UNIVERSITÀ CATTOLICA del Sacro Cuore

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Wolfgang Britz, Institute for Food and Resource Economics, University Bonn, Bonn, Germany.

Linda Arata, Department of Agricultural and Food Economics, Università Cattolica del Sacro Cuore.

wolfgang.britz@ilr.uni-bonn.de

linda.arata@unicatt.it

I quaderni possono essere richiesti a:
Dipartimento di Scienze Economiche e Sociali,
Università Cattolica del Sacro Cuore
Via Emilia Parmense 84 - 29122 Piacenza - Tel. 0523 599.342
http://dipartimenti.unicatt.it/dises

dises-pc@unicatt.it

www.vitaepensiero.it

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ABSTRACT

This study belongs to the barely explored research strand of 'Econometric Mathematical Programming' and presents a simultaneous estimation of the cost function and of the farmers' risk attitude parameters in a programming model setup. Resource and policy constraints of the model are allowed to be not binding. We use crop shares as decision variables to avoid scale bias and we consider price and crop yield variances separately. The model is formulated as a bi-level programming model and the empirical application concerns three unbalanced panels of specialized arable farms observed for at least three consecutive years in Northern Italy, in the Cologne-Aachen area in Germany and in the Grandes-Cultures area in France over the time period 1995-2007. We achieve a quite satisfactory fit in the estimation exercise and find own and cross price elasticities from sensitivity experiments in reasonable ranges.

Keywords: econometric mathematical programming, risk behaviour, cost function estimation

JEL code: C61, Q12

1. INTRODUCTION

Mathematical programming models are widely applied in agricultural economics analysis as they allow for a detailed representation of technology as well as for the introduction of new activities or resource and policy constraints. However, classical linear programming models lead to overspecialized solutions and are hard to calibrate against observed data. The seminal paper of Howitt (1995) on Positive Mathematical Programming (PMP) propose the fundamental idea of introducing non-linearities into a programming model such that first and second order conditions hold at the observed levels of decisions variables. The so-called standard PMP approach of Howitt first generates shadow prices of resource and of calibration constraints based on the original linear model. These shadow prices are subsequently used to parameterize a non-linear yield or dual cost function such that the final non-linear model replicates the observed behaviour. Besides allowing for perfect calibration, the non-linearities also lead to a smoother supply response in simulation exercises compared to linear models. Since its formalisation by Howitt, the original PMP setup has been improved and extended in various directions (Paris and Howitt, 1998, Helming, 2001, Rohm and Dabbert, 2003, Kanellopoulos et al., 2010, Graveline and Mérel, 2014).

The original PMP approach uses one observation only for each (aggregated) agent. While calibration requires that the derivatives of the Lagrangian towards all the decision variables are zero at their observed values, information on their change when moving away from that point cannot be deduced from a single observation. Accordingly, the calibration problem itself is ill-posed such that the early specific rules can be considered *ad hoc* (see Heckelei and Britz 2005 for a discussion). A more convincing way to face the ill-posed nature of PMP problems is the use of exogenous supply elasticities. Mérel and Bucaram (2010), Mérel et al. (2011) and Garnache and Mérel (2015) discuss the feasibility of a calibration using exogenous elasticities and accounting for possible changes in shadow values of resource constraints. They derive closed form expressions for the implied supply elasticities of a constant elasticity of substitution (CES) programming model with one constraint along with the necessary and sufficient conditions to allow for exact calibration against given supply elasticities. Furthermore, they analyse the elasticity ranges to which different types of cost and yield specifications of PMP based models can be calibrated.

The use of exogenous supply elasticities requires some estimation exercise and it is best performed using disaggregated observations on the agents depicted by the programming model. Lacking such information, PMP models that employ exogenous elasticity rely on elasticity estimates at regional or even national level to parameterize models working at more disaggregate levels of farm group or even single farms. There is however no convincing argument why farmers differing for instance in resource endowments should all exhibit the same supply elasticities.

Heckelei and Wolff (2003) propose therefore as an alternative to the standard PMP methodology the simultaneous estimation of all the parameters of the programming model, including shadow values, by exploiting its first order conditions (FOCs). That not only avoids the aforementioned weaknesses of the standard PMP (e.g. arbitrary specification rules on the second order parameters, the use of regional/national elasticity to calibrate individual farm behaviour), but also removes inconsistent parameter estimates resulting from using shadow prices derived from the uncalibrated model (Heckelei 2002). Such an estimation approach, termed by Buysse et al. (2007a) "Econometric Mathematical Programming", bridges the gap between econometrics and mathematical programming and opens new avenues to estimate and simulate farmers' behaviour. It allows to consistently exploit information from multiple observations to estimate technological and behavioural parameters, while benefitting from a programming set-up with explicit constraints. The latter feature is firstly useful to depict policy changes, for instance the newly introduced so called "Greening" requirements under the Common Agricultural Policy. Secondly, agri-environmental relationships are more easily analysed in a programming set-up with an explicit production function. Standard econometric approaches such as duality based estimations are hardly useful to simulate impacts of (so far unobserved) technology restrictions. Furthermore, as argued by Heckelei and Wolff (2003), Econometric Mathematical Programming does not require closed form equations during estimation which gives flexibility in depicting technology including multiple inequalities constraints.

This flexibility can render Econometric Mathematical Programming estimators computationally demanding, especially when inequality constraints such as production quotas or set-aside obligations as well as non-negativity conditions on decision variables imply estimating equations formulated as complementary slackness conditions. Consequently, empirical works that combine econometrics and mathematical programming are still scarce. Buysse et al. (2007b) apply a Maximum-Entropy estimator to a set of farm level models with farm specific parameters in a dual cost function to analyse sugar beet reform on Belgian farms. They reduce computational complexity by first assuming that all constraints are binding which avoids complementary slackness. Secondly, their cost function only has diagonal elements, i.e. marginal production costs of an activity depend only on its own level. Jansson and Heckelei (2011) instead perform a large scale estimation of the regional parameters of the agricultural sectoral model CAPRI across 219 EU regions and 23 crop activities by means of a Bayesian estimator, allowing for cross-cost effects between crop groups besides diagonal effects. Cortignani and Severini (2012) report on the estimation of an expected utility maximisation programming model for 27 Italian farms where gross margin risk is included. The paper however does not report details such as which constraints are considered or if slackness is allowed. In another paper, Jansson and Heckelei (2009) implement a bi-level program to estimate the parameters of a transport programming model accounting also for zero trade flows by the complementary slackness conditions. A bi-level program consists of a programming model (outer problem) with the first order conditions (FOCs) of another programming model (inner problem) as constraints. They test for consistency of the parameter estimates based on randomly generated samples and find that the bi-level estimates are superior in terms of precision to the traditional calibration method applied in transport models. Recently, the development of the so-called Extended Mathematical Programming (EMP, Ferris et al., 2009) package of the GAMS (General Algebraic Modelling System) software helps to solve some computational issues. Indeed, the package allows, inter-alia, automatic formulation of the FOCs of a bi-level program (Vicente and Calamai, 1994).

Our study presents an Econometric Mathematical Programming application based on observed, not generated, data to estimate the parameters of a farm-level programming model, depicting several resource and institutional constraints as inequalities. In addition, the programming model accounts for production and market risk in an expected utility-variance (E-V) framework. Specifically, we estimate simultaneously the parameters of a quadratic cost function, the parameters of the technical progress and of the efficiency differences between farms, the farmer risk's aversion coefficient as a function of acreage and family labour and, implicitly, the dual values of the constraints. The estimator is applied to three large unbalanced panels of single arable farm observations over the period 1995-2007 from three European Union (EU) agricultural intensive areas: Northern Italy, the Cologne-Aachen area in Germany and the Grandes-Cultures area in Northern France.

To the best of our knowledge there is no study that estimates the parameters of a risk programming model allowing for non-binding constrains while using large panels of farms observed over a relatively long time span. Moreover, we introduce an expectation model for crop prices and yields in order to avoid the bias from using the realized (ex-post) values under uncertainty (Pope and Chavas, 1994).

2. MODEL AND ESTIMATION APPROACH

We assume expected utility maximisation of the farmer in a classical mean-variance approach where the farmer chooses crop shares to maximise profit per ha under risk subject to a set of resource and institutional constraints:

$$\max_{\mathbf{s}} EU \big/ L = E(\tilde{\mathbf{p}} \odot \mathbf{yld})'\mathbf{s} + \mathbf{sub}'\mathbf{s} + dsub - \\ c(\mathbf{s}, E(\mathbf{yld}), px, \frac{lab}{L} \mid \boldsymbol{\beta}_1) - 0.5\alpha(L, lab \mid \boldsymbol{\beta}_2)\boldsymbol{\sigma}_R^2(\mathbf{s})$$

subject to $\mathbf{As} \leq \mathbf{b}/L \quad (\boldsymbol{\gamma})$

$$\mathbf{0} \leq \mathbf{s} \leq \mathbf{1}$$

where, EU is the expected utility and $\tilde{\mathbf{p}}$ and $\mathbf{yld}$ are vectors of random exogenous output prices and yields and $\circ$ is the Hadamard or element-wise product. E is the expectation operator, $\mathbf{s}$ is the vector of crop shares as decision variables, $\mathbf{sub}$ is a vector of related coupled subsidies per hectare and $dsub$ is the value of the farm decoupled subsidy per hectare. L and lab are given land and family labour endowment respectively. Note that crop output, $\mathbf{x}$, is hence the product of the given land endowment, L, the given yields, $\mathbf{yld}$, and the crop shares decision variables, $\mathbf{s}$, such that the decision problem can alternatively be interpreted as choosing utility maximal output quantities, or, crop acreages.

$c(\mathbf{s}, E(\mathbf{yld}), px, \frac{lab}{L} \,|\, \boldsymbol{\beta}_1)$ is the per ha cost function with $\boldsymbol{\beta}_1$ the vector of its parameters to be estimated. The per hectare cost function depicts total farm specific variable costs, farm overheads, the cost of hired labour and capital depreciation, but excludes the cost of land rents and family labour. The cost function is expressed as a function of crop shares, expected yields, a general farm input price index and family labour per hectare. $\alpha(L, lab \,|\, \boldsymbol{\beta}_2)$ represents the farmer's relative coefficient of risk aversion as a function of farm endowments: farmland, L, and farm's family labour, lab. The parameters of that function ($\boldsymbol{\beta}_2$) are simultaneously estimated with the parameters of the cost function and the shadow values of the constraints ($\boldsymbol{\gamma}$).

σ_R^2 is the variance of farm revenue per ha expressed as a function of crop shares and where we disentangle the contribution of prices and yields to this variance. $\mathbf{A}$ and $\mathbf{b}$ are respectively the matrix of resource use per unit of crop land and the vector of right hand side value indicating either resource endowment or institutional bounds with associated shadow prices $\boldsymbol{\gamma}$.

Note that our model is specified and estimated on a per hectare basis to avoid scale bias as detailed below. Consequently, the decision variables are the crop shares. The parameters of the cost function and the ones defining the individual risk aversion coefficients as well as the shadow values of the constraints are simultaneously estimated based on the FOCs of the above model. As we allow for non-binding constraints a set of Karush-Kuhn-Tucker (KKT) conditions represents the estimation framework (section 2.2).

Before detailing the empirical model, we present a small example to illustrate the rationale behind the use of crop shares instead of acreages as decision variables. Consider two farms A and B with land considered as a fixed factor, farm B's acreage being doubled compared to farm A and assume the two farms have the same crop yields. In addition, assume that the farms share the technology expressed by the quadratic cost function $TC = \mathbf{d}'\boldsymbol{\upsilon} + 0.5\boldsymbol{\upsilon}'\mathbf{Q}\boldsymbol{\upsilon}$ where $\boldsymbol{\upsilon}$ is the vector of crop quantities (=crop yield*crop acreages) as decision variable at given yields. The marginal costs $\mathbf{MC} = \mathbf{d} + 0.5\mathbf{Q}\boldsymbol{\upsilon}$ are a linear function of crop quantities. If both farms choose the same crop shares, each element in $\boldsymbol{\upsilon}$ for farm B is double as large as for farm A. Consequently, the vector of marginal costs of the twice larger farm B is increased by $0.5\mathbf{Q}\boldsymbol{\upsilon}$ compared to A. $\mathbf{Q}\boldsymbol{\upsilon}$ is positive due to the necessary positive definiteness of $\mathbf{Q}$. Using a model written in crop acreages superimposes hence by construction decreasing returns to scale across farms, i.e. the larger the farm size the larger the farm marginal cost. As the supply cost function is the inverse of the marginal cost function, we have that $\mathbf{p} = \mathbf{Q}\boldsymbol{\upsilon}$ where $\mathbf{p}$ is the price vector of outputs. Assuming the off diagonal element of the matrix $\mathbf{Q}$ equal to zero, an increase of output price of crop *i* by one unit leads to a change in output supply of that crop equal to Q_{ii}^{-1} for farm A and B independent of their size. As a consequence, the functional form implies by construction that the larger farm B shows lower supply elasticities compared to the smaller farm A and vice versa. That is why we normalise by farmland and turn to a per unit cost function. As our decision variables are acreages and we consider yields as fixed and exogenous, the per unit cost refers to per hectare costs.

The quadratic form of the variance-covariance component implies also that, when the decision variables are the crop quantities the variance of farm's B revenue is four times larger as in farm A at same crop shares. A standard E-V model with the same risk aversion coefficient for both farmers means a higher impact of risk relative to revenue for the larger farm. Consequently, larger farms would be forced by the model's construction to choose farm programs which decrease risk more compared to small farms, i.e. to behave more risk aversely. As larger farmer can be assumed (at least in average of larger samples) to be wealthier, literature rather suggests the opposite. The normalisation by farmland corrects for this.

2.1 Empirical Model

Many PMP applications use observed yields and prices as expected ones potentially leading to estimation bias which we avoid by applying an expectation models (Pope and Chavas, 1994). Specifically, we apply the adaptive expectations for crop prices using each sample's mean prices (Chavas and Holt, 1990; Sckokai and Moro, 2006). Consequently, expected prices do not differ across farms as the unbalanced nature of our panel does not allow constructing price series at farm level. Furthermore, the use of sample price series also avoids potentially endogeneity problems related to quality choice. Under the adaptive expectation hypothesis the farmer makes an expectation at time t about the crop price at time $t+1$ based on the price observed at year t (naive expectation) plus the mean error under last years' naive expectations. That is:

$$E_t(p_{t+1}) = p_t + E(p_t - p_{t-1})$$

where the second term on the right hand side of the equation is the sample mean of the errors made in the previous years in making a naive expectation. We aggregate the individual crop expected prices into groups by means of Törnqvist indices.

Yearly yield expectations for each farm f are modelled as:

$$E(yld_{ft}) = E(yld_t) * \frac{E(yld_f)}{E(yld^{\wedge})}$$

where the sample expected yields $E(yld_t)$ in each year are adjusted by the ratio between average farm yields $E(yld_f)$ and sample yields for those years where the crop is observed in that farm $E(yld\char`^)$. Hence, differently from the expected prices, the expected yields are specific to each farm.

We introduce a dual quadratic cost function which summarizes all farm variable costs (besides land rent), plus the cost of hired labour and the farm capital depreciation reported in FADN[1]. The cost function depends on both crop shares as decision variables[2] and given expected yields. We assume that farmers apply a two-stage decision process where first expected yields are decided upon[3]. Hence,

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[1] We exclude renting cost of land as the land constraint comprises both rented and owned land. The cost of family labour is not observed in the database. The wage farm household decide to reserve may be largely different from one household to another. Hence we refrain from identify some proxies for the price of family labour such as reported wages for hired one. Instead, we include family labour units as an explanatory variables of the total farm costs (i.e. higher amount of family labour units should have a negative effect on total farm cost).

[2] Heckelei and Britz, (2000) use a somewhat different approach to avoid scale bias as they normalize the whole Q matrix element wise by the square root of the observed production quantities. In our setting it is preferable to normalise by farmland as the observed crop shares are assumed to be reported with an error (see Section 2.2).

[3] Estimating an alternative model where yield decisions are endogenous did not yield satisfactory results. Already a simple visual comparison of the development of average yields, output and input prices index in the sample suggest that there is little relation between these. Modeling yield decisions at the single farm would require

expected yields enter exogenously in our cost function to account for cost differences stemming from yield and not from crop share differences between farms. The quadratic part of the cost function, cq_{ft}, for farm f in year t before the normalisation on a per hectare basis, is

$$(1)^4 \quad cq_{ft} = \mathbf{x}_{ft}'\mathbf{Q}_1\mathbf{yld}_{ft} + \mathbf{x}_{ft}'\mathbf{Q}_2\mathbf{yld}^2_{ft} + \mathbf{x}_{ft}'\mathbf{Q}_3\mathbf{s}_{ft}$$

where, $\mathbf{x}$ is the expected output quantity in a year computed from the exogenously given expected yield, the endogenous crop share choice and the given farmland. $\mathbf{Q}$ are symmetric matrices of dimension $I \times I$, being I the number of crops grown in the area where the farm is located. All the three $\mathbf{Q}$ matrices are estimated inside the model. Specifically, $\mathbf{Q_1}$ and $\mathbf{Q_2}$ are diagonal matrices of parameters which measure the linear and quadratic effect of expected yields on crop marginal costs while $\mathbf{Q_3}$ is a full matrix of parameters

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information on farm characteristics affecting the relation between input use and yields, such as soil type and micro-climate. Introducing a farm specific constant for each crop to capture the impact of such non-observable factors would potentially overfit the whole model as we have only between 5 and 7 observations per farm on average in each sample, and even fewer for individual crops in a farm. We therefore opted to rather treat expected yields as fixed.

[4] For notation easiness we omit the E operator in all the following equations and we use p, yld and x to indicate expected price, yield and quantity respectively if not differently specified.

which accounts for own and cross effects of crop shares on crop marginal costs. A Cholesky decomposition of $\mathbf{Q}_3$ ensures positive definiteness and hence convexity of the cost function with respect to the crop shares (Paris and Howitt, 1998). $\mathbf{Q}_3$ is hence expressed as a product between a lower triangular matrix and its transpose matrix, $\mathbf{LL}'$. The definiteness of the other matrices, $\mathbf{Q}_1$ and $\mathbf{Q}_2$, is not required as yields are not treated as decision variables; thus marginal costs might also decrease in expected crop yields, for instance, if larger expected yields occur in farms with favourite climatic or soil conditions which result in a reduction of irrigation, fertilizer or crop protection costs per unit. The latter cannot be excluded beforehand as we cannot control for soil and climatic properties separately. All the **Q** matrices are regional, not farm specific, and identical for all years.

The linear part of the costs function, still before normalisation by farmland, consists of regional parameters, estimated inside the model, which account for (1) costs at farm level independent of the production program such as certain accounting cost, $cfix$, (2) variable costs per unit of crop produced $\mathbf{cv}$,(3) costs per ha of land independent of the production program and its square, ch_1 and ch_2, due to basic field operations, (4) the effect of family labour and its square on costs, δ_1 and δ_2 and (5) the cross effect between land and family labour on costs, ν:

(2) $$cl_{ft} = cfix + \mathbf{cv'x}_{ft} + ch_1\, L_{ft} + ch_2\, L^2_{ft} + \delta_1 lab_{ft} + \delta_2 lab^2_{ft} + \nu L_{ft} * lab_{ft}$$

The linear and the quadratic terms terms are multiplied by a yearly specific general farm input price index, px, taken from Eurostat database and by a technical progress term $1+\delta t$ where the slope term, δ , is estimated. The farm input price index summarizes changes in crop variable inputs and farm overheads, farm buildings and machinery[5]. We also add a farm specific scaling factor, cf_f, which measures cost efficiency differences across farms and which is also estimated (equation (3)). The normalisation of the cost function by farmland leads to:

$$(3) \quad c_{ft} = \left[(cl_{ft} + cq_{ft}) * px_t * (1+\delta t) * cf_f \right] / L_{ft}$$

As we are not distinguishing between different inputs, the total marginal costs per hectare with respect to crop share can be derived by taking the derivative of (3) towards the crop shares. Hence, the per hectare marginal cost of a crop (see (4)) depend on (a) the estimated variable cost per crop share unit cv_i of that crop, (b) the given expected yield of that crop according to the diagonal elements of $\mathbf{Q_1}$ and $\mathbf{Q_2}$, and the crop share mix according to $\mathbf{Q_3}$. In addition, marginal costs change from one year to another according to the product of px and $(1+\delta t)$ while accounting for a farm specific efficiency multiplier according cf.

$$(4) \quad \frac{dc_{ft}}{ds_{fti}} = \begin{pmatrix} cv_i yld_i + yld_{ift}\mathbf{Q}_{1i\cdot}\mathbf{yld}_{ft} + yld_{ift}\mathbf{Q}_{2i\cdot}\mathbf{yld}^2_{ft} + \\ 2yld_{ift}Q_{3ii}s_{ift} + \sum_{j \neq i} s_{jft}Q_{3ij}(yld_{ift} + yld_{jft}) \end{pmatrix} px_t\,(1+\delta t)\,cf_f$$

[5] The index from Eurostat does not include hired labor. However, we have checked that on average the share of hired labor cost on our total cost in each sample is rather small, 6%, 4.5% and 5.3% in the Italian, German and French sample respectively. In addition, as changes in the production costs of agricultural inputs also reflect the changes in wages, we consider the index as well suited for our purpose.

Where, $\mathbf{Q}_{1i\cdot}$ indicates the i^{th} row of the matrix $\mathbf{Q_1}$, $\mathbf{Q}_{2i\cdot}$ indicates the i^{th} row of the matrix $\mathbf{Q_2}$ and Q_{3ii} indicates the element of the i^{th} row and i^{th} column of matrix $\mathbf{Q_3}$.

The reader should note here some relevant differences to PMP applications. Firstly, the cost function in (3) and its derivative in (4) are driven by *crop shares* and not, as usual in PMP, by acreages. Perhaps more important, there are no 'unobserved' costs in our model, (4) is an estimator of all costs reported by the farms with the exemption of land rents. The difference between revenues plus subsidies minus these per hectare costs multiplied by farmland defines the return to land, family labour and to binding constraints in the model, such as the set-aside requirement and production quotas. This return defines the expected profits from agriculture available to each farmer to remunerate family labour and capital including land.

The variance of farm revenues in year t per ha, $\sigma^2_{R_{ft}}$, is modelled by separating the variance of yields (production risk) from the variance of prices (market risk) according to Coyle (1999):

$$\sigma^2_{R_{ft}} = (\mathbf{s}_{ft} \odot E(\mathbf{yld}_{ft}))' \mathbf{V}_p (\mathbf{s}_{ft} \circ E(\mathbf{yld}_{ft})) + \quad (5)$$
$$\mathbf{p}'_{ft} \mathbf{V}_{yld} \mathbf{p}_{ft} + \sum_{i=1}^{5} \sum_{j=1}^{5} V_{p_{ij}} V_{yld_{ij}}$$

where $\mathbf{V}_p$ is the variance-covariance matrix of crop prices and $\mathbf{V}_{yld}$ is the variance-covariance matrix of yields. The variance-covariance matrix of prices is computed from sample mean series of market prices over the period 1995-2007 after deflating by the Consumer Price Index. The variance-covariance of crop yields is computed based on the farm level data in order to avoid the underestimation of yield variation due to the use of aggregate data (Just and Weninger, 1999). The first step in the computation of the crop yield variance-covariance consists in de-trending the farm yields. In order to estimate the yield trend, the regional yields are regressed on a time component starting from a quadratic specification and scaling it down to linear or even no trend specification according to the statistical significance of the higher order coefficient. Farm level yields are then de-trended by using the coefficient estimates from the regional de-trending regression. Next, the farm level de-trended yields are subtracted by the crop mean yields specific for that farm; as a consequence, if the farm grows a crop only in one year, that farm does not contribute to the computation of the yield variance for that crop as the subtraction leads to a zero value which is dropped by the computation of the variance. Finally, the de-trended and mean corrected yields are used to compute the variance-covariance matrix. We assume independence between crop price and yield variability; this assumption seems reasonable given that we mostly consider internationally traded crops (Serra et al., 2006).

The constraints of the programming model relate to the land balance and, where applicable, to compulsory set aside and sugar beet quotas. All constraints are expressed as inequality constraints thus we allow the farm to use less land than the amount available on the farm, to have voluntary set aside and to not employ all the sugar beet quota. All the constraints are normalised by farmland.

All the parameters of the cost function and of the risk term as well as the shadow values of the constraints are simultaneously estimated by the use of the FOCs (see section 2.2). The FOCs for the farmer's optimal land allocation of our model are equations (6) - (8)below.

$$(6)\quad \left(\begin{array}{l} p_{ti} yld_{fti} + sub_{ift} - 0.5 * \dfrac{dc_{ft}\left(\mathbf{s}_{ft}, yld_{fti}, \dfrac{lab_{ft}}{L_{ft}}, px_t \mid \boldsymbol{\beta}_1 \right)}{d(s_{fti})} - \\ \dfrac{\alpha(L_{ft}, lab_{ft} \mid \boldsymbol{\beta}_2) d\left[\sigma_R^2 \left(\mathbf{s}_{ft} \right) \right]}{d(s_{fti})} - \mathbf{A}^T \boldsymbol{\gamma}_{ft} \end{array} \right) \odot s_{fti} \geq 0$$

$$(7)\quad \left(\frac{\mathbf{b}_{ft}}{L_{ft}} - \mathbf{A}'\mathbf{s}_{ft} \right) \odot \boldsymbol{\gamma}_{ft} \geq 0$$

$$(8)\quad \sum_i \mathbf{s}_{fti} \leq 1 \quad [\gamma]$$

As crop shares are non-negative, the FOCs of the model with respect to crop shares are expressed as complementary slackness conditions. The complementary slackness expressed in equation (6) states that at the optimum either marginal revenue is equal to the marginal cost or the crop share is equal to zero. Equation (7) indicates that either a constraint is binding or its shadow value is equal to zero. Equation (8) guarantees the sum of crop shares to be no larger than 1.

2.2 Technical implementation of the estimator

The Cholesky decomposition, cost function, risk term and further elements render the estimation of the cost function parameters and of the risk coefficient parameters highly non-linear. In addition, as we allow for (a) not using all the land available on the farm, (b) set-aside more land than legally required, the optimality conditions are KTTs, such that no closed form representation exists (see equations (6) - (8)). Similar to most PMP related work, we estimate in GAMS as standard econometric packages do not offer solvers for this class of estimation problems. We benefit from the EMP package of GAMS to formulate and solve our model as a bi-level programming model. Applications to water allocation problems based on bi-level model formulation and estimation are discussed by Britz et al. (2013) and Kuhn et al. (2014). A bi-level programming model is a programming model (outer problem) that has another optimisation model (inner problem) as a constraint. The optimal solution of the inner problem is parameterized by the variables of the outer problem. EMP formulates the bi-level problem as a Mathematical Program with Complementarity Constraints (MPCC) by replacing the lower level optimization problem by its first order optimality conditions. The program in this version is then solved by one of the solver available in GAMS such as CONOPT. In our bi-level programming setting, the outer optimization problem is the statistical estimator which searches for the optimal parameters minimizing the sum of squared error terms, while the inner optimization problem depicts the maximization of expected utility problem for each farm in each year the farm is observed subject to the endowment and institutional constraints. The inner problem thus determines at upper-level determined parameters the optimal crop allocation and resulting costs which in turn define the error terms as a function of the given parameters. Differently from the standard PMP application where only one observation is used and where the model is calibrated to that observation, our exercise employs multiple observations (the unbalanced samples of farms from the three EU areas) and thus it is an estimation exercise with error terms. Our estimation framework considers measurement errors on the allocated crop shares and on the total costs per hectare, but does not assume allocation errors of the farmer.

Specifically,
Outer problem

(9)
$$\min_{\boldsymbol{\beta}_1,\boldsymbol{\beta}_2} \varepsilon = \sum_{i=1}^{I} w_i \frac{\sum_{f=1}^{F}\sum_{t=1}^{T}\left(s_{fti} - s_{fti}^{obs}\right)^2}{\sum_{f=1}^{F}\sum_{t=1}^{T}\left(s_{fti}^{obs} - s_{i}^{SMobs}\right)^2} + \frac{\sum_{f=1}^{F}\sum_{t=1}^{T}\left(c_{ft}(\mathbf{s}_{ft}, \mathbf{yld}_{ft}, \frac{lab_{ft}}{L_{ft}}, px \mid \boldsymbol{\beta}_1) - c_{ft}^{obs}\right)^2}{\sum_{f=1}^{F}\sum_{t=1}^{T}\left(c_{ft}^{obs} - c^{SMobs}\right)^2}$$

subject to

Inner problem (for farm f in year t)

$$\max_{\mathbf{s}|\boldsymbol{\beta}_1,\boldsymbol{\beta}_2} EU\big/L = E(\tilde{\mathbf{p}} \circ \mathbf{yld})'\mathbf{s} + \mathbf{sub}'\mathbf{s} + dsub - c(\mathbf{s}, E(\mathbf{yld}), px, \frac{lab}{L} \mid \boldsymbol{\beta}_1) - 0.5\boldsymbol{\alpha}(L, lab \mid \boldsymbol{\beta}_2)\boldsymbol{\sigma}_R^2\left(\mathbf{s}\right)$$

(10) subject to
$$\mathbf{A}\mathbf{s}_{ft} \le \mathbf{b}_{ft}\big/L_{ft} \quad (\boldsymbol{\gamma})$$

$$\mathbf{0} \le \mathbf{s}_{ft} \le \mathbf{1}$$

$$\sum_i \mathbf{s}_{fti} \le 1 \ [\gamma]$$

where, the superscripts *obs* and *SMobs* indicate observed values for each farm and the sample mean, respectively, w_i is a weighting factor which depends on the number of observations where a crop was observed. The panels are not only unbalanced, but some crops are also more frequently observed than others. The weighting factor accounts for the share of observations where a crop is observed to avoid that crops with limited information receive the same weight as frequently observed ones.

The outer problem aims at minimising the sum of squared disturbances normalised by the corresponding total sum of squares by choosing $\boldsymbol{\beta}_1$ and $\boldsymbol{\beta}_2$. The inner problem takes the parameters given by the outer problem and finds for each farm f in each year t the farmer's crop share decisions maximising the farmer's expected utility subject to the constraints and it determines the disturbances as a function of the estimated parameters.

EMP automatically generates the FOCs of the inner problems, while GAMS offers transparent interfaces to performing Non-Linear Programming solvers such as CONOPT (Drud A. 1985 and 1992). As we estimate on unbalanced panels over many years, we have enough degrees of freedom to refrain from using a priori-information e.g. on supply elasticities.

3. DATA

The model is estimated for three different sets of farm-level data observed over the period 1995-2007 from three European Union (EU) agricultural intensive regions: Northern Italy, Cologne-Aachen area in Germany and the Grandes-Cultures area in Northern France. We consider farms which stay at least three consecutive years in the sample and which manage at least 30 hectares. The latter removes almost no observations in Germany and France, but quite some in Italy and ensures that the three samples comprise commercial farms of similar size and are thus comparable. We focus on arable farms which produce cereals, oilseeds, sugar beet and, for Germany only, potatoes[6]. These crop categories represent the dominant production system in the regions under analysis. Farms producing specialty crops such as vegetables or rice are excluded from the analysis as their technology is rather different from the crops considered. We also exclude farms which are classified as specialized arable, but have some animals or produce fodder. The data are from the FADN (Farm Accountancy Data Network) database, the most widely used farm-level database in the European Union and the only source of microeconomic data harmonized at the E.U. level (European Commission 2008). The FADN reports yearly technical and economic data of a large sample of commercial farms from each member state. Our final samples include 620 observations (the combinations of farms and years) in Northern Italy, 693 observations in Cologne-Aachen area and 1,698 in the French Grandes-Cultures area (Table 1). The number of farms is 140, 112 and 282 respectively in each sample. The French and German farms stay on average for a longer period (around 6 years) in the sample compared to Italians (4.4 years). We group the crops grown on these farms into five crop categories: wheat, corn, other cereals, oilseeds and sugar beet; in Germany, potatoes are added. We also consider the set aside area, making a distinction between compulsory and voluntary set aside.

Italian farms in the sample grow on average 2.6 crops on their farmland and are thus less diversified compared to the German and French farms in the sample which grow 3.2 and 3.7 crops on average respectively. In addition, 19% of Italian farms are specialized on one crop only, namely corn, while in the German sample there is only one

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[6] The share of potato production in the other two regions is small and only a limited number of farms in the two samples grow potatoes.

farm growing only one crop and none is observed in the French sample. Corn is the most widespread crop in the Italian sample (93.7% of observations and 55% in terms of crop share) followed by oilseeds and wheat. It should be however noticed that agricultural policy has affected the crop shares over the period considered, by changing administrative prices, introducing and changing coupled subsidies and then replacing coupled by decoupled subsidies, by changing the threshold of set aside required: oilseeds crop share has decreased in Italy by 12% after 1999 while wheat crop share has increased by 7% in the same period. The higher specialisation of Italian farms may be also related to the smaller average size (78 hectares) compared to German farms (102) and French farms (131). In both the German and French samples, wheat has the highest share of farmland and it is grown in almost all farms. In Germany sugar beet has the second highest crop share, and is grown by almost all farms followed by other cereals. In France oilseeds takes the second place both in terms of share and in terms of adoption among farms. A larger share of the French farms grows also other cereals and sugar beet besides wheat and oilseeds.

Italian farms employ the largest amount of family labour per hectare: 2.68/100 ha compared to 1.48 in the German sample and 1.14 in the French sample. However, hired labour use differs: while it is employed in 20.6% of the Italian farms only, it is used in 36% of the French and in more than half of the German farms. The total costs per hectare, excluding land rents, is higher in Germany (1,149 euro) compared to Italy (862 euro) and France (849 euro) which might be explained by the larger employment of hired labour, by the large percentage of German farms that grow sugar beet and by the inclusion of potato cropping. However, if family labour was remunerated at the rate of hired one and included in the cost per hectare, especially Italian farms would experience a large increase in cost as they show the highest amount of family labour per hectare. The average profit per hectare of Italian farmers over the period considered is 818 euro, larger than German (530) and the French (440) and this rank has been constant over the years. That might reflect the lower share of hired labour and the different land-man-ratio such that the reported profits

TABLE 1

Descriptive statistics of the three samples

	Italy				Germany				France			
Number of observations		620			693				1698			
Number of farms		140			112				282			
Average number of crops grown on the farm		2.6			3.2				3.7			
Observations that grow only one crop	numb er	118			1				0			
	%	19			0.1				0			
	mean	sd*	observations		mean	sd	observations		mean	sd	observations	
Crop share (%)			numb er	%			number	%			number	%
Common wheat	24.5	15.8	298	48.1	47.3	13.7	686	99.0	45.6	10.4	1698	100.0
Corn	55.0	25.1	581	93.7	8.0	8.3	9	1.3	11.6	7.7	702	41.3
Other cereals	19.2	12.6	83	13.4	18.4	10.4	558	80.5	18.0	9.9	1393	82.0
Oilseeds	26.7	15.7	409	66.0	11.1	6.4	142	20.5	22.5	9.4	1599	94.1
Sugar beet	18.3	8.6	273	44.0	24.8	7.9	685	98.8	13.2	8.0	959	56.4
Potatoes	-	-	-	-	21.4	17.1	118	17.0	-	-	-	-
Fallow land set aside	9.1	3.4	574	92.6	8.2	3.2	665	96.0	7.1	4.7	1482	87.3
Farm size (ha)	77.8	78.8			102.2	53.6			131.4	67.1		
Family labour (units/ha)	1.43	0.75			1.2	0.47			1.2	0.45		

*sd indicates standard deviation. Source: FADN data for Italian farms need to remunerate more labour. The corn price has the highest variance among the crops considered in Italy and France, while potatoes rank the first in Germany (Table A1 and A2). In Italy, all crop prices move together as the covariance between any two crops is positive. In Germany, a negative covariance between sugar beet and all other crops but corn is observed, similar to France. The covariance of the yields is positive for all crops in France, while the yield of other cereals is negatively related to corn yields in Italy and Germany. A negative relationship is also detected for sugar beet yields with respect to the yields of potatoes and oilseeds and for oilseeds yields with respect to potato yields in Germany. In Italy, oilseed and wheat yields are negatively correlated. These observations indicate that the scope of using crop choice to manage risk is limited, larger gains would results from stronger negative correlations, while we mostly observe positive ones.

4. EMPIRICAL RESULTS

4.1 Goodness of fit

Table 2 reports the goodness of fit of the estimated model for all the three samples. The overall goodness of fit is measured by $(1-\varepsilon)*100$ (i.e. the R^2 of a standard regression analysis) and it is decomposed according to equation 7 into goodness of fit for each crop shares and for total farm variable cost per hectare. The R^2 of both the estimated crop shares and total farm costs per hectare in our panel analysis is relatively high, despite the fact that we have only one parameter in the estimation, the efficiency multiplier, which is specific for each farm. The latter is certainly a reason why the fit for the costs per hectare is relatively high. The estimated crop shares are driven by the cost function parameter. We estimate the fit for crop shares and not for the observed acreage, hence there is no artificial increase in the explanatory power by adding a land balance which would help to explain the variance in observed acreages and total cost linked to farm size in hectares and not linked to differences in managing the land.

The Italian sample presents the best fit for the cost per hectare compared to the other two countries. This may be partly driven by the shorter time Italian farms stay in the panel on average such that farm specific efficiency multipliers can capture more of the variance. There is not clear ranking across countries with regard to the fit for crop shares. As expected, in each region sugar beet and set aside are the two crops that have the highest goodness of fit. This is clearly due to constraints largely driving the share of these crops (sugar beet quota and set aside obligation respectively). For the remaining crops, no uniform picture arises across the samples.

TABLE 2

*Values of each component of the overall goodness of fit (1-ε)*100 for each*

	Italy	Germany	France
Crop share			
wheat	38.3	60.2	37.2
corn	70.4	80.7	31.6
other cereals	21.2	39.0	24.5
oilseeds	34.9	33.2	58.2
sugar beet	97.0	82.6	98.4
potatoes	-	35.1	-
Set aside	67.8	63.2	77.1
Cost	72.0	50.2	62.2
Overall	62.8	58	55.2

Source: Own calculation

4.2 Supply elasticity

Note that our estimation (as most PMP related exercises) does not aim at hypotheses testing on individual parameters in the cost function, but rather on a robust parameterization of simulation model. Accordingly, beside the fits, what matters most is plausible simulation behaviour. We hence derive price elasticities generated by simulations (Table 3): we simulate a 1% increase in the crop price and we solve the inner model (maximisation of the farmer's expected utility) for each farm in each year to find the new optimal crop shares given that price increase. The elasticity is then computed as the average in the sample. As expected by imposing curvature conditions, all own price elasticities in each region are positive. Furthermore, we find elasticities in a plausible range between 0.5 and 2 in our estimation without a priori information on the parameters. In Italy, the most frequently, grown crop, namely corn, presents the largest elasticity (1.2), while in Germany corn is the least grown crop and also displays the largest own price elasticity followed by other cereals. These two crops show the largest values for elasticity also in France. Due to quota constraints, the value of own and cross price elasticity for sugar beet in Germany is zero. Although quota constraints are considered also in Italy and France, in the two countries the quota is not binding for some farms leading to an average change in sugar beet production when price changes. The cross price elasticities are not positive for all crops in each region but one (the cross price elasticity of other cereal on oilseeds crop share) and are generally below unity.

The table also reports the reaction of selected key economic variables to a change in the crop prices. As expected the effect of a change of crop price on revenue and land rent is positive, while the effect on costs is crop dependent and related to the substitution relationships between crops. The profit (measured as revenue minus variable costs) rises when a crop price rises. As expected, in all the three regions considered the most grown crop in the region produces the largest change in revenue and profit due to a price change.

4.3 Risk aversion coefficient

The relative risk aversion coefficient is endogenously estimated in the model and it is modelled as a function of farmland and its square and family labour and its square. We use farmland as the main farm asset as a proxy for wealth to reflect the relation between risk aversion behaviour and wealth. Unfortunately, we do not have data on total family labour and off-farm income to better capture off-farm labour as a diversification strategy. Instead, we use farm labour and farm labour per hectare as a proxy.

The average risk aversion coefficient is larger in the French sample (0.06562) compared to the Italian sample (0.01267) and the German sample (0.000563). The French sample is also the one with the highest variability of the coefficient among farmers. The

TABLE 3

Own and cross price elasticity of crop shares and of some economic variables at sample mean

		common wheat	corn	other cereals	oilseeds	sugar beet	potatoes
	Share						
	common wheat	0.90	-1.48	-0.02	-0.17	0.00	n.a.
	corn	-0.17	1.22	-0.02	-0.17	-0.01	n.a.
	other cereals	-0.14	-0.83	0.49	-0.20	-0.01	n.a.
	oilseeds	-0.14	-0.73	-0.02	0.71	-0.01	n.a.
Italy	sugar beet	0.00	-0.04	0.00	0.00	0.07	n.a.
	potatoes	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.
	Economic variables						
	revenue	0.08	0.98	0.00	0.11	0.21	n.a.
	land rent	0.21	0.73	0.02	0.29	0.01	n.a.
	profit	0.11	1.33	0.01	0.13	0.30	n.a.
	Share						
	common wheat	0.72	0.00	-0.44	-0.01	0.00	-0.02
	corn	-1.53	1.43	-0.07	0.00	0.00	-0.07
	other cereals	-1.95	0.00	1.37	-0.02	0.00	-0.06
	oilseeds	-0.11	0.00	0.10	0.62	0.00	-0.07
Germany	sugar beet	-0.05	0.00	-0.05	-0.05	0.00	-0.02
	potatoes	-0.05	0.00	-0.04	-0.01	0.00	0.58
	Economic variables						
	revenue	0.39	0.00	0.01	0.00	0.45	0.18
	land rent	0.70	0.00	0.15	0.03	0.00	0.19
	profit	0.80	0.00	0.18	0.05	1.08	0.27
	Share						
	common wheat	0.63	-0.08	-0.40	-0.08	-0.01	n.a.
	corn	-0.75	1.72	-0.61	-0.33	-0.01	n.a.
	other cereals	-1.35	-0.22	1.87	-0.31	0.00	n.a.
	oilseeds	-0.20	-0.09	-0.23	0.53	0.00	n.a.
France	sugar beet	-0.01	0.00	0.00	0.00	0.06	n.a.
	potatoes	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.
	Economic variables						
	revenue	0.47	0.05	0.13	0.17	0.23	n.a.
	land rent	0.62	0.04	0.26	0.46	0.01	n.a.
	profit	1.17	0.18	0.24	0.19	0.50	n.a.

coefficient estimates indicating the effect of a change in land and family labour on the risk aversion attitude of the farmer are presented in Table 4. Italian and French farmers show an increase in the relative risk attitude linked to a rise in the land availability; however the marginal effect of land size is decreasing. Opposite, in Germany the larger the farm size measured by land the lower the risk aversion with a decreasing marginal effect. The effect of family labour on the aversion towards risk is negative in all three countries. With a close relation between farmland and family labour, it is however hard to disentangle these effects. One would need information on non-agricultural income and its sources on each farm to discuss on wealth and diversification effects. Overall, we have to conclude that data limitations prevent us from deriving a clear picture of wealth and off-farm diversification effects on risk behaviour.

TABLE 4

Risk aversion coefficient estimates

		Italy	Germany	France
Relative risk aversion coefficient	mean	0.01267	0.000563	0.06562
	Standard deviation	0.00083	0.001663	0.03088
Parameter estimate of the independent variables which explain the relative risk aversion coefficient				
Land		$2 \cdot 10^5$	$-4.3 \cdot 10^5$	$2 \cdot 10^5$
Land^2		$-4.4 \cdot 10^8$	$1.5 \cdot 10^7$	$-4.8 \cdot 10^8$
Labour		$-2.8 \cdot 10^3$	$-2.2 \cdot 10^3$	$-6.3 \cdot 10^3$
Labour^2		$5 \cdot 10^4$	$-3.3 \cdot 10^4$	$2.1 \cdot 10^3$

5. Discussion and Conclusions

In this study we present an estimation of the parameters of an expected utility programming model based on the FOCs and allowing for non-binding constraints. This study belongs to the new, and barely explored, research strand of Econometric Mathematical Porgramming which is promising in combining the advantages of econometrics and mathematical programming. We employ a quadratic cost function for total farm cost besides land rent and family labour and introduce an explicit land balance and institutional constraints (sugar beet quota and set aside obligation) in the estimation. The risk component is introduced by accounting for both price and farm-level yield variance. The model is normalized by total farmland in order to avoid scale bias and such that crop shares are the decision variables. We use large rotating panels of specialised arable farms from FADN in Northern Italy, the Cologne-Aachen region in Germany and the Grandes-Cultures region in Northern France observed over the time period 1995-2007 and we keep only farms which stay at least three consecutive years in the panels and which are larger than 30 hectares. The parameters of the programming model are estimated by means of a bi-level programming approach which allows for the inclusion of no binding constraints in the programming model. Our exercise applied to the observed farm level data shows satisfactory results. We obtained quite satisfactory fit for crop shares and costs with differences across the three regions. The values of the estimated supply elasticities are in a reasonable range and consistent with the theory. The elasticity of land rent, revenue and profit with respect to crop price exhibits a sign that is consistent with the expectation for all of the crops in each region and in all the three regions the crop with the higher shares also exhibits the larger impact of a price change on these variables. French farms in the sample display the average largest relative risk aversion coefficient followed by Italian and German farmers, but we cannot find clear and easy to interpret impacts of total land endowment as a proxy for wealth and farm labour (per hectare) as a proxy of off-farm diversification on risk aversion.

Our work opens to many future empirical and theoretical developments. It would be interesting for example to include additional resource constraints (e.g. water availability) and to use our model or a similarly estimated programming model to simulate potential policy tools. Another interesting, and so far hardly explored research strand would be the development of statistical tests for the parameters estimated in a programming model setup. Although estimations of programming model so far aim mostly at robust parameter estimates to perform simulation analysis, statistical tests on the parameter estimates value would represent a further step towards joining econometrics and mathematical programming.

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APPENDIX

Table A1. Variance-covariance matrix of crop price index (base year 2000)

	common wheat	corn	other cereals	oilseeds	sugar beet	potatoes
Italy						
common wheat	0.034	0.037	0.039	0.020	0.012	
corn	0.037	0.049	0.045	0.023	0.004	
other cereals	0.039	0.045	0.049	0.026	0.009	
oilseeds	0.02	0.023	0.026	0.021	0	
sugar beet	0.012	0.004	0.009	0	0.023	
Germany						
common wheat	0.051	0.001	0.045	0.009	-0.007	0.01
corn	0.001	0.087	-0.003	-0.005	0.018	0.008
other cereals	0.045	-0.003	0.042	0.007	-0.007	0.003
oilseeds	0.009	-0.005	0.007	0.016	-0.006	0.02
sugar beet	-0.007	0.018	-0.007	-0.006	0.017	-0.038
potatoes	0.010	0.008	0.003	0.02	-0.038	0.178
France						
common wheat	0.051	0.057	0.044	0.024	-0.001	
corn	0.057	0.07	0.05	0.023	-0.006	
other cereals	0.044	0.05	0.043	0.017	0.006	
oilseeds	0.024	0.023	0.017	0.02	-0.007	
sugar beet	-0.001	-0.006	0.006	-0.007	0.028	

APPENDIX

Table A2. Variance-covariance matrix of crop yields value ($euro^2/ha^2$ at 2000 price)

	common wheat	corn	other cereals	oilseeds	sugar beet	potatoes
Italy						
common wheat	12,019	3,132	2,800	-126	10,360	
corn	3,132	37,802	-6,317	2,994	3,950	
other cereals	2,800	-6,317	10,051	-8,076	13,937	
oilseeds	-126	2,994	-8,076	26,312	11,004	
sugar beet	10,360	3,950	13,937	11,004	191,814	
Germany						
common wheat	11,847	4,962	5,182	1,936	1,239	132
corn	4,962	14,237	-2,354		21,580	23,936
other cereals	5,182	-2,354	15,951	2,181	456	1,332
oilseeds	1,936		2,181	13,628	-2,106	-7,915
sugar beet	1,239	21,580	456	-2,106	87,857	-1,205
potatoes	132	23,936	1,332	-7,915	-1,205	198,364
France						
common wheat	8,793	2,294	4,053	2,038	4,473	
corn	2,294	18,775	1,824	1,208	12,525	
other cereals	4,053	1,824	11,382	3,198	3,774	
oilseeds	2,038	1,208	3,198	10,747	4,628	
sugar beet	4,473	12,525	3,774	4,628	124,829	

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