QUADERNI DEL DIPARTIMENTO DI SCIENZE ECONOMICHE E SOCIALI

PANEL COINTEGRATION TESTS: A REVIEW

Laura Barbieri

Serie Rossa: Economia – Quaderno N. 44 novembre 2006

UNIVERSITÀ CATTOLICA DEL SACRO CUORE

PIACENZA

I QUADERNI
Possono essere richiesti a:

Dipartimento di Scienze Economiche e Sociali, Università Cattolica, Via Emilia Parmense 84, 29100 Piacenza tel. 0523 599342. Oppure si può ottenere una copia dall'area di download del Dipartimento al seguente indirizzo: **http://www.unicatt.it/dipartimenti/ScEcoSoc/default.stm**

Panel Cointegration Tests: A Review

by

Laura Barbieri

Università Cattolica del Sacro Cuore, Dipartimento di Scienze Economiche e Sociali - Piacenza
E-mail address: laura.barbieri@unicatt.it

Abstract: Much research has been carried out recently on the topic of econometric non-stationary panel data, especially because of the availability of new data sets (e.g. the Penn World Tables by Summers and Heston, 1991) in which the time series dimension and the cross-section dimension are of the same order of magnitude. My previous working paper (Barbieri, 2006) presented a review of the most recent unit root tests in a panel framework. This paper does the same with the panel cointegration test literature. This kind of test has been developed to extend the unit root approach to a multivariate framework.

Of the panel cointegration tests, one can distinguish between those which verify the null hypothesis of no cointegration (Kao test, 1999; McCoskey and Kao test, 1999a, Pedroni's tests, 2004; Groen and Kleibergen test, 2003; Larsson and Lyhagen, 1999) and those which verify the null hypothesis of cointegration, such as the McCoskey and Kao test (1998).

JEL classification: C12, C22, C23

Key words: Non-stationary panel data, Panel cointegration tests.

Introduction

This paper follows on directly from my previous working paper on panel unit root tests (Barbieri, 2006). The aim of these works is to give a review of the recent main results in the econometric non-stationary panel data literature. Indeed, over the last two decades much research has been carried out in this area, and knowing and understanding progress in this field is essential for all econometricians who want to work with panel data. While Barbieri (2006) presented the recent main results in the field of panel unit root tests, this paper deals with the question of panel cointegration tests.

Unit root tests verify the stationarity of the series. However, empirical questions often concern multivariate relationships; it becomes essential to find out if a particular set of variables is cointegrated. In the time series framework, cointegration refers to the idea that if a set of variables is individually integrated of order one, it is possible that some linear combinations of these variables are stationary. In this case, the vector of slope coefficients is referred to as the cointegrating vector.[1]

Panel unit root tests can be adapted for residual-based cointegration tests by testing the series of residuals for stationarity. Unfortunately this adaptation is even more difficult because of the estimation procedure. The cointegration tests which test the null hypothesis of no cointegration must take into consideration the so-called "spurious regression" problem. Tests based on the null of cointegration must take into consideration an efficient estimation of a cointegrated relationship. Further, the concept of "pooled" estimation is different from pooling the cross-section testing results. In the case of unit root testing, most tests treat each individual cross-section independently. In the case of cointegration, treating each cross-section independently may translate into allowing for varying slopes and varying intercepts. This has strong implications for the model.

To extend unit root approaches to a multivariate framework, panel cointegration tests have been developed. These tests can be classified into two groups: tests assessing the null hypothesis of no cointegration (Kao, 1999; McCoskey and Kao, 1999a; Pedroni 1997, 1999, 2000, 2004; Groen and Kleibergen, 2003; Larsson and Lyhagen, 1999; Bai and Ng, 2004;

[1] This vector is generally not unique, and the question of how many cointegrating relationships exist between a certain set of variables is also an important issue. Here this question is not analysed in depth: it is assumed that the researcher, taken as given a particular normalization process among the variables, is only interested in discovering if they are cointegrated. In this case, it is necessary recall that conventional tests often suffer from very low power when applied to series of moderate length.

Choi, 2001) and tests assessing the null of cointegration (McCoskey and Kao, 1998)[2]. It should be noted that the asymptotic analysis of both approaches involves the use of sequential limit arguments $(T, N \rightarrow \infty)_{\text{seq}}$.

Greater attention is required in the interpretation of the panel cointegration test, given that it raises even more difficulties than the interpretation of panel unit root tests. For example, if we consider a vector of data y_{it} (as usual, $i = 1,...,N$, $t = 1,...,T$), with r_i cointegrating relationships such as $z_{it} = \beta_i' y_{it}$ is I(0), there are a variety of possible situations: the number of cointegrating vectors, $r_i = r$, can be the same in each group; the number of cointegrating vectors can be at least $r_{\min}$; the cointegrating vectors β_i can be identical in each group. These alternatives make it difficult to interpret the possible rejection of the cointegration hypothesis.

The pre-testing problems in cointegrating models are much more severe than in the unit root tests, where the issues are just determination of lag lengths and treatment of the deterministic elements. The large number of choices involved in cointegration analysis, even in the case of a set of time-series variables, is discussed in Pesaran and Smith (1998).

This paper is organized as follows. The first section briefly presents the spurious regression problem, while section 2 reviews the cointegration test verifying the null hypothesis of no cointegration: residual tests (Kao test, 1999; McCoskey and Kao test, 1999a; several of Pedroni's tests, 2004) and a couple of likelihood-based tests (Groen and Kleibergen, 2003; Larsson and Lyhagen, 1999) are presented. Section 3 presents the McCoskey and Kao test (1998) for the null of cointegration and section 4 compares the presented tests. My conclusions close the paper.

1. Spurious regression in panel data

The spurious regression problem holds even in the panel framework. Entorf (1997) studies spurious fixed effects regressions when the true model involves independent random walks with and without drifts and finds that as $T \rightarrow \infty$ and N is finite, the nonsense regression phenomenon is relevant for spurious fixed effects models and inference based on *t*-values can be highly misleading. Kao (1999) and Phillips and Moon (1999) derive the asymptotic distribution of the least square dummy variable (*LSDV*) estimator and various conventional

[2] See Table A.1. in the Appendix for a summary of the main characteristics of these tests.

statistics from the spurious regression in panel data. They also show that the coefficient of *OLS* estimator is consistent and the asymptotics of the *OLS* estimator are very different from those of the spurious regression in pure time series. This is important for residual-based cointegration tests in panel data, because the null distribution of residual-based cointegration tests depends on the asymptotics of the *OLS* estimator.

However, there are a number of issues in tests for unit roots and cointegration in panels and their interpretation; in this framework spurious regression seems to be less of a problem because it is reduced by averaging.

In the time series framework, it has been shown (Granger and Newbold, 1974; Phillips, 1986) that, for given i, regressing a non-stationary variable y_{it} on a vector of non-stationary variables x_{it} may lead to spurious regression results. In fact, in a spurious regression the *OLS* estimator converges to a random variable. This means that the *OLS* estimator is not consistent and the *t*-statistic diverges. As a consequence a spurious regression may show an apparently significant relationship between the variables even if they are generated independently.

In contrast, as Phillips and Moon (1999) note, if we suppose that there are panel observations of y_{it} and x_{it} with large cross-sectional and time series components, the noise can often be characterized as independent across individuals even if the noise in the time series regression is strong. Hence, by pooling the cross-section and time series observations, we may smooth the strong effect of the residuals in the regression while retaining the strength of the signal (x_{it}). In such a case, we can expect a panel-pooled regression to provide a consistent estimate of some long-run regression coefficient.

Then, Phillips and Moon (1999) show that panel data methods allow to estimate a long-run relationship between several variables even in the case where the single time series dimension will lead to a spurious regression. This new relation is a long-run average relationship over the cross-section and it is parameterized in terms of a matrix regression coefficient derived from the cross-section long-run average covariance matrix. However interpretation of such relation is ambiguous because, as in the time series framework, *t*-statistics diverge (Kao, 1999).

2. Null hypothesis of no cointegration

The tests verifying null hypothesis of no cointegration (Kao, 1999; McCoskey and Kao, 1999a; Pedroni 1997, 1999, 2000, 2004; Groen and Kleibergen, 2003; Larsson and Lyhagen, 1999; Bai and Ng, 2004; Choi, 2001) are based on the principle of deciding whether or not the error process of the regression equation is stationary. These tests can be divided into two groups, the residual-based tests and the likelihood-based tests, and are presented in the follow.

2.1. Residual-Based Cointegration Tests

The residual-based tests are constructed on the basis of the Engle and Granger (1987) test in time series framework and use residuals of the panel static regression to construct the test statistics and tabulate the distributions.

It is obvious that in this case obtaining good estimates of the residuals is essential to obtain good tests. Moreover, the asymptotic properties of these tests will depend on the asymptotics of the estimators.[3]

2.1.1. Kao test (1999)

Kao (1999) presents two tests for the null hypothesis of no cointegration in panel data: the *DF* and *ADF* type tests. He considers the special case where cointegration vectors are homogeneous between individuals, i.e. these tests do not allow for heterogeneity under alternative hypothesis and they cannot be applied to a bivariate system (where only one regressor is present in the cointegration relation).

Referring to the sequential limit theory, Kao shows that the asymptotic distribution of these statistics will converge to a standard normal one $\mathcal{N}(0,1)$.

Kao results are offered for the asymptotics of spurious regression within a panel data setting. The specification of the panel model allows for differing intercepts across cross-sections and common slopes. Further, the long-run variance covariance matrix is assumed to

[3] The tests assuming common slopes are derived under the assumption of a spurious regression and are based on *OLS* estimation.

be the same for all cross-section observations. Kao shows that in the panel data case the results for *LSDV* estimation are somewhat more encouraging. In fact, adding the cross-section dimension, an appropriate normalization of the estimated parameter converges in distribution to a normal, mean zero, random variable and, even though the model is misspecified, the *LSDV* estimator is consistent; however, the *t*-statistic keeps on diverging.

These asymptotic results on the spurious regression are essential for testing the null hypothesis of no cointegration. Under the null of no cointegration the residuals required for the test need to be estimated, by construction, from a spurious regression. Note that the residual based test is equivalent to testing for a unit root in the *LSDV* estimated residuals. Using the panel model, the *DF* and *ADF* test statistics, after appropriate normalizations will converge in distribution to random variables with normal distributions.

Kao presents two sets of specifications for the *DF* test statistics. The first set of test statistics depends directly on consistent estimation of long-run parameters. The second set of test statistics does not.

Kao considers the following model:

$$y_{it} = \alpha_i + \beta\, x_{it} + e_{it}\,,\ i = 1,\ldots,N\,,\ t = 1,\ldots,T \tag{2.1.1.1}$$

$$y_{it} = y_{it-1} + u_{it} \tag{2.1.1.2}$$

$$x_{it} = x_{it-1} + \varepsilon_{it} \tag{2.1.1.3}$$

where α_i are the fixed effects varying across the cross-section observations, β is the slope parameter common across i and u_{it} are constant terms. Note that since both y_{it} and x_{it} are random walks, under the null hypothesis of no cointegration, the residual series e_{it} should be non-stationary.

Before introducing the test, we define the lung-run covariance matrix of $w_{it} = (u_{it}, \varepsilon_{it})'$ [4], as:

$$\Omega = \lim_{T\to\infty} \frac{1}{T} E\left(\sum_{t=1}^{T} w_{it}\right)\left(\sum_{t=1}^{T} w_{it}\right)' = \Sigma + \Gamma + \Gamma' = \begin{bmatrix} \sigma_{0u}^2 & \sigma_{0u\varepsilon} \\ \sigma_{0u\varepsilon} & \sigma_{0\varepsilon}^2 \end{bmatrix},$$

where

$$\Gamma = \lim_{T\to\infty} \frac{1}{T} \sum_{k=1}^{T-1} \sum_{t=k+1}^{T} E\left(w_{it} w_{it-k}'\right) = \begin{bmatrix} \Gamma_u & \Gamma_{u\varepsilon} \\ \Gamma_{\varepsilon u} & \Gamma_{\varepsilon} \end{bmatrix}$$

[4] Note that w_{it} is a bivariate innovation process with zero mean vector. The definition of the standard long-run variance-covariance matrix is the standard definition used in the time series literature.

and

$$\Gamma = \lim_{T\to\infty}\frac{1}{T}\sum_{k=1}^{T} E\left(w_{it} w_{it}'\right) = \begin{bmatrix} \sigma_u^2 & \sigma_{u\varepsilon} \\ \sigma_{u\varepsilon} & \sigma_\varepsilon^2 \end{bmatrix}.$$

In this framework, Σ, can be thought as the contemporaneous correlation and Γ as the correlation across time. A special case of this long-run relationship is when $\Gamma = 0$ (i.e. $\sigma_u^2 = \sigma_{0u}^2 = \sigma_v^2 = \sigma_{0v}^2$) which means strong exogeneity and no serial correlation.

Kao suppose that $\left(1/\sqrt{T}\right)\sum_{t=1}^{Tr} w_{it} \Rightarrow B_i(\Omega)$ for all i as $T \to \infty$ where $B_i(\Omega)$ is a vector Brownian motion with asymptotic covariance Ω.

Both tests proposed by Kao can be calculated from the estimated residuals of (2.1.1.1), $\hat{e}_{it}$, as:

$$\hat{e}_{it} = \rho\, \hat{e}_{it-1} + \sum_{j=1}^{p} \theta_j \Delta\hat{e}_{it-j} + v_{itp} \tag{2.1.1.4}$$

where the lags are added in the specification to take care of possible autocorrelation and the number lags, p, is chosen such that the residuals v_{itp} are serially uncorrelated with past errors[5].

In order to test the null hypothesis of no cointegration, the null can be written as $H_0 : \rho = 1$ against the alternative $H_a : \rho < 1$. The *OLS* estimate of ρ is given by:

$$\hat{\rho} = \frac{\sum_{i=1}^{N}\sum_{t=2}^{T}\hat{e}_{it}\hat{e}_{it-1}}{\sum_{i=1}^{N}\sum_{t=2}^{T}\hat{e}_{it-1}^2}. \tag{2.1.1.5}$$

Asymptotically:

$$T\sqrt{N}\left(\hat{\rho}-1\right) - \sqrt{N}\frac{\mu_{3T}}{\mu_{4T}} \sim \mathcal{N}\left(0\,,3+\frac{36\sigma_v^4}{5\sigma_{0v}^4}\right) \tag{2.1.1.6}$$

and

$$t_\rho - \frac{\sqrt{N}\mu_{3T}}{s\sqrt{\mu_{4T}}} \sim \mathcal{N}\left(0\,,\frac{\sigma_{0v}^2}{2\sigma_v^2} + \frac{3\sigma_v^2}{10\sigma_{0v}^2}\right) \tag{2.1.1.7}$$

where

$$\mu_{3T} = E\left[\frac{1}{T}\sum_{t=2}^{T}\hat{e}_{it-1}\Delta\hat{e}_{it}\right],\ \mu_{4T} = E\left[\frac{1}{T^2}\sum_{t=2}^{T}\hat{e}_{it-1}^2\right]$$

[5] In the case of *DF* test all $\theta_j = 0$.

Now, let X_{ip} be the matrix of observations on the p regressors $(\Delta\hat{e}_{it-1},...,\Delta\hat{e}_{it-p})$ such that $Q_i = I - X_{ip}(X'_{ip}X_{ip})^{-1}X'_{ip}$, and let e_i be a vector of $\hat{e}_{it-1}$ with $s_v^2 = (1/NT)\sum_{i=1}^{N}\sum_{t=2}^{T}(\hat{e}_{it} - \hat{\rho}\,\hat{e}_{it-1})^2 = (1/NT)\sum_{i=1}^{N}\sum_{t=1}^{T}\hat{v}_{itp}^2$. The *ADF* test statistic for the null hypothesis of no cointegration is based on the following *t*-statistic:

$$t_{ADF} = (\hat{\rho}-1)\frac{\left[\sum_{j=1}^{N}(e'_iQ_ie_i)\right]^{1/2}}{s_v}. \tag{2.1.1.8}$$

Kao shows that asymptotically:

$$t_{ADF} - \frac{\sqrt{N}\mu_{5T}}{s_v\sqrt{\mu_{6T}}} \sim \mathcal{N}\left(0\,,\frac{\sigma_{0v}^2}{2\sigma_0^2} + \frac{3\sigma_v^2}{10\sigma_{0v}^2}\right) \tag{2.1.1.9}$$

with

$$\mu_{5T} = E\left[\frac{1}{T}e'_iQ_iv_i\right],\ \mu_{6T} = E\left[\frac{1}{T^2}e'_iQ_ie_i\right].$$

The limiting distributions in (2.1.1.6), (2.1.1.7) and (2.1.1.9) are normal distributions with zero mean and contain nuisance parameters ($\mu_{3T},\mu_{4T},\mu_{5T},\mu_{6T},\sigma_v^2$ and σ_{0v}^2) that represent possible long-run weak exogeneity and serial correlation in the errors. As in the time series literature, consistent estimates of these long-run parameters are required and they would be based on the long-run variance-covariance matrix of w_{it}. Let $\hat{\sigma}_v^2$ and $\hat{\sigma}_{0v}^2$ be the estimates of σ_v^2 and σ_{0v}^2 respectively and note that, as $(T,N\to\infty)_{\mathrm{seq}}$:

$$\mu_{3T} \xrightarrow{p} -\sigma_v^2/2,\ \mu_{4T} \xrightarrow{p} -\sigma_{0v}^2/6$$

$$\mu_{5T} \xrightarrow{p} -\sigma_{0v}^2/2,\ \mu_{6T} \xrightarrow{p} \sigma_{0v}^2/6.$$

Now, it is possible to define some new statistics:

$$DF_\rho^* = \frac{T\sqrt{N}(\hat{\rho}-1) + \frac{3\sqrt{N}\hat{\sigma}_v^2}{\hat{\sigma}_{0v}^2}}{\sqrt{3 + \frac{36\hat{\sigma}_v^4}{5\hat{\sigma}_{0v}^4}}} \sim \mathcal{N}(0,1), \tag{2.1.1.10}$$

$$DF_t^* = \frac{t_\rho + \frac{\sqrt{6N}\hat{\sigma}_v}{2\hat{\sigma}_{0v}}}{\sqrt{\frac{\hat{\sigma}_{0v}^2}{2\hat{\sigma}_v^2} + \frac{3\hat{\sigma}_v^2}{10\hat{\sigma}_{0v}^2}}} \sim \mathcal{N}(0,1) \quad (2.1.1.11)$$

$$ADF^{\mathrm{K}} = \frac{t_{ADF} + \frac{\sqrt{6N}\hat{\sigma}_v}{2\hat{\sigma}_{0v}}}{\sqrt{\frac{\hat{\sigma}_{0v}^2}{2\hat{\sigma}_v^2} + \frac{3\hat{\sigma}_v^2}{10\hat{\sigma}_{0v}^2}}} \sim \mathcal{N}(0,1) \quad (2.1.1.12)$$

whose limiting distributions by the sequential limit $(T, N \to \infty)_{\mathrm{seq}}$ do not depend on the nuisance parameters.

When the case of strong exogeneity and no serial correlation (i.e. $\sigma_u^2 = \sigma_{0u}^2 = \sigma_v^2 = \sigma_{0u}^2$) is considered, the (2.1.1.6) and (2.1.1.7) can be re-written as:

$$T\sqrt{N}(\hat{\rho}-1) - \sqrt{N}\frac{\mu_{5T}}{\mu_{6T}} \sim \mathcal{N}\left(0, \frac{51}{5}\right) \quad (2.1.1.13)$$

and

$$\sqrt{\frac{5}{4}}\left(t_\rho - \frac{\sqrt{N}\mu_{5T}}{\sqrt{\mu_{6T}}}\right) \sim \mathcal{N}(0,1) \quad (2.1.1.14)$$

and the bias-corrected test statistics become

$$DF_\rho = \frac{T\sqrt{N}(\hat{\rho}-1) + 3\sqrt{N}}{\sqrt{10.2}} \sim \mathcal{N}(0,1), \quad (2.1.1.15)$$

$$DF_t = \sqrt{1.25}t_\rho + \sqrt{1.875}N \sim \mathcal{N}(0,1). \quad (2.1.1.16)$$

These tests do not require estimates of the long-run variance-covariance matrix as the others do.

Kao compares the previous five residual based tests through Monte Carlo simulation using one-sided standard normal critical values and shows that:

- when T and N are small, all tests have little power;

- when T is small (e.g. $T = 10$) and N is large, all tests have a large size distortion and little power;

- when T increases to at least 25 for all N, the size distortion begins to disappear quickly and DF_ρ^* test dominates DF_t^* and ADF tests in terms of power.

In general, DF_{ρ}^{*} and DF_{t}^{*} tests outperform the other tests in terms of size and power properties and DF_{ρ} and DF_{t} are substantially robust despite the model misspecification due to their independence from the estimation of long-run parameters. Specifically, in terms of robustness of the tests across different specifications, *ADF* and *DF* statistics do not perform very well and their distribution can be far from the standard normal distributions predicted by the theory. This is because of the dependence of this statistics on the long-run parameters and of the difficulty to obtain good results for the long-run estimates in the sample sizes feasible for applied research work.

These problems are a consequence of the additional cross-section dimension, that however allows to "smooth" the limiting distribution into a normal distribution.

Kao proposes a non-parametric correction to the *ADF* *t*-statistic test which can take the advantages of the normal distribution, but also cleanse the limiting distribution. He observes that t_{ADF} can be re-written as:

$$t_{ADF}^{\bullet} = \frac{\sqrt{N}\xi_{5T}}{s_v \sqrt{\xi_{6T}}}$$

where $\xi_{5NT} = (1/N)\sum_{i=1}^{N} \zeta_{5iT}$, $\zeta_{5iT} = (1/T)\left(e_i^{*'} Q_i^{*} v_i\right)$, $\xi_{6NT} = (1/N)\sum_{i=1}^{N} \zeta_{6iT}$ and $\zeta_{6iT} = \left(1/T^2\right)\left(e_i^{*'} Q_i^{*} e_i^{*}\right)$.

It is then possible to make the following adjustments:

$$\frac{\sigma_v}{s_v}\left(\frac{\sqrt{N}(\xi_{5T} - \lambda)}{\sigma_{ov}\sqrt{\xi_{6T}}} + \frac{\sqrt{6N}}{2}\right) \sim \mathcal{N}\left(0, \frac{4}{5}\right) \tag{2.1.1.17}$$

where $\lambda = \left(\sigma_{0v}^2 - \sigma_v^2\right)/2$. Note that the distribution in (2.1.1.17) does not depend on nuisance parameters.

2.1.2. McCoskey and Kao (1999a) test of no cointegration

In his models Kao (1999) hypothesizes common slopes across the cross-sections. McCoskey and Kao (1999a) relax this assumption and propose two tests (an average *ADF* test and an average Phillips Z_t statistic) for the null of no cointegration with varying slopes and intercepts across the cross-sectional observations.

For the *average ADF test*, they follow the Im, Pesaran and Shin (2003) –*IPS* thereafter- approach to the unit root test and present a cointegration test based on the average of the *ADF* statistics of the cross-sections. They consider the model (2.1.1.1):

$$y_{it} = \alpha_i + x'_{it}\beta_i + e_{it}\,,\ i = 1,...,N\,,\ t = 1,...,T \tag{2.1.2.1}$$

with

$$y_{it} = y_{it-1} + u_{it} \tag{2.1.2.2}$$

$$x_{it} = x_{it-1} + \varepsilon_{it}\,. \tag{2.1.2.3}$$

Each cross-section regression (which allows its individual cointegrating vector) is estimated separately. Then, single statistics are constructed under the assumption of cross-sections independence of each other statistic and heteroskedasticity across the cross-sections. The pooling from the panel is done in the final step where the panel test statistic is constructed averaging the individual cross-section statistics.

The *ADF* test can be constructed as:

$$\hat{e}_{it} = \rho_{\mathrm{i}}\hat{e}_{it-1} + \sum_{j=1}^{p}\theta_{ij}\Delta\hat{e}_{it-j} + v_{itp}\,, \tag{2.1.2.4}$$

where $\hat{e}_{it}$ are *OLS* residuals from (2.1.2.1). The (2.1.2.4) equation can be also written as:

$$\Delta\hat{e}_{it} = \rho_{\mathrm{i}}\hat{e}_{it-1} + \sum_{j=1}^{p}\theta_{ij}\Delta\hat{e}_{it-j} + v_{itp}\,.^{6} \tag{2.1.2.5}$$

The null hypothesis of interest is $H_0 : \rho_i = 0$ and the *t*-statistic for each i results:

$$t_{ADF\,i} = \frac{\left(\hat{u}'_{-1}Q_{xp}\hat{u}_{-1}\right)^{1/2}\hat{\rho}_i}{s_v}\,, \tag{2.1.2.6}$$

where $\hat{u}_{-1}$ is the vector of observations of $\hat{u}_{t-1}$, $Q_{X_p} = I - X_p\left(X'_pX_p\right)X'_p$ with X_p matrix of observations on the p regressors $\left(\Delta\hat{u}_{t-1},...,\Delta\hat{u}_{t-p}\right)$, and $s_v^2 = \frac{1}{T}\sum_{t=1}^{T}\hat{v}_{tp}^2$.

Since, the $t_{ADF\,i}$ converges to a functional of Brownian motion (Phillips and Ouliaris, 1990) and are cross-sectionally independent, it is possible to calculate:

$$\bar{t}_{ADF} = \frac{1}{N}\sum_{i=1}^{N}t_{ADF\,i}\,. \tag{2.1.2.7}$$

Using the logic from Phillips and Moon (1999), McCoskey and Kao (1999a) show that:

$$ADF_{\bar{t}}^{\mathrm{MK}} = \sqrt{N}\left(\bar{t}_{ADF} - \mu_{ADF}\right) \sim \mathcal{N}\left(0\,,\sigma_{ADF}^2\right), \tag{2.1.2.8}$$

[6] See Phillips and Ouliaris (1990).

that is, the limiting distribution of the ADF_t^{MK} test statistic is free of nuisance parameters and depends only in the number of regressors.

The moments μ_{ADF} and σ_{ADF}^2 can be obtained through simulation following the Phillips and Ouliaris (1990) logic in the time series case.

The second test proposed by McCoskey and Kao (1999a) is based on the average, across the cross-sections, of the Phillips Z_t statistics. This statistic is, by definition, for the varying intercepts and varying slopes model.

In order to calculate the Phillips Z_t test, McCoskey and Kao follow the procedure illustrated by Phillips and Ouliaris (1990). Firstly, as for the *ADF* test, they calculate the estimated residuals from the original regression equation (2.1.2.1) using *OLS*. Then, these estimated residuals, $\hat{e}_{it}$, are used to perform the following regression:

$$\hat{e}_{it} = \alpha_i \hat{e}_{it-1} + v_{it}, \tag{2.1.2.9}$$

which is similar to the *ADF* test without lagged terms and whose error terms v_{it} can depend on cross-correlation and autocorrelation.

If we define:

$$s_{iv}^2 = \frac{1}{T}\sum_{t=1}^{T} \hat{v}_{it}^2 \tag{2.1.2.10}$$

and

$$s_{iTl}^2 = \frac{1}{T}\sum_{i=1it}^{T} \hat{v}_{it}^2 + \frac{2}{T}\sum_{s=1}^{l} w_{sl} \sum_{t=s+1}^{T} \hat{v}_{it}\hat{v}_{it-s}, \tag{2.1.2.11}$$

the final statistic can be expressed as:

$$\overline{Z}_{it} = \frac{(\hat{\alpha}-1)}{\dfrac{s_{iTl}}{\left(\sum_{t=2}^{T} \hat{e}_{it-1}^2\right)^{\frac{1}{2}}}} - \frac{\frac{1}{2}\left(s_{iTl}^2 - s_{iv}^2\right)}{s_{iTl}\left(\dfrac{1}{T^2}\sum_{t=2}^{T} \hat{e}_{it-1}^2\right)^{\frac{1}{2}}}. \tag{2.1.2.12}$$

This statistic converges in distribution to the same functional of Brownian motion as the *ADF t*-statistic (Phillips and Ouliaris, 1990) and uses the same simulated moments.

Now, define as $\overline{Z}_t = (1/N)\sum_{i=1}^{N} \overline{Z}_{it}$ the average of the cross-section $\overline{Z}_{it}$ statistics, it can be shown that:

$$ADF_{\bar{Z}}^{\mathrm{MK}} = \sqrt{N}\left(\bar{Z}_t - \mu_{ADF}\right) \sim \mathcal{N}\left(0\,, \sigma_{ADF}^2\right).^{7} \qquad (2.1.2.13)$$

2.1.3. Pedroni tests (2004)

Pedroni (1999, 2004) proposes a residual-based test for the null of cointegration for dynamic panels with multiple regressors in which the short-run dynamics and the long-run slope coefficients are permitted to be heterogeneous across individuals. The test allows for individual heterogeneous fixed effects and trend terms and no exogeneity requirements are imposed on the regressors of the cointegrating regressions.

In earlier versions of his work Pedroni (1995, 1997) also studied the properties of tests for the null of no cointegration in panels with homogeneous cointegrating vectors β_i. He showed that in this case and with strictly exogenous regressors, under the null the distribution for residual-based tests is asymptotically equivalent to the distribution for raw panel unit root tests even if the residuals are estimated. With endogenous regressors, the asymptotic equivalence result falls and a correction is needed for the asymptotic bias induced by the estimated regressor effect[8].

The difficulty with this approach arises when a common slope coefficient is hypothesized despite the fact that the true slopes are heterogeneous. In this case, the estimated residuals for any member of the panel will be non-stationary, even if in truth they are cointegrated and it is not easy to interpret the resulting test for no cointegration

This is why, Pedroni (2004) considers a set of residual-based test statistics for the null of no cointegration in the general case of fully endogenous regressors, no pooled slope coefficients and varying dynamics. The advantage of these tests is that they pool only the information regarding the possible existence of the cointegrating relationship that comes from the statistical properties of the estimated residuals.

Specifically, the tests ask for the residuals estimation from static cointegrating long-run relation for a time series panel of observables y_{it}:

$$y_{it} = \alpha_i + \delta_i t + \beta_{1i} x_{1it} + \beta_{2i} x_{2it} + \ldots + \beta_{Ki} x_{Kit} + e_{it} \qquad (2.1.3.1)$$

$$i = 1,\ldots,N\ ,\ t = 1,\ldots,T\ ,\ k = 1,\ldots,K$$

[7] Also Pedroni (1997) considers a version of the average of the $\bar{Z}_{it}$ statistics.

[8] Kao (1999) examines the properties of such a test for the special case of homogeneous slope estimates and short-run dynamics.

where as usual T is the number of observations over time, N is the number of units in the panel and K is the number of regressors. It is possible to interpret the model (2.1.3.1) as N different equations, each of which has K regressors. The variables y_{it} and x_{it} are assumed to be I(1), for each member i of the panel, and under the null of no cointegration the residual e_{it} will also be I(1). α_i and δ_i are scalars denoting fixed effects and unit-specific linear trend parameters, respectively and β_i are the cointegration slopes; note that all this coefficients are permitted to vary across individuals, so that considerable heterogeneity is allowed by this specification.

Pedroni considers the use of seven residual-based panel cointegration statistics, four based on pooling the data along the within-dimension (denoted 'panel cointegration statistics') and three based on pooling along the between-dimension (denoted 'group mean cointegration statistics').

Computationally, the former are constructed by summing both the numerator and the denominator terms over the N dimension separately (i.e. they are based on estimators that effectively pool the autoregressive coefficient across different members for the unit root tests on the estimated residuals), whereas the latter are constructed by first dividing the numerator by the denominator prior to summing over the N dimension (i.e. they are based on estimators that simply average the individually estimated coefficients for each member i).

Another distinction between the two sets of test is based on the alternative hypothesis specification. In fact, even if both sets of test verify the null hypothesis of no cointegration:

$$H_0 : \rho_i = 1 \; \forall i ,$$

where ρ_i is the autoregressive coefficient of estimated residuals under the alternative hypothesis ($\hat{e}_{it} = \rho_i \hat{e}_{it-1} + u_{it}$), alternative hypothesis specification is different:

- the panel cointegration statistics impose a common coefficient under the alternative hypothesis which results:

$$H_a^w : \rho_i = \rho < 1 , \; \forall i$$

- the group mean cointegration statistics allow for heterogeneous coefficients under the alternative hypothesis and it results:

$$H_a^b : \; \rho_i < 1 \;\; \forall i \; .$$

It is evident that the tests based on the between dimension are more general allowing for cross-section heterogeneity[9].

Defining $\hat{e}_{it}$ the estimated residuals from (2.1.3.1), the seven Pedroni's statistics are:

1. Panel v-Statistic: $$Z_{\hat{v}_{NT}} = \frac{1}{\left(\sum_{i=1}^{N}\sum_{t=1}^{T}\hat{L}_{11i}^{-2}\hat{e}_{it-1}^{2}\right)}$$

2. Panel ρ-Statistic: $$Z_{\hat{\rho}_{NT}-1} = \frac{\sum_{i=1}^{N}\sum_{t=1}^{T}\hat{L}_{11i}^{-2}\left(\hat{e}_{it-1}\Delta\hat{e}_{it} - \hat{\lambda}_i\right)}{\left(\sum_{i=1}^{N}\sum_{t=1}^{T}\hat{L}_{11i}^{-2}\hat{e}_{it-1}^{2}\right)}$$

3. Panel t-Statistic (non-parametric): $$Z_{t_{NT}} = \frac{\sum_{i=1}^{N}\sum_{t=1}^{T}\hat{L}_{11i}^{-2}\left(\hat{e}_{it-1}\Delta\hat{e}_{it} - \hat{\lambda}_i\right)}{\sqrt{\tilde{\sigma}_{NT}^{2}\left(\sum_{i=1}^{N}\sum_{t=1}^{T}\hat{L}_{11i}^{-2}\hat{e}_{it-1}^{2}\right)}}$$

4. Panel t-Statistic (parametric): $$Z_{t_{NT}}^{*} = \frac{\sum_{i=1}^{N}\sum_{t=2}^{T}\hat{L}_{11i}^{-2}\hat{e}_{it-1}^{*}\Delta\hat{e}_{it}^{*}}{\sqrt{\tilde{s}_{NT}^{*2}\left(\sum_{i=1}^{N}\sum_{t=2}^{T}\hat{L}_{11i}^{-2}\hat{e}_{it-1}^{*2}\right)}}$$

5. Group ρ-Statistic: $$\tilde{Z}_{\hat{\rho}_{NT}-1} = \sum_{i=1}^{N}\frac{\sum_{t=1}^{T}\left(\hat{e}_{it-1}\Delta\hat{e}_{it} - \hat{\lambda}_i\right)}{\left(\sum_{t=1}^{T}\hat{e}_{it-1}^{2}\right)}$$

6. Group t-Statistic (non-parametric): $$\tilde{Z}_{t_{NT}} = \sum_{i=1}^{N}\frac{\sum_{t=1}^{T}\left(\hat{e}_{it-1}\Delta\hat{e}_{it} - \hat{\lambda}_i\right)}{\sqrt{\hat{\sigma}_i^{2}\left(\sum_{t=1}^{T}\hat{e}_{it-1}^{2}\right)}}$$

7. Group t-Statistic (parametric): $$Z_{t_{NT}}^{*} = \sum_{i=1}^{N}\frac{\sum_{t=1}^{T}\hat{e}_{it-1}^{*}\Delta\hat{e}_{it}^{*}}{\sqrt{\sum_{t=2}^{T}\tilde{s}_i^{*2}\hat{e}_{it-1}^{*2}}}$$

The nuisance parameter estimator $\hat{L}_{11i}^{2}$ is the member-specific long-run conditional variance for the residuals. If $\Omega_i = \lim_{T\to\infty} E\left[T^{-1}\left(\sum_{t=1}^{T}\Delta z_{it}\right)\left(\sum_{t=1}^{T}\Delta z_{it}\right)'\right]$ is the long-run covariance matrix for the partitioned vector of differenced unit root series $\Delta z_{it} = \left(\Delta y_{it}, \Delta x_{it}'\right)'$, $\hat{L}_i$ is the lower triangular Cholesky composition of $\hat{\Omega}_i$ and $\hat{L}_{11i}^{2}$ is given as $\hat{L}_{11i}^{2} = \hat{\Omega}_{11i} - \hat{\Omega}_{21i}\hat{\Omega}_{22i}^{-1}\hat{\Omega}_{21i}'$, where $\hat{\Omega}_i$ is any consistent estimator of Ω_i[10].

---

[9] Note the analogy with the Levin, Lin and Chu (2002) and *IPS* tests in terms of the heterogeneity permitted under the alternative hypothesis: in the former case for the root of the raw time series, the autoregressive coefficient in the estimated residuals for the latter.

[10] $\hat{L}_{11i}^{2}$ can also be interpreted as a conditional asymptotic variance based on the projection of Δy_{it} onto Δx_{it}. Consequently, $\hat{L}_{11i}^{2}$ can be estimated by regressing Δy_{it} onto the vector Δx_{it} and computing the asymptotic variance of the residuals of this regression, using -for example- the Newey-West (1987) estimator.

The remaining nuisance parameter estimators are respectively defined as $\hat{\lambda}_i = (1/2)(\hat{\sigma}_i^2 - \hat{s}_i^2)$, $\tilde{\sigma}_{NT}^2 = N^{-1}\sum \hat{L}_{11i}^{-2}\hat{\sigma}_i^2$, $\tilde{s}_{NT}^{*2} = N^{-1}\sum_{i=1}^{N}\hat{s}_i^{*2}$ and, being $\hat{\sigma}_i^2$ and $\hat{s}_i^2$ ($\hat{s}_i^2 = T^{-1}\sum_{t=1}^{T}\hat{u}_{it}^2$) the individual long-run and contemporaneous variances respectively of the residuals $\hat{u}_{it}$ ($\hat{u}_{it} = \hat{e}_{it} - \rho_i\hat{e}_{it-1}$) and $\hat{s}_i^*$ the standard contemporaneous variance of the residual from the *ADF* regression (then $\tilde{s}_{NT}^{*2}$ is simply the contemporaneous panel variance estimator).

It is straightforward to observe that the first category of four statistics includes a type of non-parametric variance ratio statistic, a panel version of a non-parametric Phillips and Perron (1988) ρ-statistic, a non-parametric form of the average of the Phillips and Perron *t*-statistic and an *ADF* type *t*-statistic.

The second category of panel cointegration statistics is based on a group mean approach and includes a Phillips and Perron type ρ-statistic, a Phillips and Perron type *t*-statistic and an *ADF* type *t*-statistic. The comparative advantage of each of these statistics will depend on the underlying data-generating process.

To obtain the test statistics, following steps are required.

- Firstly, the residuals $\hat{e}_{it}$ for the estimated panel cointegration regression (2.1.3.1) are calculated.
- In the second step, the residuals for the differenced regression $\Delta y_{it} = b_{1i}\Delta x_{1it} + b_{2i}\Delta x_{2it} + ... + b_{Ki}\Delta x_{Kit} + \eta_{it}$ are obtained to compute $\hat{L}_{11i}^2$ as the long-run variance of $\hat{\eta}_{it}$ using any kernel estimator (i.e. the Newey-West (1987) estimator).
- Then the residuals $\hat{e}_{it}$ are used to estimate the appropriate autoregression:

(a) for the non-parametric statistics, the regression $\hat{e}_{it} = \rho_i\hat{e}_{it-1} + \hat{u}_{it}$ is estimated to compute the long-run variance $\hat{\sigma}_i^2$ (i.e. the simple variance of the residuals $\hat{u}_{it}$). The term $\hat{\lambda}_i$ can then be computed as $\hat{\lambda}_i = (1/2)(\hat{\sigma}_i^2 - \hat{s}_i^2)$ where $\hat{s}_i^2$ is just the simple variance of $\hat{u}_{it}$[11].

(b) For the parametric statistics, the regression $\hat{e}_{it} = \hat{\rho}_i\hat{e}_{it-1} + \sum_{k=1}^{K}\hat{\rho}_{ik}\Delta\hat{e}_{it-k} + \hat{u}_{it}^*$ is estimated; its residuals are used to compute the simple variance of $\hat{u}_{it}^*$ denoted $\hat{s}_{it}^{*2}$.

Finally, the appropriate mean and variance adjustment terms reported in Pedroni (1999) have to be applied.

[11] These error correction terms are the same as the usual correction terms that enter into the conventional single-equation Phillips-Perron (1988) tests.

To discuss the distribution of the tests, the $(K \times 1)$-dimensional vector of observations $z_{it} = (y_{it}, x_{it})'$ is considered, such that under the null hypothesis of no cointegration, the true process z_{it} is generated as:

$$z_{it} = z_{it-1} + \xi_{it} . \tag{2.1.3.2}$$

The error term in the above equation $\xi'_{it} = \left(\xi^{y}_{it}, \xi^{x'}_{it}\right)'$ is a $(K \times 1)$-dimensional stationary ARMA process which satisfies $\left(1/\sqrt{T}\right)\sum_{t=1}^{[Tr]}\xi_{it} \Rightarrow B_i(\Omega_i)$[12] for each i as $T \to \infty$, where $B_i(\Omega_i)$[13] is a vector Brownian motion. The $(K \times K)$-dimensional asymptotic covariance matrix Ω_i of this Brownian motion is (partitioned conformably) such that the $(K-1)\times(K-1)$ lower diagonal block $\Omega_{22i} > 0$ (to rule out cointegration amongst the regressors). The ξ_{it} process specification therefore imposes cross-sectional independence (excepting any common aggregate disturbances) but allows for a wide range of temporal dependence in the data. In particular, no exogeneity requirements are imposed on the regressors in (2.1.3.1).

Under these assumptions, Pedroni (1995, 1997) shows that, under appropriate normalisations based on Brownian movement functions, each test statistic has standard normal asymptotic distributions under the null hypothesis as $(T, N \to \infty)_{\text{seq}}$. The author computes critical values for the tests (Pedroni, 1999) as well.

Under the alternative hypothesis, the panel v-statistic diverges to positive infinity, and the right tail of the normal distribution is used to reject the null hypothesis of no cointegration. For the remaining six statistics, the left tail of the normal distribution is used to reject the null hypothesis.

Pedroni (1997) simulations shows that, when $T > 100$, statistics have the same power. For little samples ($T < 20$), the most powerful test is the *ADF* test based on the between dimension (*group t-statistic*).

---

[12] $r \in [0,1]$ and $[Tr]$ denotes the integer part of Tr. Note that under this condition the standard functional central limit theorem holds individually for each member series as T grows large.
[13] The $B_i(\Omega_i)$ are taken to be defined on the same probability space for all *i*.

2.2. Likelihood-Based Cointegration Tests

Now two *LR* tests for the null of no cointegration, inspired at the Johansen (1991, 1996) work, are presented: the Groen and Kleibergen (2003) test and the Larsson and Lyhagen (1999) test.

2.2.1. Groen and Kleibergen test (2003)

Groen and Kleibergen (2003) propose a likelihood-based framework for the panel cointegration analysis of a fixed number of vector error correction (VEC) models. They calculate maximum likelihood estimators of the cointegrating vectors using iterated *GMM* estimators and construct likelihood ratio statistics to test for a common cointegration rank across the individual VEC models, both with heterogeneous and homogeneous cointegrating vectors.

The test is based on the *trace test* proposed by Johansen (1991, 1996).

Firstly, they stack VEC models of the different individuals into a joint panel VEC model. Then, considering the standard time series framework for cointegration testing, they construct a restricted VEC model:

$$\Delta y_t = \begin{pmatrix} \Pi_1 & 0 & \cdots & 0 \\ 0 & \Pi_2 & & 0 \\ \vdots & & \ddots & \vdots \\ 0 & 0 & \cdots & \Pi_N \end{pmatrix} y_{t-1} + \varepsilon_t = \Pi_{\mathrm{A}}\, y_{t-1} + \varepsilon_t \tag{2.2.1.1}$$

where Δy_t, y_t and ε_t are $(Nk \times 1)$ vectors –with k equal to the number of variables-, the coefficient matrix Π_{A} has dimension $(Nk \times Nk)$. The submatrices Π_i are of dimension $(k \times k)$ for $i = 1,...,N$ and relate Δy_{it} to y_{it-1}. The disturbance vector ε_t contains the $(k \times 1)$ disturbance vectors ε_{it} for each individual VEC model and $\varepsilon_t \sim \mathcal{N}(0\,,\Omega)$ [14] with nondiagonal covariance matrix

[14] This normality assumption can be relaxed (Groen and Kleibergen, 2003).

$$\Omega = \begin{pmatrix} \Omega_{11} & \Omega_{12} & \cdots & \Omega_{1N} \\ \Omega_{21} & \Omega_{22} & \dots & \Omega_{2N} \\ \vdots & & \ddots & \vdots \\ \Omega_{N1} & \Omega_{N2} & \dots & \Omega_{NN} \end{pmatrix}.$$

The submatrix Ω_{ij} is of dimension $(k \times k)$ and $\Omega_{ij} \equiv \mathrm{cov}(\varepsilon_{it}, \varepsilon_{jt}) \neq 0$ for $i, j = 1,..., N$ [15].

Note that this specification for the model does not allow for cross-sectional dependence through the covariance matrix of the disturbance terms Ω, i.e. transitory cross-sectional dependence. However as the off-diagonal elements of the Π_{A} matrix are set equal to zero, the model is a restricted version of the model in which the off-diagonal elements are left to be estimated. These restrictions on the Π_{A} matrix impose that there is no linear dependence between the variables of individual *i* and lags of the variables of individual *j*, for $j \neq i$, i.e. no persistent cross-sectional dependence. By imposing the restrictions, cross-dependence in the panel is only allowed through the non-diagonal covariance structure of the error term Ω. Larsson and Lyhagen (1999) do allow for non-zero off-diagonal elements, which also implicitly allows for the possibility of cointegration between series of different individuals in the panel. The reason why these restrictions are imposed here is that if these restrictions are not imposed, all the off-diagonal elements of the Π_{A} matrix would also have to be estimated. This is very likely not to be efficient due to the large number of parameters that have to be estimated in the estimation procedure.

Then, Groen and Kleibergen impose also a rank reduction on the different Π_i's assuming that the cointegration rank is identical for all *N* individuals cointegration. This reduction allows for the following reduced-rank specification of the model (2.2.1.1):

$$\Delta y_t = \begin{pmatrix} \alpha_1 \beta_1' & 0 & \cdots & 0 \\ 0 & \alpha_2 \beta_2' & & 0 \\ \vdots & & \ddots & \vdots \\ 0 & 0 & \cdots & \alpha_N \beta_N' \end{pmatrix} y_{t-1} + \varepsilon_t = \Pi_{\mathrm{B}} y_{t-1} + \varepsilon_t \qquad (2.2.1.2)$$

In this framework, panel cointegration testing is identical to verifying $H_0 : \Pi_{\mathrm{B}}$ versus $H_1 : \Pi_{\mathrm{A}}$. The null is tested using *LR* test statistics denoted by $LR(\Pi_{\mathrm{B}} | \Pi_{\mathrm{A}})$. If the panel VEC models (2.2.1.1) and (2.2.1.2) are considered as being composed of *N* standard time series *k*-

[15] Because of the large number of parameters and the presence of spurious correlations, an unrestricted VEC model is not efficient estimate, even for moderate sizes of *N* and *k*: in fact, the number of parameter of the covariance matrix should be $(Nk)^2$.

variate VEC models, the asymptotic behavior of $LR(\Pi_B | \Pi_A)$ can be based on the asymptotic behavior of standard time series-based likelihood ratio cointegration rank tests defined as:

$$LR_i(r|k) \Rightarrow tr\left(\int dB_{k-r,i} B'_{k-r,i} \left[\int dB_{k-r,i} B'_{k-r,i}\right]^{-1} \int dB_{k-r,i} B'_{k-r,i}\right)$$

where $LR_i(r|k)$ is the Johansen's *LR* statistic for each of the restrictions $r = 0,...,k-1$ versus full rank k referred to the *i*-th individuals and B_{k-r} is a $(k-r)$-dimensional vector Brownian motion with an identity covariance matrix.

Groen and Kleibergen show that the limiting distribution of $LR(\Pi_B | \Pi_A)$ is invariant to the covariance matrix of the error terms (i.e. $LR(\Pi_B | \Pi_A)$ is robust with respect to the choices of the covariance matrix).

Moreover, they show that for fixed *N*:

$$LR(\Pi_B | \Pi_A) \Rightarrow \sum_{i=1}^{N} tr\left(\int dB_{k-r,i} B'_{k-r,i} \left[\int dB_{k-r,i} B'_{k-r,i}\right]^{-1} \int dB_{k-r,i} B'_{k-r,i}\right) = \sum_{i=1}^{N} LR_i(r|k)$$

as $T \to \infty$, i.e. nothing is lost by assuming that the covariance matrix has zero non-diagonal covariances and the tests based on the cross-independence ($\sum_{i=1}^{N} LR_i(r|k)$), will perform just as well (asymptotically) as the tests based on the cross-dependence, ($LR(\Pi_B | \Pi_A)$).[16]

Now, let $\overline{LR}(r|k)$ be the average of the N individual trace statistics $LR_i(r|k)$:

$$\overline{LR}(r|k) = \frac{1}{N} \sum_{i=1}^{N} LR_i(r|k).$$

Then it is possible to build:

$$\frac{\overline{LR}(r|k) - E\left[\overline{LR}(r|k)\right]}{V\left[\overline{LR}(r|k)\right]} \qquad (2.2.1.3)$$

which is asymptotically standard normally distributed as $(T, N \to \infty)_{\text{seq}}$; a central limit theorem provided that $E\left[\overline{LR}(r|k)\right]$ and $Var\left[\overline{LR}(r|k)\right]$ are bounded.

Groen and Klaibergen also show that $\overline{LR}(r|k)$ and $\overline{LR}(\Pi_B | \Pi_A)$ are equivalent for large *N* and T, where:

$$\overline{LR}(\Pi_B | \Pi_A) = \frac{1}{N} LR(\Pi_B | \Pi_A) \qquad (2.2.1.4)$$

[16] This Groen and Kleibergen’s result is fundamental for econometrics and applied econometrics. In fact, it means that there exists tests/estimators based on the cross-independence which are equivalent to tests/estimators based on the cross-dependence in nonstationary panel time series.

is the Larsson *et al.* (2001) test when N is fixed and T is large.

If a fixed N is considered:

$$\overline{LR}(\Pi_B|\Pi_A) = \frac{1}{N} LR(\Pi_B|\Pi_A)$$

$$\Rightarrow \frac{1}{N}\sum_{i=1}^{N} tr\left(\int dB_{k-r,i} B'_{k-r,i} \left[\int dB_{k-r,i} B'_{k-r,i}\right]^{-1} \int dB_{k-r,i} B'_{k-r,i} \right) = \frac{1}{N}\sum_{i=1}^{N} Z_{ki} \tag{2.2.1.5}$$

where

$$Z_{ki} = tr\left(\int dB_{k-r,i} B'_{k-r,i} \left[\int dB_{k-r,i} B'_{k-r,i}\right]^{-1} \int dB_{k-r,i} B'_{k-r,i} \right) \tag{2.2.1.6}$$

as $T \to \infty$. Since $B_{k-r,i}$ and $B_{k-r,j}$ are independent for $i \neq j$:

$$\frac{\frac{1}{N}\sum_{i=1}^{N} Z_{ki} - E\left[\frac{1}{N}\sum_{i=1}^{N} Z_{ki}\right]}{Var\left[\frac{1}{N}\sum_{i=1}^{N} Z_{ki}\right]} \sim \mathcal{N}(0,1) \tag{2.2.1.7}$$

as $N \to \infty$, which implies that:

$$\frac{\overline{LR}(\Pi_B|\Pi_A) - E\left[\overline{LR}(\Pi_B|\Pi_A)\right]}{Var\left[\overline{LR}(\Pi_B|\Pi_A)\right]} \sim \mathcal{N}(0,1) \tag{2.2.1.8}$$

as $(T, N \to \infty)_{\text{seq}}$.

2.2.2. Larsson and Lyhagen test (1999)

The Larsson *et al.* (2001) test is a *LR* panel test for the existence of a common cointegrating rank in heterogeneous panel models. This test is based on the average of the individual rank trace statistics developed by Johansen (1996) and is very similar to the *IPS*-bar statistic. In Monte Carlo simulation, Larsson *et al.* investigate the small sample properties of the standardized *LR*-bar statistic and find that the proposed test requires a large time series dimension. Even if the panel has a large cross-sectional dimension, the size of the test will be severely distorted.

Groen and Kleibergen (2003) verified that the Larsson *et al.* test is robust with respect to the cross-dependence in panel data.

Larsson and Lyhagen (1999) extend previous work by Larsson *et al.* (2001) and Groen and Kleibergen (2003). They focus on multivariate cointegration and present a more general panel cointegration model which corresponds to a restricted cointegrated vector autoregression (VAR) model. They consider the model:

$$\Delta y_t = \alpha\beta' y_{t-1} + \sum_{k=1}^{m-1} \Gamma_k \Delta y_{t-k} + \varepsilon_t$$

where the vector y_{it} of dimension $Np \times 1$ is given by stacking the *N* vectors

$$y_{it} = \left(y_{i1t}, y_{i2t,}, ..., y_{ipt}\right)', \; i = 1,...,N \;,\; t = 1,...,T \;, \tag{2.2.2.1}$$

being *N* and *T* defined as usual and being *p* the dimension of each unit. Hence y_{ijt} denotes the *i*th group, the *j*th variable at time *t*, where $j = 1,..., p$. In the same way:

$$\varepsilon_{it} = \left(\varepsilon_{i1t}, \varepsilon_{i2t,}, ..., \varepsilon_{ipt}\right)', \; i = 1,...,N \;,\; t = 1,...,T \;, \tag{2.2.2.2}$$

and the $Np \times 1$ dimensional vector ε_t is obtained by stacking the N $p \times 1$ vectors ε_{it}. The error process ε_t has a multivariate normal distribution as $\varepsilon_t \sim \mathcal{N}_{Np}(0, \Omega)$ with covariance matrix $(Np \times Np)$ given by

$$\Omega = \begin{pmatrix} \Omega_{11} & \Omega_{12} & \cdots & \Omega_{1N} \\ \Omega_{21} & \Omega_{22} & \dots & \Omega_{2N} \\ \vdots & & \ddots & \vdots \\ \Omega_{N1} & \Omega_{N2} & \dots & \Omega_{NN} \end{pmatrix}.$$

Each of the individual Ω_{ij} matrices (denoting the covariance matrix of ε_{it} with ε_{jt}) is of dimension $p \times p$. The matrices of fixed coefficients Γ_k have dimension $Np \times Np$ while the matrices α and β are both of order $Np \times \sum_{i=1}^{N} r_i$, where r_i is the rank of each unit, $0 \le r_i \le p$ and are given by:

$$\alpha = \begin{pmatrix} \alpha_{11} & \alpha_{12} & \cdots & \alpha_{1N} \\ \alpha_{21} & \alpha_{22} & \dots & \alpha_{2N} \\ \vdots & & \ddots & \vdots \\ \alpha_{N1} & \alpha_{N2} & \dots & \alpha_{NN} \end{pmatrix}, \; \beta = \begin{pmatrix} \beta_{11} & \beta_{12} & \cdots & \beta_{1N} \\ \beta_{21} & \beta_{22} & \dots & \beta_{2N} \\ \vdots & & \ddots & \vdots \\ \beta_{N1} & \beta_{N2} & \dots & \beta_{NN} \end{pmatrix}.$$

α contains the short-run coefficients (adjustment parameters) and β the lung-run coefficients each of rank r_i.

The general model (2.2.2.1) allows for simultaneous long-run relations between several variables for a panel of groups and for heterogeneous long-run cointegration relations within each group. But Larsson and Lyhagen impose the restriction $\beta_{ij} = 0 \, \forall i \neq j$ that rules out cointegrating relationships across the groups. However the model allows for an important short-run dependence between the panel groups, since α_{ij} is not restricted to zero for the $\forall i \neq j$. Specifically, the elements $\alpha_{ij}\beta'_j$ represent the short-run dependences of the changes in the series for group i that are due to the long-run equilibrium deviations in group j; the elements $\alpha_{ii}\beta'_j$ represent the short-run adjustments in group i resulting from a deviation from long-run equilibrium in group i[17]. Then, while the short term dynamics is allowed to vary over individuals, the long term dynamics is assumed to be constant.

Larsson and Lyhagen impose also that r_i is the same for each panel group, i.e. there is a common maximum rank.

It is very important to recall these two restrictions in empirical applications, since they often turn out to be inconsistent with both theory and data.

The estimation[18] of the individual cointegrating relations consists in an iterative procedure of reduced rank regressions in which each β_{ii} is estimated by concentrating out the remaining $N-1$ matrices in the β matrix. Hence, β_{11} to β_{NN} are estimated at each step, and the procedure is repeated until convergence[19].

After the estimation step, the distribution of estimated parameters and the likelihood ratio test for the cointegrating rank are derived.

Larsson and Lyhagen extend the Johansen (1996) trace statistic and propose a panel test which uses the information available in the panel data. This test is a standardized *LR*-bar statistic given by the average of the *N* individual trace statistics. Specifically, the following rank hypotheses are considered:

$$\begin{cases} H_0 : r_i = q & 0 \leq q < p \\ H_a : r_i = p & \text{for each } i \end{cases}$$

---

[17] Larsson *et al.* (2001) consider a similar heterogeneous panel data model under cointegrating restrictions, with the added assumption that no dependencies are allowed between the panel groups. This makes the model completely heterogeneous. Groen and Kleibergen (2003) relax this assumption.

[18] As *N* grows for given *T*, it may be not possible to estimate the parameters of the model. In fact, in this case only some degrees of freedom are saved, with respect to unrestricted VAR, by restricting the parameters in the off-diagonal terms of β to zero. For example, if *m*=1, the number of parameters is at most N^2p^2. With Np equations, each equation must have Np observations to give an exactly identified system. Since the right hand of equation (2.4.2.1) consists of lagged left side variables, one observation is lost and the number of time units used must be at least $T = Np + 2$.

[19] For starting values, Larsson and Lyhagen propose using the β_{ii} estimated from a standard cointegration analysis on each unit separately.

These hypothesis are verified by a likelihood type test under the block-diagonality restriction on β. The asymptotic distribution of this panel trace test statistic is a convolution of the distribution of a well known *DF* type distribution (the standard Johansen trace test statistic) and an independent χ^2 variable with $N(N-1)(p-q)q$ degrees of freedom. This means that if $r_i = 0$ the panel trace test is equal to the Johansen test; if $r_i > 0$ an additional component appears in the distribution of the panel trace statistic, which accounts for the additional zero restrictions imposed on β.

In this framework, testing for $r_i = q$ versus $r_i = p$ corresponds to testing for rank Nq versus Np in a Johansen context, whereas testing for rank in the Johansen framework would allow for all the intermediate possibilities.

Then, Larsson and Lyhagen show that the asymptotic distribution of the log *LR* test of the homogeneity hypothesis:

$$\begin{aligned} &H_0 : \beta_1 = \beta_2 = \ldots = \beta_N = \beta \\ &H_a : \beta_i \neq \beta_j \quad \text{for some } i, j \end{aligned}$$

results to be a χ^2 with $(N-1)q(p-q)$ degrees of freedom, under the null hypothesis and given the rank r, (as $T \to \infty$).[20]

Monte Carlo simulations show that the test for common cointegrating space has sufficiently good size and power properties while the test for common cointegrating rank does not. This is why, they propose to use a Bartlett (1937) corrected test statistic[21] which is found to have the desired properties, i.e. a size very close to the nominal one.

3. <u>Null hypothesis of cointegration</u>

Tests of null hypothesis of cointegration were introduced in the times series literature as a response to some critiques of the null hypothesis of no cointegration (Harris and Inder, 1994, and Shin, 1994). The null of no cointegration has the disadvantage that rejection could be caused, in many cases, by the low power of the test rather than by the true underlying nature

[20] Obviously because this is the difference of the numbers of free parameters under the different hypotheses.

[21] A simple expression for the Bartlett corrected statistic is $C_T^* = EC_T(C_\infty / EC_\infty)$, where C_T is the statistic for sample size T, C_∞ denote the asymptotic one and E is the expectation operator.

of the data. The null of cointegration is often preferred in applications where cointegration is predicted a priori by economic theory.

3.1. McCoskey and Kao test (1998)

McCoskey and Kao (1998) propose a residual-based test for null hypothesis of cointegration in the heterogeneous panels framework[22]. This test can be view as an extension of the *LM* test and the locally best invariant test for an MA unit root for time series (Harris and Inder, 1994; Shin, 1994; Leybourne and McCabe, 1994, and Kwiatowski *et al.*, 1992).

For the residual based test of the null of cointegration, an efficient estimation technique of cointegrated variables is required. In the time series framework there exist a variety of asymptotically efficient methods as the fully modified (*FM*) estimator of Phillips and Hansen (1990) and the dynamic least squares (*DOLS*) estimator (Saikkonen, 1991; Stock and Watson, 1993).

In the panel data framework, Kao and Chiang (2000) show that both the *FM* and *DOLS* methods can give asymptotically normally distributed estimators with zero means.

The considered model allows for varying slopes and intercepts[23]:

$$y_{it} = \alpha_i + x'_{it}\beta_i + e_{it}\,,\; i = 1,\ldots,N\,,\; t = 1,\ldots,T \tag{3.1.1}$$

$$x_{it} = x_{it-1} + \varepsilon_{it} \tag{3.1.2}$$

$$e_{it} = \gamma_{it} + u_{it}\,. \tag{3.1.3}$$

e_{it} is composed of two separate terms, γ_{it} and u_{it} where:

$$\gamma_{it} = \gamma_{it-1} + \theta\, u_{it} \tag{3.1.4}$$

and u_{it} are $i.i.d.\left(0\,,\sigma_{\mathrm{u}}^2\right)$. The null of hypothesis of interest is $\theta = 0$.

Independence across cross-sectional units and no cointegration amongst the regressors are assumed.

Let Ω be the long-run variance-covariance matrix[24] of $w_{it} = \left(u_{it}\,,\varepsilon'_{it}\right)'$ defined as

[22] In a similar vein, Hadri (2000) develops a test of the null of trend versus difference stationarity.

[23] The framework can be generalized to include unit-specific trend terms δ_i (McCoskey and Kao, 1999a).

[24] A critical point in this analysis is the assumption of a constant variance-covariance matrix across the cross-sectional units. As McCoskey and Kao (1998) note, the analysis could be generalized but the relaxation of this restriction, associated to the assumption of independence across *i*, will imply that the method will be equivalent to equation-by-equation estimation of the system.

$$\Omega = \begin{bmatrix} \varpi_1^2 & \varpi_{12} \\ \varpi_{21} & \Omega_{22} \end{bmatrix}.$$

McCoskey and Kao propose the test statistic which follows:

$$\overline{LM} = \frac{\frac{1}{N}\sum_{i=1}^{N}\frac{1}{T^2}\sum_{t=1}^{T} S_{it}^{+2}}{s^{+2}}, \tag{3.1.5}$$

where:

$$S_{it}^{+} = \sum_{j=1}^{t}\hat{e}_{ij} = \sum_{j=1}^{t}\left(y_{ij}^{+} - \alpha_i - \hat{\beta}_i'^{+} x_{ij}\right)$$

$$s^{+2} = \frac{1}{NT}\sum_{i=1}^{N}\sum_{t=1}^{T}\hat{e}_{it}^{+2},$$

$$y_{ik}^{+} = y_{ik} - \hat{\varpi}_{12}\hat{\Omega}_{22}^{-1}\omega_{ik},$$

$\hat{\beta}_i'^{+}$ is the *FM* estimator of β_i and $\hat{\varpi}_{1.2}^2 = \varpi_1^2 - \varpi_{12}\Omega_{22}^{-1}\varpi_{21}$ is a consistent estimator of the long-run conditional variance under the null, σ_v^2, which is used in place of s^{+2} if the residuals are estimated using the *FM* estimator.

The construction of the statistic (3.1.5) requires a consistent estimation of Ω in order to implement the non-parametric corrections. If these corrections are made, the *FM* estimator takes account of serial correlation of the residuals in (3.1.2) and the endogeneity of the regressors and provides an asymptotically unbiased estimator (Kao and Chiang, 2000)[25].

The asymptotic result for the test is:

$$\sqrt{N}\left(\overline{LM} - \mu_v\right) \sim \mathcal{N}\left(0, \sigma_v^2\right);$$

since the statistic diverges under the alternative hypothesis, large values imply rejections of the null hypothesis.

The correction factors μ_v and σ_v^2 are moments of a complex functional of Brownian motion defined in Harris and Inder (1994), which depend only on the number of regressors and can be found through Monte Carlo simulation.

The limiting distribution of $\overline{LM}$ is then free of nuisance parameters and robust to heteroscedasticity.

The asymptotics of the panel tests take advantage of the sequential limit theory which allows for indices across the two dimensions of *T* and *N*. Note that for the panel *LM* test an

[25] The *DOLS* estimator uses lagged and future differences of x_{it} to correct for these effects.

additional dimension is considered to create the partial sums of the residuals. The fact that the model here allows for varying intercepts means that each cross-section is actually estimated individually, thus the additional dimension is manageable in the asymptotics.

The McCoskey and Kao (1998) test is in the spirit of the IPS analysis, since it involves the averaging of the individual *LM* statistics across the cross-sections. Analogies with Pedroni tests can also be found (i.e. the use of nonparametric corrections) as with a number of other presented tests for the use of the mean and variance correction factors.

McCoskey and Kao (1999b) conclude that using their *LM* test does not compromise the ability of the researcher in determining the underlying nature of the data when the null hypothesis of cointegration is *more* logical than the null of no cointegration.

4. A comparison between the presented tests

McCoskey and Kao (1999b) simulations compare the size and power of different residual based tests for cointegration in heterogeneous panel data, with varying slopes and varying intercepts.

Three of the tests under consideration, based on the average *ADF* test ($ADF_{\bar{t}}^{\mathrm{MK}}$ and $ADF_{\bar{Z}}^{\mathrm{MK}}$) and Pedroni's pooled tests ($Z_{t_{NT}}$), are constructed under the null hypothesis of no cointegration. The fourth test, based on the McCoskey and Kao *LM* test ($\overline{LM}$), is constructed under the null of cointegration.

Within the former four tests (H_0: no cointegration), $ADF_{\bar{t}}^{\mathrm{MK}}$ seemingly performs the best in terms of empirical size in the varying slopes and varying intercepts case. Its size has a small range across the nominal size (0.05) for all *N* and *T*. $ADF_{\bar{Z}}^{\mathrm{MK}}$ has a strong tendency to over-reject when $T \leq 25$ and tendency to under-reject for $T \geq 25$. $Z_{t_{NT}}$ behaves as $ADF_{\bar{Z}}^{\mathrm{MK}}$ but in neither cases the problem is severe. The under-rejection problem of $Z_{t_{NT}}$ is bigger for $T > 25$ and $N > 25$. These results highlight that relative sizes of *T* and *N* can significantly impact the characteristics of the test in panel data.

In terms of power $Z_{t_{NT}}$ is the best. Power of all the tests increases as ρ decreases. Moreover, the power of the tests increases more when T grows than when *N* grows.

Then McCoskey and Kao (1999b) consider the $\overline{LM}$ test and note that, in general for fixed T and increasing N, the size of this test decreases. In terms of power, in all cases when a most powerful test could be determined, $\overline{LM}$ was the best[26].

McCoskey and Kao conclude that of the two reasons for the introduction of the test of the null hypothesis of cointegration, low power and attractiveness of the null, the introduction of the cross-section dimension of the panel solves one: all of the tests show sufficient power when used with panel data.

Banerjee *et al*. (2004) compare the Larsson and Lyhagen (1999) and Pedroni (2004) tests. Their simulations indicate that when the hypothesis of no cross-unit cointegration (i.e. block-diagonal β matrix) and equal rank across the units are satisfied, the Larsson-Lyhagen test has good size and power properties, and often yields gains in efficiency relative to the full-system analysis for the estimation of the cointegrating parameters. However, when the previous assumptions are violated, both univariate and multivariate tests displaying size distortions. When N is small the presence of cross-unit cointegration is less harmful for the single-equation tests than for the Larsson-Lyhagen test statistic.

For empirical analysis, Banerjee *et al*. (2004) suggest to use full-system estimation whenever it is possible. If this is not feasible, they recommend to conduct a preliminary unit-by-unit cointegration analysis and to test for the presence of cross-unit cointegration by the Gonzalo and Grenger test. If the null hypothesis of no cross-unit cointegration is accepted, and the unit-by-unit analysis does not indicate the presence of different rank across units, then the Larsson-Lyhagen or Pedroni test (depending on the size of N) can be applied, yielding efficiency gains in terms of higher power and lower standard errors for the estimated cointegrating coefficients.

Gutierrez (2003) considers Kao's DF_ρ and DF_ρ^* tests and Pedroni's Panel ρ-statistic and Group ρ-statistic tests which provide the best power inside the group of tests proposed by Kao (1999) and Pedroni (1999). Gutierrez (2003) shows that for an homogeneous panel, Kao's DF_ρ and DF_ρ^* tests outperform Pedroni's Panel ρ-statistic and Group ρ-statistic tests when T is small, while Pedroni's tests have higher power than Kao's when T increases. Thus when the sample grows large, the power of Pedroni tests outperforms Kao tests and both tests show better performance than *LR*-bar test proposed by Larsson *et al*. (2001).

[26] This result is obvious being $\overline{LM}$ the only test of the four derived under the null hypothesis of cointegration.

Generally the power of these tests increases when *N* or *T* or the proportion of cointegrated relationships in the panel increases.

In addition, depending on a *T*-dimension of the panel, cointegration tests can have high power when a small or high fraction of the relationships are cointegrated. This result suggests that when rejecting the null hypothesis of no cointegration for the whole panel not *all* the relationships can be really cointegrated.

Conclusions

This work is strictly related to my previous working paper on panel unit root tests (Barbieri, 2006). In this second paper I discuss the panel cointegration question and several tests are critically analyzed. Firstly, the spurious regression problem is briefly recalled. Then, the panel cointegration tests are introduced. Specifically, the tests are split into two groups: tests which verify the null hypothesis of no cointegration and tests which verify the null of cointegration. In fact, as we saw in the panel unit root case[27], it is a well known fact that the classical-hypothesis tests accept the null hypothesis unless there is strong evidence to the contrary. Then, in order to decide whether economic series are cointegrated, it is necessary to perform a test of the null hypothesis of no cointegration as well as of cointegration.

Amongst the no cointegration tests, we can distinguish between residual-based tests and likelihood-based tests. It is important to recall that in the cointegration test framework, tests are based on severe restrictions. The way forward in the longer term is either to develop tests which do not impose such severe restrictions, or to find more reasonable and testable ways of incorporating restrictions within the maximum-likelihood framework.

The panel cointegration test area is developing very rapidly. However, researchers should bear it in mind that the mechanical application of panel unit-root or cointegration tests is to be avoided. Their application requires that the hypotheses involved be interesting in the context of the empirical application, which is a question of theory rather than statistics.

[27] See the "stationary test" section in Barbieri (2006).

References

[1] Bai, J. and S. Ng (2004), "A Panic Attack on Unit Roots and Cointegration", *Econometrica*, 72, 1127-1177.

[2] Barbieri, L. (2006), "Panel Unit Root Tests: A Review", Quaderni del Dipartimento di Scienze Economiche e Sociali, Serie Rossa, n.43, Università Cattolica del Sacro Cuore, Piacenza.

[3] Banerjee, A., M. Marcellino and C. Osbat (2004), "Some Cautions on the Use of Panel Methods for Integrated Series of Macro-Economic Data", *Econometrics Journal*, 7, 322-340.

[4] Bartlett, M.S. (1937), "Properties of sufficiency and statistical tests", *Proceedings of the Royal Statistical Society of London*, Series A, 160, 268-282.

[5] Choi, I. (2001), "Unit Root Tests for Panel Data", *Journal of International Money and Finance*, 20, 249–272.

[6] Engle, R. and G. Granger (1987), "Cointegration and Error-Correction: Representation, Estimation and Testing", *Econometrica*, 55, 251-76.

[7] Entfor, H. (1997), "Random Walks with Drifts: Nonsense Regression and Spurious Fixed-Effect Estimation", *Journal of Econometrics*, 80, 287-296.

[8] Granger, C. and P. Newbold (1974), "Spourious Regressions in Econometrics", *Journal of Econometrics*, 2, 111-120.

[9] Groen, Jan J.J., and F. Kleibergen (2003), "Likelihood-Based Cointegration Analysis in Panels of Vector Error-Correction Models", *Journal of Business and Economic Statistics*, 21, 295-318.

[10] Gutierrez, L. (2003), "Panel Unit Roots Tests for Cross-Sectionally Correlated Panels: A Monte Carlo Comparison", *Economics Letters*, 80, 105-111.

[11] Hadri, K. (2000), "Testing for Unit Roots in Heterogeneous Panel Data", *Econometrics Journal*, 3, 148-161.

[12] Harris, R.D.F. and B. Inder (1994), "A Test of the Null Hypothesis of Cointegration", in *Nonstationary Time Series Analysis and Cointegration*, Hargreaves C. (ed), Oxford University Press.

[13] Im K.S., M.H. Pesaran and Y. Shin (2003), "Testing for Unit Roots in Heterogeneous Panels", *Journal of Econometrics*, 115, 53-74.

[14] Johansen, S. (1991), "Estimation and Hypothesis Testing of Cointegrating Vectors in Gaussian Vector Autoregressive Models", *Econometrica*, 59, 1551-80.

[15] Johansen, S. (1996), *Likelihood-Based Inference in Cointegrated Vector Autoregressive Models* (2nd ed.), Oxford, U.K.: Oxford University Press.

[16] Kao, C. (1999), "Spurious Regression and Residual-Based Tests for Cointegration in Panel Data", *Journal of Econometrics*, 90, 1-44.

[17] Kao, C. and M.H. Chiang (2000), "On the Estimation and Inference of a Cointegrated Regression in Panel Data", in B. Baltagi (ed.), *Nonstationary Panels, Panel Cointegration, and Dynamic Panels*, Advances in Econometrics, 15, 179-222, Elsevier Science.

[18] Kwiatkowski, D., P.C.B. Phillips, P. Schmidt and Y. Shin (1992), "Testing the Null Hypothesis of Stationarity Against the Alternative of a Unit Root", *Journal of Econometrics*, 54, 91-115.

[19] Larsson, R. and J. Lyhagen (1999), "Likelihood-based Inference in Multivariate Panel Cointegration Models", *Working Paper Series in Economics and Finance*, 331, Stockholm School of Economics.

[20] Larsson, R. J. Lyhagen and M. Lothgren (2001), "Likelihood-based Cointegration Tests in Heterogeneous Panels", *Econometrics Journal*, 4, 109-142.

[21] Levin, A., C.F. Lin and C.S.J. Chu (2002) "Unit root tests in panel data: asymptotic and finite-sample properties", *Journal of Econometrics*, 108, 1-24.

[22] Leybourne, S.J. and B.P.M. McCabe (1994), "A Consistent Test for a Unit Root", *Journal of Business and Economic Statistics*, 12, 157-166.

[23] McCoskey, S. and C. Kao (1998), "A Residual-based of the Null Hypothesis of Cointegration in Panel Data", *Econometrics Reviews*, 17, 57-84.

[24] McCoskey, S. and C. Kao (1999a), "A Monte Carlo Comparison of Tests for Cointegration in Panel Data", *Journal of Propagation in Probability and Statistics*, 1, 165-198.

[25] McCoskey, S. and C. Kao (1999b), "Testing the Stability of a Production Function with Urbanization as a Shift Factor", *Oxford Bulletin of Economics and Statistics*, 61, 671-690.

[26] Newey, W. and K. West (1987), "A Simple Positive Semi-Definite, Heteroscedasticity and Autocorrelation Consistent Covariance Matrix", *Econometrica*, 50, 703-708.

[27] Pedroni, P. (1995), "Panel Cointegration; Asymptotic and Finite Sample Properties of Pooled Time Series Tests, with an Application to the PPP Hypothesis", Indiana University Working Papers in Economics, No. 95-013, June.

[28] Pedroni, P. (1997), "Panel Cointegration: Asymptotic and Finite Sample Properties of Pooled Time Series Tests With an Application to the PPP Hypothesis: New results", *Working Paper in Economics*, Indiana University.

[29] Pedroni, P. (1999), “Critical Values for Cointegration Tests in Heterogeneous Panels with Multiple Regressors”, *Oxford Bulletin of Economics and Statistics*, 61, 653-670.

[30] Pedroni, P. (2000), “Fully Modified *OLS* for Heterogeneous Cointegrated Panels”, in B. Baltagi (ed.), *Nonstationary Panels, Panel Cointegration, and Dynamic Panels*, Advances in Econometrics, 15, Elsevier Science.

[31] Pedroni, P. (2004), “Panel Cointegration: Asymptotic and Finite Sample Properties of Pooled Time Series Tests With an Application to the PPP Hypothesis”, *Econometric Theory*, 20, 597–625.

[32] Pesaran, M.H. and R.P. Smith (1998), “Structural Analysis of Cointegrating VARs”, *Journal of Economic Surveys,* 12, 471-505.

[33] Phillips, P.C.B. (1986), “Understanding Spurious Regressions in Economics”, *Journal of Econometrics*, 33, 311-340.

[34] Phillips, P.C.B. and B.E. Hansen (1990), “Statistical Inference in Instrumental Variables Regression with I(1) Processes”, *Review of Economic Studies*, 57, 99-125.

[35] Phillips, P.C.B. and H.R. Moon (1999), “Linear Regression Limit Theory for Nonstationary Panel Data”, *Econometrica*, 67, 1057-1111.

[36] Phillips, P.C.B. and S. Ouliaris (1990), “Asymptotic Properties of Residual Based Tests for Cointegration”, *Econometrica*, 58, 165-193.

[37] Phillips, P.C.B. and P. Perron (1988), “Testing for a Unit Root in Time Series Regression”, *Biometrika*, 75, 335-346.

[38] Saikkonen, P. (1991), “Asymptotically Efficient Estimation of Cointegrating Regressions”, *Econometric Theory*, 58, 1-21.

[39] Shin, Y. (1994), “A Residual-based Test of the Null of Cointegration Against the Alternative of No Cointegration”, *Econometric Theory*, 19, 91-115.

[40] Stock, J. and M. Watson (1993), “A Simple Estimator of Cointegrating Vectors in Higher Order Integrated System”, *Econometrica*, 61, 783-820.

[41] Summers, R. and A. Heston (1991), “The Penn World Table (Mark 5): An expanded set of international comparisons 1950-1988”, *Quarterly Journal of Economics*, 106, 327-368.

APPENDIX

Table A.1. Main characteristics of the panel cointegration tests.

Test	Hypotesis test	Model specification	Advantages (+) /disadvantages (-)	Properties
KAO (1999)	• null hypothesis of no cointegration • homogeneous alternative (it cannot be applied to a bivariate system)	• varying intercepts • common slopes • identical lung-run variance-covariance matrix for all of the cross-section observations • *LSDV* estimator	+ DF_ρ and DF_t are substantially robust despite the model misspecification due to their independence from the estimation of long-run parameters + a non-parametric correction for ADF^{K} takes the advantages of the normal distribution and cleanses the limiting distribution	• they are residual tests • they are *DF* and *ADF* type tests • they have standard normal limiting distributions • when *T* and *N* are small, all tests have little power • when *T* is small (e.g. *T* =10) and *N* is large, all of the tests have a large size distortion and little power • when *T* increases to at least 25 for all *N* , the size distortion begins to disappear quickly and DF_ρ^* test dominates DF_t^* and ADF^{K} in terms of power • in general, DF_ρ^* and DF_t^* tests outperform the other tests in terms of size and power properties
McCOSKEY & KAO (1999)	• null hypothesis of no cointegration • heterogeneous alternative	• varying slopes • varying intercepts • *OLS* estimator	+ the normal limiting distributions of the tests are free of nuisance parameters - the moments of the two statistics can be obtained through simulation	• they are average residual *ADF*/Phillips tests • $ADF_{\bar{t}}^{\mathrm{MK}}$ seemingly performs the best. Its size has a small range across the nominal size (0.05) for all *N* and *T* • $ADF_{\bar{Z}}^{\mathrm{MK}}$ has a strong tendency to over-reject when $T \leq 25$ and tendency to under-reject for $T \geq 25$
PEDRONI (2004)	• null hypothesis of no cointegration • heterogeneous alternative is also considered (tests based on the between dimension)	• varying dynamics • heterogeneous fixed (individual and time) effects • heterogeneous trend terms	+ no exogeneity requirements are imposed on the regressors of the cointegrating regressions + they pool only the information regarding the possible existence of the cointegrating relationships	• they are residual tests • they have standard normal limiting distributions • T>100 all statistics have the same power • T<20 most powerful is the group *t*-statistic
GROEN & KLEIBERGEN (2003)	• null hypothesis of no cointegration (test for a common cointegrating rank) • homogeneous and heterogeneous cointegrating vectors	• VEC panel model • *GMM* estimator • cointegration relationships across the groups are not allowed	- the cointegration rank is assumed to be identical for all individuals cointegration	• it is a *LR* based test • it has a normal standard limiting distribution • it is robust with respect to the choices of the covariance matrix

LARSSON & LYHAGEN (1999)	• null hypothesis of no cointegration (test for a common cointegrating rank) • heterogeneous alternative	• restricted cointegrated VAR model • heterogeneous panel data models • cointegration relationships across the groups are not allowed • short term dynamics allowed to vary over individuals • constant long term dynamics	**-** it requires large *T*: if it is not, the size is very distorted **+** robust with respect to the cross-dependence in panel data **-** a common maximum rank is assumed (these restrictions can be inconsistent with both theory and data)	• it is a *LR* based test (based on the average of the individual rank trace statistics – similar to the *IPS* test) • it has a chi squared limiting distribution • it has sufficient good size and power • using a Bartlett corrected test statistic it is possible to have a size very close to the nominal one also for the common rank test
McCOSKEY & KAO (1998)	• null hypothesis of cointegration	• varying slopes • varying intercepts • FM or *DOLS* estimators for the cointegrated variables • independence across cross-sectional units • no cointegration amongst the regressors	**+** if non-parametric corrections are implemented, the *FM* estimator takes account of serial correlation of the residuals and the endogeneity of the regressors and provides an asymptotically unbiased estimator	• it is a residual test (based on the average of the individual *LM* statistics – similar to the *IPS* test) • the limiting distribution of the test is normal, free of nuisance parameters and robust to heteroskedasticity • for fixed *T* and increasing *N*, the size of this test decreases