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OPTIMAL INFLATION WEIGHTS FOR EU COUNTRIES

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Abstract: What is the appropriate stabilization goal of monetary policy in the European Union? We would like to answer this question considering 10 EU countries, heterogeneous in the degree of price stickiness, affected by asymmetric real disturbances both on the demand and the supply side. According to the academic literature, in this environment, the optimal target is an asymmetric price index in which higher weight is given to the region where the level of price stickiness is higher. This paper has the aim of verifying this result in a more realistic European setting in which we consider a 10-region model and we use the country micro data on price stickiness.

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1 Introduction

The desirable monetary policy, according to a fair amount of consensus in the academic literature, is one that achieves a low expected value of a discounted loss function where the losses each period are a weighted average of terms quadratic in the deviation of inflation from a target rate and in some measure of output relative to potential.

The importance given to inflation stabilization is in line, not only with the ECB primary objective, imposed by the Maastricht Treaty, but also with the academic literature, according to which, within the class of sticky price models, the optimality of monetary policy, that aims at complete price stability, is found to be robust (Woodford 2003). The introduction of asymmetric shocks and heterogeneity in the degree of price stickiness opens another question to the evaluation of alternative monetary policy rules: what inflation to target. Relatively to this issue there is a discrepancy between the ECB choice, aggregate inflation, and academic results, according to which, aggregate inflation is no longer the optimal target. In the case of asymmetries, the stabilization of an appropriately defined asymmetric price index (that puts more weight on the stickier prices) is a better policy, even though is not fully optimal.

Aoki (2001) and Benigno (2004) promote these results. Aoki (2001) constructs a two-sector dynamic general equilibrium model with a flexible-price sector and a sticky-price sector and shows that the optimal monetary policy, characterized as an inflation targeting regime, stabilizes core inflation, rather than a broader measure of inflation, where core inflation is identified as an index of inflation in the sticky-price sector. Benigno (2004) focuses on the optimal policy in a currency area and develops a two region model. Benigno's main conclusions are that, in a context of asymmetric shocks and different level of price rigidities between the regions, the optimal plan is not feasible but it can be approximated by a second best solution that consists in an inflation targeting policy in which higher weight is given to the inflation in the region with higher degree of nominal rigidity. Benigno (2004) shows that the deadweight loss reduction from using the optimal inflation target is always above 97.8 p.c. These two important contributions have given the main intuition to understand the optimal policy conduct in the case of asymmetries of shock and heterogeneity in the level of price rigidities, but their models are highly stylized.
We follow this mainstream, but, differently from the previous studies, our main objective is to use this setup to verify whether we can draw the same conclusions in a more realistic environment in which we focus on 10 of the 14 regions in Europe and we use real data on price rigidities. Moreover, we are able to calculate inflation weights for each country and to quantify the reduction in the deadweight loss that society can obtain by using an asymmetric inflation target instead of an aggregate inflation target in this more realistic European setting. The work is organized as follows. Sections 1, 2 present the model, which is taken from Benigno (2001) K-region extension, section 3 turns to the welfare comparisons between the two different inflation targets and the optimal plan. Section 4 shows the main results of the simulation and section 5 concludes.

2 The model

Following Benigno (2001) we develop a K-region model. The whole economy is populated by a continuum of agents on the interval [0,1]. Each agent is both consumer and producer. Consumer of all the goods produced within the economy, producer of a single differentiated product. In each region a measure n_i of goods is produced, with i= 1,2,...,K. We have that $\sum_{i=1}^{K} n_i = 1$.

2.1 Consumer problem

Preferences of the generic household j are given by

$$U_t^j = E_t \sum_{s=t}^{\infty} \left[U(C_s^j) + L\left(\frac{M_s^j}{P_s}, \xi^i\right) - V(y_s^j, z_s^j) \right], \tag{1}$$

where the upper index j denotes a variable that is specific to agent j, while the upper index i denotes a variable specific to region i. E_t denotes the expectation conditional on the information set at date t, while β is the intertemporal discount factor 0$\leq$ β $\geq$1. Agents obtain utility from consumption and from the liquidity services of holding money, while they receive disutility from producing goods. The utility function is separable in these three factors. U is increasing concave function of the index C^j defined by

$$C^j \equiv \frac{\prod_{i=1}^{K} (C^j)^{n_i}}{\prod_{i=1}^{K} n_i} \tag{2}$$

and C_i^j is an index of goods produced in region i. Specifically,

$$C_i^j \equiv \left[\left(\frac{1}{n^i}\right)^{\frac{1}{\sigma}} \int_{u \in i} c^j(u)^{\frac{\sigma-1}{\sigma}} du \right]^{\frac{\sigma}{\sigma-1}} \tag{3}$$

for i=1,2,...,K. We have that σ, which is assumed to be greater than one, is the elasticity of substitution across goods produced within a region, while the elasticity of substitution between the bundles C_i is 1. The parameter n denotes both the population size and the economic size of the region. L is an increasing concave function of the real money balances, while ξ^i is a region-specific shock to the liquidity preference, an exogenous disturbance to money demand. Agents obtain utility from the purchasing power of money, where M_t^j is the agent j's money balance at the end of date t,while P is the appropriate region-specific price index. We can drop the upper index i (which indicates the regions in which the goods are sold) from the price index because, given the assumptions of the model, the purchasing power parity holds. P is defined as

$$P \equiv \prod_{i=1}^{K} (P_i)^{n_i} \tag{4}$$

and

$$P_i \equiv \left[\left(\frac{1}{n_i}\right) \int_{u \in i} p(u)^{1-\sigma} du \right]^{\frac{1}{1-\sigma}} \tag{5}$$

where u are the goods produced in each country. We can define the relative price of region i with respect to the overall price index as $P_i^R \equiv P_i/P$ for i=1,2,...,K. Finally V is an increasing convex function of agent j's supply of its product y^j. z^i is a region-specific stochastic disturbance that we will interpret as a productivity shock. Applying the two stage budgeting problem we can derive the demand of good u produced in region i as

$$c^j(u) = \left(\frac{p(u)}{P_i}\right)^{-\sigma} (P_i^R)^{-1} C^W \tag{6}$$

where the union aggregate consumption C^W is defined as

$$C^W \equiv \int_0^1 C^j dj. \tag{7}$$

We assume that each fiscal authority allocates government purchases only among the goods produced in the region of its sovereignty. The public expenditure production functions are such that they imply the following demand of the generic good u.

$$g(u) = \left(\frac{p(u)}{P_i}\right)^{-\sigma} (G_i) \tag{8}$$

Combining the two demand functions we can write the total demand for good u produced in region i as

$$y^d(u) = \left(\frac{p(u)}{P_i}\right)^{-\sigma} [(P_i^R)^{-1} C^W + G_i] \tag{9}$$

Considering the first order conditions and the aggregate budget constraint of each region we use Benigno (2004) result of perfect risk sharing of consumption between regions at any time and state, in other words we state the stationarity of assets and consumption. To complete the demand side of the economy we compute aggregate demand for the generic region by using the appropriate Dixit - Stiglitz aggregator

$$Y_i \equiv \left[\left(\frac{1}{n_i}\right) \int_0^1 y^d(u)^{\frac{\sigma-1}{\sigma}} du\right]^{\frac{\sigma}{\sigma-1}} \tag{10}$$

After applying (10) to (9)we obtain

$$Y_i = (P_i^R)^{-1} C + G_i \tag{11}$$

While consumption is completely insured, aggregate production can vary between regions. From (11), it follows that changes in the relative prices create divergences in output.

2.2 Firms problem

Sellers are monopolist competitors, demand is not taken as given, but sellers are small respect to the overall market and they take as given the indexes P, P_i and C. Prices are subject to changes at random intervals as in Calvo (1983). In each period a seller faces a fixed probability $1 - \alpha$ of adjusting its price, irrespective on how long it has been since the seller has changed its price. It is important to note that all the sellers that belong to the same region and that can modify their price at a certain time will face the same discounted future demands and future marginal costs under the hypothesis that the new price is maintained. Thus they will set the same price. We denote $\tilde{p}_t(u)$ the price of good u chosen at date t and with $\tilde{y}_{t,t+k}(u)$ the total demand of good u at time $t+k$ under the circumstances that the price $\tilde{p}_t(u)$ still applies. The function the seller maximizes is the following

$$E_t \sum_{k=0}^{\infty} (\alpha^i \beta)^k [\lambda_{t+k}(1-\tau^i)\tilde{p}_t(u)\tilde{y}_{t,t+k}(u) - V(\tilde{y}_{t,t+k}(u), z^i_{t+k})] \tag{12}$$

where revenues are evaluated using the marginal utility of nominal income[2]which is the same for all consumers belonging to the union, because of both the hypothesis of complete markets within each region and the result of redundancy of interregional bonds (Benigno 2004). Using the demand relations derived in the previous paragraph we have

$$\tilde{y}_{t,t+k}(u) = \left(\frac{\tilde{p}_t(u)}{P_{i,t+k}}\right)^{-\sigma} [(P^R_i)^{-1} C_{t+i} + G^i_{t+k}] \tag{13}$$

The seller maximizes (12) with respect to $\tilde{p}_t(u)$, the optimal choice of $\tilde{p}_t(u)$ is

$$\tilde{p}_t(u) = \frac{\sigma}{(\sigma-1)(1-\tau^i)} \frac{E_t \sum_{k=0}^{\infty} (\alpha^i \beta)^k V_y(\tilde{y}_{t,t+k}(u), z^i_{t+k}) \tilde{y}_{t,t+k}(u)}{E_t \sum_{k=0}^{\infty} (\alpha^i \beta)^k \lambda_{t+k} \tilde{y}_{t,t+k}(u)} \tag{14}$$

Calvo-price setting implies the following state equation for prices

$$P^{1-\sigma}_{i,t} = \alpha^k P^{1-\sigma}_{i,t-1} + (1-\alpha^k)\tilde{p}_t(u)^{1-\sigma} \tag{15}$$

because in each region the fraction $(1-\alpha)$ of sellers, that is chosen to adjust the price, sets the same price. The model is closed by stating the instrument of monetary policy in terms of the one period risk-free interest rate on the nominal bond denominated in the common currency. In terms of our equilibrium conditions this means that the money market equilibrium condition can be neglected.

2.3 Equilibrium

We can describe equilibrium by combining the aggregate demand block with the aggregate supply block. Our model is not solvable in a closed form solution for this reason we focus on equilibria where the state variables follow paths that are close to a deterministic stationary equilibrium, in which the inflation rates and the stochastic shocks are zero at all dates and the interest rate rule is set in order to anchor the nominal interest rate to the inverse of the inter temporal discount factor in the consumer preferences. As it is common in models with monopolistic competition, the marginal utility of consumption is not equated to the marginal disutility of producing output.

3 Log-linear equilibrium fluctuations

In this section, we first focus on the fluctuations around the steady state in the case in which prices are flexible, then we will analyze the case in which prices are sticky. Given a variable X_t we denote with $\tilde{X}_t$ the deviation of the logarithmic of that variable from its

[1] $\lambda_{t+k} = U_C(C_{t+k}/P_{t+k})$

steady state in the case prices were flexible, while with $\hat{X}_t$we denote the deviation of the same variable under sticky prices. In addition, given a generic variable X a world variable X^W is defined as follows

$$X^W \equiv \sum_{i=1}^{K} n_i X_i, \tag{16}$$

while a relative variable X_i^R is defined as

$$X_i^R \equiv X_i - X^W \tag{17}$$

while $X_{i,j}^R$ as

$$X_{i,j}^R \equiv X_i - X^j. \tag{18}$$

3.1 Flexible prices

The flexible price solution is

$$\tilde{C}_t^W = \frac{\eta}{\rho+\eta}(\bar{Y}_t^W - g_t^W), \tag{19}$$

$$\tilde{Y}_t^W = \frac{\eta}{\rho+\eta}\bar{Y}_t^W + \frac{\rho}{\rho+\eta}g_t^W, \tag{20}$$

$$\tilde{P}_{i,t}^R = \frac{\eta}{1+\eta}(g_{i,t}^R - \bar{Y}_{i,t}^R), \tag{21}$$

where $\bar{Y}_{i,t}^R$ and $g_{i,t}^R$ are respectively supply and government purchase shocks specific to region i, while η and ρ are the inverse respectively of elasticity of producing the goods and the inter temporal elasticity of substitution of consumption. Union consumption and output depend only on union supply and government purchase shocks. Instead, the relative prices are affected only by relative disturbances. Whenever there are asymmetric disturbances that induce the households in a region to work more, changes in the relative prices optimally shift part of the burden to the household in the other region. A larger government purchase shock in one region worsen the relative prices in that region, while a larger supply shock leads to an improvement. In an equilibrium in which the union inflation rate is zero, the implied path of the nominal interest rate $\tilde{R}_t$, which we call the natural interest rate, is

$$\tilde{R}_t = \frac{\rho\eta}{\rho+\eta}E_t[(\bar{Y}_{t+1}^W - \bar{Y}_t^W) - (g_{t+1}^W - g_t^W)], \tag{22}$$

This natural interest rate is only function of union disturbances.

3.2 Sticky prices

Here we discuss the log-linear approximation of the equilibrium under the hypothesis of sticky prices. We obtain the log-linear version of the Euler equation and of aggregate outputs as

$$E_t\hat{C}_{t+1}^W = \hat{C}_t^W + \rho^{-1}(\hat{R}_t - E_t\pi_{t+1}^W), \tag{23}$$

$$\hat{Y}_{i,t} = -\hat{P}^R_{i,t} + \hat{C}^W_t + g^i_t, \tag{24}$$

for each i=1,2,...,K. Our set of AS equations will be

$$\pi^i_t = -k^i_P(\hat{P}^R_{i,t} - \tilde{P}^R_{i,t}) + k^i_C(\hat{C}^W_t - \tilde{C}^W_t) + \beta E_t \pi^i_{t+1}, \tag{25}$$

where the region-specific inflation rates depend on the expectations of future price setting behavior as well as on the deviations of the union output gap from zero and the relative prices from their natural rates.[3]Furthermore the definition of relative price implies

$$\hat{P}^R_{i,t} = \hat{P}^R_{i,t-1} + \pi^i_t - \pi^W_t. \tag{26}$$

From this relation it follows that the relative prices is a state variable. If monetary policy is not able to eliminate the link between the inflation rate and the relative prices, inflation itself will be a function of its past values.

4 Welfare comparison

The actual main goal of the European Central Bank is to stabilize the MUICP (Monetary Union Index of Consumer Prices), which is a weighted sum of the single countries HICP index weighted by the economic size of each country, but as Benigno 2004 shows, in the case of price rigidity asymmetries, an aggregate inflation target is sub-optimal.
In order to define optimality Benigno (2004), following Rotemberg and Woodford (1997), King and Wolman (1998) and Woodford (1999) uses a model founded upon private-sector optimization to analyze the consequences of alternative policy rules.
We follow this literature and thus we define the welfare criterion of the Central Bank as the discounted value of a weighted average of the average utility flows of all the households,

$$W = E_0 \sum_{j=0}^{\infty} \sum_{i=1}^{K} \beta^j n_i w^i_{t+j} \tag{27}$$

We can simplify the welfare function to

$$W_t = -\Omega \sum_{j=0}^{\infty} \beta^j L_{t+j} \tag{28}$$

where

$$L_{t+j} = \Lambda[c^W_{t+j} - \bar{c}^W]^2 + \Gamma[\sum_{i=1}^{K} n_i(\hat{P}^R_{i,t} - \tilde{P}^R_{i,t})^2] + \sum_{i=1}^{K} \gamma_i(\pi^i_{t+j})^2 + t.i.p. + o(\|\xi\|^3), \tag{29}$$

and

$$\Omega \equiv \frac{1}{2} U_C \bar{C} (\sum_{i=1}^{K} n_i d^i) \sigma(1 + \sigma\eta), \tag{30}$$

[2]We have defined $k^i_C \equiv [(1 - \alpha^i\beta)(1 - \alpha^i)/\alpha^i][(\rho + \eta)/(1 + \rho\eta)]$ and $k^i_P = k^i_C[(1 + \eta)/(\rho + \eta)]$

$$\Lambda \equiv \frac{1}{\sigma}(\sum_{i=1}^{K} n_i (k_C^i)^{-1})^{-1}, \tag{31}$$

$$\Gamma \equiv \frac{1}{\sigma}(\sum_{i=1}^{K} n_i (k_P^i)^{-1})^{-1}, \tag{32}$$

$$\gamma_i \equiv \frac{n_i d^i}{(\sum_{i=1}^{K} n_i d^i)}, \tag{33}$$

$$d^i \equiv \frac{\alpha^i}{(1-\alpha^i)(1-\alpha^i\beta)}. \tag{34}$$

The Central Bank loss function summarizes the three different sources of inefficiencies present in a currency union economy: the inefficient level of output, the inefficient dispersion of prices and the non-efficient path of the relative prices in response to asymmetric shocks caused by the price stickiness.
We note that when the degree of rigidity are the same, γ_i coincides with n_i. Given this structure, efficiency can be obtained only if K-1 regions have flexible prices, in this case monetary policy should target the inflation rate in the sticky price region. If all the regions have the same degree of nominal rigidity, then the optimal policy is to target to zero π_t^W, which corresponds to the actual ECB target, the MUICP index.
The optimal plan is one in which the Central Bank minimizes the loss function (all the inefficiencies in the economy). If the degree of price stickiness is different between regions, the optimal plan is infeasible because of the mismatch between objectives and instruments and can only be considered as a benchmark.

In the case in which the level of price rigidity is heterogeneous, the Central Bank can commit only to the class of inflation targeting rules which are not fully optimal.
In accordance with Benigno (2004) we will show that the equilibrium paths of the variables $\pi_{t,i}$, y_t^W and $\hat{P}_{t,i}^R - \tilde{P}_{t,i}^R$ can be described by particular linear combinations of the variables $\hat{P}_{t,i}^R$ and $\tilde{P}_{t,i}^R$ which is assumed to follow a Markovian process. The two variables represent the smallest set of state variables that contains all the information needed for the evaluation of welfare, which is a characteristic of discretionary rules. The optimal plan, on the other hand, is a function not only of current values but also of past values. The inertia that drives this plan is caused not only by the intrinsic inertia of relative prices but also by the gain in credibility that monetary policy can achieve.

4.1 The optimal plan

In order to calculate the reduction in welfare that derives from the optimal plan we have to minimize the loss function:

$$\min_{y_t^W, \pi_t^i, \hat{P}_t^i} L_{t+j} = \Lambda[c_{t+j}^W - \bar{c}^W]^2 + \Gamma[\sum_{i=1}^{K} n_i(\hat{P}_{i,t}^R - \tilde{P}_{i,t}^R)^2] + \sum_{i=1}^{K} \gamma_i (\pi_{t+j}^i)^2, \tag{35}$$

subject to equations (25) and (26).
From this minimization we obtain the following process:

$$A \cdot E_t \begin{bmatrix} y_{t+1}^W \\ P_{t+1,i}^{\hat{R}} - P_{t+1,i}^{\tilde{R}} \\ \pi_{t+1,i} \\ \phi_{t+1,K+i} \\ P_{t,i}^{\tilde{R}} \\ P_{t,i}^{\hat{R}} \\ \phi_{t,i} \end{bmatrix} = B \cdot \begin{bmatrix} y_t^W \\ P_{t,i}^{\hat{R}} - P_{t,i}^{\tilde{R}} \\ \pi_{t,i} \\ \phi_{t,K+i} \\ P_{t-1,i}^{\tilde{R}} \\ P_{t-1,i}^{\hat{R}} \\ \phi_{t-1,i} \end{bmatrix} \tag{36}$$

This system can be reduced to the following state space representation:

$$\begin{bmatrix} y_t^W \\ P_{t,i}^{\hat{R}} - P_{t,i}^{\tilde{R}} \\ \pi_{t,i} \\ \phi_{t,K+i} \\ P_{t-1,i}^{\tilde{R}} \\ P_{t-1,i}^{\hat{R}} \\ \phi_{t-1,i} \end{bmatrix} = D \cdot \begin{bmatrix} P_{t-1,i}^{\tilde{R}} \\ P_{t-1,i}^{\hat{R}} \\ \phi_{t-1,i} \end{bmatrix} \tag{37}$$

$$\begin{bmatrix} P_{t,i}^{\tilde{R}} \\ P_{t,i}^{\hat{R}} \\ \phi_{t,i} \end{bmatrix} = G \cdot \begin{bmatrix} P_{t-1,i}^{\tilde{R}} \\ P_{t-1,i}^{\hat{R}} \\ \phi_{t-1,i} \end{bmatrix} + H \cdot \epsilon_{t,i} \tag{38}$$

We can calculate the autocovariance function of the VAR(1) explained by (38) and then find the autocovariance function of (37). We can finally calculate the reduction in Welfare, caused by an asymmetric shock to the Union, as a permanent percentage shift of units of steady state consumption in the following way:

$$WRopt = -100 \cdot \Omega \cdot \left[\Lambda var\left(y_t^W\right) + \Gamma \sum_{i=1}^{K} n_i var(P_{t,i}^{\hat{R}} - P_{t,i}^{\tilde{R}}) + \sum_{i=1}^{K} \gamma_i var(\pi_{t,i}) \right] \tag{39}$$

4.2 Pure inflation targeting

We assume that the Central Bank targets inflation of each country in Europe according to the economic sizes of the single countries. In this context the constraint that the Central Bank has is the following:

$$\pi_t^W = \sum_{i=1}^{K} n_i \pi_t^i = 0 \tag{40}$$

After some algebraic manipulation we can write all the arguments of the loss function as functions of the variables $P_{t,i}^{\hat{R}}$ and $P_{t,i}^{\tilde{R}}$ as follows:

$$\begin{bmatrix} \tilde{P^R_{t,i}} \\ \pi_{t,i} \\ y^W_t \\ \hat{P^R_{t,i}} - \tilde{P^R_{t,i}} \\ \tilde{P^R_{t,i}} - P^{\tilde{R}}_{t-1,i} \\ \tilde{P^R_{t,i}} \end{bmatrix} = \begin{bmatrix} R & 0 & 0 & 0 & 0 & 0 \\ D_{11} & 0 & 0 & 0 & 0 & D_{16} \\ D_{21} & 0 & 0 & 0 & 0 & D_{26} \\ D_{31} & 0 & 0 & 0 & 0 & D_{36} \\ D_{41} & 0 & 0 & 0 & 0 & D_{46} \\ D_{51} & 0 & 0 & 0 & 0 & D_{56} \end{bmatrix} \cdot \begin{bmatrix} P^{\tilde{R}}_{t-1,i} \\ \pi_{t-1,i} \\ y^W_{t-1} \\ P^{\hat{R}}_{t-1,i} - P^{\tilde{R}}_{t-1,i} \\ P^{\tilde{R}}_{t-1,i} - P^{\tilde{R}}_{t-1,i} \\ P^{\tilde{R}}_{t-1,i} \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \cdot \Sigma_t$$

The above system can be written in this way:

$$\begin{bmatrix} \tilde{P^R_{t,i}} \\ Y_{t,i} \end{bmatrix} = M \cdot \begin{bmatrix} P^{\tilde{R}}_{t-1,i} \\ Y_{t-1,i} \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} \cdot \Sigma_{t,i} \tag{41}$$

We can now calculate the variance covariance matrix for this VAR(1) process.

$$vec\Gamma(0) = (I - M \otimes M)^{-1} \cdot vec\Sigma_{t,i} \tag{42}$$

In the principal diagonal of this matrix we can find the variances of the variables that are argument of the Central Bank's loss function:$\pi_{t,i}$, y^W_t and $\hat{P^R_{t,i}} - \tilde{P^R_{t,i}}$. We can finally calculate the reduction in Welfare, caused by an asymmetric shock to the Union, as a permanent percentage shift of units of steady state consumption.

$$WRinf = -100 \cdot \Omega \cdot \left[\Lambda var\left(y^W_t\right) + \Gamma \sum_{i=1}^{K} n_i var(\hat{P^R_{t,i}} - \tilde{P^R_{t,i}}) + \sum_{i=1}^{K} \gamma_i var(\pi_{t,i}) \right] \tag{43}$$

4.3 Optimal inflation targeting

In the optimal inflation targeting context, the Central Bank has an inflation target, but the weights are unknown and they have to be chosen optimally. They have to be chosen in order to minimize the reduction in welfare caused by an asymmetric shock. The problem is similar to the pure inflation targeting except that this time the inflation constraint is the following:

$$\sum_{i=1}^{K} \gamma_i \pi^i_t = 0 \tag{44}$$

After some algebraic manipulation we can write all the arguments of the loss function as functions of the variables $\hat{P^R_{t,i}}$ and $\tilde{P^R_{t,i}}$ as follows:

$$\begin{bmatrix} \tilde{P^R_{t,i}} \\ \pi_{t,i} \\ y^W_t \\ \hat{P^R_{t,i}} - \tilde{P^R_{t,i}} \\ \tilde{P^R_{t,i}} - P^{\tilde{R}}_{t-1,i} \\ \tilde{P^R_{t,i}} \end{bmatrix} = \begin{bmatrix} R & 0 & 0 & 0 & 0 & 0 \\ D^*_{11} & 0 & 0 & 0 & 0 & D^*_{16} \\ D^*_{21} & 0 & 0 & 0 & 0 & D^*_{26} \\ D^*_{31} & 0 & 0 & 0 & 0 & D^*_{36} \\ D^*_{41} & 0 & 0 & 0 & 0 & D^*_{46} \\ D^*_{51} & 0 & 0 & 0 & 0 & D^*_{56} \end{bmatrix} \cdot \begin{bmatrix} P^{\tilde{R}}_{t-1,i} \\ \pi_{t-1,i} \\ y^W_{t-1} \\ P^{\hat{R}}_{t-1,i} - P^{\tilde{R}}_{t-1,i} \\ P^{\tilde{R}}_{t-1,i} - P^{\tilde{R}}_{t-1,i} \\ P^{\tilde{R}}_{t-1,i} \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \cdot \Sigma_t$$

The above system can be written in this way:

$$\begin{bmatrix} \tilde{P^R_{t,i}} \\ Y_{t,i} \end{bmatrix} = M^* \cdot \begin{bmatrix} \tilde{P^R_{t-1,i}} \\ Y_{t-1,i} \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} \cdot \Sigma_{t,i} \tag{45}$$

We can now calculate the variance covariance matrix for this VAR(1) process.

$$vec\Gamma(0) = (I - M^* \otimes M^*)^{-1} \cdot vec\Sigma_{t,i} \tag{46}$$

In the principal diagonal of this matrix we can find the variances of the variables that are argument of the Central Bank's loss function as a function of the vector of weights (γ_i) with i=1,...,K:$\pi_{t,i}$, y_t^W and $\hat{P^R_{t,i}} - \tilde{P^R_{t,i}}$. We can finally calculate the reduction in Welfare, caused by an asymmetric shock to the Union, as a permanent percentage shift of units of steady state consumption that is a function of the vector (γ_i).

$$WR(\gamma) = 100 \cdot \Omega \cdot \left[\Lambda var\left(y_t^W\right) + \Gamma \sum_{i=1}^{K} n_i var(\hat{P^R_{t,i}} - \tilde{P^R_{t,i}}) + \sum_{i=1}^{K} \gamma_i var(\pi_{t,i}) \right] \tag{47}$$

At this point we find the vector of γ_i that minimizes the following expression:

$$\min_{\gamma_i} \left[\Lambda var\left(y_t^W\right) + \Gamma \sum_{i=1}^{K} n_i var(\hat{P^R_{t,i}} - \tilde{P^R_{t,i}}) + \sum_{i=1}^{K} \gamma_i var(\pi_{t,i}) \right] \tag{48}$$

s.t.

$$0 < \gamma_i > 1 \tag{49}$$

$$\sum_{i=1}^{K} \gamma_i = 1 \tag{50}$$

Once we have the optimal γ_i^* we can calculate the reduction in welfare and compare it with the pure inflation target welfare reduction.

4.4 Calibration

Most of the free parameters of the model are taken from Benigno (2004): the intertemporal discount factor (β) is set to 0.99, the degree of monopolistic competition (σ) is set equal to 7.66, the risk aversion coefficient (ρ) is set equal to 1/6 and the elasticity of producing differentiated goods (η) is set equal to 0.67. We assume that the shock $\tilde{P}_{i,t}^R$ follows a Markovian process, a first order autoregressive process of the kind $\tilde{P}_{i,t}^R = \phi_i \tilde{P}_{i,t-1}^R + \epsilon_{i,t}$, where $\epsilon_{i,t}$ is white noise and for i=1,2,...K. For now we are assuming that all the shocks are the same so we set $\phi_i = 0.95$ and the variance of $\epsilon_{i,t} = 0.0086^2$. The level of price stickiness (α) and the economic sizes (n), differently from Benigno (2004), are accurately reported for each country. Eurostat provides the share of the single country private consumption relative to the Euro area private consumption (economic size) while the Euro System Inflation Persistence Network calculates the unconditional degree of price stickiness for each country.

Table 1 reports the frequency of price changes as an average over the period 1996-2001 on the basis of a common sample of 50 product categories for each country and for 5 of the main components of the CPI (unprocessed food, processed food, energy, non energy and services). The frequency of price changes for the Euro area, using the European National Central Banks studies, is equal on average to 15.1 p.c., while for the US, the estimation of price changes is equal to 24.8 p.c. according to Bills and Klenow (2004) calculations on a sub-sample of 50 products.

Table 1: *Frequency of price changes by product type*

	Un.Food	Pr.Food	Energy	Non Energy	Services	Total
Austria	37.5	15.5	72.3	8.4	7.1	15.4
Belgium	31.5	19.1	81.6	5.9	3.0	17.6
Germany	25.2	8.9	91.4	5.4	4.3	13.5
Spain	50.9	17.7	n.a.	6.1	4.6	13.3
Finland	52.7	12.8	89.3	18.1	11.6	20.3
France	24.7	20.3	76.9	18.0	7.4	20.9
Italy	19.3	9.4	61.6	5.8	4.6	10.0
Luxembourg	54.6	10.5	73.9	14.5	4.8	23.0
Netherlands	30.8	17.3	72.6	14.2	7.9	16.2
Portugal	55.3	24.5	15.9	14.3	13.6	21.1
EURO AREA	28.3	13.7	78.0	9.2	5.6	15.1
US	47.7	27.1	74.1	22.4	15.0	24.8

There exists a sizable variation both across countries and across sectors. Energy prices and unprocessed food prices change very often (78.0 p.c. and 28.3 p.c.), compared to other sectors, while for countries, the level of price rigidity ranges on average between 10 p.c. in Italy and 23.0 p.c. in Luxembourg. According to Dhyne et al. (2005) the source of the cross-country variation is likely to be both structural (consumption structure, outlet composition) methodological (the treatment of sales and of quality adjustment by each NSI) or reflects differences in the relative importance of regulated prices across countries.

Table 2 reports the average duration of price spells in quarters, the price rigidity in quarters, which is the free parameter α used in our model and the economic size, the parameter n. The average duration of price spells is calculated considering the simple relation between duration and frequency in discrete time $f = \frac{1}{D}$.

5 Results

In this section we report the vector of optimal weights that result from the optimization exercise described above and the welfare comparison between the optimal plan, the pure inflation targeting and the optimal inflation targeting. As we explained in the previous section the weights of the last inflation target are calculated optimally in order to minimize the reduction in welfare caused by an asymmetric shock. For this task we tried different minimization approaches: grid search, local minimization and global minimization with the simulated annealing algorithm.

Grid search is the most primitive method to minimize a function and it consists in specifying a grid of points, in our case a grid of all the possible combination of gammas, evaluate

Table 2: *Average duration, Price rigidity(α), Economic size(n)*

	Average duration in quarters	Price rigidity(α)	Economic size(n)
Austria	1.6	38.4	3.25
Belgium	1.4	29.6	3.51
Germany	1.8	46.0	30.01
Spain	1.9	46.8	12.49
Finland	1.2	18.8	1.65
France	1.2	16.4	21.19
Italy	2.5	60.0	19.89
Luxembourg	1.1	8.0	0.28
Netherlands	1.5	35.2	5.44
Portugal	1.2	15.6	2.29

the function at each of these points and pick the minimum. Benigno (2004) uses this method to calculate the optimal gammas, but, while he was dealing with only a vector of 100 points, we have to consider 9 vectors of one hundred points each and all the possible combinations. Grid search is very useful to understand how the function behaves, but in our case this method is highly inefficient.

For this reason we used SQP (Sequential Quadratic Programming) methods to calculate the local minimum. This method, based on the work of Biggs (1975), Hann (1977) and Powell (1978) allows to closely mimic Newton's method for constrained optimization just as is done for unconstrained optimization. The principal idea is the formulation of a Quadratic Programming subproblem, based on a quadratic approximation of the Lagrangian function where the nonlinear constraints are linearized. The solution of the subproblem is used to form a new iterate. From the local minimization results we can point out two main outcomes. On one hand, the results vary by choosing different starting values, leading to the conclusion that there might be multiple local minima. On the other hand, we have noticed that the difference in terms of welfare reduction between the pure inflation target and the optimal inflation target is extremely low. This last result can be due to the fact that the function is very ill-behaved and the local minimization algorithm is unable to find the global minimum, or that the function is extremely flat and for this reason there is no difference between the two inflation targets.

To solve this dilemma, we tried a third approach which seeks the global minimum using the simulated annealing algorithm. The simulated annealing algorithm proposed by Kirkpatrick, Gelatt and Vecchi (1983) is based on random evaluations of the cost function. First it reaches an area in the function of the global domain where a global minimum should be present, it then develops finer details, finding a good near optimal local minimum, if not the global minimum itself. After a long computational time the minimum is found, but, in accordance with the results of the local minimization, there is still a small difference between the two different inflation targeting policies.

In table 3 we report the optimal inflation weights that derive from the minimization problem, the starting values, which are the pure inflation target weights, and the level of price stickiness. From the table we can naturally compare the weights in the two different inflation targets. We notice that some countries, such as Spain and Germany, preserve the same weight, Italy has a much higher weight caused by the extremely high level of price rigidity and all the other countries have a lower weight. France, in particular, which is

21 p.c. of the size of the whole Union, receives an optimal weight of 3 p.c. In table 4 we report the reduction in welfare in the three different policies and the deadweight loss reduction passing from the pure inflation targeting to the optimal inflation targeting. We can notice that the reduction in welfare in the case of the optimal plan is much lower, but the difference between the two inflation targets is very low, in fact the deadweight loss reduction is 12.4 p.c. which differs from Benigno's result of 97.8 p.c.

Table 3: *Average duration, starting values, optimal weights(γ)*

	Average duration in months	Starting values	Optimal weights
Austria	6.5	3.3	2.1
Belgium	5.7	3.5	1.4
Germany	7.4	30.0	30.0
Spain	7.5	12.5	13.1
Finland	4.9	1.7	0.3
France	4.8	21.2	3.0
Italy	10.0	19.9	47.0
Luxembourg	4.3	0.3	0.0
Netherlands	6.2	5.4	2.9
Portugal	4.7	2.3	0.3

Table 4: *Welfare comparisons and deadweight loss reduction*

	Optimal Plan	Pure Inflation	Optimal Inflation	DR
percentage shift	-0.6953	-1.3967	-1.3164	12.36

6 Conclusions

In this work we have verified Benigno (2004) results in a more general and realistic framework, in which we consider 10 of the 14 countries of the EU, considering micro data on the level of price stickiness. Using the K-region extension, which can be found in Benigno (2001) appendix, and following the taxation approach, we have calculated the optimal weights for the inflation target according to which higher weight should be given to the inflation of the regions with higher degree of nominal rigidity. The results show that higher weight, compared to the MUICP index, should be given to Italy and lower weight should be given to France. Even if the weights are calculated optimally the gain in welfare reduction from choosing an aggregate inflation target (pure inflation target) compared to an asymmetric inflation target is very low. From these results it follows that the Central Bank, given that the optimal plan is infisible and that it can commit only to the class of inflation targeting rules, cannot do much better than choosing an aggregate inflation targeting.

References

Aoki K. (2001) Optimal monetary responses to relative price changes, *Journal of Monetary economics*, 48, 55–80.

Benigno P. (2001) Optimal monetary policy in a currency area, *CEPR Discussion Paper*, , 2755.

Benigno P. (2004) Optimal monetary policy in a currency area, *Journal of International Economics*, 63, 293–320.

Biggs M. (1975) Constrained minimization using recursive quadratic programming, in: *Towards Global Optimization*, Dixon L. and Szergo G., eds., North-Holland, 341–349.

Bils M. and Klenow P. (2004) Some evidence on the importance of sticky prices, *Journal of Political Economy*, 112, 947–985.

Calvo G. (1983) Staggered prices in a utility-maximizing framework, *Journal of Monetary Economics*, , 12, 383–398.

Corana A., Marchesi M., Martini C. and Ridella S. (1987) Minimizing multimodal functions of continuous variables with the ”simulated annealing” algorithm, *ACM Transactions on Mathematical Software*, 13, 3, 262–280.

Dhyne E., Alvarez L., Veronese G., Dias D., Hoffman J., Jonker N., Lunnemann P., Rumler F. and Vilmunen J. (2005) Price setting in the euro area: Some stylized facts from individual consumer price data, *ECB Working Paper Series*, 524.

Han S. (1977) A globally convergent method for nonlinear programming, *Journal of Optimization Theory and Applications*, 22, 297.

Judd K. (1998) *Numerical Methods in Economics*, MIT Press, Cambridge.

King R. and Wolman A. (1998) What should monetary authority do when prices are sticky, in: *Monetary Policy Rules*, Taylor J., ed., University of Chicago Press, Chicago, 349– 404.

Kirkpatrick S., Gelatt C. and Vecchi M. (1983) Optimization by simulated annealing, *Science*, 220, 4598, 671–680.

Rotemberg J. and Woodford M. (1997) An optimization-based econometric framework for the evaluation of monetary policy, in: *NBER Macroeconomic Annual*, Bernanke B. and Rotemberg J., eds., MIT Press, Cambridge, 297–346.

Woodford M. (1999) Optimal monetary policy inertia, *NBER Working Paper*, 7261.

Woodford M. (2003) *Interest and Prices*, Princeton University Press.