,(s, R, h) > 0$$

and then

$$M\,(s, R, h) = \frac{\dfrac{\delta\nu\,(w - c\,(h) - s)}{\delta c_1}}{\mu \cdot p\,(h) \cdot \dfrac{\delta\nu\,(s \cdot R)}{\delta c_2} \cdot s} - \frac{R}{s} > 0 \qquad (16)$$

Consider two degrees of myopia such that $\mu^b < \mu^a$ and the annuity contract $(\bar{s}, \bar{R})$ with $\bar{s} > s^*$ for all individuals. Given contract $(\bar{s}, \bar{R})$ both individuals μ^b and individuals μ^a maximize their indirect utility with respect to h:

$$-\frac{\delta c\,(h^b)}{\delta h} \cdot \frac{\delta\nu\,(w - c\,(h^b) - \bar{s})}{\delta c_1} + \mu^b \cdot \frac{\delta p\,(h^b)}{\delta h} \cdot \nu\,(\bar{s} \cdot \bar{R}) = 0$$

and

$$-\frac{\delta c\,(h^a)}{\delta h} \cdot \frac{\delta\nu\,(w - c\,(h^a) - \bar{s})}{\delta c_1} + \mu^a \cdot \frac{\delta p\,(h^a)}{\delta h} \cdot \nu\,(\bar{s} \cdot \bar{R}) = 0$$

Since $\dfrac{\delta^2 c(h)}{\delta h \delta h} = 0$ and $\dfrac{\delta^2 p(h)}{\delta h \delta h} = 0$ we have $\dfrac{\delta c\left(h^b\right)}{\delta h} = \dfrac{\delta c(h^a)}{\delta h} = \dfrac{\delta c(h)}{\delta h}$ and $\dfrac{\delta p\left(h^b\right)}{\delta h} = \dfrac{\delta p(h^a)}{\delta h} = \dfrac{\delta p(h)}{\delta h}$. Then we can write

$$\frac{\delta \nu\left(w - c\left(h^b\right) - \bar{s}\right)}{\delta c_1} = \mu^b \cdot \frac{\frac{\delta p(h)}{\delta h}}{\frac{\delta c(h)}{\delta h}} \cdot \nu\left(\bar{s} \cdot \bar{R}\right)$$

and

$$\frac{\delta \nu\left(w - c(h^a) - \bar{s}\right)}{\delta c_1} = \mu^a \cdot \frac{\frac{\delta p(h)}{\delta h}}{\frac{\delta c(h)}{\delta h}} \cdot \nu\left(\bar{s} \cdot \bar{R}\right)$$

we can argue that $\mu^b < \mu^a$ implies $h^b < h^a$ (and then $p\left(h^b\right) < p(h^a)$). If we replace the last two equations in the $M(s, R, h)$ equation (equation 16) we obtain

$$\frac{\cfrac{\frac{\delta p(h)}{\delta h}}{\frac{\delta c(h)}{\delta h}} \cdot \nu\left(\bar{s} \cdot \bar{R}\right)}{p\left(h^b\right) \cdot \cfrac{\delta \nu\left(\bar{s} \cdot \bar{R}\right)}{\delta c_2} \cdot \bar{s}} - \frac{\bar{R}}{\bar{s}} > \frac{\cfrac{\frac{\delta p(h)}{\delta h}}{\frac{\delta c(h)}{\delta h}} \cdot \nu\left(\bar{s} \cdot \bar{R}\right)}{p(h^a) \cdot \cfrac{\delta \nu\left(\bar{s} \cdot \bar{R}\right)}{\delta c_2} \cdot s} - \frac{\bar{R}}{\bar{s}}$$

Since $M(s, R, h) > 0$ we have that the slope of μ^b is greater than the slope of μ^a: the slope is declining in μ. Then the utility level curve of μ^a crosses the utility level curve of μ^b from above (see figure 2).

For each value of s annuity supplying firms offer the rate of return $R(p(h)) = \dfrac{1}{p(h)}$ which solves the following equation

$$-\frac{\delta c(h)}{\delta h} \cdot \frac{\delta \nu(w - c(h) - s)}{\delta c_1} + \mu \cdot \frac{\delta p(h)}{\delta h} \cdot \nu\left(s \cdot \frac{1}{p(h)}\right) = 0$$

Since with $\mu^b < \mu^a$ we have $h^b < h^a$ (and then $p\left(h^b\right) < p(h^a)$), annuity contract curve of individual μ^b is higher than annuity contract curve of individual μ^a. Then for each value of s the rate of return offered to individual μ^b is higher than the rate of return offered to individual μ^a: $R\left(p\left(h^b\right)\right) < R(p(h^a))$ (see figures 3 and 4).

The utility level curve of individual μ^a tangent to the annuity contract curve of individual μ^a at (s^a, R^a) and the utility level curve of individual μ^b tangent to the annuity contract curve of individual μ^b at $\left(s^b, R^b\right)$ are such that $R^a < R^b$ (see figure 3).

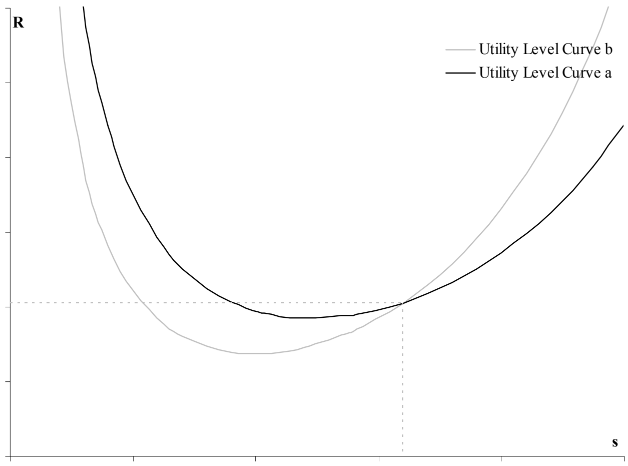

Figure 2: Utility level curves and $M(s, R, h) > 0$.

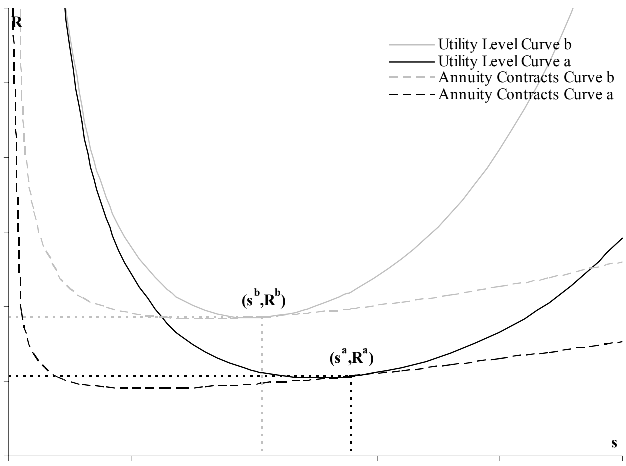

Figure 3: Utility level curves and annuity contracts curves.

Consider the utility level curve of individual μ^a which is tangent to annuity contract curve of individual μ^a. This utility level curve of individual μ^a crosses the annuity contract curve of individual μ^b in two points: (s_l, R_l) and (s_r, R_r) with $s_l < s^b$ and $s_r > s^b$. Now consider the two utility level curves of individual μ^b which crosses the annuity contract curve of individual μ^b at (s_l, R_l) and (s_r, R_r). Since the slope of the utility level curve is declining in μ (at (s_l, R_l) the utility level curve of μ^b crosses the utility level curve of μ^a from above and at (s_r, R_r) the utility level curve of μ^b crosses the utility level curve of μ^a from below), the utility level curve of individual μ^b which crosses the annuity contract curve at (s_l, R_l) is higher than the utility level curve of individual μ^b which crosses the annuity contract curve at (s_r, R_r) (see figure 4).

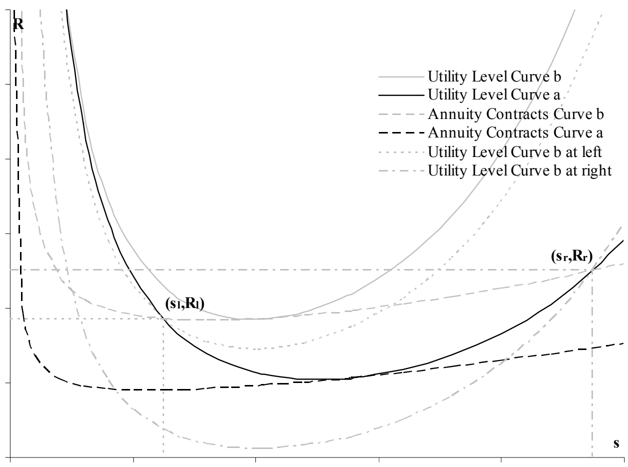

Figure 4: For individual μ^b the utility level curve at (s_l, R_l) is higher than the utility level curve at (s_r, R_r).

7 Separating Equilibrium

Because individuals can take hidden actions to affect their longevity, competitive firms offer the utility-maximizing actuarially fair annuity contracts $\left(\tilde{s}^i, R\left(p\left(\tilde{h}^i\right)\right)\right)$ subject to the constraint that individuals choose the optimal level of $\tilde{h}^i$ in response to these contracts (incentive constraint) [2].

$$\underset{\tilde{s}^i,\tilde{h}^i}{Max} \nu^i\left(w - c\left(\tilde{h}^i\right) - \tilde{s}^i\right) + \mu^i \cdot p\left(\tilde{h}^i\right) \cdot \nu^i\left(\tilde{s}^i \cdot R\left(p\left(\tilde{h}^i\right)\right)\right)$$

$$s.t. \;\; \tilde{h}^i \text{ solves} \underset{\breve{h}^i}{Max} \nu^i\left(w - c\left(\breve{h}^i\right) - \tilde{s}^i\right) + \mu^i \cdot p\left(\breve{h}^i\right) \cdot \nu^i\left(\tilde{s}^i \cdot R\left(p\left(\tilde{h}^i\right)\right)\right) \quad \left[\lambda^i_{mh}\right]$$

	$\mu^L = 0.4$	$\tilde{\mu}^L = 0.4$	$\mu^M = 0.7$	$\tilde{\mu}^M = 0.7$	$\mu^H = 1.0$	$\tilde{\mu}^H = 1.0$
s^i	80.16	82.66	124.86	126.66	142.74	144.06
h^i	283.52	351.90	441.63	484.75	504.87	535.80
p^i	35.44%	43.99%	55.20%	60.59%	63.11%	66.98%
R^i	2.82	2.27	1.81	1.65	1.58	1.49
V^i	7.1061	7.0898	7.8729	7.8548	8.8426	8.8240
	$\mu^L = 0.5$	$\tilde{\mu}^L = 0.5$	$\mu^M = 0.7$	$\tilde{\mu}^M = 0.7$	$\mu^H = 0.9$	$\tilde{\mu}^H = 0.9$
s^i	101.02	103.32	124.86	126.66	138.10	139.56
h^i	357.30	415.11	441.63	484.75	488.47	522.65
p^i	44.66%	51.89%	55.20%	60.59%	61.06%	65.33%
R^i	2.24	1.93	1.81	1.65	1.64	1.53
V^i	7.3251	7.3078	7.8729	7.8548	8.5059	8.4874
	$\mu^L = 0.6$	$\tilde{\mu}^L = 0.6$	$\mu^M = 0.7$	$\tilde{\mu}^M = 0.7$	$\mu^H = 0.8$	$\tilde{\mu}^H = 0.8$
s^i	114.92	116.96	124.86	126.66	132.31	133.92
h^i	406.49	455.99	441.63	484.75	467.98	506.12
p^i	50.81%	57.00%	55.20%	60.59%	58.50%	63.27%
R^i	1.97	1.75	1.81	1.65	1.71	1.58
V^i	7.5849	7.5671	7.8729	7.8548	8.1815	8.1632

Table 7: Separating Equilibrium and Incentive Constraints

$$0 \leq \tilde{h}^i \leq h^{\max}$$
$$\tilde{s}^i \geq 0$$

Since we have a continuous set of possible actions ($\tilde{h}^i \in [0, h^{\max}]$), we have an infinity of incentive constraints. One trick used in this case is to replace incentive constraint with first-order condition (first-order condition approach) [5]:

$$-\frac{\delta c\left(\tilde{h}^i\right)}{\delta h} \cdot \nu^i_{c_1} + \mu^i \cdot \frac{\delta p\left(\tilde{h}^i\right)}{\delta h} \cdot \nu^i\left(\tilde{s}^i \cdot R\left(p\left(\tilde{h}^i\right)\right)\right) = 0$$

The results of simulation are summarized in table 7.

The simulation shows that $V^L\left(\tilde{h}^L\right) < V^L\left(h^L\right)$, $V^M\left(\tilde{h}^M\right) < V^M\left(h^M\right)$ and $V^H\left(\tilde{h}^H\right) < V^H\left(h^H\right)$: when there is moral hazard in the annuity market all individuals are affected by negative externalities.

The contract $\left(\tilde{s}^L, R\left(p\left(\tilde{h}^L\right)\right)\right)$ (low-risk contract), the contract preferred by members of group L, the contract $\left(\tilde{s}^M, R\left(p\left(\tilde{h}^M\right)\right)\right)$ (medium-risk contract), the contract preferred by members of group M and the contract $\left(\tilde{s}^H, R\left(p\left(\tilde{h}^H\right)\right)\right)$ (high-risk contract), the contract preferred by members of group H, are characterized by $\tilde{s}^H > \tilde{s}^M > \tilde{s}^L$ and $R\left(p\left(\tilde{h}^H\right)\right) < R\left(p\left(\tilde{h}^M\right)\right) < R\left(p\left(\tilde{h}^L\right)\right)$.

For the case $\mu^L = 0.4$, $\mu^M = 0.7$ and $\mu^H = 1.0$ the results of simulation are also described in figure 5.

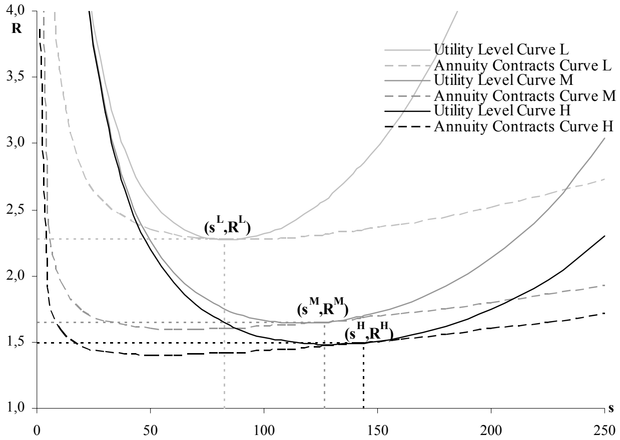

Figure 5: Separating Equilibrium and Incentive Constraints: Case $\mu^L = 0.4$, $\mu^M = 0.7$ and $\mu^H = 1.0$.

The level curve of M at $\left(\tilde{s}^L, R\left(p\left(\tilde{h}^L\right)\right)\right)$ is higher than this one of M at $\left(\tilde{s}^M, R\left(p\left(\tilde{h}^M\right)\right)\right)$ and the level curve of H at $\left(\tilde{s}^L, R\left(p\left(\tilde{h}^L\right)\right)\right)$ is higher than this one of H at $\left(\tilde{s}^H, R\left(p\left(\tilde{h}^H\right)\right)\right)$: both members of group M and members of group H prefer contract $\left(\tilde{s}^L, R\left(p\left(\tilde{h}^L\right)\right)\right)$ to contracts $\left(\tilde{s}^M, R\left(p\left(\tilde{h}^M\right)\right)\right)$ and $\left(\tilde{s}^H, R\left(p\left(\tilde{h}^H\right)\right)\right)$. Since medium-risk individuals (members of group M) and high-risk individuals (members of group H) have hidden information about their longevity, they purchase contract $\left(\tilde{s}^L, R\left(p\left(\tilde{h}^L\right)\right)\right)$. The maximization problem of high-risk agents M and this one of high-risk agents H are

$$\underset{\hat{h}^M}{Max} \hat{\nu}^M\left(w - c\left(\hat{h}^M\right) - \tilde{s}^L\right) + \mu^M \cdot p\left(\hat{h}^M\right) \cdot \hat{\nu}^M\left(\tilde{s}^L \cdot R\left(p\left(\tilde{h}^L\right)\right)\right) \quad (17)$$

$$s.t. \;\; 0 \leq \hat{h}^M \leq h^{\max}$$

and

$$\underset{\hat{h}^H}{Max} \hat{\nu}^H\left(w - c\left(\hat{h}^H\right) - \tilde{s}^L\right) + \mu^H \cdot p\left(\hat{h}^H\right) \cdot \hat{\nu}^H\left(\tilde{s}^L \cdot R\left(p\left(\tilde{h}^L\right)\right)\right) \quad (18)$$

	$\mu^L = 0.4$	$\tilde{\mu}^L = 0.4$	
s^i	80.16	**82.66**	
h^i	283.52	351.90	
p^i	35.44%	43.99%	
R^i	2.82	**2.27**	
V^i	7.1061	7.0898	
	$\mu^M = 0.7$	$\tilde{\mu}^M = 0.7$	$\hat{\mu}^M = 0.7$
s^i	124.86	126.66	**82.66**
h^i	441.63	484.75	515.60
p^i	55.20%	60.59%	64.45%
R^i	1.81	1.65	**2.27**
V^i	7.8729	7.8548	7.9711
	$\mu^H = 1.0$	$\tilde{\mu}^H = 1.0$	$\hat{\mu}^H = 1.0$
s^i	142.74	144.06	**82.66**
h^i	504.87	535.80	581.08
p^i	63.11%	66.98%	72.64%
R^i	1.58	1.49	**2.27**
V^i	8.8426	8.8240	9.0554

Table 8: Separating Equilibrium and Incentive Constraints - Adverse Selection - Case 0.4, 0.7 and 1.0

$$s.t.\ \ 0 \leq \hat{h}^H \leq h^{\max}$$

The results of simulation are summarized in tables 8, 9 and 10.

If a firm offered the pooling contract $\left(\tilde{s}^L, R\left(p\left(\tilde{h}^L\right)\right)\right)$, individuals of groups M and H would purchase this contract instead of contracts $\left(\tilde{s}^M, R\left(p\left(\tilde{h}^M\right)\right)\right)$ and $\left(\tilde{s}^H, R\left(p\left(\tilde{h}^H\right)\right)\right)$.

For the case $\mu^L = 0.4$, $\mu^M = 0.7$ and $\mu^H = 1.0$ the results of simulation are also described in figure 6.

Since contract $\left(\tilde{s}^L, R\left(p\left(\tilde{h}^L\right)\right)\right)$ is actuarially fair for members of group L only, if members of groups M and H purchased it profits of firms would necessarily be negative. Thus from the point of view of firms contract of separating equilibrium for group L must not be more attractive to members of group M than contract $\left(\tilde{s}^M, R\left(p\left(\tilde{h}^M\right)\right)\right)$ and contracts of separating equilibrium for groups L and M must not be more attractive to members of group H than contract $\left(\tilde{s}^H, R\left(p\left(\tilde{h}^H\right)\right)\right)$ (incentive-compatibility or self-selection constraints) [3].

	$\mu^L = 0.5$	$\tilde{\mu}^L = 0.5$	
s^i	101.02	**103.32**	
h^i	357.30	415.11	
p^i	44.66%	51.89%	
R^i	2.24	**1.93**	
V^i	7.3251	7.3078	
	$\mu^M = 0.7$	$\tilde{\mu}^M = 0.7$	$\hat{\mu}^M = 0.7$
s^i	124.86	126.66	**103.32**
h^i	441.63	484.75	501.46
p^i	55.20%	60.59%	62.28%
R^i	1.81	1.65	**1.93**
V^i	7.8729	7.8548	7.9207
	$\mu^H = 0.9$	$\tilde{\mu}^H = 0.9$	$\hat{\mu}^H = 0.9$
s^i	138.10	139.56	**103.32**
h^i	488.47	522.65	549.43
p^i	61.06%	65.33%	68.68%
R^i	1.64	1.53	**1.93**
V^i	8.5059	8.4874	8.6188

Table 9: Separating Equilibrium and Incentive Constraints - Adverse Selection - Case 0.5, 0.7 and 0.9

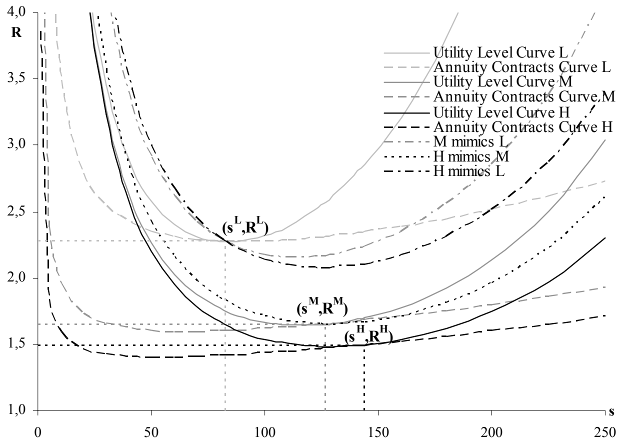

Figure 6: Separating Equilibrium and Incentive Constraints - Adverse Selection: Case $\mu^L = 0.4$, $\mu^M = 0.7$ and $\mu^H = 1.0$.

	$\mu^L = 0.6$	$\tilde{\mu}^L = 0.6$	
s^i	114.92	**116.96**	
h^i	406.49	455.99	
p^i	50.81%	57.00%	
R^i	1.97	**1.75**	
V^i	7.5849	7.5671	
	$\mu^M = 0.7$	$\tilde{\mu}^M = 0.7$	$\hat{\mu}^M = 0.7$
s^i	124.86	126.66	**116.96**
h^i	441.63	484.75	491.77
p^i	55.20%	60.59%	61.47%
R^i	1.81	1.65	**1.75**
V^i	7.8729	7.8548	7.8831
	$\mu^H = 0.8$	$\tilde{\mu}^H = 0.8$	$\hat{\mu}^H = 0.8$
s^i	132.31	133.92	**116.96**
h^i	467.98	506.12	518.60
p^i	58.50%	63.27%	64.83%
R^i	1.71	1.58	**1.75**
V^i	8.1815	8.1632	8.2197

Table 10: Separating Equilibrium and Incentive Constraints - Adverse Selection - Case 0.6, 0.7 and 0.8

With the self-selection constraints we have to consider the two points where the utility level curve H intersects the annuity contracts curve M (at $\left(s_l^{HM}, R_l^{HM}\right)$ with $s_l^{HM} < s^M$ and at $\left(s_r^{HM}, R_r^{HM}\right)$ with $s_r^{HM} > s^M$), the two points where the utility level curve M intersects the annuity contracts curve L (at $\left(s_l^{ML}, R_l^{ML}\right)$ with $s_l^{ML} < s^L$ and at $\left(s_r^{ML}, R_r^{ML}\right)$ with $s_r^{ML} > s^L$) and the two points where the utility level curve H intersects the annuity contracts curve L (at $\left(s_l^{HL}, R_l^{HL}\right)$ with $s_l^{HL} < s^L$ and at $\left(s_r^{HL}, R_r^{HL}\right)$ with $s_r^{HL} > s^L$).

We have seen that for an individual with a smaller degree of myopia the utility level curve which crosses the annuity contract curve at (s_l, R_l) is higher than the utility level curve which crosses the annuity contract curve at (s_r, R_r) (see figure 4), then in the following we can consider only the points $\left(s_l^{HM}, R_l^{HM}\right)$, $\left(s_l^{ML}, R_l^{ML}\right)$ and $\left(s_l^{HL}, R_l^{HL}\right)$.

Finally since $M(s, R, h) < 0$ and the slope of the utility level curve is declining in μ (see figure 1), we can restrict the analysis to point $\left(s_l^{HM}, R_l^{HM}\right)$ where the utility level curve H intersects the annuity contracts curve M (with $s_l^{HM} < s^M$) and to point $\left(s_l^{ML}, R_l^{ML}\right)$ where the utility level curve M intersects the annuity contracts curve L (with $s_l^{ML} < s^L$).

Thus competitive firms offer to members of groups L and M the utility-maximizing actuarially fair annuity contracts $\left(\check{s}^L, R\left(p\left(\check{h}^L\right)\right)\right)$ and $\left(\check{s}^M, R\left(p\left(\check{h}^M\right)\right)\right)$ subject to the constraints that individuals choose the optimal level of $\check{h}^L$ and $\check{h}^M$ in response to these contracts (incentive constraints) and subject to the constraints that contract of separating equilibrium for group L must not be more attractive to members of group M than contract $\left(\check{s}^M, R\left(p\left(\check{h}^M\right)\right)\right)$ and that contract of separating equilibrium for group M must not be more attractive to members of group H than contract $\left(\tilde{s}^H, R\left(p\left(\tilde{h}^H\right)\right)\right)$ (self-selection constraints):

$$\underset{\check{s}^L,\check{h}^L}{Max}\nu^L\left(w - c\left(\check{h}^L\right) - \check{s}^L\right) + \mu^L \cdot p\left(\check{h}^L\right) \cdot \nu^L\left(\check{s}^L \cdot R\left(p\left(\check{h}^L\right)\right)\right)$$

$$s.t.\ \check{h}^L \text{ solves} \underset{\check{h}^L}{Max}\nu^L\left(w - c\left(\check{h}^L\right) - \check{s}^L\right) + \\ + \mu^L \cdot p\left(\check{h}^L\right) \cdot \nu^L\left(\check{s}^L \cdot R\left(p\left(\check{h}^L\right)\right)\right) \quad \left[\lambda_{mh}^L\right]$$

$$\nu^M\left(w - q_h \cdot \check{h}^M - \check{s}^M\right) + \mu^M \cdot p\left(\check{h}^M\right) \cdot \nu^M\left(\check{s}^M \cdot R\left(p\left(\check{h}^M\right)\right)\right) \\ \geq \hat{\nu}^M\left(w - q_h \cdot \hat{h}^M - \check{s}^L\right) + \mu^M \cdot p\left(\hat{h}^M\right) \cdot \hat{\nu}^M\left(\check{s}^L \cdot R\left(p\left(\check{h}^L\right)\right)\right) \quad \left[\lambda_{as}^L\right]$$

$$0 \leq \check{h}^L \leq h^{\max} \\ \check{s}^L \geq 0$$

and

$$\underset{\tilde{s}^M,\tilde{h}^M}{Max}\nu^M\left(w - c\left(\check{h}^M\right) - \check{s}^M\right) + \mu^M \cdot p\left(\check{h}^M\right) \cdot \nu^M\left(\check{s}^M \cdot R\left(p\left(\check{h}^M\right)\right)\right)$$

$$s.t.\check{h}^M \text{ solves} \underset{\check{h}^M}{Max}\nu^M\left(w - c\left(\check{h}^M\right) - \check{s}^M\right) + \\ + \mu^M \cdot p\left(\check{h}^M\right) \cdot \nu^M\left(\check{s}^M \cdot R\left(p\left(\check{h}^M\right)\right)\right) \quad \left[\lambda_{mh}^M\right]$$

$$\nu^H\left(w - q_h \cdot \tilde{h}^H - \tilde{s}^H\right) + \mu^H \cdot p\left(\tilde{h}^H\right) \cdot \nu^H\left(\tilde{s}^H \cdot R\left(p\left(\tilde{h}^H\right)\right)\right) \\ \geq \hat{\nu}^H\left(w - q_h \cdot \hat{h}^H - \check{s}^M\right) + \mu^H \cdot p\left(\hat{h}^H\right) \cdot \hat{\nu}^H\left(\check{s}^M \cdot R\left(p\left(\check{h}^M\right)\right)\right) \quad \left[\lambda_{as}^M\right]$$

$$0 \leq \check{h}^M \leq h^{\max}$$
$$\check{s}^M \geq 0$$

Because the self-selection constraints are clearly binding[2] (and then we have not $\lambda_{as}^i \geq 0$ but $\lambda_{as}^i > 0$), we may replace the weak inequality with equality [3, Eckestein, Eichenbaum and Peled (1985)]:

$$\begin{aligned} &\nu^M \left(w - q_h \cdot \check{h}^M - \check{s}^M\right) + \mu^M \cdot p\left(\check{h}^M\right) \cdot \nu^M \left(\check{s}^M \cdot R\left(p\left(\check{h}^M\right)\right)\right) \\ &= \hat{\nu}^M \left(w - q_h \cdot \hat{h}^M - \check{s}^L\right) + \mu^M \cdot p\left(\hat{h}^M\right) \cdot \hat{\nu}^M \left(\check{s}^L \cdot R\left(p\left(\check{h}^L\right)\right)\right) \qquad [\lambda_{as}^L] \end{aligned}$$

and

$$\begin{aligned} &\nu^H \left(w - q_h \cdot \tilde{h}^H - \tilde{s}^H\right) + \mu^H \cdot p\left(\tilde{h}^H\right) \cdot \nu^H \left(\tilde{s}^H \cdot R\left(p\left(\tilde{h}^H\right)\right)\right) \\ &= \hat{\nu}^H \left(w - q_h \cdot \hat{h}^H - \check{s}^M\right) + \mu^H \cdot p\left(\hat{h}^H\right) \cdot \hat{\nu}^H \left(\check{s}^M \cdot R\left(p\left(\check{h}^M\right)\right)\right) \qquad [\lambda_{as}^M] \end{aligned}$$

The results of simulation are summarized in tables 11, 12 and 13.

The simulation shows that $V^L\left(\check{h}^L\right) < V^L\left(\tilde{h}^L\right) < V^L\left(h^L\right)$ and $V^M\left(\check{h}^M\right) < V^M\left(\tilde{h}^M\right) < V^M\left(h^M\right)$: when there is adverse selection in the annuity market two groups of individuals (low risk individuals L and medium risk individuals M) are affected by negative externalities.

For the case $\mu^L = 0.4$, $\mu^M = 0.7$ and $\mu^H = 1.0$ the results of simulation are also described in figure 7.

Hence in the case of separating equilibrium there are two kinds of negative externalities.

1. The fact that firms consider the possibility that individuals modify the investment in health care in first period in response to separating contracts offered to them is a first negative externality: $V^L\left(\tilde{s}^L, \tilde{h}^L\right) < V^L\left(s^L, h^L\right)$, $V^M\left(\tilde{s}^M, \tilde{h}^M\right) < V^M\left(s^M, h^M\right)$ and $V^H\left(\tilde{s}^H, \tilde{h}^H\right) < V^H\left(s^H, h^H\right)$.

2. The presence of high-risk individuals (group H) exerts a second negative externality on agents of group M and the presence of medium-risk individuals (group M) and high-risk individuals (group

[2] Any s^i which satisfies self-selection constraint with strict inequality cannot be a solution to the maximization problem since the derivative of the maximand with respect to s^i at any point that satisfies self-selection constraint is non-zero.

	$\mu^L = 0.4$	$\tilde{\mu}^L = 0.4$	$\check{\mu}^L = 0.4$		
s^i	80.16	82.66	**41.37**		
h^i	283.52	351.90	332.24		
p^i	35.44%	43.99%	41.53%		
R^i	2.82	2.27	**2.41**		
V^i	7.1061	7.0898	7.0621		
	$\mu^M = 0.7$	$\tilde{\mu}^M = 0.7$	$\check{\mu}^M = 0.7$	$\hat{\mu}^M(L) = 0.7$	
s^i	124.86	126.66	**88.39**	**41.37**	
h^i	441.63	484.75	498.62	518.53	
p^i	55.20%	60.59%	62.33%	64.82%	
R^i	1.81	1.65	**1.60**	**2.41**	
V^i	7.8729	7.8548	7.8257	7.8257	
	$\mu^H = 1.0$	$\tilde{\mu}^H = 1.0$		$\hat{\mu}^H(M) = 1.0$	$\hat{\mu}^H(L) = 1.0$
s^i	142.74	144.06		**88.39**	**41.37**
h^i	504.87	535.80		567.82	593.04
p^i	63.11%	66.98%		70.98%	74.13%
R^i	1.58	1.49		**1.60**	**2.41**
V^i	8.8426	8.8240		8.8240	8.7923

Table 11: Separating Equilibrium, Incentive Constraints and Self-Selection Constraints - Case 0.4, 0.7 and 1.0

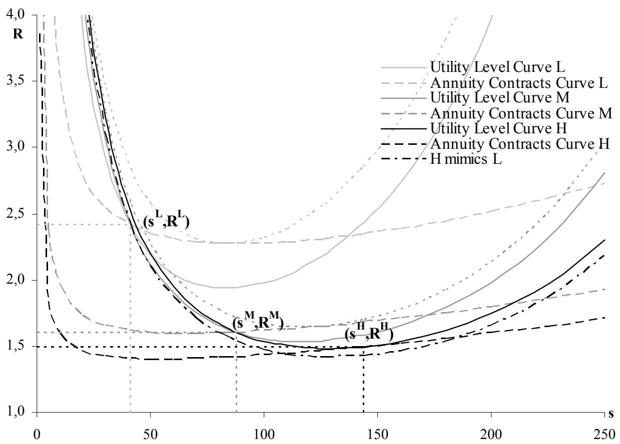

Figure 7: Separating Equilibrium, Incentive Constraints and Self-Selection Constraints: Case $\mu^L = 0.4$, $\mu^M = 0.7$ and $\mu^H = 1.0$.

	$\mu^L = 0.5$	$\tilde{\mu}^L = 0.5$	$\check{\mu}^L = 0.5$		
s^i	101.02	103.32	**59.87**		
h^i	357.30	415.11	415.13		
p^i	44.66%	51.89%	51.89%		
R^i	2.24	1.93	**1.93**		
V^i	7.3251	7.3078	7.2751		
	$\mu^M = 0.7$	$\tilde{\mu}^M = 0.7$	$\check{\mu}^M = 0.7$	$\hat{\mu}^M(L) = 0.7$	
s^i	124.86	126.66	**91.96**	**59.87**	
h^i	441.63	484.75	497.68	511.41	
p^i	55.20%	60.59%	62.21%	63.93%	
R^i	1.81	1.65	**1.61**	**1.93**	
V^i	7.8729	7.8548	7.8314	7.8314	
	$\mu^H = 0.9$	$\tilde{\mu}^H = 0.9$		$\hat{\mu}^H(M) = 0.9$	$\hat{\mu}^H(L) = 0.9$
s^i	138.10	139.56		**88.39**	**59.87**
h^i	488.47	522.65		548.51	564.90
p^i	61.06%	65.33%		68.56%	70.61%
R^i	1.64	1.53		**1.61**	**1.93**
V^i	8.5059	8.4874		8.4874	8.4728

Table 12: Separating Equilibrium, Incentive Constraints and Self-Selection Constraints - Case 0.5, 0.7 and 0.9

H) exerts a second negative externality on agents of group L: $V^M\left(\check{s}^M, \check{h}^M\right) < V^M\left(\tilde{s}^M, \tilde{h}^M\right) < V^M\left(s^M, h^M\right)$ and $V^L\left(\check{s}^L, \check{h}^L\right) < V^L\left(\tilde{s}^L, \tilde{h}^L\right) < V^L\left(s^L, h_1^L\right)$.

These negative externalities are purely destructive because members of groups L, M and H are worse off than they would be in the absence of private information.

8 Pooling Equilibrium

With private information firms don't know whether any particular individual belongs to group L, M or H, then in the case of pooling equilibrium firms offer to the members of three groups not only the same rate of return, but also the same quantity of annuity: a pooling contract is characterized not only by $R_t^L = R_t^M = R_t^H = \bar{R}_t$, but also by $s_t^L = s_t^M = s_t^H = \bar{s}_t$ and $s_{t+1}^L = s_{t+1}^M = s_{t+1}^H = \bar{s}_{t+1}$. Then the condition of zero

	$\mu^L = 0.6$	$\tilde{\mu}^L = 0.6$	$\check{\mu}^L = 0.6$		
s^i	114.92	116.96	**79.27**		
h^i	406.49	455.99	465.29		
p^i	50.81%	57.00%	58.16%		
R^i	1.97	1.75	**1.72**		
V^i	7.5849	7.5671	7.5415		
	$\mu^M = 0.7$	$\tilde{\mu}^M = 0.7$	$\check{\mu}^M = 0.7$	$\hat{\mu}^M(L) = 0.7$	
s^i	124.86	126.66	**98.41**	**79.27**	
h^i	441.63	484.75	495.75	504.05	
p^i	55.20%	60.59%	61.97%	63.01%	
R^i	1.81	1.65	**1.61**	**1.72**	
V^i	7.8729	7.8548	7.8398	7.8398	
	$\mu^H = 0.8$	$\tilde{\mu}^H = 0.8$		$\hat{\mu}^H(M) = 0.8$	$\hat{\mu}^H(L) = 0.8$
s^i	132.31	133.92		**98.41**	**79.27**
h^i	467.98	506.12		523.94	533.12
p^i	58.50%	63.27%		65.49%	66.64%
R^i	1.71	1.58		**1.61**	**1.72**
V^i	8.1815	8.1632		8.1632	8.1588

Table 13: Separating Equilibrium, Incentive Constraints and Self-Selection Constraints - Case 0.6, 0.7 and 0.8

profits is given by

$$\begin{aligned}&\bar{s}_{t+1} + \gamma_M \cdot \bar{s}_{t+1} + \gamma_H \cdot \bar{s}_{t+1} + \\ &-\bar{R}_t \cdot \left[p_t\left(\bar{h}^L\right) \cdot \bar{s}_t + \gamma_M \cdot p_t\left(\bar{h}^M\right) \cdot \bar{s}_t + \gamma_H \cdot p_t\left(\bar{h}^H\right) \cdot \bar{s}_t\right] = 0\end{aligned}$$

In a stationary equilibrium $\bar{R}_t = \bar{R}$ and $\bar{s}_t = \bar{s}$ for all t and then the rate of return of the pooling contract is

$$\bar{R} = \frac{1 + \gamma_M + \gamma_H}{p\left(\bar{h}^L\right) + \gamma_M \cdot p\left(\bar{h}^M\right) + \gamma_H \cdot p\left(\bar{h}^H\right)} \tag{19}$$

Suppose that annuity-supplying firms offer a pooling contract characterized by a quantity of annuity $\bar{s}$ that doesn't maximize the utility of low-risk individuals. Given this case, if a firm offered a pooling contract characterized by a quantity of annuity $\bar{s}$ that maximizes the utility of low-risk individuals, individuals of group L would purchase this second contract and the profits of firms that offer the first contract would become negative. Thus the quantity of annuity of a pooling contract which assures non-negative profits is

$$\bar{s} = \arg \underset{s}{Max} \;\; v^L\left(w - c\left(\bar{h}^L\right) - s\right) + \mu^L \cdot p\left(\bar{h}^L\right) \cdot v^L\left(s \cdot \bar{R}\right)$$

$\gamma_M = 0.50$ and $\gamma_H = 0.50$							
	0.4		0.7		1.0		
	$\check{\mu}^L$	$\bar{\mu}^L$	$\check{\mu}^M$	$\bar{\mu}^M$	μ^H	$\tilde{\mu}^H$	$\bar{\mu}^H$
$\bar{s}$		84.76					
s^i	41.37		88.39		142.74	144.06	
$\bar{R}$		1.83					
R^i	2.41		1.60		1.58	1.49	
V^i	**7.0621**	7.0523	7.8257	7.8774	8.8426	8.8240	8.9054
	0.5		0.7		0.9		
	$\check{\mu}^L$	$\bar{\mu}^L$	$\check{\mu}^M$	$\bar{\mu}^M$	μ^H	$\tilde{\mu}^H$	$\bar{\mu}^H$
$\bar{s}$		104.98					
s^i	59.87		91.96		138.10	139.56	
$\bar{R}$		1.72					
R^i	1.93		1.61		1.64	1.53	
V^i	7.2751	**7.2787**	7.8314	7.8726	8.5059	8.4874	8.5517
	0.6		0.7		0.8		
	$\check{\mu}^L$	$\bar{\mu}^L$	$\check{\mu}^M$	$\bar{\mu}^M$	μ^H	$\tilde{\mu}^H$	$\bar{\mu}^H$
$\bar{s}$		117.80					
s^i	79.27		98.41		132.31	133.92	
$\bar{R}$		1.67					
R^i	1.72		1.61		1.71	1.58	
V^i	7.5415	**7.5507**	7.8398	7.8627	8.1815	8.1632	8.1952

Table 14: Pooling Equilibrium with gamma M 0.50 and gamma H 0.50

Because individuals can take hidden actions to affect their longevity, competitive firms offer the utility-maximizing actuarially fair annuity contracts $\left(\bar{s}, \bar{R}\right)$ subject to the constraint (incentive constraint) that individuals choose the optimal level of h^i in response to this contract [2]:

$$\underset{\bar{s},\bar{h}^L,\bar{h}^M,\bar{h}^H}{Max} v^L\left(w - c\left(\bar{h}^L\right) - \bar{s}\right) + \mu^L \cdot p\left(\bar{h}^L\right) \cdot v^L\left(\bar{s}\cdot\bar{R}\right) \tag{20}$$

$$s.t.\ \ \bar{h}^L \text{ solves} \underset{\check{h}^L}{Max} \nu^L\left(w - c\left(\check{h}^L\right) - \bar{s}\right) + \mu^L \cdot p\left(\check{h}^L\right) \cdot \nu^L\left(\bar{s}\cdot\bar{R}\right) \quad \left[\lambda^L_{mh}\right]$$

$$s.t.\ \ \bar{h}^M \text{ solves} \underset{\check{h}^M}{Max} \nu^M\left(w - c\left(\check{h}^M\right) - \bar{s}\right) + \mu^M \cdot p\left(\check{h}^M\right) \cdot \nu^M\left(\bar{s}\cdot\bar{R}\right) \quad \left[\lambda^M_{mh}\right]$$

$$s.t.\ \ \bar{h}^H \text{ solves} \underset{\check{h}^H}{Max} \nu^H\left(w - c\left(\check{h}^H\right) - \bar{s}\right) + \mu^H \cdot p\left(\check{h}^H\right) \cdot \nu^H\left(\bar{s}\cdot\bar{R}\right) \quad \left[\lambda^H_{mh}\right]$$

$$0 \le \bar{h}^L, \bar{h}^M, \bar{h}^H \le h^{\max}$$

$$\bar{s} \ge 0$$

$\gamma_M = 1.00$ and $\gamma_H = 1.00$							
	0.4		0.7		1.0		
	$\check{\mu}^L$	$\bar{\mu}^L$	$\check{\mu}^M$	$\bar{\mu}^M$	μ^H	$\tilde{\mu}^H$	$\bar{\mu}^H$
$\bar{s}$		85.31					
s^i	41.37		88.39		142.74	144.06	
$\bar{R}$		1.71					
R^i	2.41		1.60		1.58	1.49	
V^i	**7.0621**	7.0411	7.8257	7.8487	8.8426	8.8240	8.8593
	0.5		0.7		0.9		
	$\check{\mu}^L$	$\bar{\mu}^L$	$\check{\mu}^M$	$\bar{\mu}^M$	μ^H	$\tilde{\mu}^H$	$\bar{\mu}^H$
$\bar{s}$		105.49					
s^i	59.87		91.96		138.10	139.56	
$\bar{R}$		1.66					
R^i	1.93		1.61		1.64	1.53	
V^i	**7.2751**	7.2695	7.8314	7.8574	8.5059	8.4874	8.5304
	0.6		0.7		0.8		
	$\check{\mu}^L$	$\bar{\mu}^L$	$\check{\mu}^M$	$\bar{\mu}^M$	μ^H	$\tilde{\mu}^H$	$\bar{\mu}^H$
$\bar{s}$		118.07					
s^i	79.27		98.41		132.31	133.92	
$\bar{R}$		1.65					
R^i	1.72		1.61		1.71	1.58	
V^i	7.5415	**7.5453**	7.8398	7.8560	8.1815	8.1632	8.1873

Table 15: Pooling Equilibrium with gamma M 1.00 and gamma H 1.00

The results of simulation are summarized in tables 14, 15 and 16.

The simulation shows that

- when the distance among the degrees of myopia of three groups μ^i is small, the pooling equilibrium dominates the separating equilibrium;
- when the number of type M agents and the number of type H agents are relatively small with respect to the number of type L agents (γ_M and γ_H are small), the pooling equilibrium dominates the separating equilibrium.

If the distance among the degrees of myopia of three groups μ^i is small, the effect of survival probabilities of medium-risk individuals and of high-risk individuals ($p\left(h^H\right)$ and $p\left(h^M\right)$) on the rate of return of the pooling contract $\bar{R}$ is small ($\bar{R}$ is relatively large): then

$V^L\left(\bar{s},\bar{h}^L\right) > V^L\left(\check{s}^L,\check{h}^L\right)$ and the pooling equilibrium dominates the separating equilibrium.

For small values of γ_M and γ_H (for a relatively small number of type M agents and type H agents) the decreasing effect on the rate of return of the pooling contract $\bar{R}$ caused by the existence of medium-risk individuals and high-risk individuals is small ($\bar{R}$ is relatively large): then $V^L\left(\bar{s},\bar{h}^L\right) > V^L\left(\check{s}^L,\check{h}^L\right)$ and the pooling equilibrium dominates the separating equilibrium.

Hence in the case of pooling equilibrium there are two kinds of negative externalities.

1. The fact that firms consider the possibility that members of all groups modify the investment in health care in the first period in response to the pooling contract offered to them is a first negative externality. However when a pooling equilibrium is offered by the annuity supplying firms, for members of H this negative externality is completely compensated by $\bar{R} > R\left(p\left(h^H\right)\right)$.

2. The presence of high-risk individuals (group H) exerts a second negative externality on other agents (groups L and M) and the presence of medium-risk individuals (group M) exerts a second negative externality on low-risk agents (group L) because $p\left(h^H\right) > p\left(h^M\right) > p\left(h^L\right)$.

These negative externalities are not purely destructive because while members of group L and members of group M are worse off than they would be in the absence of private information, members of group H are better off.

9 Conclusions and Plan for Future Research

In this work I have shown that both in separating equilibrium and in pooling equilibrium there are negative externalities.

In the case of separating equilibrium there are two kinds of negative externalities: the fact that firms consider the possibility that individuals modify the investment in health care in the first period in response to separating contracts offered to them is a first negative externality and the presence of higher risk individuals exerts a second negative externality on lower risk individuals. These negative externalities are purely destructive because all individuals are worse off than they would be in the absence of private information.

In the case of pooling equilibrium there are two kinds of negative externalities: the fact that firms consider the possibility that members of all groups modify the investment in health care in the first period in

$\gamma_M = 2.00$ and $\gamma_H = 2.00$							
	0.4		0.7		1.0		
	$\check{\mu}^L$	$\bar{\mu}^L$	$\check{\mu}^M$	$\bar{\mu}^M$	μ^H	$\tilde{\mu}^H$	$\bar{\mu}^H$
$\bar{s}$		85.71					
s^i	41.37		88.39		142.74	144.06	
$\bar{R}$		1.62					
R^i	2.41		1.60		1.58	1.49	
V^i	**7.0621**	7.0325	7.8257	7.8266	8.8426	8.8240	8.8236
	0.5		0.7		0.9		
	$\check{\mu}^L$	$\bar{\mu}^L$	$\check{\mu}^M$	$\bar{\mu}^M$	μ^H	$\tilde{\mu}^H$	$\bar{\mu}^H$
$\bar{s}$		105.89					
s^i	59.87		91.96		138.10	139.56	
$\bar{R}$		1.62					
R^i	1.93		1.61		1.64	1.53	
V^i	**7.2751**	7.2623	7.8314	7.8454	8.5059	8.4874	8.5137
	0.6		0.7		0.8		
	$\check{\mu}^L$	$\bar{\mu}^L$	$\check{\mu}^M$	$\bar{\mu}^M$	μ^H	$\tilde{\mu}^H$	$\bar{\mu}^H$
$\bar{s}$		118.28					
s^i	79.27		98.41		132.31	133.92	
$\bar{R}$		1.63					
R^i	1.72		1.61		1.71	1.58	
V^i	**7.5415**	7.5411	7.8398	7.8507	8.1815	8.1632	8.1809

Table 16: Pooling Equilibrium with gamma M 2.00 and gamma H 2.00

response to the pooling contract offered to them is a first negative externality (for high-risk individuals this negative externality is completely compensated by the fact that the rate of return of the pooling contract is higher than the rate of return of the separating contract) and the presence of higher risk individuals exerts a second negative externality on lower risk individuals. These negative externalities are not purely destructive because while low-risk and medium-risk individuals are worse off than they would be in the absence of private information, high-risk individuals are better off.

Given these negative externalities the investigation of possible Pareto improving policies which either Social Planner or individuals through a Majority Rule can undertake is interesting.

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