UNIVERSITÀ CATTOLICA del Sacro Cuore

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Abstract. This study introduces a new automated approach, we have called "Volaxivity Algorithm", to detect risk-on and risk-off signals across several asset classes, starting from observing the related Volatility Indices. The algorithm is based on the definition and construction of the new set of risk sentiment Indices discussed in this paper, including the "Volaxivity Index".

The theoretical background of our model is defined by the Reflexivity Theory and by an event-driven definition of time.

In order to assess the validity of our methodology from a risk management point of view, we compare the signals originated by the algorithm with their realizations over 19 quarters between January 2016 and September 2020.

The empirical results discussed in the backtesting section of the paper confirm the validity of our approach, even during stressed market periods. We also found that the predictive capability of our model is higher for risk-off signals.

Keywords. Reflexivity; Volatility Indices; Risk Management; Central Banks.

J.E.L. **classification.** E58, G12, G17, G32, G40.

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1. Introduction

The purpose of the present study is to build a model to interpret and predict risk sentiment turning points in financial markets. To do that, we leverage the historical time series of a panel of Volatility Indices, representative of the most traded asset classes.

As discussed in our previous work, the risk perception from market participants plays a vital role in anticipating shock events and managing their effects (Bagato L., Gioia A. and Mandelli E., 2018). In this respect, several studies have highlighted that Volatility Indices are widely seen as a market proxy for risk aversion and uncertainty (see Adrian et al., 2017). Our study argued that volatility spikes, due to equity investors' panic conditions, appear as a quick reverse reaction to previous periods, dominated by positive perceptions about Goldilocks perspectives. Time compression and perception of incumbent disasters exacerbate the volatility trends due to over-hedging trading activity and the interactions among exchange-traded instruments.

Risk perception could also be altered during low volatility regimes. "*Volatility in markets is at low levels…to the extent that low levels of volatility may induce risk-taking behavior… is a concern to me and to the Committee.*" (Janet Yellen, former Federal Reserve Chair, June 18, 2014.)

The existence of a connection between Volatility Indices and the Reflexivity Theory has been empirically documented in Bagato L., Gioia A. and Mandelli E. (2018).

According to Reflexivity Theory, market prices are subject to so-called feedback loops (Bookstaber et al., 2015), i.e. prices are influenced by fundamentals, which in turn influence the expectations and behavior of market players leading to new price patterns.

It is worth recalling that Reflexivity Theory rejects the basic assumption of the classical theory that financial markets totally and instantaneously absorb the information flow, expressing an equilibrium price for each asset class. According to EMH (Efficient Market Hypothesis), in the event of "strong form" efficiency, the market tends to an equilibrium (Fama, 1969).

In this study we build an algorithmic risk assessment tool to investigate further the dynamics outlined by the Reflexivity Theory. We extend our previous analysis to a panel of 30 Volatility Indices belonging to different asset classes, with the aim to early detect risk-on and risk-off turning points for each asset class, as defined in paragraph 3.

The theoretical assumptions of our model are the predominance of Reflexivity behaviours in modern financial markets and the capability of Volatility Indices to detect risk sentiment reversal signals.

We also believe that time perception plays an important role in the succession of critical events. For this reason, we abandon the concept of natural passing of time and we base our analysis on an event-driven framework, recalling the methodology proposed by Golub A., Glattfelder J.B. and Olsen R.B. (2017).

As a result, our model produces daily risk-on and risk-off signals for each monitored Volatility Index or asset class.

We also introduce a set of new risk sentiment indicators. We named these indicators "Volaxivity Indices" to recall the underlying concepts of Volatility and Reflexivity. Our automated risk assessment system is called "Volaxivity Algorithm".

With the aim to test the predictive capability of this tool, we simulate and backtest the historical time series of the signals originated by our model and analyse the empirical results.

The rest of the paper is organized as follows: the next two sections provide respectively a brief literature review and further insights concerning the theoretical background of our model; section 4 describes the building blocks of the Volaxivity Algorithm; section 5 provides empirical and backtesting results; the last paragraph concludes our work.

2. Literature Review

Over the last few years, some empirical studies have significantly contributed to the analysis of market timing and risk signals based on volatility indices. In this section, we recall the most recent contributions.

The idea of the interrelation between market volatility and real economy is not new: "*Long-term swings in asset prices may generate large negative externalities in the form of severe real economic disruptions, whereas the short-term volatility more generally increases uncertainty and may reduce the level of investment and consumption*" (Romer, 1988).

In 2017, a major study on the subject provided an extensive review of previous literature about the idea to exploit volatility as a market predictor or as a risk signal (Cloutier R. et al., 2017). The authors of this study developed a tactical allocation strategy based on the value of the VIX Index, designed to reduce portfolio risk when market risk increases following high volatility levels.

In the same year, another important contribution is provided by Obayashi Y. et al. (2017). The authors define the object of prediction as drawdown events, which coincide with periods of high realized volatility. While recognizing that a simple volatility regime framework may produce robust market timing signals, they recommend using their methodology in applications based on the solid economic rationale that justifies its use.

More recently (2020), a study by Harwood V. notes that markets can stay at low volatility for a long time, while high volatility periods tend to be transitory.

A relevant implication of the above analysis is represented by the effects of volatility expectations on financial stability. BIS (Bank for International Settlements) has extensively analysed this topic between 2006 and 2021. BIS noticed that improvements in monetary policymaking, such as greater gradualism and transparency, "*may have played a major role in the reduction of money market volatility observed since 2004, and perhaps some role in the reduction of volatility at longer horizons and in other markets*" (BIS, 2006). Another relevant finding of BIS is that shocks in stock market volatility have more sizeable effects than shocks in bond market volatility, both pre-and post-crisis.

Lavin et al. (2021) explored the predictive power of VIX, VSTOXX, and VXJ indices, both separately and together, to dynamically represent the broad phenomenon of synchronization of financial markets, which is also relevant from a financial regulator's point of view for the consequences of the risk of financial contagion across markets.

From a risk monitoring perspective, the recent study of Banco de España (2021) reveals that central banks actively leverage on implied volatility indices as indicators for market risks. Indeed, central banks must develop and maintain risk identification tools to detect potential threats to financial stability early and address them with the most appropriate policy tools.

In this context, our goals and contributions with respect to the previous literature are twofold: firstly, we develop a standard methodology for assessing risk signals that can be applied to different asset classes, such as equity markets, commodities, interest rates, FX, and exchange-traded volatility products. Secondly, we create a new set of indicators that can be used to build a completely automated risk assessment tool across asset classes.

Our methodology is built on an event-driven framework based on the definition of intrinsic time provided by Golub, Glattfelder and Olsen (2017), rather than to the natural passing of time. The benefit of this approach is to eliminate the noise between events. Our algorithm and its implication are illustrated in depth in the rest of this paper.

In addition, our model is fully configurable and its parameters can be calibrated to incorporate risk limits or meet yield enhancement targets.

3. Background and definitions

In financial markets, the risk is measurable and refers to the probability of adverse events. In contrast, uncertainty denotes a situation where future events and possible outcomes are unknown, indicating a situation of ambiguity about the probability distribution parameters of the events or the actual distribution governing future paths of state variables. In this sense, risk can be considered as a subset of uncertainty.

When investors' sentiment is optimistic about the economy and the geopolitical conditions, riskier assets and high-yielding instruments tend to get pricier. This scenario is known as "risk-on", while the opposite scenario is defined as "risk-off". The underlying assumption is that price behaviour is governed by changes in investors' risk tolerance.

As discussed in the previous paragraphs, Volatility Indices such as the VIX index are often used as risk or uncertainty indicators. In this respect, it is worth noting that indices based on implied volatility look ahead and represent future volatility over a specific time frame. The 30-day measure is the most widely used because it reflects the higher liquidity of options with that time to maturity.

In this work we leverage on Volatility Indices to model market risk in the short term and generate risk-on and risk-off signals. Our algorithm is therefore designed to assess near-term market risk, rather than to assess uncertainty about the economical contest in general.

As a first step, we create an event-driven framework based on the definition of intrinsic time (Golub, Glattfelder, and Olsen, 2017). In essence, we map the historical time series of a Volatility Index, $\boldsymbol{x(t)}$, to a discrete set of events $\boldsymbol{\Omega}[\boldsymbol{x(t)}, \boldsymbol{\delta}]$, where $\boldsymbol{\delta}$ is a threshold that defines the granularity of intrinsic time.

This approach benefits from filtering out the noise between two consecutive events. As a result, clock time ceases to exist between events.

Applying the methodology to a given Volatility Index, such as the VIX, we state that an event occurs when the log return of the Index - respect to the Index level at the previous event - exceeds the threshold $\boldsymbol{\delta}$.

Events can be classified based on their direction. "Upper" events occur when log return between events is above $+\boldsymbol{\delta}$, whereas "lower" ones occur when log return is below - $\boldsymbol{\delta}$.

Events are also classified as directional changes (DC) if the direction of the event is different from the direction of the previous event, whereas are classified as overshoots (OS) when the direction of the event is the same as the direction of the previous one, i.e. there is a sequence of at least two upper or lower events.

In our work, the number of calendar days between two consecutive events is referred to as time-to-event (TTE).

The intrinsic time series of the events is called Coastline, which can be represented as a price curve made of segments.

The dependent variable of our model is the type of the next event (OS or DC). This feature cannot be observed for the current intrinsic time and, therefore, must be forecasted by the algorithm.

4. Empirical strategy

In this section, we provide a high-level description of the Volaxivity Algorithm. There are up to 30 Volatility Indices currently monitored by the algorithm.

The following steps are executed for each monitored Volatility Index:

1) construction of the Coastline curve;
2) logistic regression estimation on *k* rolling observations of the Coastline, splitting the sample between training (80%) and testing data (20%);
3) prediction of the next DC probability, based on the regression parameters and on the value of scaled input features;
4) calculation of diagnostic statistics and confusion matrix on the testing sample;
5) calculation of cut-off probabilities through a cost function;
6) extrapolation of long and short volatility signals based on the cut-off probabilities of the logistic model;
7) identification of risk-on and risk-off signals;
8) construction of the Partial Volaxivity Index (PVI), as defined in the following sections.

Once the above steps are completed for each Volatility Index, the PVIs can be aggregated by asset class. We call such measure Cluster Volaxivity Index (CVI).

Then we compute a global level index including all CVIs. We call the latter measure Standardized Global Volaxivity Index (SGVI or $\boldsymbol{\zeta}$).

4.1 Logistic model

The logistic models are trained on the last *k* rolling observations of each Volatility Index Coastline. For each model, we define $\boldsymbol{P(DC)}$ as the probability that the following event will be a DC:

$$\boldsymbol{P(DC) = \frac{1}{1+e^{-X}}} \tag{1}$$

where X is a linear combination of the following input features: INDEX_LAST (last fixing of the Volatility Index), INDEX_LN (natural logarithm of the Volatility Index fixing), INDEX_LN_DIFF (log return of the Volatility Index between two consecutive events), TTE (number of calendar days between two consecutive events), SUM2TTE (sum of the last two TTE), dummy_OS (dummy variable with value 1 in case of OS event), dummy_UPPER (dummy variable with value 1 in case of upper event), dummy_DC_OS (dummy variable with value 1 in case of DC-OS sequence in the last two observed events), dummy_OS_DC (dummy variable with value 1 in case of OS-DC sequence in the last two observed events), dummy_OS_OS (dummy variable with value 1 in case of OS-OS sequence in the last two observed events).

We use logistic model parameters to predict the next event type starting from the scaled input variables. In other words, aim of the model is to predict the probability that the next event in the price pattern of the Volatility Index will be a directional change.

We also compute the following diagnostics for each logistic regression: precision, recall, accuracy, F1, ROC curve (Receiver Operating Characteristic, i.e. true positive rate vs false positive rate chart), AUC score (Area Under ROC curve, which is an indicator of model performance). Model diagnostics are computed on a daily basis to be able to monitor the goodness of fit over time.

The rolling approach used in our work benefits from maintaining under continuous control the stability of the parameters estimated by the model, as further discussed in section 5.

The forecast of the next event type is performed through three configurations of the logistic model. According to the model configuration, each design maximizes the difference between the true positive rate (*tpr*) and the false positive rate (*fpr*), adding a penalty for the false-positive rate. In particular, the cut-off level is computed as the value of the linear combination of X that maximizes $(\boldsymbol{tpr - i \cdot fpr})$, where $\boldsymbol{i} > 0$.

We compute the respective cut-off probability ξ using equation (1) for each cut-off level.

Suppose the predicted directional change probability for the current observation exceeds ξ. In that case, the model predicts a DC signal, which is interpreted as long or short volatility signal depending on the previous event direction. It is also interpreted as a risk-off or risk-on signal based on the Volatility Index type.

4.2 The Volaxivity Index

To build the Volaxivity Index, we introduce the following additional definitions:

$\boldsymbol{w_{ij}} =$ weight assigned to Volatility Index i within the cluster j, where $\boldsymbol{w_{ij} > 0}\ \forall\ \boldsymbol{i, j}$;

$\boldsymbol{\sum_{i=1}^{N} w_{ij} = 1}$ where N is the number of Volatility Indices in the Volatility Cluster j;

C = number of Volatility Clusters;

M = number of models implemented;

$\boldsymbol{s_{ij}}$ = volatility signal: (1 = predicted increase in volatility; -1 = predicted reduction in volatility; 0 = no signal);

$\boldsymbol{d_{ij}} =$ contribution of a predicted increase in volatility ("1" signal) on the Volatility Index i respect to Cluster j (the contribution can be 1 or -1);

$\boldsymbol{g_{ij}}$ = signal strength.

The Standardized Global Volaxivity Index (SGVI), is defined as follows:

$$SGVI = \zeta = \frac{100}{C \cdot M} \cdot \sum_{j=1}^{C} \sum_{i=1}^{N} w_{ij} \cdot d_{ij} \cdot s_{ij} \cdot g_{ij} \tag{2}$$

The SGVI spans from -100 (maximum risk-off indicator) to +100 (maximum risk-on indicator).

The product $\boldsymbol{w_{ij} \cdot d_{ij} \cdot s_{ij} \cdot g_{ij}}$ is the non-standardized Partial Volaxivity Index for a single Volatility Index (or PVI):

$$PVI = w_{ij} \cdot d_{ij} \cdot s_{ij} \cdot g_{ij} \tag{3}$$

$\boldsymbol{\sum_{i=1}^{N} w_{ij} \cdot d_{ij} \cdot s_{ij} \cdot g_{ij}}$ is the non-standardized Volaxivity Index for a single Volatility Cluster (or CVI):

$$CVI = \sum_{i=1}^{N} w_{ij} \cdot d_{ij} \cdot s_{ij} \cdot g_{ij} \tag{4}$$

5. Data and discussion

5.1 Data

We applied the Volaxivity Algorithm to the daily historical time series of the Volatility Indices reported in the Appendix (Table 1).

The sample period spans from January 2011 to September 2020. Some Indices are not available for the whole sample because their calculation has been discontinued. If a Volatility Index is discontinued, the weights of the remaining indices in the same Volatility Cluster are rebalanced accordingly.

5.2 Estimation

Logistic regressions are estimated for each trading day and Volatility Index of the sample period. We ran about 17,000 models in total, including in each model all the events that occurred within a rolling window of 1,000 calendar days.

As an example, Figure 1 shows the ROC Curve for the VIX Index. This specific logistic model is estimated on the events that occurred on the VIX Index during the period July 6, 2018 – April 1, 2021. The picture refers to the model's application on the testing sample (20%) after having estimated the parameters over the training sample (80%). Table 2 reports the diagnostic statistics for the above model.

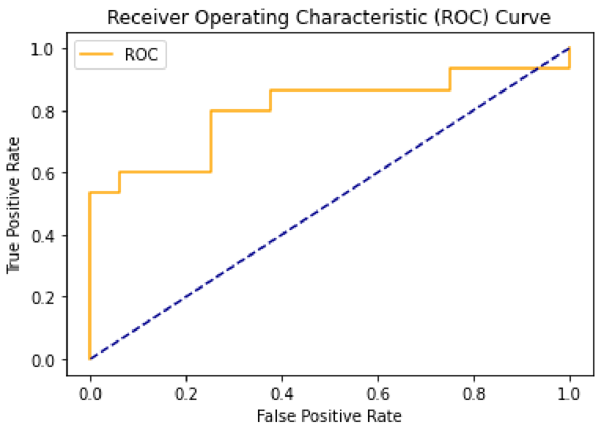

Figure 1: ROC Curve on the testing sample (20% of the full sample)

Statistic	*Value*
AUC (Area Under Curve)	0.80
Precision = tp / (tp + fp)	0.82
Recall = tp / (tp + fn)	0.60
F1 (armonic mean of precision and recall)	0.69
Accuracy = (tp + tn) / (tp + tn + fp + fn)	0.74

(tp = true positives, fp = false positives, tn = true negatives, fn = false negatives)

Table 2: Diagnostic statistics of the logistic model on the testing sample.

5.3 Parameters stability

Table 2 statistics are computed every day for each Volatility Index to assess the stability of the parameters and the predictive power of the whole algorithm over time. For this purpose, Figure 2 shows the historical time series of Precision and Recall statistics computed on the rolling logistic regression for the VIX Index.

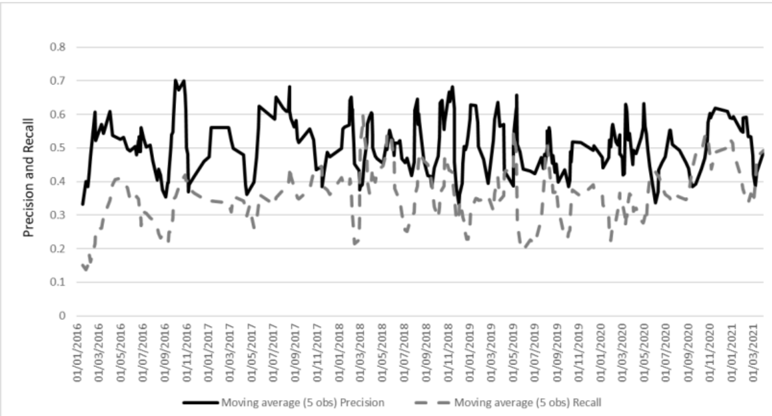

Figure 2: Rolling Precision and Recall of the logistic models estimated for the VIX Index (moving averages on 5 obs).

5.4 Backtesting

In the rest of this section, we describe the backtesting results of the risk monitoring system based on Volaxivity Algorithm signals. The main assumptions are described in Table 3.

Feature	*Value or range*
Period of application	Jan 2016 - Aug 2020
δ	10%
Logistic regression rolling window	1,000 obs
Number of monitored Volatility Indices	Between 28 and 30
Number of Indices belonging to each Vol. Cluster	Between 3 and 5

Table 3: Risk monitoring system assumptions.

We also assume that:

- Volatility Indices are tradeable assets, whereas some of them do not actually have liquid futures markets. This assumption could be relaxed by replacing some of the Indices with liquid futures or other exchange-traded products;
- signals on different Indices are treated independently even if the Indices belong to the same Volatility Cluster.

Table 4 (Appendix) displays the multivariate frequency distribution of the actual event types (directional changes or overshoots) and the forecasted values of the model. The results are split between risk-off (table 4.1) and risk-on (table 4.2) signals.

Table 4.3 provides summary statistics on the performance of the whole algorithm. The underlying idea is to reduce the number of unnecessary portfolio adjustments that can be interpreted as false positives (*fp*). The high number of false negatives (*fn*) is due to the chosen configuration of the cost function, which adds a penalty for the false-positive rate. Ideally, it is possible to modify the cost function configuration to reduce the *fn* rate. With such design, risk signals tend to be less frequent, and *fn* increases. In other words, in this model configuration, we tend to maximize algorithm precision, as shown in the values presented in tables 4.1 and 4.2.

We also found that model precision tends to be higher for risk-off signals concerning risk-on signals.

Another approach to test our risk monitoring system is to simulate a portfolio strategy whose decisions are strictly based on risk signals originated by the model. Tables 5.1 and 5.2 (Appendix) show the results of such an approach.

Both approaches validate the forecasting capability of our method and its applicability in the financial industry.

6. Concluding remarks

As known, Volatility Indices are broadly used by risk managers and central banks as market risk indicators on several asset classes, such as equities, fixed income, exchange rates, and commodities. There are several contributions in the recent literature concerning the ability of implied volatility indicators to lead market uncertainty and risk perception, thanks to the fact that implied volatility reflects the market expectation about the short-term volatility on the underlying asset class.

This paper introduced a novel automated risk monitoring algorithm based on the Volatility Indices available on equity, commodity, exchange rates, and fixed income markets. The analysis on the equity asset class is split among equity indices, emerging markets, and single stocks, depending on the underlying type of the single Volatility Index.

The theoretical framework of our study is represented by the Reflexivity Theory and by the assumption that under stressed market conditions, the volatility shocks can propagate across asset classes. Our model's output consists of creating a new set of risk indicators that can be exploited to early detect sentiment reversal signals. The indicators are designed to be automatically adjusted daily to constantly track and monitor the changing dynamics of the implied volatility levels. The same indicators can also be used to detect early "risk-on" or "risk-off" signals on single markets or as a proxy of global risk aversion.

We named these indicators "Volaxivity Indices" to recall the main concepts of the theoretical framework, Volatility, and Reflexivity.

To validate and test our approach, we backtested the Volaxivity Algorithm over 19 quarters between January 2016 and September 2020.

As documented in the empirical section of this paper, backtesting results confirm the predictive capability of our risk monitoring system, revealing a higher precision of risk-off signals with respect to risk-on signals. Moreover, our model can be easily configured to apply to additional asset classes and incorporate risk or asset allocation limits. These features are fundamental to using the tool to monitor the risk of financial contagion across markets.

We believe that institutional investors could profitably use our risk monitoring framework as decision support, yield enhancement, or risk management methodology. Central banks and other financial market authorities could also benefit from our model as a market risk indicator or monitoring tool.

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Appendix

Id	Symbol	Index Description
1	BPVIX	Cboe/CME FX British Pound Volatility Index
2	EUVIX	Cboe/CME FX Euro Volatility Index
3	EVZ	Cboe EuroCurrency ETF Volatility Index
4	GVZ	Cboe Gold ETF Volatility Index
5	JYVIX	Cboe/CME FX Yen Volatility Index
6	OVX	Cboe Crude Oil ETF Volatility Index
7	RVX	Cboe Russell 2000 Volatility Index
8	SRVIX	Cboe Interest Rate Swap Volatility Index
9	TYVIX	Cboe/CBOT 10-y U.S. Treas. Note Vol. Index
10	VIX	Cboe Volatility Index
11	VIX1Y	Cboe 1-Year Volatility Index
12	VIX3M	Cboe 3-Month Volatility Index
13	VIX6M	Cboe S&P 500 6-Month Volatility Index
14	VIX9D	Cboe S&P 500 9-Day Volatility Index
15	VVIX	Cboe VIX of VIX Index
16	VXAPL	Cboe Equity VIX on Apple
17	VXAZN	Cboe Equity VIX on Amazon
18	VXD	Cboe DJIA Volatility Index
19	VXEEM	Cboe Emerging Markets ETF Volatility Index
20	VXEFA	Cboe EFA ETF Volatility Index
21	VXEWZ	Cboe Brazil ETF Volatility Index
22	VXFXI	Cboe China ETF Volatility Index
23	VXGDX	Cboe Gold Miners ETF Volatility Index
24	VXGOG	Cboe Equity VIX on Google
25	VXGS	Cboe Equity VIX on Goldman Sachs
26	VXIBM	Cboe Equity VIX on IBM
27	VXN	Cboe NASDAQ Volatility Index
28	VXO	Cboe S&P 100 Volatility Index
29	VXSLV	Cboe Silver ETF Volatility Index
30	VXXLE	Cboe Energy Sector ETF Volatility Index

Table 1.1: Volatility Indices used in the analysis. Descriptive statistics are available in Tables 1.2 and 1.3.

Id	From date	To date	# Obs	Mean
1	01/01/2011	14/05/2020	2358	9.04
2	01/01/2011	14/05/2020	2358	9.12
3	03/01/2011	30/09/2020	2450	9.24
4	03/01/2011	30/09/2020	2450	17.00
5	01/01/2011	14/05/2020	2318	9.83
6	03/01/2011	30/09/2020	2450	36.07
7	28/09/2019	30/09/2020	253	32.21
8	18/06/2012	30/09/2020	2148	80.39
9	03/01/2011	14/05/2020	2040	5.32
10	03/01/2011	30/09/2020	2450	17.33
11	03/01/2011	30/09/2020	2533	21.56
12	03/01/2011	30/09/2020	2451	19.04
13	03/01/2011	30/09/2020	2451	20.35
14	28/09/2019	30/09/2020	253	26.31
15	03/01/2011	30/09/2020	2533	37.24
16	03/01/2011	30/09/2020	2451	29.20
17	03/01/2011	30/09/2020	2451	32.70
18	28/09/2019	30/09/2020	253	26.66
19	17/03/2011	30/09/2020	2399	23.44
20	03/01/2011	30/09/2020	2533	18.83
21	16/03/2011	30/09/2020	2481	34.53
22	16/03/2011	30/09/2020	2399	25.74
23	16/03/2011	30/09/2020	2399	36.76
24	03/01/2011	30/09/2020	2451	25.59
25	03/01/2011	30/09/2020	2451	29.01
26	03/01/2011	30/09/2020	2451	22.88
27	03/01/2011	30/09/2020	2450	19.61
28	03/01/2011	30/09/2020	2533	16.87
29	16/03/2011	30/09/2020	2399	29.76
30	17/03/2011	30/09/2020	2399	24.84

Table 1.2: Volatility Indices used in the analysis. Descriptive statistics.

Id	Min	Max	StdDev	Coeff. of variation
1	4.33	29.10	2.73	30%
2	3.99	20.25	2.88	32%
3	4.13	19.87	2.87	31%
4	8.88	48.98	5.27	31%
5	4.29	23.06	2.68	27%
6	14.50	236.80	19.26	53%
7	13.35	83.19	15.38	48%
8	60.68	112.30	10.12	13%
9	3.16	16.39	1.36	26%
10	9.14	82.69	7.42	43%
11	15.56	45.86	4.71	22%
12	11.85	72.98	6.48	34%
13	13.75	61.11	5.65	28%
14	9.04	106.66	18.04	69%
15	14.49	207.59	27.38	74%
16	12.52	101.69	8.40	29%
17	5.13	72.66	8.98	27%
18	11.46	67.07	13.21	50%
19	13.28	92.46	7.90	34%
20	7.62	75.17	8.24	44%
21	16.67	144.42	11.67	34%
22	15.09	69.28	7.04	27%
23	15.40	118.75	10.67	29%
24	9.21	78.07	7.18	28%
25	16.16	123.83	11.10	38%
26	13.23	96.65	7.15	31%
27	10.31	80.08	7.11	36%
28	6.32	93.85	8.04	48%
29	14.89	100.66	11.21	38%
30	11.71	130.61	11.63	47%

Table 1.3: Volatility Indices used in the analysis. Descriptive statistics. Data source of Tables 1.2 and 1.3: our analysis of CBOE web site data. Sample period January 2011 - September 2020. Please note that some Indices are not available for the whole sample because their calculation has been discontinued.

	Volatility Cluster	Actual Event DC (tp)	OS (fp)	Total	% Successful predictions (precision)
Predicted Directional Change Event (Risk-off)	Commodities	36	31	67	54%
	Emerging Markets	28	13	41	68%
	Equity Indices	41	24	65	63%
	Eq. Single Stocks	39	25	64	61%
	Rates & FX	45	38	83	54%
	VIX	19	14	33	58%
	No. of risk-off signals	**208**	**145**	**353**	**59%**

Table 4.1: Risk monitoring system backtesting results. Risk-off signals (tp = true positive, fp = false positive).

	Volatility Cluster	Actual Event DC (tp)	OS (fp)	Total	% Successful predictions (precision)
Predicted Directional Change Event (Risk-on)	Commodities	28	21	49	57%
	Emerging Markets	18	9	27	67%
	Equity Indices	19	11	30	63%
	Eq. Single Stocks	45	59	104	43%
	Rates & FX	52	36	88	59%
	VIX	17	12	29	59%
	No. of risk-off signals	**179**	**148**	**327**	**55%**

Table 4.2: Risk monitoring system backtesting results. Risk-on signals (tp = true positive, fp = false positive).

Event	No Risk Signal	Risk-on or Risk-off signals
Directional Change	2201 (*fn*)	387 (*tp*)
Overshoot	1668 (*tn*)	293 (*fp*)
Total # of Events	**3869**	**680**

Statistic	*Value*
Precision = tp / (tp + fp)	0.57
Recall = tp / (tp + fn)	0.15
F1 (armonic mean of precision and recall)	0.24
Accuracy = (tp + tn) / (tp + tn + fp + fn)	0.45

Table 4.3: All coastline events. Summary statistics (tp = true positive, fp = false positive, tn = true negative, fn = false negative).

Statistic	Gross Results
Theoretical return since inception (Jan 2016 – Sept 2020)	134%
Compounded annual return	20%
Sharpe Ratio (yearly)	1.19
# Winning / # Losing signals	387/293
# Winning / # Losing quarters	15/4
Quarterly returns correlation vs S&P 500	0.20 (p-value 0.41)

Table 5.1: Summary statistics of a theoretical strategy based on risk signals, period Jan 2016 – Sept 2020.

Year	Theoretical strategy gross returns	S&P 500 returns
2016	45%	10%
2017	17%	19%
2018	0%	-6%
2019	21%	29%
2020 (until Q3)	13%	4%

Table 5.2: Yearly performance of the theoretical strategy, period Jan 2016 – Sept 2020. In Tables 5.1 and 5.2 we assume no transaction costs. The goodness of the results has proven to be confirmed also assuming positive transaction costs.

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