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INDUSTRIAL STRUCTURE AND THE MACROECONOMY THEORETICAL PREMISES FOR A MACROMODEL WITH SOCIAL MOBILITY, OLIGOPOLY, ENTRY/EXIT AND CYCLE

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Industrial structure and the macroeconomy

Theoretical premises for a macromodel with social mobility, oligopoly, entry/exit and cycle

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Abstract. This work introduces a new theoretical macroeconomic framework for an oligopolistic economy with heterogeneous agents and wage rigidity where the macroeconomic fluctuations can be determined not only by technology shocks, but also by the process of entry/exit of oligopolistic firms, potentially interacting with distributional shocks. In this framework, microfundation is interpreted in a peculiar way, where agents have the same preferences, modelled with a conventional CRRA utility function, are heterogeneous in their budget constraint and may change their social status in each period according to a stochastic process interacting with labour market and entry/exit. This theoretical framework may be employed for further research focused on entry/exit and its potential interactions with monetary policy.

Keywords: Macroeconomic Industrial structure, aggregate demand, Industrial Organization and Macroeconomics.

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1 Introduction

The purpose of this work is introducing a new theoretical macroeconomic framework for an oligopolistic economy with heterogeneous agents and social mobility where the macroeconomic fluctuations can be determined not only by technology shocks, but also by the process of entry/exit of oligopolistic firms, potentially interacting with distributional shocks.

The standard New-Keynesian framework for monetary policy analysis departed from the Real Business Cycle literature by introducing monopolistic competition à la Dixit and Stiglitz (1977), where market structure was exogenous and the firms, producing differentiated goods, were modelled as a continuum in the space [0,1].[1] In this way each of the firms was assumed to be infinitesimal and this implied that entry/exit of such infinitesimal elements, by definition, could not possibly affect the production capacity. The monetary policy could affect the "output gap", i.e. the gap between the actual price equilibrium and the benchmark case of flexible prices, although price rigidity and price behaviour should be related, in principle, to the market structure. Etro (2009) extensively discusses an innovative approach to model endogenous market structure in macroeconomics, international economics, growth and economic policy, while Etro and Colciago (2010) introduce a complete model of business cycle with differentiated goods, endogenous market structure at the sectorial level, full employment, and different industrial sectors, where the two separate cases of price (Bertrand) competition and quantity (Cournot) competition are extensively analyzed. They show that with no product differentiation and with a unique homogeneous good, mark ups only survive in the case of quantity (Cournot) competition, while they vanish in the case of price (Bertrand) competition, which degenerates into the conventional case of perfectly competitive real business cycle. In their model the

[1]See, for instance, the seminal works by Mc Callum and Nelson (1999), Galì (2002) and Walsh (2003), ch. 5.

interacton between business cycle and market structure goes as follows: an exogenous technology shocks affects output and consumption, increases the profits and, as a consequence, triggers entry. They do not explicitly refer to oligopoly (the word "oligopoly" never actually appears in their paper), and introduce instead a more general framework of "imperfect competition", that may include several sub-cases according to the pricing mechanism and/or to the value of the elasticity of substitution among commodities. In that context, the assumption of full employment and intrasectorial competition, in an economy whose production capacity and potential output is still only driven by technology shocks, basically amplifies the original (and conventional) technology shocks, wich imply changes in firms' markups and profits and, only as a consequence, entry/exit and market structure endogeneity. However Etro and Colciago provide an appealing explanation for a number of empirical regularities, such as countercyclical mark ups and pro-cyclical business creation.

The model I introduce here is simpler and analyzes the macroeconomic fluctuations by explicitly formalizing an economy with an oligopolistic industrial sector with an homogeneous good, where each individual agent cannot be at the same time worker and entrepreneur. Introducing in a macromodel the assumption of quantity (Cournot) competition might raise the problem of how are prices determined without referring to an auctioneer or, equvalently, what prevents the firms from implementing price undercutting and price competition. A possible way to deal with these problems is assuming some kind of quantity precommitment à la Kreps and Scheinkman (1983), whose results can be extended under fairly general conditions (see in this regard Madden, 1998).

Introducing oligopoly and entry/exit in a macromodel with business cycle obsiously means attaching a certain relevance to the role of demand expectations in agents' decisions, which is, of course the object of extensive research. For instance, Lorenzoni (2009) introduces a model of business cycles driven by shocks to consumer expectations regarding aggregate productivity and where agents are hit by heterogeneous productivity shocks: they observe their own productivity and a noisy public signal regarding aggregate productivity and this “noise shocks,” mimicks the features of aggregate demand shocks. News shocks (together with other shocks) are the focus of Jaimovich and Rebelo (2009) model, that generates both aggregate and sectoral comovement in response to both contemporaneous shocks and news shocks about fundamentals.

This paper introduces a general equilibrium, dynamic, stochastic and microfounded model. However, the microfoundation is based on a peculiar interpretation of the notion of representative agent, which requires a few comments.

A potential source of problems in the conventional use of the representative agent lies in the aggregation of heterogeneous agents, as pointed out by Forni and Lippi (1997), who show that many statistical features associated to the dynamic structure of a model (like Granger causality and cointegration), when derived from the micro theory do not, in general, survive aggregation. This means that the parameters of a macromodel do not usually bear a simple relationship to the corresponding parameters of the micromodel. Of course, this kind of problem cannot be solved without explicitly formalizing a statistical aggregation process and individuals' externalities and a first purpose of this model is to introduce a preliminary and simplified form of aggregation of heterogeneous agents with potential conflicts and externalities.

Another criticism to the representative-agent methodology has been raised a long time ago by Blinder (1986), who pointed out that mocrofounded models with a representative agent, by assuming that the observable choices of optimizing individuals are "internal solutions" may yield biased econometric estimates when the choices of a relevant portion of individuals are actually corner solutions: "For many goods, the primary reason for a downward sloping market demand curve may be that more people drop out of the market as the price rises, not that each individual consumer reduces his purchases" (Blinder, 1986, p. 76).

Finally, a last criticism that I am raising here, is associated to the interpretation of the representative agent utility function: Logically speaking, what does the representative agent utility function represent? If we look at it with the criteria of "hard sciences" can it really be interpreted as a proper microfoundation of a macroeconomic system composed by a high number of heterogeneous individuals without formalizing any statistical law of aggregation that accounts for externalities and agents' rational interactions? Is it not instead a sort of "aggegate utility function", and if so, is it not rather a macroeconomic "preference" function? In other words, if the utility function of the representative agent is metaphorically meant to model all the consumers of an economy, is it not subject to the Lucas critique? Why can we not explicitly model agents' (rational) interactions by means of some statistical principles of aggregation? In this regard, Aoki and Yoshikawa (2007, p. 28) point out that "the standard approach in 'micro-founded' formulates complicated intertemporal optimization problems facing the representative agent. By so doing, it ignores interactions among nonidentical agents. Also, it does not examine a class of problems in which several types of agents simultaneously attempt to solve fimilar but slightly different optimization problems with slightly different sets of contraints. When these sets of constraints are not consistent, no truly optimal solution exists". And furthermore, for what concerns the role of microfoundation, "Roughly speaking, we deemphasize the role of precise optimization of an individual unit while emphasizing the importance of proper aggregation for understanding the behaviour of the macroeconomy. The experiences in disciplines outside economics such as physics, population genetics and combinatorial stochastic processes that deal with a large number of interacting entities amply demonstrate that detalis of specification of optimizing agents (units) frequently diminish as the mumber of agents become very large. only certain key features of parameters such as correlations among agents matter in determining aggregate behaviour". (Aoki and Yoshikawa (2007, pp.28-29).

Of course, one may reply that even an "aggregate utility function" still allows to build the aggregate behaviour on some rigorous, logical and consistent axioms of preference.Therefore, in this paper, the utility function of the representative agent, which is the basis for the derivation of the aggregate demand, shall be employed as the basis for the microfoundation of the aggregate demand. However it is interpreted as an aggregate object and its budget constraint contains a principle of aggregation of heterogeneous agents, interactons, conflicts and esternalities. In this sense, this paper follows Aoki and Yoshikawa approach, for what concerns a few modelling tools employed to formalize the entry process of new firms and the presence of heterogeneous individuals with different (and sometimes conflicting) targets, but still builds the aggregate demand on a utility function and on a set of consistent axioms of preference and optimizing behaviour.

All individuals can hold financial assets, but, in our simplifying formalization of the inancial sector, the suppliers of "external finance", as opposed to the individuals holding the control of the firm, do not take part into the firm's decision process and may only be remunerated at the market interest rate. On the other hand, in this model, the activity of "investing in share", is rendered by the decision to undertake the (time consuming) monitoring activity on the firm's decisions, i.e., being an entrepreneur. The entrepreneur controls the firm in the sense that she fully controls the allocation of the cash flow. Furthermore, being an entrepreneur is assumed to be "full time job" and to absorb all the available time. All the individuals may have the chance to become entrepreneurs, but for at least one period they are fully committed to their job. The stylized fact captured by this assumption is that, on the one hand, a prevailing activity exists for each individual, on the other hanbd, workers and entrepreneurs might have diverging incentives and be in conflict: In this way, a form of unconventional heterogeneity (in budget constaints and sources of income) is introduced among agents with the same preferences. The status of worker, entrepreneur or unemployed may stochastically change in each period, generating in this way distrubutional shocks on the aggregate demand. Entry/exit is associated to the transition process from being a worker to being entrepreneur or unemployed. In particular, entry/exit, by modifying the number of firms, affect the production capacity and may potentially interact with monetary policy.

Finally, a last important detail that characterizes this model as a general equilibrium model is that the wage setting rule and the entry/exit decisions interact, since the workers are perceived by the firms as potential entrepreneurs, therefore potential rivals and potential entrants for the incumbents. In other words, while explicitly formalizing the interactions between endogenous market structure and the macroeconomy, instead of assuming that the representative agent is at the same time worker and entrepreneur (therefore in conflict with herself while setting wages), I assume that workers *may become* entrepreneurs (or unemployed) and entrepreneurs may become workers (or unemployed) in the future, at some entry or exit cost. In this way, labour market and entry decision are connected. Entry is obviously not simply a matter of substituting the firms which leave the market (although the interdependence between the sectorial rates of entry and exit is a well established empirical fact in the applied research on industry dynamics, as shown recently, among others by Manjón-Antolín, 2010, who investigates some empirical features of such interdependence) and the issue of the interaction between market structure and entry/exit decisions lead by the agents' expectation is not an exclusive concern of large industries and large firms. For instance, Dunne *et al.* (2009) empirically analyze the short run and long run dynamics of an oligopolistic sector and the role of entry costs and toughness of short-run price competition, by using micro data for the U.S. dentists and chiropractors industries, certainly not two sectors characterized by giant firms...

2 Consumers

The consumers choose their optimal consumption path by allocating their permanent income and financial assets over time. We assume, for the sake of simplicity, that financial assets are risk free and include Government bonds and deposits (i.e. an aggregate roughly corresponding to M3), that deposits are remunerated and that the interest rate on risk free Government bonds is equal to the interest rate on deposits for the sake of simplicity, since they are both assumed to be risk free financial assets. Monetary policy is described in this model as interest rate setting by the monetary authorities. Deposits, of course, can be thought of as a function of the money base, but exogenous changes in the money stock will not be considered in our analysis, although they could easily be formalized in this model as changes in the nominal amount of risk free financial assets. In the rest of the model we refer to the aggregate that includes Government bonds and M3 as the generic "financial asset". The financial asset is risk free, since, for the sake of simplicity, in this model, the decision of investing in risky assets is equivalent to the decision of starting a new firm, i.e. allocating human capital into "being a new entrant".

We also assume that the deposits are issued by a perfectly competitive aggregate financial and banking sector, which perfectly diversifies its lending risk to industrial firms, so that we only consider a generic interest rate r, exogenously set by the policy makers. The specific simplifying assumption of a unique generic interest rate is also common to most conventional newkeynesian models for policy analysis.

Agents' heterogeneity might not change as regularly and predictably as age. What I am arguing here is that qualitative differences in budget contraints can be a potential source of heterogeneity and a configuration with one huge firm and many workers might be different in many regards (not just for commodity pricing) from a configuration with many ologopolists competing in many dimensions (not only in pricing). Agents share the same utility function, while their main source of earnings may be given either by wages, or profits or transferrals from the public sector to the unemployed individuals. These transferrals are, in aggregate terms and, for the sake of simplicity, they are assumed to be proportional to the income (i.e. profits or wages).

The financial sector is very simplified: the suppliers of "external finance", as opposed to the individuals holding the control of the firm, do not take part into the firm's decision process.

We can think of the banking sector as an operator that instantaneously perform all the transactions among individuals, with no specific need of cash, provided that all budget constrains are satisfied. These transactions are proportional to the aggregate income, who pay a commission on them, say ς. The cost of banks' intermediation is exogenous and equally distributed on firms. Furthermore, it is assumed to be equal to the income perceived by the individuals working in banks, so that it does not carry any aggregate effect on the aggregate income. The entrepreneurs can be incumbent, earning at time $t+i$ the incumbent profits π_{t+i}^{in} or new entrants, earning the new entrant profits π_{t+i}^{e} which, in general, diverge from π_{t+i}^{in} since the new entrants have to pay the entry costs to enter the market. The entrepreneurs hire the workers, pay them the wages ω_t at time t. They pay themselves the same wage ω_t, and keep the residual profits, so that the remuneration for the entrepreneurial activity is given by ω_t plus π_{t+i}^{in} if the entrepreneur is an incumbent or ω_t plus π_{t+i}^{e} if she is a new entrant. When $\pi_{t+i}^{in} < 0$ and $\pi_{t+i}^{e} < 0$, respectively, the incumbent and the new entrant go bankrupt (which happens with a given probabilty, to be specified later), the entrepreneur and the workers become unemployed and, until they are hired again by a new firm, they receive the unemployment subsidy.

Each entrepreneur hires a certain number of workers to be employed in the production process and yo be paid ω_t, for the period t. Each entrepreneur is remunetated for her "full time" entrepreneurial activity by paying to herself the wage ω_t(like any worker) out of the company cash flow and, in addition, receives the remaining profits, π_t^{in} if she is an incumbent, π_t^e if she is a new entrant. Being entrepreneur requires some skill that can be acquired after working at least one period and is lost by not working and being unemployed for at least one period. Being a worker, on the other hand does not requires any particular skill. The distribution of the aggregate income can be formalized as follows

$$\begin{aligned} Q_{t+i} &= n_{t+i}(\omega_{t+i} + h_{t+i}^e \pi_{t+i}^e + h_{t+i}^{in}\pi_{t+i}^{in})(1-\tau-\varsigma) + \\ &+ (n_{t+i}\omega_{t+i} + n_{t+i}h_{t+i}^{in}\pi_{t+i}^{in} + n_{t+i}h_{t+i}^e\pi_{t+i}^e)\tau + \\ &+ (n_{t+i}\omega_{t+i} + n_{t+i}h_{t+i}^{in}\pi_{t+i}^{in} + n_{t+i}h_{t+i}^e\pi_{t+i}^e)\varsigma \qquad (1) \\ Q_{t+i} &= n_{t+i}(\omega_{t+i} + h_{t+i}^e\pi_{t+i}^e + h_{t+i}^{in}\pi_{t+i}^{in}) \end{aligned}$$

where Q_{t+i} is the aggregate nominal income at time "$t+i$", ω_{t+i} is the wage per worker before taxes, assuming that the labour contract is such that each worker receives the wage ω_{t+i} at time "$t+i$" for a fixed amount of hours of labour, π_{t+i}^{in} the profits of the incumbent entrepreneurs before taxes, π_{t+i}^e the profits of the new entrants before taxes, τ the tax on labor and profits that determine the income of the unemployed individuals, n_{t+i} the number of employed individuals at time *t+i*, h_{t+i}^{in}, (with $0 < h_{t+i}^{in} < 1$) the portion of incumbent entrepreneurs at time *t+i*, h_{t+i}^e (with $0 < h_{t+i}^e < 1$) the portion of new entrants at time *t+i* (with $h_{t+i} = h_{t+i}^{in} + h_{t+i}^e$ and $0 < h_{t+i} < 1$), τ is a tax, assumed to be proportional for the sake of simplicity, $(n_{t+i}\omega_{t+i}+n_{t+i}h_{t+i}^{in}\pi_{t+i}^{in}+n_{t+i}h_{t+i}^e\pi_{t+i}^e)\tau$ the transferral to unemployed at time *t+i*, $(n_{t+i}\omega_{t+i}+n_{t+i}h_{t+i}^{in}\pi_{t+i}^{in}+n_{t+i}h_{t+i}^e\pi_{t+i}^e)\varsigma$ the commissions to the banking system at time *t+i*. We assume that τ and ς are constant and very small compared to the other varibles. It has . Finally, let us define:

$\xi = n/l;$

dividing both sides of the last row of the equation by *l,* we get the income in per capita terms, with ξ_{t+i} , $\xi_{t+i}h_{t+i}^{in}$, and $\xi_{t+i}h_{t+i}^e$ normalized:

$$q_{t+i} = \omega_{t+i}\xi_{t+i} + \pi^{in}_{t+i}\xi_{t+i}h^{in}_{t+i} + \pi^{e}_{t+i}\xi_{t+i}h^{e}_{t+i}$$

If l is the number of individuals composing the labour force, then the per capita transferral to unemployed individuals is lower during recessions, when the income is lower and there are less firms and less employed workers.

We are now enabled to define the problem of the representative consumer with his budget constraint, while keeping at the same time in the model a specific notion of agents' heterogeneity.

We consider a CRRA utility function, based on the premises contained in the introduction. In addition, we assume here that the agents may rationally formulate commonly shared expectations on the relevant future variables of the model, although this detail will be better specified later. The consumer problem is the following:

$$\max U_t = E_t\left[\sum_{i=0}^{\infty}\left(\frac{1}{1+\rho}\right)^i u(c_{t+i})\right] \tag{2}$$

$$c_{t+i}, i = 0, ...\infty \tag{3}$$

for each $i = 0, 1, ...\infty$ where ρ is the subjective rate of intertemporal preference for the consumers, subject to the following constraint in real terms:

$$E_t(a_{t+i+1}) = (1 + r_{t+i})E_{t+i-1}(a_{t+i}) + E_{t+i-1}(y_{t+i}) - E_{t+i-1}(c_{t+i}) \tag{4}$$

and

$$c_{t+i} \geq 0$$

at every time $t + i$ from $i = 0, ..., \infty$

where a_{t+i} is the financial asset in real terms on which the consumer can invest its wealth at time "$t + i$", $y_{t+i} = q_{t+i}/P_{t+i}$ is the real per capita income at time "$t + i$" and r is the real interest rate on the financial asset and controlled by the central bank.

The financial asset in which the agents may invest their wealth do not include shares: in this simplified model, investing in shares is a time consuming activity and implies undertaking the monitoring and organizational activity of being an entrepreneur.

The budget constraint 4 also holds for $i = 0, ..., \infty$. the transversality condition is the following

$$\lim_{j \longrightarrow \infty} a_{t+j} \left(\frac{1}{1 + r_{t+j}} \right)^j \geq 0 \tag{5}$$

If the marginal utility of consumption is always positive (i.e. not in the case of a quadratic utility function, but, for instance in a CRRA utility funcion, which is log-linear) the above transversality condition 5 is always satisfied in terms of equality.

We define the total wealth of the consumer as composed by the financial wealth and the human wealth H_t, given, at each time t, by the present discounted value of the expected future income, i.e.

The financial wealth a_t and the human capital H_t are assumed to be valued at the beginning of period t, while $W_t = (1+r_t)(a_t+H_t)$ represents the overall wealth, which is valued at the end of period t, but before consumption c_t, that absorbs part of the available resources. We also assume that both profits and wages are paid at the end of the period, when consumption takes place. The human wealth valued at the beginning of time t is the following:

$$H_t = \frac{1}{(1 + r_t)} \sum_{i=0}^{\infty} \left(\frac{1}{1 + E_t(r_{t+i})} \right)^i E_t(y_{t+i}) \tag{6}$$

and, as above

$$W_t = (1 + r_t)(a_t + H_t) \tag{7}$$

hence

$$E_t(W_{t+1}) = (1 + r_t)\left[E_t(a_{t+1}) + \frac{1}{(1 + r_t)}\sum_{i=0}^{\infty}\left(\frac{1}{1 + E_t(r_{t+i})}\right)^i E_t(y_{t+1+i})\right] \tag{8}$$

Substituting in 8 for the definition of a_{t+1} we get:

$$\begin{aligned} E(W_{t+1}) &= (1 + r_t)\left[(1 + r_t)a_t + y_t - c_t + \right. \\ &\left. + \frac{1}{(1 + r_t)}\sum_{i=0}^{\infty}\left(\frac{1}{1 + E_t(r_{t+i})}\right)^i E_t(y_{t+1+i})\right] \end{aligned}$$

Hence

$$\begin{aligned} E(W_{t+1}) &= [(1 + r_t)(a_t + H_t) - c_t] \\ &= (1 + r_t)(W_t - c_t) \end{aligned}$$

and, generalizing

$$E(W_{t+i+1}) = (1 + r_{_{t+i}})(W_{t+i} - c_{t+i})$$

Where W_{t+i+1} is the state variable

Let us assume now that the instantaneous utility be represented by the following function:

$$u_t = \frac{c_t^{1-\gamma}}{1 - \gamma} \tag{9}$$

with $0 < \gamma < 1$

With no new information, each individual is expecting to keep on doing the same job from period to period. However, informational shocks might turn out into shocks on the composition of the aggregate output and, hence, on the aggregate demand.

Therefore the consumer problem boils down into the following Bellman and Euler equations respectively:

$$V(W_t) = \max_{c_t} \left[\frac{c_t^{1-\gamma}}{1-\gamma} + \left(\frac{1}{1+\rho} \right) E(V(W_{t+1})) \right] \tag{10}$$

$$u'(c_t) = \frac{1+r_t}{1+\rho} E_t u'(c_{t+1}) \tag{11}$$

Applying the standard dynamic programmic techniques yields the following consumption function (see appendx 1 for the algebraic details), which, in our model without fixed capital, also represents the aggregate expenditure:

$$d(W_t) = \left[1 - (1+r_t)^{\frac{1-\gamma}{\gamma}} (1+\rho)^{-\frac{1}{\gamma}} \right] (a_t + H_t) \tag{12}$$

Let us define

$$\Xi = \left[1 - (1+r_t)^{\frac{1-\gamma}{\gamma}} (1+\rho)^{-\frac{1}{\gamma}} \right]$$

Since ρ is constant and $0 < \gamma < 1$, then $\partial C(\cdot)/\partial r_t < 0$

Let ι_t be the inflation rate. Considering the definition of real per capita income $y_{t+i} = q_{t+i}/P_{t+i}$, the definition of real financial wealth $a_{t+i} = A_{t+i}/P_{t+i}$,and the link between price level and inflation $P_{t+n} = P_t(1+E(\iota_t))^n$, based on the assumption that (with no unexpected random shocks) the best predictor for future inflation is the current inflation, then we get[2]:

[2] Of course, in this case, all the random shocks affecting the future inflation would also affect the aggregate demand

$$\begin{aligned}(a_t + H_t) &= \left(a_t + \frac{1}{1+r_t} \sum_{i=0}^{\infty} \left(\frac{1}{(1+E\left(r_{t+i}\right))} \right)^i E(y_{t+i}) \right) \\ &= \left(\frac{A_t}{P_t} + \frac{1}{P_t} \frac{1}{1+r_t} \sum_{i=0}^{\infty} \left(\frac{1}{(1+E\left(r_{t+i}\right))(1+E(\iota_{t+i}))} \right)^i E(q_{t+i}) \right)\end{aligned}$$

Hence, rearranging 12 we get the following equation, which yields the consumption function in per capita terms

$$d(W_t) = \frac{\Xi(r_t)}{P_t} \left(A_t + \frac{1}{1+r_t} \sum_{i=0}^{\infty} \left(\frac{1}{(1+E\left(r_{t+i}\right))(1+E(\iota_{t+i}))} \right)^i E(q_{t+i}) \right) \tag{13}$$

or, equivalently, if we want to explicitly formalyze the income distribution between labour and capital, we get:

$$\begin{aligned}d(P_t, W_t) &= \frac{\Xi(r_t)}{P_t} \{ A_t + (1+r_t)^{-1} \sum_{i=0}^{\infty} [(1+E\left(r_{t+i}\right))(1+E(\iota_{t+i}))]^{-i} \cdot \\ &\quad \cdot E(\omega_{t+i}\xi_{t+i} + \pi_{t+i}^{in}\xi_{t+i}h_{t+i}^{in} + \pi_{t+i}^{e}\xi_{t+i}h_{t+i}^{e}) \}\end{aligned} \tag{14}$$

and, if we want the consumption function in aggregate terms:

$$D(P_t, W_t) = \frac{\Xi(r_t)}{P_t} \left(A_t + \frac{1}{1+r_t} \sum_{i=0}^{\infty} \left(\frac{1}{(1+E\left(r_{t+i}\right))(1+E(\iota_{t+i}))} \right)^i E(Q_{t+i}) \right) \tag{15}$$

or, again, if we want to explicitly formalize income distribution

$$\begin{aligned}D(P_t, W_t) &= (\Xi(r_t)/P_t) \cdot \\ &\quad \cdot \{ A_t + [1/(1+r_t)] \sum_{i=0}^{\infty} [(1+E\left(r_{t+i}\right))(1+E(\iota_{t+i}))]^{-i} \cdot \\ &\quad \cdot E(n_{t+i}(\omega_{t+i} + h_{t+i}^{e}\pi_{t+i}^{e} + h_{t+i}^{in}\pi_{t+i}^{in})) \}\end{aligned} \tag{16}$$

Each individual is maximizing her utility in an intertemporal framework with infinte horizon. With no informative shocks, we assume that for each individual, the present expected value of each variable is the best predictor for its future income. However, shocks of various nature might force some entrepreneurs to become workers or unemployed, or might induce some of the the workers to become entrepreneurs or force them to become unemployed. The nature of these shocks will be analyzed in the next sections.

Monetary policy, by modifying r in its present and expected value, generates an overall effect on d, while the behaviour of the expected inflation is more complex and depends on the way monetary policy interacts with the market structure.

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3 Labour market and firms

First of all, we have to introduce a further assumption by assuming that, when prices are null, there is no economic activity, no production and, as a consequence the market revenue function of each individual firm $\varkappa(P_t, n_t, h_t)$ is null.

All the firms are identical, use the same producton technology to produce the same generic good in regime of oligopoly. Having defined φ_t as the individual output produced by each individual firm at time t, the production technology of each individual firm is summarized by the following function:

$$\varphi_t = \Lambda L_t^{\alpha} \tag{17}$$

Labour is the only production factor. we assume a labour contract that establishes *ex ante* a fixed number of hours to be worked. We also assume that starting a new firm involves entrepreneurial and organisational skills and exogenous sunk costs of entry F. The model can be extended by also including capital, but at this stage we assume that only labour is employed in the production process.

Entry/exit, by increasing/reducing the number of existing firms, affects the production capacity. The workers that remain unemployed for at least one period loose their skill to potentially become entrepreneurs, but they can stll become workers at no cost if they are hired by a firm. Only those who have been workers for at least one period can turn entrepreneurs.

At any generic point t in time, a portion $(h_t^e + h_t^{in})$ of the employed labour force n_t is composed by entrepreneurs (incumbents and new entrants). In our oligopolistic economy there are exogenous fixed costs of entry F. F can be though of as a kind of setting up and organizational sunk cost. Differently from Etro and Colciago (2010) and due to the assumption of this model, the entry costs are not given by the stock price of a generic incumbent firm: they are exogenous instead and consist of all the cost that need to be implemented to start the economic activity[3].

Since the workers may decide to become entrepreneurs, the wage and entry decisions are connected and they both depend on an incentive compatibility constraint: Labour market does not necessarily clear due to wage rigidity.

Between time $t-1$ and time t the wages that apply for the next period (from time t to time $t+1$) are set. This wage setting process involves the incumbents firms and the employed workers and is radically different in the cases of unemployment or full employment. To explain it we need to introduce a few conecepts and definitions.

[3] Apart from unemployment, which is caused by wage rigidity, the existence of different social groups can be thought of as being initally determined by a random initial distribution of extra resources allowing a subset h_0 of the labour force l at the initial time *0*, to cover,once for all, the initial exogenous sunk costs, at the initial instant of the whole economic process.

The oligopolistic entrepreneurs have incentive to keep the wages low, but not so low to trigger entry: in fact with no idyosincratic information shocks, individuals share the same expectations, if the wages were set so low that their expected future value would be lower than the expected future value of the profits of an entrant weighted with the (generally high) probability of bankruptcy, then the workers would prefer to bear the risk of entering the market as entrepreneurs. Since this is common knowledge, all the workers would have the same incentive to enter the market, large scale entry would take place until profits vanish out and entreprenuers would only earn the wage they pay themselves, but would get zero profits. We call the wage that does not *ex ante* trigger entry the "incentive compatible wage" and denote it ω^*. Let us assume that τ is very small (and basically only exists to guarantee the bare survival of unemplyed individuals): with positive unemployment, all the incumbent firms have exactly the same incentive not to offer any wage that is higher than the "incentive compatible wage". At the same time, each and every employed worker who is offered the "incentive compatible wage" by her firm between time $t-1$ and t, knows that, by rejecting that offer, she would be substituted by an uneployed worker and become uneployed for the next period.

In case of full employment, the nature of wage setting is radically different. First, the incumbents do not have any credible way to induce their workers to accept a "no entry wage", no matter how this is defined, because there is no longer any credible threat to offer the same contract to unemployed workers, in case of rejection. Second, as a consequence, there is no longer a common incentive for the incumbents in coordinating themselves in the process of wage setting. Therefore the incumbents are not able, in this case, to publically announce any optimal wage ω^*.On the other hand, rivalry among firms still exists and each rival firm can push a competitor out of the market by "stealing" its workers and offering them marginally higher wages (and, of course, not renewing the contract to its previous workers). The only way to prevent this, for each incumbent firm is offering wages that eliminate the extra profits, so that the entrepreneurs are only remunerated by the wage they pay to themselves. Let us call this wage ω^0.Any lower wage offered by a firm to its workers would expose the firm to the risk of being pushed out of the market by its competitors, who could potentially steal its workers (after firing their own workers) by offering them ω^0. It can be shown that $\omega^0$is Nash equilibrium in a stage game among the oligopolistic incumbent firms. [4].

[4] It can also be shown that ω^0 is a subgame perfect equilibrium in a bargaining game with alternating offers between the workers and entrepreneurs.

For the proof of the existence of a subgame perfect equilibrium in a bargaining game with alternating offers, see Osborne and Rubinstein (1994, pp.118-123, proposition 122.1). In particular, all the assumptions of the proof by Osborne and Rubinstein apply to this model, if we assume that at time t both the entrepreneurs and the workers elect a representative who negotiate with the countepart the wages that will apply from time t to time *t+1,* that all the assumptions of "Proposition 122.1 in Osborne and Rubinstein (1994) apply, with the following additional assumptions:

1) the preferences of workers and entrepreneurs are their utility function, increasing and monotonic in wealth, which, in its turn, is increasing and monotonic in the income that is negotiated by the representatives of the two social groups;

2) the representative of the workers and the representative of the entrepreneurs share the same information, expectations and have full knowledge of the "best agreement" of each other;

3) the set of possible agreements X (which is a compact and connected subset of a Euclidean space) degenerates to the right interval $[w^*, w^*+\epsilon)$ of point w^* *(*with ϵ arbitrary), for the assumptions earlier introfuced in this paper.

The compact and convex set of possible agreements degenerates to an arbitrarily small interval around w^*, which is common knowledge for the representative of the workers and the representative of the entrepreneurs.

Under the above assumptions, proposition 122.1 in Osborne and Rubinstein (1994) prooves the existence of a subgame perfect equilibrium of a bargaining game with alter-

Let us consider first the case of unemployment.

The "incentive compatible wage" is an *ex ante* variable, subject to unexpected random and policy shocks: it is jointly set and publically announced by the incumbent firms between time $t-1$ and time t. Immediately after, but still between time $t-1$ and time t, entry takes place and workers are hired.

We assume that the incumbents have the incentive not to delay the announcement of the "incentive compatible wage" and that delaying such an announcement would convey the signal that there is a positive probability that the incumbent is not able to announce the "incentive compatible wage" and that would trigger unlimited entry.

Once ω^* is announced and the process of hiring workers and production organization has begun, some existing workers may receive a random informational shock that generates a certain amount of entry, i.e. the decisions of entry and hiring/firing workers takes place in the time interval *(t-1, t)* for the period t.. Unexpected monetary policy taking place after the announcement of ω^* may also generate entry by affecting the entry cost, as explained below.

Entry/exit, by affecting the number of existing firms, affects production capacity and aggregate employment. Furthermore the mark up is subject to emerging firms' coalitions in the goods markets and/or mergers.

The risk free interest rate r, for the sake of simplicity is assumed to be exogenous and under the control of the monetary authorities, who are assumed to handle the possible risk in the banking sector and, at the same time, monitor its efficiency and, by the assumption of perfectly competitive banking sector, is the same that banks charge on their loans[5]. However, in

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anting offers.

[5] This means that for a new entrant (i.e. for a worker who decides to become entrepreneur), if it were not for the risk of bankruptcy, it would be indifferent to cover the fixed costs of entry F at time t by borrowing money form the banking system at the interest rate r_{t-1} or (partly or entirely) cover it by selling the portion F of her financial wealth A_t

case of bankrupcty, the entrepreneur would loose her job, would not have the right to start a new firm next period, but would keep all of her financial assets A_t safe from risk and for herself. This point is also particularly relevant for the interpretation of the microfoundation of the aggregate demand that will be shown later. At the same time, there is no advese selection because the new entrant who goes bankruptcy would still be penalized by earning (for an indefinitely long leght of time) the mere unemployment subsidies. Therefore we assume, that any new entrant has to borrow from the banking and financial system in order to cover the cost of entry.

Incumbent firms have incentive to make entry unappealing to the potential entrants (i.e. workers), therefore the labor market is not characterized here by a conventional labour demand being set equal to the marginal productivity of labour in value, but instead wages are set at a level that would not create (on average) the incentive for the existing workers to entry the market as entrepreneurs.

The expected remuneration of the new entrant is given by the profits π_t^e plus the wage ω_t^* that the entrepreneur pays to herself.

The expeced (at time $t-1$) remuneration of the new entrant for time t is $E_{t-1}((\pi_t^e + \omega_t^*)(1-\tau-\varsigma))$ i.e. the new entrant profits plus the wages that she pays to herself, where

$$
\begin{aligned}
&E_{t-1}((\pi_t^e + \omega_t^*)(1-\tau-\varsigma)) = \\
&= \{[E_{t-1}(P_t^e)\, E_{t-1}(c_t^e) - E_{t-1}(\omega_t^*)\, E_{t-1}(L_t^*) - (1+r_{t-1})F_t + E_t(\omega_t^*)] \cdot \\
&\cdot (1-\tau-\varsigma)\}
\end{aligned}
$$

On the other hand, the *ex ante* probability at time t for the generic new entrant to stay in the market is:

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and giving up the interests $r_t F$.

$$\begin{aligned}\Pr(\pi_t^e &\geq 0) = \Pr[E_{t-1}\left(P_t^e\right)E(c_t^e) - E_{t-1}\left(\omega_t^*\right)E_{t-1}(L_t^*) - \\ -(1-r_{t-1})F]\left(1-\tau-\varsigma\right) &\geq 0 \end{aligned} \qquad (18)$$

where $E_{t-1}\left(P_t^e\right)$ is the expected price, in case of entry, in a game with Bertrand competition with quantity pre-commitment, $E_{t-1}(c_t^e)$ the expected quantity sold by the new entrant and $E_{t-1}\left(\omega_t^*\right)$. In general their price in case of entry will be different from the price without entry, even from an individual entrant point of view. The definition of $E_{t-1}((\pi_t^e + \omega_t^*)\left(1-\tau-\varsigma\right))$ and 18 show that a reduction in the interest rate r_{t-1} at time $t-1$ decided by the monetary authorities, would affect both 18 and $E_{t-1}((\pi_t^e + \omega_t^*)\left(1-\tau-\varsigma\right))$.

The *ex ante* probability of bankruptcy of the generic new entrant at time t is defined as follows :

$$1 - \Pr(\pi_t^e \geq 0)$$

or

$$\begin{aligned}\Pr(\pi_t^e &< 0) = \qquad (19)\\ &= \Pr\left\{\left[E_{t-1}\left(P_t^e\right)E(c_t^e) - E_{t-1}\left(\omega_t^*\right)E_{t-1}(L_t^*) - (1+r_{t-1})F\right]\left(1-\tau-\varsigma\right) < 0\right\}\end{aligned}$$

Let us define now the *ex ante* probability for the generic incumbent to stay in the market at time t. .

$$\Pr(\pi_t^{in} \geq 0) = \Pr\left\{\left[E_{t-1}\left(P_t^j\right)E(c_t^j) - E_{t-1}\left(\omega_t\right)E_{t-1}(L_t^*)\right]\left(1-\tau-\varsigma\right) \geq 0\right\}$$

The remuneration of a generic incumbent at time t is given by the profits π_t^{in} and the wage ω_t^* that the entrepreneur pays to herself.

Incumbent remuneration =

$$= E_{t-1}((\pi_t^{in} + \omega_t^*)(1 - \tau - \varsigma))$$
$$= \left[E_{t-1}\left(P_t^j\right)E(c_t^j) - E_{t-1}\left(\omega_t^*\right)E_{t-1}(L_t^*) + \omega_t^*\right](1 - \tau - \varsigma)$$

where P_t^j and $E(c_t^j)$, the price level and the expected output sold by the incumbent, would be, in general, different in case of entry or in case of non entry. .

Let us define then the *ex ante* probabilty of bankruptcy for a generic incumbent at time t.

$$1 - \Pr\left\{\pi_t^{in} \geq 0\right\}$$

or

$$\Pr\left\{\pi_t^{in} < 0\right\} = \Pr\left\{\left[E_{t-1}\left(P_t^j\right)E(c_t^j) - E_{t-1}\left(\omega_t\right)E_{t-1}(L_t^*)\right](1 - \tau - \varsigma) < 0\right\} \tag{20}$$

When a firm goes bankrupt both the entrepreneur and the workers lose their job and get unemployed. Therefore an entrepreneur who goes bankrupt at time t, is unemployed at time $t+1$ and can only hope to be hired as a worker at time $t+2$. This means that we do not need to assume "ad hoc" bankruptcy costs.

At time t the new entrant will survive with probability $\Pr(\pi_t^e \geq 0)$ and the income to be weighted with the probability $\Pr(\pi_t^e \geq 0)$ is π_t^e; on the other hand, with probability $[1 - \Pr(\pi_t^e \geq 0)]$ the firm will go bankrupt and the new entrant will get the unemployment subsidy $(n_t\omega_t + h_t^{in}\pi_t^{in} + h_t^e\pi_t^e)\tau\left(l - n_t + h_t\right)^{-1}$. At time t, the new entrant will have 2 possible outcomes, or "stories". At time *t+1,* if successful, the new entrant will be an incumbent and survive with probability $\Pr(\pi_{t+1}^{in} \geq 0)$ or fail with probability $[1 - \Pr(\pi_{t+1}^{in} \geq 0)]$; still at time *t+1*, the unsuccessful new entrant will be unemployed and have a certain probability of being still unemployed and a probability of being hired as a worker, and so on. In other words, at time *t=1* there will be 2 possible outcomes (or stories) for the new entrant, at time *t=2* there will be 4 possible stories, at time *t=3*, there will be 8 possible stories, at time t=n there will be 2^n possible stories. Similarly, the worker who decides not to enter the market as an entrepreneur, with probability $\Pr(\pi_t^{in} \geq 0)$ will earn the wage ω_t and with probability $[1 - \Pr(\pi_t^{in} \geq 0)]$ will loose the job because her firm will go bankrupt and earn the unemployment subsidy $(n_t\omega_t + h_t^e\pi_t^e + h_t^{in}\pi_t^{in})\tau\,(l - n_t)^{-1}$. However, if we move on in time, for instance, at time *t+2*, the surviving entrant will get with probability $\Pr\left\{\pi_{t+1}^{in} \geq 0\right\}$ the profit of the incumbent π_{t+1}^{in} and with probability $\left[1 - \Pr(\pi_{t+1}^{in} \geq 0)\right]$ the unenployment subsidy. Valuating the expectation of future profits for the new entrant means valuating a tree of probabilities where from *t+1* onwards, in each period the firm can survive (with a certain probability) or going bankrupt (with the complementary probability). Having gone bankrupt in period *t+1* can be followed by the event of being hired as a worker by a new firm or remaning unemployed. Having succeded in period $t + 1$ can be followed by a further success or by a failure; and the failure can be followed by the event of being hired as a worker or remaining unemployed again, an so on. In other word, the rational forward lookng decision maker that decides at time *t=1,2,3..n,* i.e. for all the future periods from t onwards, faces 2^t different stories for each and every t periodo in its future. For instance, when *t=3*, i.e. 3 periods ahead from the moment where the decision is taken, there will be $2^3 = 8$ possibile stories, each of them with a given sequence of conditional probabilities. So, for instance, the probability that the firm of the new entrant will survive after the entry and for 2 periods ahead after the entry is given by $\Pr(\pi_t^e \geq 0) \cdot \Pr(\pi_{t+1}^{in} \geq 0) \cdot \Pr(\pi_{t+2}^{in} \geq 0)$. The probability that a new entrant will survive 3 periods and then go bankrupt is given by $\Pr(\pi_t^e \geq 0) \cdot \Pr(\pi_{t+1}^{in} \geq 0) \cdot \Pr(\pi_{t+2}^{in} \geq 0) \cdot \left[1 - \Pr(\pi_{t+3}^{in} \geq 0)\right]$.

The further away the expectations formulated at time "t" the higher the number of combinations of possible future stories that characterizes the future of the decision maker. This boils down into an increasing degree of on-going uncertainty associated to each and every expected variable, since at every future time, each agent can be in one out of several states that depend on the decisions simultaneously taken by all the other agents. Obvously this does not prevent the agents from formulating expectations on the future relevant variables, even though the variance of such expectations might be higher the further away in the future is the forecast.[6]. What we need to assume here, for the determination of the incentive-compatibility constraint in wage setting are just two precise restrictions:

We approximate and define then the expected future stream of income from time $t+1$ onwards for the successful entrant at time t as

$$J_{t+1} = J_{t+1}(\overbrace{E(h_{t+1}^{in})}^{-}, \overbrace{E(h_{t+1}^{e}(\overbrace{r_{t+1}}^{-})}^{-}), \overbrace{\Pr(\pi_t^e \geq 0)}^{+}) \tag{21}$$

J_{t+1} positively depends on r_{t+1} because a higher interest rate would reduce the number of entrants and reduce the income of the incumbents i.e.

$$J_{t+1} = J_{t+1}(\overbrace{E(h_{t+1}^{in})}^{-}, \overbrace{r_{t+1}}^{+}), \overbrace{\Pr(\pi_t^e \geq 0)}^{+})$$

J_{t+1} is the expected future stream of possible incomes of the successful entrant at time t (who has become incumbent from time $t+1$ onwards) that also takes into account the future probability of the future incombent to

[6] The influence of externality in individuals' choices and in expectation formulation is certainly a very relevant issue. In this regard, the rational beliefs assumption (Kurz, 1994a, 1994b) could be an interesting approach that could be profitably applied to this model too. However, questioning the assumption of rational expectations is far beyond the purpose of this draft.

go bankrupt (due to the simultaneous decisions and interactions with other agents) and get the unemployment subsidy.[7]

Similarly we can approximate in the following way the the expected future stream of income from time $t+1$ onwards for the worker employed by an incumbent surviving at the beginning of time $t+1$

$$\Gamma_{t+1} = \Gamma_{t+1}(\overbrace{\Pr(u=0)}^{+}, \overbrace{\Pr(\pi_t^{in} \geq 0)}^{+}) \tag{22}$$

Both J_{t+1} and Γ_{t+1} reflect the expectations of the individuals, however, idiosyncratic information shocks may take place all the time adn determine entry and exit.

Since the firms are price setters with quantity precommitments, what matter for the workers is the expected rivalry among firms, the probability of survival of the firm they work for and whether or not the economy will be in full employment. We can think of the probability of full employment (or zero unemployment), defined as $pr(u=0)$ as a positive function of the expected number of existing firms $E(h_{t+1}^{in}) + E(h_{t+1}^{e}(r_{t+1}))$, so that we can rewrite

$\Pr(u=0) = \upsilon(E(h_{t+1}^{in}) + E(h_{t+1}^{e}(r_{t+1})))$

therefore

[7] Furthermore, the profits of the new entrant are higher the lower the interest rate and the higher the probability of survival. Indeed, there is no future income for the new entrant if it fails before time $t+1$. Therefore, from time $t+2$ onwards, in the event of unemployment, the stream of future expected income of an unemployed individual (taking into account the probability to be hired again as a worker) will be exactly the same, no matter whether the unemployed individual has been a worker or an entrepreneur before.

$$\Gamma_{t+1} = \Gamma_{t+1}(\overbrace{E(h_{t+1}^{in}) + E(h_{t+1}^{e}(r_{t+1}))}^{+}, \overbrace{\Pr(\pi_t^{in} \geq 0)}^{+}) \tag{23}$$

Similarly to J_{t+1}, $\Gamma_{t+1}$is the expected future stream of possible incomes of the worker at time t; this takes into account the future probability of the worker to loose her job (due to the simultaneous decisions and interactions of other agents) and get the unemployment subsidy.

In addition to the variables introduced earlier, let us define Υ_{t+2} as the expected stream of income from time $t+2$ onwards of an individual unemployed at time $t+1$. We are now enabled to write the incentive compatibility

constraint for wage setting under unemployment.

In this case the wage is set by the oligopolistic firms in such a way to discourage entry, therefore it has to satisfy the incentive compatibility constraint saying that the expected future discounted stream of income from time $t+1$ onwards for the worker employed by an incumbent surviving at the beginning of time $t+1$ has to be greater or equal to the expected future discounted stream of income from time $t+1$ onwards for the new entrant.

$$\begin{aligned}
&\Pr(\pi_t^e \geq 0)(1+\rho)^{-1}\{[E_{t-1}(\pi_t^e) + E_{t-1}(\omega_t)](1-\tau-\varsigma)+\\
&+J_t(\cdot)\} + [\Pr(\pi_t^e < 0)](1+\rho)^{-1}\cdot\\
&\cdot E_{t-1}[(n_t(\omega_t + h_t^e\pi_t^e + h_t^{in}\pi_t^{in}))\tau(l-n_t)^{-1}+\\
&+\Upsilon_{t+1}] \leq (1+\rho)^{-1}\cdot\Pr(\pi_t^{in}\geq 0)\cdot\\
&\cdot[E_{t-1}(\omega_t)(1-\tau-\varsigma)+\Gamma_t(\cdot)] + (1+\rho)^{-1}\cdot\\
&\cdot\{\Pr(\pi_t^{in}<0)\cdot E_{t-1}[(n_t(\omega_t + h_t^e\pi_t^e + h_t^{in}\pi_t^{in}))\tau(l-n_t)^{-1} + \Upsilon_t]\}
\end{aligned}$$

as we said, for τ very small, the term $[n_t(\omega_t + h_t^e\pi_t^e + h_t^{in}\pi_t^{in}))\tau(l-n_t)^{-1}]$ will be very small and negligible.

Let us define it "*subsidies*". Even smaller will be the term

$$[\Pr(\pi_t^e < 0)](1+\rho)^{-1}[n_t(\omega_t + h_t^e\pi_t^e + h_t^{in}\pi_t^{in}))\tau(l-n_t)^{-1}]$$

and the term
$[\Pr(\pi^{in}_{t+1} < 0)](1+\rho)^{-1}[n_t(\omega_t + h^e_t\pi^e_t + h^{in}_t\pi^{in}_t))\tau(l-n_t)^{-1}]$

Therefore we may write

$$\begin{aligned}\omega^*_t \;\; \geq \;\; & \frac{\Pr(\pi^e_t \geq 0)\cdot E_{t-1}\left(\pi^e_t\right)}{\Pr(\pi^{in}_t \geq 0) - \Pr(\pi^e_t \geq 0)} + \\ & + \frac{\Pr(\pi^e_t \geq 0)\cdot J_{t+1}(\cdot)}{[\Pr(\pi^{in}_t \geq 0) - \Pr(\pi^e_t \geq 0)]\cdot(1-\tau-\varsigma)} + \\ & + \frac{J_{t+1}(\cdot)\cdot\Pr(\pi^e_t \geq 0) - \Gamma_{t+1}(\cdot)\Pr(\pi^{in}_t \geq 0)}{[\Pr(\pi^{in}_t \geq 0) - \Pr(\pi^e_t \geq 0)]\cdot(1-\tau-\varsigma)} + \\ & + \frac{\Upsilon_{t+1}[\Pr(\pi^e_t < 0) - \Pr(\pi^{in}_t < 0)]}{[\Pr(\pi^{in}_t \geq 0) - \Pr(\pi^e_t \geq 0)]\cdot(1-\tau-\varsigma)} + \\ & + \frac{[\Pr(\pi^e_t < 0) - \Pr(\pi^{in}_t < 0)]subsidies}{[\Pr(\pi^{in}_t \geq 0) - \Pr(\pi^e_t \geq 0)]\cdot(1-\tau-\varsigma)} \end{aligned} \tag{24}$$

If the incentive-compatibility constraint 24 on wage setting were not respected, then all the workers would have incentive to leave their jobs and start a new firm. Next period there would be full employment, all the firms would be incumbent and would have lost their bargaining power and the wages, which, as we said, would be set at a level where expected profits would be zero. Therefore the wage setting oligopolistic firms have incentive not tu push the wages below the incentive compatibility constraint. In particular, considering 24 as a binding equality and neglecting the last fraction in the last row (who is positive and whose magnitude is very small compared to the rest of the inequality),ω^*_t is a function of the following variables:

$$\begin{aligned}\omega^*_t \;\; = \;\; & \omega^*_t(\overbrace{E_{t-1}\left(\pi^e_t \mid \Omega_{t-1}\right)}^{+}, \overbrace{\Pr\left[(\pi^e_t \geq 0) \mid \Omega_{t-1}\right]}^{+}, \overbrace{\Pr\left[(\pi^{in}_t \geq 0) \mid \Omega_{t-1}\right]}^{-}, \\ & \overbrace{E_{t-1}(h^{in}_t \mid \Omega_{t-1})}^{-}, \overbrace{E_{t-1}(h^e_t \mid \Omega_{t-1})}^{-}, \overbrace{r_{t-1}}^{-})\end{aligned} \tag{25}$$

The higher the expected profits and the probability of survival of new entrants, the higher the wages; The higher the probability of survival of the incumbent and the expected number of firms, the lower the wages (since a safer job and a higher number of firms makes entry less attractive). Furthermore, for the same reason associated to the incentive compatibility constraint, they are affected (although with one lag) by the interest rate r_t

The notation of 25 specifies that all the probabilities and expected variables are, of course, conditional on the information set Ω_{t-1} available at time $t-1$. Defining the right-hand side of inequality 24 as Φ_t, we can introduce an object that turns out to be useful in aggregating the behaviour of heterogeneous agents: **the probability of entry,** $\Pr(entry)_t$, which may be interpreted as **the ratio between the integral (over the whole population of workers $n_t(1-h_t)$ at time t) of the generic individuals "i" for whom, due to idiosyncratic informational shocks,** $w_t < E_{t-1,i}(\Phi_t)$ **,and the total amount of workers** $n_t(1-h_t)$.

$$\Pr(entry)_t = \left[\int_0^{n_t(1-h_t)} (\Pr(w_t < E_{t-1,i}(\Phi_t))_i di \right] / n_t(1-h_t) \qquad (26)$$

Having defined $E_i(\Phi_t)$ as the optimal wage expected by the worker i for time t, The difference $w_t - E_{t-1,i}(\Phi_t) < 0$ can only be determined by random idiosyncratic shocks. In this sense, ideally, $\int_0^{n_t(1-h_t)} (\Pr(w_t < E_{t-1,i}(\Phi_t))_i di$ can be thought of as the sum of the absolute values of all the negative idiosyncratic shocks for all the individual workers for time t. They reflect the preception of the workers who have an expected optimal wage higher than the one set in the "incentive compatible" wage constraint defined in 24 for the period about to begin. Let us define $\varepsilon_{t,j}$ the generic idiosyncratic shock (which may be both positive and negative) and $\varepsilon_{t,j}^-$ the negative idiosyncratic shock, i.e. the shock generating the incentive to enter the market. Let us define the sum of the absolute value of these negative socks at time t as

$S^{-}(\varepsilon_{t,j}^{-})$ and let us define $Var(\varepsilon_{t,j})$ as the variance of **all the shocks (both positive and negative)** at time t

We introduce here 3 assumptions for potential empirical implementations and numerical simulations.

We assume that:

a) all the individuals share a common information set on the trend of the aggregate output $E_{t+i-1}(Q_{t+i})$ **(such as defined in 15) that may be estimated with no bias as an AR(k);**

b) let us define an operator $T_{t,k}^{*}$ **as s a function of the information available at time** t $T_{t,k}^{*} = T_{t,k}^{*}(\rho_1, \rho_2, ...\rho_k \mid Q_0, Q_1, ..., Q_{t-1})$ **that contains the trend information at time t, such that** $E_t(Q_{t+1}) = T_{t,1}^{*}Q_t$ ... , $E_t(Q_{t+k}) = T_{t,k}^{*}Q_t$

c) The variance $Var(\varepsilon_{t,j})$ **of all the (both positive and negative) idiosyncratic shocks, for any time t is an increasing function of** $T_{t,1}^{*}$

Then $S^{-}(\varepsilon_{t,j}^{-})$ **is also an increasing function of** $T_{t,1}^{*}$

Assumptions "a)", "b)" and "c)" hold under rational expectations, although they do nut need such a restrictive assumption as rational expectations.

Assumptions "a)", "b)" and "c)" basically say that when the macroeconomic trend of the aggregate output and market size unexpectedly increases, then $S^{-}(\varepsilon_{t,j}^{-})$ also increases.

For those who like the rational expectation assumption, this simply means that even is the overall market expectations are correct, each individual is more likely to make idiosyncratic missperception mistakes in interpreting how would the oligopolists react to each other when the market size (aggregate output) modifies its pattern of change.

For those who prefere other approaches, like behavioural economics or rational beliefs theory[8], this simply means that the diversity of opinions on

[8] See, for instance, Kurz (1994a, 1994b)

how would the oligopolists react to each other increases whn the market size changes its pattern of growth.

However, to put it another way, since expectations are based on the available information set, when the pattern of behaviour of this set changes, the uncertainty and prediction volatility increase, since all the individuals take all their decisions simultaneously, even though they are all rational. This is consistent with sevaral notions of rationality.

Assumptions "a)", "b)" and "c)" imply that $\Pr(entry)_{t+1}$ is an increasing functon of $T^*_{t,1}$.

Since we have defined already the probability of bankruptcy of a new entrant and an incumbent (which are $(1-\Pr(\pi_t^{in} \geq 0))$ and $(1-\Pr(\pi_t^{in} \geq 0))$ respectively) the probability of bankruptcy of a generical firm $\Pr(exit)_t$ may be expressed as follws:

$$\Pr(exit)_t = \frac{h_t^e(\Pr(\pi_t^e < 0)) + h_t^{in}(\Pr(\pi_t^{in} < 0))}{h_t} \tag{27}$$

In the event of full employment, for the reasons explained before, there are no extra-profits, all the entrepreneus are incumbent, but since they do not enjoy any market power, the remuneration of each entrepreneur is given by the wage she pays to herself. Therefore, in this case the (gross, before taxes) wage is derived by the condition $E_{t-1}(\pi_t^{in}) = 0$, which implies

$$\omega_t^{fu} = \frac{E_{t-1}\left(P_t^j \mid \Omega_{t-1}\right) E_{t-1}(c_t^j \mid \Omega_{t-1})}{E_{t-1}(L_t^* \mid \Omega_{t-1})} \tag{28}$$

or, since all firms are, in this case, identical, equivalently, in aggregate terms:

$$\omega_t^{fu} = \frac{E_{t-1}\left(P_t \mid \Omega_{t-1}\right) E_{t-1}(Q_t \mid \Omega_{t-1})}{E_{t-1}\left[n_t(1-h_t) \mid \Omega_{t-1}\right]} \tag{29}$$

Therefore the determination of the the wages has a point of discontinuity triggered by the level of full employment. In fact:

$$\omega_t = \begin{cases} \omega_t^* & if \quad n_t < l \\ \omega_t^{fu} & if \quad n_t = l \end{cases} \tag{30}$$

The situation of full employment is subject to number of unexpected shocks and, therefore, is likely to be a very temporary configuration of the system.

3.1 Modeling entry/exit in the macroeconomic equilibrium

Following Aoki and Yoshikawa (2007), we start by introducing an interpretation of how do interacting agents behave at a microeconomic level. Suppose that agents have binary choices or there are two types of agents. The two choices can be represented by two states (say state 0 and state 1). If we have n agents, the state of n agents may be represented as follows:

$$s = (s_1, s_2, ..., s_n)$$

where che choice by agent i is denoted by $s_i = 1$ or $s_i = 0$ and so on. A set of all the possible values of s is called "state space" S. This vector contains a complete description of who has chosen what. The purpose of this assumption is to describe the dynamic process of how do agents revise their choices in time, due to incentives, externalities, costs and unexpected news. Since we are interested in time evolution the states, we consider a jump Markov process, although, in our case, (differently from Aoki and Yoshikawa), in discrete time and not in continuous time. Its timing is relevant.

The process we are interested in concerns the workers who become entrepreneurs and the entrepreneurs who go bankrupt (i.e., given the number of employed individuals n_t at time t, how many of them increase or decrease the portion of existing firms h_t).

On the other hand, the increase or decrease in n_t is a mere consequence of the proess of entry/exit of new firms and may be easily modelled if we assume, for the sake of simplicity, that in each period t the number of workers employed by each firm is determined at the time when entry is decided (i.e. at time $t-1$) when the number of firms operating next period t is known and all the firms set their quantity precommitment and (given the amount of labour needed per unit of product)

Let us start by modelling entry and introduce the notion of transition rates. The agents make a binary choice between two states (in this case being a worker and being an entrepreneur) which can be interpreted as one agent changing his mind (and his state). Following the notation of Aoki and Yoshikawa, we have, between time $t-1$ and t, when entry takes place the following transition rates:

$$q(n_t h_t, n_t h_t + 1) = [n_t(1 - h_t)]\, \eta_1(h_t) \tag{31}$$

$$q(n_t h_t, n_t h_t - 1) = n_t h_t \eta_2(h_t) \tag{32}$$

Equation 31 represents the transition rate of an increase in the number of workers who were not entrepreneurs and decide to enter the market (with $0 < h_t < n_t$). The transition rate refers to a notion of feasibility (not probability in itself) of the choice to enter the market and depends on the number $n_t(1 - h_t)$ of employed people who are not entrepreneurs. On the other hand, $\eta_1(h_t)$ is a function that takes into account externality: for this reason it is a decreasing function of h_t because the decision to enter the market is discouraged by a high number of existing entrepreneurs. The higher h_t, the smaller $\eta_1(h_t)$. In the benchmark case where the economy reaches full employment, the workers will be remunerated exactly like the entrepreneurs, and there will be no incentive and no room for new entries. Empirically speaking this configuration of the economic system is not likely to last for long, since equations 28 and 29 contain many potential sources of stochastic shocks.

In each period t we assume that a given portion of the existing entrepreneurs h_t is expected to leave the market for "natural" causes and become unemployed. Let us call this expected value $E_{t-1}(\delta_t)$. Let us further assume that $E_{t-1}(\delta_t)$ is fixed and exogenous, but subject to random shocks, so that

$$\delta_t = E_{t-1}(\delta_t) + \varepsilon_t^{\delta}$$

where ε_t^{δ} is the "exit random shock", distributed as $N(0, \varepsilon_t^{\delta})$.

$\delta_t \cdot n_t \cdot h_t$ may be interpreted as the outflow of existing firms out of the market and ε_t^{δ} reflects any idiosyncratic informational shock leading to exit. The assumption of exogenous bankrupcty rate closely recalls the one made by Etro and Colciago (2010) and, of course, as they also point out, it might be improved upon, by endogenizing it. However, we keep δ_t exogenous (although subkect to a random shock) for the sake of simplicity.

Following again Aoki and Yoshikawa (2007), we define now the so-called "master equation", i.e. the Chapman-Kolmogorov equation, describing the time evolution of the probability distribution of states. For the purposes of this paper, we only need to use it here in a simplified way, to identify the stationarity or equilibrium probabilities of states, without considering the other solution tools and techniques invocked by Aoki and Yoshikawa, such as the use of the probability generating function or the Taylor expansion or the cumulant generating function. Differently from Aoki and Yoshikawa (2007), we apply the Chapman-Kolmogorov equation in discrete time and not in continuous time.

What we are interested in, for the sake of our model, is only the so-called equilibrium probabilities of states.

$$\Pr(s(s_1, s_2, ..., s_n))_{t+1} - \Pr(s(s_1, s_2, ..., s_n))_t =$$

$= \sum\limits_{s'} q(s', s) \cdot \Pr(s', t) - \Pr(s, t) \sum\limits_{s} q(s, s')$

Where the sum is taken over all states $s' \neq s$ and $q(s', s)$ is the transition rate from state s' to s. Intuitively speaking:

$\Delta \Pr(\cdot)/\Delta t =$ (inflow of probability fluxes into s) - (outflow of probability fluxes out of s). Here, of course, Δt is only a unit time interval.

In our case we can define the net inflow of probability of "being entrepreneur" $\Delta^h \Pr(\cdot)t$ as follows:

$$\begin{aligned} \Delta^h \Pr(\cdot) &= \sum_{(n-h)} q(n_t h_t, n_t h_t + 1) \cdot \Pr(entry)_{t+1} - \\ - \Pr(exit)_t \sum_h q(n_t h_t, n_t h_t - 1) &= \\ &= [n_t(1 - h_t)]\, \eta_1(h_t) \cdot \Pr(entry)_{t+1} - \Pr(exit)_t n_t h_t \eta_2(h_t) \end{aligned} \tag{33}$$

i.e.

$$\Delta^h \Pr(\cdot) = [n_t(1 - h_t)]\, \eta_1(h_t) \cdot \Pr(entry)_{t+1} - n_t h_t \delta_t \tag{34}$$

i.e. the inflow probability of firms increases with the level of employment n_t, with the probability of entry at time $t+1$ (i.e. the probability that the "no entry" incentive compatibility constraint is violated when wages are set at time t) and decreases with h_t. Then, since $[n_t(1 - h_t)]\, \eta_1(h_t) \cdot \Pr(entry)_{t+1}$ generates the new born firms (i.e. the entrants) at time t+1, we have:

$$
\begin{aligned}
\Delta^h \Pr(\cdot) &= n^e_{t+1} h^e_{t+1} - n_t \delta_t (h^e_t + h^{in}_t) \qquad (35) \\
n_{t+1} h_{t+1} - n_t h_t &= n^e_{t+1} h^e_{t+1} - n_t \delta_t (h^e_t + h^{in}_t) \\
n_{t+1} h_{t+1} - n^e_{t+1} h^e_{t+1} &= n_t h_t (1 - \delta_t) \\
n^{in}_{t+1} h^{in}_{t+1} &= n_t h_t (1 - \delta_t) \qquad (36)
\end{aligned}
$$

Setting equal to zero the left-hand side of 35 and solving the equation, we get the equilibrium probability or stationarity of the states for the entry/exit process. If we required that each pair in the right-hand side of 33 is zero, would we obtain the "detailed balance condition" (see Aoki and Yoshikawa, p. 33), which is a sufficient (not a necessary) condition. We do not need the detailed balance condition for the sake of our model because we do not need to identify each individual.

Ex ante, if the wage are set by the oligopolistic firms according to the incentive compatibility constraint 24 and if all the individuals had perfectly identical expectations (i.e. if there were no idiosyncratic informational shocks), then $\Pr(entry)_{t+1}$ be null. The ex post deviations would be those caused by all the possible stochastic shocks affecting the right-hand side of inequality 24.

If the incentive-compatible wage setting rule is not violated, if no random shock occurs, then in equilibrium no worker would have incentive for entry

Equation 34 shows that the evolution in time of the probability of "being entrepreneur" may be interpreted as the inflow probability of successful new entrants, minus the outflow probability of firms that go bankrupt. From 34 we get the dynamics for $n_t h_t$:

$$
\begin{aligned}
\Delta^h \Pr(\cdot) &= n_t h_t \left[\frac{1 - h_t}{h_t} \eta_1(h_t) \Pr(entry)_{t+1} - \delta_t \right] \qquad (37) \\
n_{t+1} h_{t+1} &= n_t h_t \left\{ 1 + \left[\frac{1 - h_t}{h_t} \eta_1(h_t) \Pr(entry)_{t+1} - \delta_t \right] \right\} \qquad (38)
\end{aligned}
$$

or, in terms of growth rate

$$\frac{n_{t+1}h_{t+1} - n_t h_t}{n_t h_t} = \left(\frac{1-h_t}{h_t} \eta_1(h_t) \cdot \Pr(entry)_{t+1} - \delta_t \right) \tag{39}$$

Obviously, in a stationary equilibrium (and only in a stationary equilibrium), when $\Delta^h \Pr(\cdot) = 0$, we have:

$$\Pr(entry)_{t+1} = \frac{h_t}{1-h_t} \cdot \frac{\delta_t}{\eta_1(h_t)} \tag{40}$$

The growth rate of $n_t h_t$ negatively depends on the level of h_t and δ_t and positively depends on the probability of entry, which is a function of $T^*_{t,k}$.

$\Delta^h \Pr(\cdot)$ in 38, together with the production function, determine the dynamics of employment, since an increase in the number of firms (under Bertrand competition with quantity pre-commitment or Cournot oligopoly) would determine, under fairly general conditions, a higher level of output and, as a consequence, given the production function 17, a higher level of employment.

$$n_{t+1} = n_t \left[1 + h_t \left(\frac{E_{t-1}(\varphi^*_t)}{\Lambda} \right)^{1/\alpha} \left(\frac{1-h_t}{h_t} \eta_1(h_t) \cdot \Pr(entry)_{t+1} - \delta_t \right) \right] \tag{41}$$

It might be interesting to note that, from 41

$$\frac{n_{t+1} - n_t}{n_{t+1}h_{t+1} - n_t h_t} = \left(\frac{E_{t-1}(\varphi^*_t)}{\Lambda} \right)^{1/\alpha}$$

i.e. the ratio between changes in employment and changes in the number of entrepreneurs increases with the expected output per firms, wich may be negatively affected by positive technology shocks, if we want to introduce them in the model.

Once the wage for time t is set (between $t-1$ and t), the entry decision are taken: both the new entrants and the incumbents decide the number of workers to hire for the next period (i.e. from t to $t+1$), on the basis of her profit expectations. In this way, since we are asssuming a labour contract that establishes *ex ante* a fixed number of hours to be worked, and given the production function, each oligopolistic firm pre-commit itself to a certain output.

Given our assumptions on the unit elastic demand function, each firm would not have incentive to increase the number of hours to be worked in each period, because an increase in the output would not increase the revenues; furtermore, in this way a firm would trigger a retaliation from the other oligopolistic firms (see Appendix 2).

Therefore, each firm that will be in the market at time t employs L_t^* units of labour (in our case workers), where

$$L_t^* = \left(\frac{E_{t-1}(\varphi_t^*)}{\Lambda} \right)^{1/\alpha} \tag{42}$$

Where φ_t^* and, as a consequence, L_t^* are determined at the pre-commitment stage in the oligopolistic game (see appendix 2).

Between time $t-1$ and t but before time t, all the new entrant and the existing firms plan and arrange the labour contracts (which start at time t), hire workers and organize entry, which actually takes place at time t. Then, at time "t" the firms (both incumbents and new entrants, i.e. the former workers) are binded with contracts to their workers and to the lenders who lent them the money to cover the fixed entry cost $(1+r_t)F$, set the amount of output to produce, i.e. precommit themselves to the quantity to be produced and sold from t to $t+1$. Madden (1998) extends the well known Kreps-Scheinkman result (Bertrand competition with quantity pre-commitment yields Cournot-Nash equilibrium in oligopoly) to fairly general conditions. This is briefly discussed in the the next section.

4 Interpretation and implications of the Cournot equilibrium

This section is only meant to recall that, under fairly general assumptions (and under the assumptions of our model), with quantity precommitments by the incumbents and new entrants, our model display a Cournot-Nash equilibrium even with price competition. This is due to an extension provided by Madden (1998) of the famous Kreps-Scheinkman (1983) result. The results by Madden (1998) that are relevant for this work are shown in Appendix 2. of Between time t-1 and t the existing firms set and announce the wages that apply between time t and t+1 and, immediataly after that, the process of entry and output determination begins. We assume that the entry process is sequential and takes place in two stages, all of them happening after time t-1 and before time t. At stage 1, after the wage is set by the incumbents, the entry/not entry decision is taken by the potential entrants (who are the workers with at least one year of seniority). Then the incumbents and the new entrants hire the workers by setting one-year labour contracts that also specify the amount of hours to be supplied by each worker for the coming period. Given the production function, this also implicitly determines the quantity precommitment. The new entrants hav to bear the fixed entry costs F, and bank loans to cover the fixed costs F, so that, at time $t+1$, the entrants will have to repay $(1+r_t)F$. All the costs are incurred at stage 1 and all of them, including labour, are sunk. Since, as we said, all incomes (profits, labour and unemployment subsidies) for time t are received by all the individuals at the end of time t, labour costs are first accounted as debt of the firms toward the workers, at the beginning of time t, and then paid out to workers at the end of time t. However, they occur (since they are recorded as debt) at the beginning of time t

The assumptions of this model, with the addition of **Assumption D2** (see Appendix 2), meet the requirements of Madden (1998) theorem shown in appendix 2, therefore the oligopolistic firms of our model, having defined capacity and output at stage 1 of the game, have a cournot payoff $\pi_i^c(c^1, ..., c^h)$ at stage 2, which is sequential to stage 1 but still takes place between time $t-1$ and time t. We assume in fact that stage 2 takes place just immediately before time t. For this reason it is known at time t.

Furthermore, Madden shows that if the demand is uniformly elastic (and constant elasticity is a sub case) and asymptotic (witht he vertical and horizontal axes as asymptotes) and the firms' costs can be represented by a convex and strictly increasing function and if the assumptions on quantity determination and rationing rules reported in Appendix 2 hold, then there exists at least one pure strategy Cournot-Nash equilibrium. In our case this equilibrium might not necessarily be unique, since unicity would require symmetric costs, i.e. identical costs for each firm (see Madden, 1998, theorem 3, p. 205).

The rationed demand function at stage 2 of the Kreps-Scheinkman game lies in between two benchmark cases: the surplus-maximizing rule and the proportional rule. In the latter, the consumers first served by lower pricing firms are chosen randomly.

We introduce a few more assumptions:.

First, all the firms share common knowledge and share expectations on the aggregate demand, know the two possible rationing mechanism and price adjustment rules that take to the Cournot-Nash equilibrium and formulate expectation about their expected market share.

Second, all firms have identical technology and identical production functions.

Third, wages, as we said, are pre-determined; the sunk costs do not only include the costs of entry, but they also include the exact amount of labour at time t, L_t^* that both the new entrants and the incumbents have employed, on the basis of their *ex ante* expectations. We call it the optimal *ex ante* amount of labour, associated to the optimal *ex ante* individual firm output φ_t^* at time t.

Fourth, on the basis of the assumptions of the model, the game among the oligopolistic firms, with Bertrand competition and quantity precommitments, admits **the exact Cournot reduced form (See Madden's, 1998 theorem in appendix 2)** and, in addition**,** a Cournot-Nash equilibrium exists. The exact determination of the quantity produced by each firm is an empirical matter.

All firms (no matter if they are incumbents of new entrants) share the same marginal cost finction. The new entrants, after taking their entry/non entry decision, they have already discounted the probability of bankruptcy, which might take the form of unexpected rationing.

Therefore, the price at time t is determined by the equality between marginal revenues an marginal costs, under Cournot oligopoly is given by

$$P_t = \frac{\omega_t^*}{\left(1 + \frac{1}{n_t h_t \cdot \varepsilon_D}\right)} \frac{1}{\alpha} (E_{t-1}(\varphi_t^*))^{\frac{1-\alpha}{\alpha}} \Lambda^{-\frac{1}{\alpha}} \tag{43}$$

In 43, $n_t h_t$ is the number of existing firms (always strictly greater than 1), ε_D the (constant) demand elasticity, φ_t^* is the individual firm output, set in the first stage of the quantity precommitment Kreps-Scheinkman game briefly described in appendix 2. We can think of φ_t^* as determined by the firms expectations at time $t-1$, according to the proportional rationing rule, which also detemines the unique market clearing price 43. Next sections contains a few more comments about the determination of φ_t^* for the sake of the empirical analysis.

5 Summarizing the theoretical model and setting up the premises for empirical analyses and numerical simulations...

All the previous sections contain a detailed explanation of the theoretical model, which boils down into the aggregate demand 16, pricing equation 43, wage determination (in equations 30, 25 and 29) as well as dynamics of the new entrants $n_t h_t^e$ and incumbent, to be explained below in this section.

Therefore, the purpose of this section is to summarize the equations of the model, in order to provide a basis for further empirical analysis and/or numerical simulation. Since this is a new and unconventional model, there is no previous literature we may refer to, for the values of the parameters for further research with possible numerical simulations, therefore all the relevant parameters have to be estimated.

The first detail to look at is dynamics of entry/exit.

For the sake of the empirical analysis, a time series estimate of δ_t may be obtained, based on the assumption that it behaves as follows:

$\delta_t = E_{t-1}(\delta) + \varepsilon_t^{\delta}$.

In particular, $E_{t-1}(\delta)$ might be estimated as as the average of the time series data for δ available from the beginning of the available time observations (say $t-j$) until $t-1$.

As we said, $\Pr(entry)_t$ is a function of $T_{t,k}^{*}$. Keeping that in mind, an easy and straightforward way to determine the dynamics of of the number of existing firms is simply by starting from 38 lagged one period and noting that the value of $n_{t-1}h_{t-1}\left[\frac{1-h_{t-1}}{h_{t-1}}\eta_1(h_{t-1}) \cdot \Pr(entry)_t\right]$ is simply the number of new born firms at time t, which is known and observable at time t, since all the labour contracts and all the quantity precommitments in the Betrand game with Cournot outcome are decided between $t-1$ and t, but just before time t. Therefore, we may write

$$n_t^e h_t^e = n_{t-1}h_{t-1}\left[\frac{1-h_{t-1}}{h_{t-1}}\eta_1(h_{t-1})\cdot \Pr(entry)_t\right] \tag{44}$$

Of course, the number of new born firms, i.e. an empirical measure for $n_t^e h_t^e$, is in commonly available statistics for most countries.

Hence, lagging one period 38, we get the value of $n_t h_t$ and, moving forwards. all the dynamics of $n_t h_t$ at time $t+1$, $t+2, ...t+n$.

Taking back one period 34 we get

$$\begin{aligned} n_t h_t^e &= (n_t h_t - n_{t-1}h_{t-1}) + n_{t-1}h_{t-1}\delta_{t-1} \\ &= n_t h_t - n_{t-1}h_{t-1}(1-\delta_{t-1}) \end{aligned} \tag{45}$$

Therefore, from 36, $n_t h_t^{in}$ is pre-determined, while $n_t h_t$, as we said, is observeble at time t, due to the assumptions of timing in the model. Obviously, the value of $\eta_1(h_t)\cdot \Pr(entry)_{t+1}$ may also be represented in a different way. From 39 we get:

$$\eta_1(h_{t-1})\cdot \Pr(entry)_t = \left(\frac{h_{t-1}}{1-h_{t-1}}\right)\left(\frac{n_t h_t - n_{t-1}h_{t-1}}{n_{t-1}h_{t-1}} - \delta_{t-1}\right) \tag{46}$$

For what concerns $L_t^* = (E_{t-1}(\varphi_t^*)/\Lambda)^{1/\alpha}$, it needs a few more words, because it is determined by the Betrand game with quantity pre-commitment yielding Cournot outcome (see appendix 2). Since Cournot equilibrium has been interpretedin literature as an evolutionary stable equilibrium (see Osborne and Rubinstein, 1994, pp.38-41) or as a perturbated game, i. e. a game subject to random shocks due to "misperception" or, in our case, informational shocks, (see, again, Osborne and Rubinstein, 1994, pp.41-42), then we may think that the quantity pre-commitment (which implicitly detemines

L_t^* given the production function) is affected by previos experience and previous observation, therefore we may think of estimating L_t^* as as the average of the time series data for L_t^* available from the beginning of the available time observations (say $t-j$) until $t-1$.[9]

The one period ahead dynamics of employment, is determined by 41

$$n_{t+1} = n_t \left[1 + h_t \left(\frac{E_{t-1}(\varphi_t^*)}{\Lambda} \right)^{1/\alpha} \left(\frac{1-h_t}{h_t} \eta_1(h_t) \cdot \Pr(entry)_{t+1} - \delta_t \right) \right]$$

For what concerns the empirical value of $\Pr\left[(\pi_t^e < 0) \mid \Omega_{t-1}\right]$ and

$\Pr\left[(\pi_t^{in} < 0) \mid \Omega_{t-1}\right]$, for each time t, we may think of using average values of the rate of mortality of the new entrant (for $\Pr\left[(\pi_t^e < 0) \mid \Omega_{t-1}\right]$) and of the incumbents (for $\Pr\left[(\pi_t^{in} < 0) \mid \Omega_{t-1}\right]$) using past data from the beginning of the available time observations (say, from $t-j$ to $t-1$,with $j > 1$.

This assumption recalls the fact that in each time t, until the data for time t are available (i.e. at time $t+1$) the agents might not be fully aware of the short run dynamics (e.g. when exactly the economy is in a rurning point of a recession or ad the end of an expansion), therefore they might tend to formulate expectations on the basis of long run expected values, estimated on the basis of the available information set.

Having estimated $\Pr\left[(\pi_t^e < 0) \mid \Omega_{t-1}\right]$ and $\Pr\left[(\pi_t^{in} < 0) \mid \Omega_{t-1}\right]$, we also know, of course, their complements to 1,

$\Pr\left[(\pi_t^e \geq 0) \mid \Omega_{t-1}\right]$ and $\Pr\left[(\pi_t^{in} \geq 0) \mid \Omega_{t-1}\right]$.

We may think of estimating the value $E_{t-1}\left(\pi_t^e \mid \Omega_{t-1}\right)$ and $E_{t-1}\left(\pi_t^{in} \mid \Omega_{t-1}\right)$ at time t as the average of their time series data available from the beginning of the available time observations (say $t-j$) until $t-1$.

[9] In a sense, this procedure recalls the empirical notion of "natural rate of unemployment", usually calculated in empirical analyses as moving average of the level of unemployment.

To perform empirical analyses or numerical simulations in further research, we make use of the aggregate demand (that includes distributional shocks), the pricing equation and the wage determination.

The aggregate demand may be written as a function of aggregate income (15, the first equation below) or by explicitly accounting for the distributional shocks, like in 16, the second equation below:

$$D(P_t, W_t) = \frac{\Xi(r_t)}{P_t}\left(A_t + \frac{1}{1+r_t}\sum_{i=0}^{\infty}\left(\frac{1}{(1+E\left(r_{t+i}\right))(1+E(\iota_{t+i}))}\right)^i E(Q_{t+i})\right)$$

or

$$\begin{aligned} D(P_t, W_t) &= (\Xi(r_t)/P_t)\cdot \\ & \cdot\{A_t + [1/(1+r_t)]\sum_{i=0}^{\infty}[(1+E\left(r_{t+i}\right))(1+E(\iota_{t+i}))]^{-i}\cdot \\ & \cdot E(n_{t+i}(\omega_{t+i} + h^e_{t+i}\pi^e_{t+i} + h^{in}_{t+i}\pi^{in}_{t+i}))\} \end{aligned}$$

The second formulation of the demand equation shows that entry/exit generates distributional shocks affecting the aggregate demand. The price level is determined by plugging equation 43.

The inflation rate ι_t is defined as $\frac{P_t - P_{t-1}}{P_{t-1}}$

For the sake of possible empirical analyses, having assumed that

$\varphi^*_t = E_{t-1}(\varphi^*_t) + \varepsilon^{\varphi}_t$, then the aggregate output is simply $n_t h_t \varphi^*_t$

The value of the future forward-looking variables $E\left(r_{t+i}\right)$ and $E(\iota_{t+i})$ is assumed to be induced by their current value, i.e. their current value is the best predictor and the best expectation for its future values), although, for what concerns the interest rate, it is possible to formalize announced changes in future monetary policy and, therefore, the time structure of the interest rate may be modelled in a more complex way that takes into account credibility and time lenght of policy regimes.

The exogenous and contemporaneous variables appearing in the equation are the liquid assets A_t (positively affecting the level of consumption) and the interest rate r_t (negatively affecting consumption).

P_t is determined by the pricing equation 43:

$$\begin{aligned} P_t &= \frac{\omega_t^*}{\left(1+\frac{1}{n_t h_t \cdot \varepsilon_D}\right)} \frac{1}{\alpha}(\varphi_t^*)^{\frac{1-\alpha}{\alpha}} \Lambda^{-\frac{1}{\alpha}} \\ &= \frac{\omega_t^*}{\left(1+\frac{1}{n_t h_t \cdot \varepsilon_D}\right)} L_t^* \frac{1}{\alpha}(\varphi_t^*)^{1-\alpha} \end{aligned}$$

We know h_t and ε_D (since we have a unit elastic aggregate demand function), while ω_t^* is determined in the wage equations, as recalled below. For what concerns $\frac{1}{\alpha}(\varphi_t^*)^{1-\alpha}$, while α may be determined from the esimates of the labour productivity available from the statistics of most country, φ_t^*, like L_t^*, is affected by the Betrand game with quantity pre-commitment yielding Cournot outcome (see appendix 2). Therefore, similarly to what we did for L_t^*, we may estimate is as as the average of the time series data for L_t^* available from the beginning of the available time observations (say $t-j$) until $t-1$.

Wages are predetermined at time t by equations 30, 25 and 29

$\omega_t = \{ {}^{\omega_t^* \;\; if \quad n<l}_{\omega_t^{fu} \;\; if \;\; n=l}$

where :

$$\begin{aligned} \omega_t^* = \omega_t^*(&\overbrace{E_{t-1}\left(\pi_t^e \mid \Omega_{t-1}\right)}^{+}, \overbrace{\Pr\left[(\pi_t^e \geq 0) \mid \Omega_{t-1}\right]}^{+}, \overbrace{\Pr\left[(\pi_t^{in} \geq 0) \mid \Omega_{t-1}\right]}^{-}, \\ &\overbrace{E_{t-1}(h_t^{in} \mid \Omega_{t-1})}^{-}, \overbrace{E_{t-1}(h_t^e \mid \Omega_{t-1})}^{-}, \overbrace{r_{t-1}}^{-}) \end{aligned}$$

and

$$\omega_t^{fu} = \frac{E_{t-1}\left(P_t\right) E_{t-1}(Q_t)}{E_{t-1} n_t (1 - h_t)}$$

For the sake of the empirical implementation, ω_t^* may be estimated by regressing it on r_{t-1} and the other dependent variables, whose value may be calculated as explained above.

The model can be extended by introducing a monetary policy rule, consisting of determining the interest rate r_t . r_t impacts with no lags the aggregate demand and with one period lag the wage determination, when there is unemployment (no impact if there is full employment). The impact of monetary policy on the value of the new born firms $\frac{1-h_t}{h_t}\eta_1(h_t) \cdot \Pr(entry)_{t+1}$ may be tested by setting $\Theta_t = \frac{1-h_t}{h_t}\eta_1(h_t) \cdot \Pr(entry)_{t+1}$ and regressing Θ_t on a set of regressors including its lagged values and the current and lagged values of r_t .A possible way to build the tests would be by applying the "general-to-specific" methodology. If r_t turned out to affect Θ_t, then the monetary policy would generate a rather rich dynamics in the economy, since it would affect the aggregate demand, the wage determination and the dynamics of entry.

It might be useful to rewrite below all the eqautions of the model, while the exogenous varible, as well as random shocks have been briefly described earlier in this section.

$$n_t h_t = n_{t-1} h_{t-1} \left[1 + \left(\frac{1 - h_{t-1}}{h_{t-1}} \eta_1(h_{t-1}) \cdot \Pr(entry)_t - \delta_t\right)\right]$$

$$n_t h_t^e = n_t h_t - n_{t-1} h_{t-1}(1 - \delta_{t-1})$$

$$n_t h_t^{in} = n_{t-1} h_{t-1}(1 - \delta_{t-1})$$

$$n_{t+1} = n_t \left[1 + h_t \left(\frac{E_{t-1}(\varphi_t^*)}{\Lambda}\right)^{1/\alpha} \left(\frac{1-h_t}{h_t}\eta_1(h_t) \cdot \Pr(entry)_{t+1} - \delta_t\right)\right]$$

$$\omega_t = \{^{\omega_t^* \;\; if \quad n<l}_{\omega_t^{fu} \;\; if \;\; n=l}$$

$$\begin{aligned} \omega_t^* \quad &= \quad \omega_t^*(\overbrace{E_{t-1}\left(\pi_t^e \mid \Omega_{t-1}\right)}^{+}, \overbrace{\Pr\left[\left(\pi_t^e \geq 0\right) \mid \Omega_{t-1}\right]}^{+}, \overbrace{\Pr\left[\left(\pi_t^{in} \geq 0\right) \mid \Omega_{t-1}\right]}^{-}, \\ &\overbrace{E_{t-1}(h_t^{in} \mid \Omega_{t-1})}^{-}, \overbrace{E_{t-1}(h_t^e \mid \Omega_{t-1})}^{-}, \overbrace{r_{t-1}}^{-}) \end{aligned}$$

$$\omega_t^{fu} = \frac{E_{t-1}\left(P_t\right) E_{t-1}(Q_t)}{E_{t-1} n_t (1-h_t)}$$

$$\begin{aligned} D(P_t, W_t) \quad &= \quad (\Xi(r_t)/P_t) \cdot \\ &\cdot \{A_t + [1/(1+r_t)] \sum_{i=0}^{\infty} [(1 + E\left(r_{t+i}\right))(1 + E(\iota_{t+i}))]^{-i} \cdot \\ &\cdot E(n_{t+i}(\omega_{t+i} + h_{t+i}^e \pi_{t+i}^e + h_{t+i}^{in} \pi_{t+i}^{in}))\} \end{aligned}$$

$$P_t = \frac{\omega_t^*}{\left(1 + \frac{1}{n_t h_t \cdot \varepsilon_D}\right)} L_t^* \frac{1}{\alpha} (\varphi_t^*)^{1-\alpha}$$

6 Concluding remarks

We have introduced here a theoretical macroeconomic framework for an oligopolistic economy with heterogeneous agents and wage rigidity where the macroeconomic fluctuations can be determined not only by technology shocks, but also by the process of entry/exit of oligopolistic firms, potentially interacting with distributional shocks. In this framework, microfundation is interpreted in a peculiar way, where agents have the same preferences, modelled with a conventional CRRA utility function, are heterogeneous in their budget constraint and may change their social status in each period according to a jump Markov process which interacts with labour market and with the process of entry/exit. This theoretical framework may be employed for further research focused on the process of entry/exit and its potential interactions with monetary policy, which may trigger higher entry or exit of oligopolistic firms and be associated to macroeconomic fluctuations.

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Appendix 1 - Microfoundation of consumption and aggregate demand

Derivation of the aggregate expenditure function

Let us recall the consumer problem:

$$\begin{aligned} \max U_t &= E_t\left[\sum_{i=0}^{\infty}\left(\frac{1}{1+\rho}\right)^i u(c_{t+i})\right] \\ c_{t+i}, i &= 0, ...\infty \end{aligned}$$

for each $i = 0, 1, ...\infty$ where $\left(\frac{1}{1+\rho}\right)$ is the subjective discount factor for the consumers

subject to the following constraint in real terms:

$$E(a_{t+i+1}) = (1 + r_{t+i})E(a_{t+i}) + E(y_{t+i}) - c_{t+i}$$

and

$$c_{t+i} \geq 0$$

Having choosen the following analytical form for consumers' preferences:

$$u_t = \frac{c_t^{1-\gamma}}{1-\gamma}$$

Then we can define the following Bellman equation:

$$V(W_t) = \max_{c_t}\left[\frac{c_t^{1-\gamma}}{1-\gamma} + \left(\frac{1}{1+\rho}\right)E(V(W_{t+1}))\right] \tag{47}$$

Subject to

$$E(W_{t+1}) = (1 + r_t)(W_t - c_t) \tag{48}$$

Where W_{t+i+1} is the state variable.

Now we assume (and later verify) that the value function has the same analytical form of the utility function, i.e.

$$V(W_t) = K\frac{W_t^{1-\gamma}}{1-\gamma} \tag{49}$$

Where K is a positive constant whose exact value will be shown later. By using the definition of $V(W_t)$49, the Bellman equation can be rewritten as follws:

$$K\frac{W_t^{1-\gamma}}{1-\gamma} = \max_{c_t}\left[\frac{c_t^{1-\gamma}}{1-\gamma} + \frac{1}{1+\rho}E\left(K\frac{W_t^{1-\gamma}}{1-\gamma}\right)\right] \tag{50}$$

hence, using the constraint 48 and deriving with respect to c_t, we get the F.O.C:

$$c_t^{-\gamma} = \frac{1+r_t}{1+\rho}K\left[(1+r_t)(W_t - c_t)\right]^{-\gamma}$$

and solving for c_t we get the consumption (demand) function:

$$c_t = \frac{1}{1 + (1+r_t)^{\frac{1-\gamma}{\gamma}}(1+\rho)^{-\frac{1}{\gamma}}K^{\frac{1}{\gamma}}}W_t$$

where K is the constant to be determined.

To complete the solution, we still use the Bellman equation 50, substitute the consumption function in it and we set:

$$M \equiv (1+r_t)^{\frac{1-\gamma}{\gamma}}(1+\rho)^{-\frac{1}{\gamma}}$$

just to simplify the notation. Then we get:

$$K\frac{W_t^{1-\gamma}}{1-\gamma} = \frac{1}{1-\gamma}\overbrace{\left(\frac{W_t}{1+MK^{\frac{1}{\gamma}}}\right)}^{c_t}{}^{1-\gamma} + \\ +\frac{1}{1+\rho}\frac{K_t}{1-\gamma}\underbrace{\left[(1+r_t)\frac{MK^{\frac{1}{\gamma}}}{1+MK^{\frac{1}{\gamma}}}W_t\right]^{1-\gamma}}_{W_{t+1}} \tag{51}$$

The value of K satisfying 51 can be obtained by equting the coefficients of $W_t^{1-\gamma}$ in the two sides of the equation and solving for K:

$$K = \left(\frac{1}{1-M}\right)^{\gamma}$$

Under the condition $M < 1$ the consumption (expenditure) function is fully specified:

$$V(W_t) = \left(\frac{1}{1-(1+r_t)^{\frac{1-\gamma}{\gamma}}(1+\rho)^{-\frac{1}{\gamma}}}\right)^{\gamma}\frac{W_t^{1-\gamma}}{1-\gamma}$$

and

$$c(W_t) = \left[1-(1+r_t)^{\frac{1-\gamma}{\gamma}}(1+\rho)^{-\frac{1}{\gamma}}\right]W_t$$

i.e.

$$c(W_t) = \left[1-(1+r_t)^{\frac{1-\gamma}{\gamma}}(1+\rho)^{-\frac{1}{\gamma}}\right](a_t+H_t)$$

Lookng at 15, having defined then:

$$\Psi_t = \Xi(r_t)\left(A_t + \frac{1}{1+r_t}\sum_{i=0}^{\infty}\left(\frac{1}{(1+E(r_{t+i}))(1+E(\iota_{t+i}))}\right)^i E(Q_{t+i})\right)$$

which allows to define the aggregate demand:

$$P_t = \Psi_t / D(W_t) \tag{52}$$

and its inverse

$$D(W_t) = \Psi_t / P_t \tag{53}$$

we have obtained then a unit elastic aggregate demand function. Similarly, if we want to define consumption in per capita terms, we can define:

$$\phi_t = \Xi(r_t)\left(A_t + \frac{1}{1+r_t}\sum_{i=0}^{\infty}\left(\frac{1}{(1+E\left(r_{t+i}\right))(1+E(\iota_{t+i}))}\right)^i E(q_{t+i})\right)$$

or

$$\begin{aligned}\phi_t \quad &= \quad \Xi(r_t)\{A_t + (1+r_t)^{-1}\sum_{i=0}^{\infty}[(1+E\left(r_{t+i}\right))(1+E(\iota_{t+i}))]^{-i} \cdot \\ &\quad \cdot E(\omega_{t+i}\xi_{t+i} + \pi_{t+i}^{in}\xi_{t+i}h_{t+i}^{in} + \pi_{t+i}^{e}\xi_{t+i}h_{t+i}^{e})\}\end{aligned}$$

and obtain a constant elastic demand function defined in per capita terms:

$$P_t = \phi_t / d(W_t) \tag{54}$$

or its inverse

$$d(W_t) = \phi_t / P_t \tag{55}$$

Appendix 2 - Entry and output determination: the existence of a Cournt-Nash equilibrium

Since the optimal amount of labour $L_t^*(c_t^i)$ of generic firm $"i"$ is a monotonic function of the homogenous good produced by firm $"i"$, we can define the generic cost funcion of firm $"i"$ as $\varkappa_i : R_+ \to R_+$. Since all costs occur at the beginning of stage 1 (i.e., at the end of time t-1 and just before time t), they all are sunk costs, and, as a consequence, there is no need to distinguish between capacity and output decision. A few assumptions guarantee the existence of the aggregate equilibrium in the goods market. These assunmptions correspond to those contained in Madden (1998), showing that with uniformly elastic demand function, the Kreps-Schenkman two-stage quantity-price game reduces to the Cournot model with any rationing mechanism between the efficient and proportinoal extremes and if all costs are sunk at the first stage.

Assumption D1

a) The aggregate demand function $D : R_{++} \to R_{++}$ **is** C^2 **with** $D'(P) < 0$ **everywhere,** $\lim_{P \to 0} D(P) = +\infty$ **and** $\lim_{P \to \infty} D(P) = 0$**.**

b) Having defined the market revenue function for firm $"i"$ **in terms of price as** $\varkappa_t^i(P_t, n_t h_t) = P_t \varpi_t^i(n_t h_t) D(P_t)$**, (where** $\varpi_t^i(n_t, h_t)$ **is the fraction of the aggregate demand satisfied by firm "i", which is a function of the firms** $n_t h_t$ **operating in the market at time** t**) there exists** $a \geq 0$ **such that the market revenue function** $\varkappa_t^i : R_{++} \to R_{++}$ **is strictly increasing on** $(0, a)$ **and non-increasing on** (a, ∞).

Madden also introduces the equivalent assumption, applying to the inverse demand (in his paper, Assumption 2), that we won't consider here, since it is equivalent and, therefore, unnecessary.

As Madden points out, Part (a) of assumption D1 ensures that the market demand curve is well-behaved, downward sloping and therefore asymptoting to the axes (like our demand function 52). Part (b) is obviously satisfied, since we are dealing with a unit elastic demand function and, given an arbitrary small "a", on (a, ∞), the revenues are constant and, therefore, non-increasing. Furthermore, by considering an arbitrary small $a \geq 0$, going from zero prices (where there is no economic activity and revenues are zero) to $a \geq 0$ would strictly increase the revenues $\varkappa_t^i(P_t, n_t h_t)$.

Madden argues that the role of part "b" of assumption D1 is to introduce the assumption of zero revenues at zero prices and, furthermore, points out that "a well-known special case of the uniform elastic demand specification is provided by the constant elasticity demand function" and, at the same page of the paper, "We remark that the specification of this draft has been stretched to accomodate uniform unit elasticity, since common examples give rise to this case" (Madden 1998, p 201).

In the Cournot model firms choose output levels c^i simultaneously, producing an aggregate output $D = \sum_{i=1}^{n} c^i$.

Furthermore, we define the Cournot payoff functions π_i^c for the generic firm i as:

$$\pi_i^c(c^1, ..., c^h) = \begin{cases} \varkappa(P_t, n_t, h_t) - k_i(c^i) & if \ c^i > 0 \\ 0 & if \ c^i = 0 \end{cases} \tag{56}$$

Where $\varkappa(P_t, n_t, h_t)$ is the revenue of the individual firm (assumed to be null if prices are null and there is no production), $k_i(c^i)$ is the cost function. The main result by Madden (1998) is the following. In the Kreps-Scheinkman model frms choose output levels simultaneously at stage 1. Then, with production costs sunk and with production levels common knowledge, firms choose prices simultaneously at stage 2.In Kreps and Scheinkman (1983), and in Osborne and Pitchik (1986) and Vives (1986) demand at stage 2 is rationed amongst firms according to the so-called efficient (or surplus-maximizing) rule; the following is the demand faced by firm i following the production vector c^i if the announced, stage 2 prices are p;

$$\Delta_{iE}(D,P) = \max\left\{0, \left[D(P_i). \sum_{p_k<p_i} c^k\right] \frac{c^i}{\sum_{p_k=p_i} c^k}\right\} \tag{57}$$

With this rationing rule, firms charging less than firm i serve those consumers with the highest valuation of the good and the term in square bracket is shared among the firms charging p_i, in proportion to their production level. At an opposite extreme is the proportional (or Beckmann, 1967) rule, used by Allen and Hellwig (1986):

$$\Delta_{iP}(D,P) = \max\left\{0, \left[1 - .\frac{c^k}{\sum_{p_k<p_i} D(P_k)}\right] D(P_i) \cdot \frac{c^i}{\sum_{p_k=p_i} c^k}\right\} \tag{58}$$

In this case the consumers served by lower priced firms are chosen randomly; $c^k/D(P_k)$ is the fraction of consumers served by k. Then we can define assumption D3 (corresponding to assumption 3 in Madden, 1998):

Assumption D2

The rationed demand function at stage 2 of the Kreps-Scheinkman game for firm $i, i = 1, ...h$ is $\Delta_i : R^n_+ \times R^n_+ \to R_+$ and satisfies

i) $\Delta_{iE}(D,P) \leq \Delta_i(D,P) \leq \Delta_{iP}(D,P), (D,P) \in R^n_+ \times R^n_{++}$

ii) Δ_i **only depends on these** P_i **for which** $c^i > 0$

Furthermore, Madden (1998), defines the correspondence between $\pi^c_i(c^1, ..., c^h)$ **and the "exact Cournot reduced form", meaning that if the quantities are chosen at stage 1 of the Kreps-Scheinkman game, then the second stage subgame Nash equilibrium that follows, always induce expected payoffs equal to the Cournot payoffs. Following Madden, we can characterize the equilibrium as follows:**

Theorem (Madden, 1998). **If assumptions D1 and D2 hold and the quantity** $D(P) = \sum c^i$ **is given at stage 1 of the Kreps-Scheinkman game, if demand is elastic at** $D(P)$**, then the Kreps-Scheinmnam**

model has the exact Cournot reduced form (See theorem 2 in Madden, 1998, p. 204 for the proof).

Appendix 3 - Dynamics of the entry rate and employment

The dynamics implied by 37 is the following:

$$\begin{aligned} n_{t+1}h_{t+1} &= n_t h_t + \Delta^h \Pr(\cdot) \\ &= n_t h_t + [n_t(1-h_t)]\,\eta_1(h_t)\cdot \Pr(w_t < \Phi_t) - n_t h_t \delta_t \end{aligned}$$

Which allows us to express the dynamics of $n_t h_t$ as follows

$$n_{t+1}h_{t+1} = n_t h_t \left[1 + \left(\frac{1-h_t}{h_t}\eta_1(h_t)\cdot \Pr(entry)_{t+1} - \delta_t\right)\right]$$

Hence

$$\frac{n_{t+1}}{n_t}\cdot\frac{h_{t+1}}{h_t} = 1 + \frac{1-h_t}{h_t}\eta_1(h_t)\cdot \Pr(entry)_{t+1} - \delta_t$$

Or, in terms of rate of variation of $n_t h_t$

$$\frac{n_{t+1}h_{t+1} - n_t h_t}{n_t h_t} = \frac{1-h_t}{h_t}\eta_1(h_t)\cdot \Pr(entry)_{t+1} - \delta_t$$

Or, in terms of index number of $n_t h_t$

$$\frac{n_{t+1}h_{t+1}}{n_t h_t} = 1 + \frac{1-h_t}{h_t}\eta_1(h_t)\cdot \Pr(entry)_{t+1} - \delta_t$$

Or, in terms of differences of $n_t h_t$

$$n_{t+1}h_{t+1} - n_t h_t = n_t h_t \left(\frac{1-h_t}{h_t} \eta_1(h_t) \cdot \Pr(entry)_t - \delta_t \right)$$

Similarly, we may define the dynamics for the level of employment

$$n_{t+1} = n_t \left[1 + h_t \left(\frac{E_{t-1}(\varphi_t^*)}{\Lambda} \right)^{1/\alpha} \left(\frac{1-h_t}{h_t} \eta_1(h_t) \cdot \Pr(entry)_{t+1} - \delta_t \right) \right]$$

The dynamics of employment too may be expressed in terms of differences

$$n_{t+1} - n_t = n_t h_t \left(\frac{E_{t-1}(\varphi_t^*)}{\Lambda} \right)^{1/\alpha} \left[\frac{1-h_t}{h_t} \eta_1(h_t) \cdot \Pr(entry)_{t+1} - \delta_t \right]$$

The ratio between changes in employment and changes in the number of entrepreneurs increases with the expected output per firms and decreases with positive technology shocks.

$$\frac{n_{t+1} - n_t}{n_{t+1}h_{t+1} - n_t h_t} = \left(\frac{E_{t-1}(\varphi_t^*)}{\Lambda} \right)^{1/\alpha}$$

Hence, given equations 37 and 42, the dynamics of employment may be formalized in several possible ways:

$$
\begin{aligned}
n_{t+1} &= n_t + \Delta^h \Pr(\cdot) L_t^* && (59)\\
n_{t+1} &= n_t + \Delta^h \Pr(\cdot) \left(\frac{E_{t-1}(\varphi_t^*)}{\Lambda}\right)^{1/\alpha} && (60)\\
n_{t+1} &= n_t + \\
& + \left[n_t(1-h_t)\eta_1(h_t) \cdot \Pr(entry)_{t+1} - n_t h_t \delta_t\right] \left(\frac{E_{t-1}(\varphi_t^*)}{\Lambda}\right)^{1/\alpha} && (61)\\
\frac{n_{t+1}-n_t}{n_t} &= \left[(1-h_t)\eta_1(h_t) \cdot \Pr(entry)_{t+1} - h_t \delta_t\right] \left(\frac{E_{t-1}(\varphi_t^*)}{\Lambda}\right)^{1/\alpha} && (62)\\
\frac{n_{t+1}-n_t}{n_t} &= h_t \left[\frac{1-h_t}{h_t}\eta_1(h_t) \cdot \Pr(entry)_{t+1} - \delta_t\right] \left(\frac{E_{t-1}(\varphi_t^*)}{\Lambda}\right)^{1/\alpha} && (63)\\
\frac{n_{t+1}-n_t}{n_t} &= h_t \left(\frac{E_{t-1}(\varphi_t^*)}{\Lambda}\right)^{1/\alpha} \frac{n_{t+1}h_{t+1} - n_t h_t}{n_t h_t} && (64)\\
n_{t+1}-n_t &= n_t h_t \left(\frac{E_{t-1}(\varphi_t^*)}{\Lambda}\right)^{1/\alpha} \left[\frac{1-h_t}{h_t}\eta_1(h_t) \cdot \Pr(entry)_{t+1} - \delta_t\right] && (65)
\end{aligned}
$$