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Abstract

The misestimation of rating transition probabilities may lead banks to lend money incoherently with borrowers' default trajectory, causing both a deterioration in asset quality and higher system distress. Applying a Mover-Stayer model to determine the migration risk of small and medium enterprises, we find that banks are over-estimating their credit risk resulting in excessive regulatory capital. This has important macroeconomic implications due to the fact that holding a large capital buffer is costly for banks and this in turn influences their ability to lend in the wider economy. This conclusion is particularly true during economic downturns with the consequence of exacerbating the cyclicality in risk capital that therefore acts to aggravate economic conditions further. We also explain part of the misevaluation of borrowers and the actual relevant weight of non-performing loans within banking portfolios: prudential prescriptions cannot be considered as effective as expected by regulators who have designed the "new" regulation in response to the most recent crisis. The Mover-Stayers approach helps to reduce calculation inaccuracy when analyzing the historical movements of borrowers' ratings and, consequently improves the efficacy of the resource allocation process and banking industry stability.

Keywords: credit risk; Markov chains; absorbing state; rating migration

1. Introduction

Rating transition probabilities are the key factor allowing credit risk management to become forward looking and, to a certain extent, reliable for investors and commercial banks. This is true for listed bonds whose risk is generally estimated by rating agencies (Lando and Skødeberg, 2002) and for loans, that are more frequently analyzed through banks' internal models. The deterioration of mortgage and loan became a contributing source to different phases of the most recent financial crisis. Unreliable migration metrics can be considered as a serious model risk factors with systemic consequences, particularly due to the fact that most of the rating models are based on Markov chain assumptions to estimate transition matrices.

One of the issues embedded in pure Markov chains is the possible presence of an absorbing state, a situation that, once included, cannot be avoided. In a chain applied to credit risk estimation, the absorbing state means a default. This raises the interesting question: why should banks trust models based on the assumption that all their counterparts will fail sooner or later? The issue is based on the incoherency affecting credit risk transition matrix estimates when they based on this pure Markovian assumption, since if the absorbing state is incorporated, then all the borrowers move, at least in the long run, to the default state. A second relevant question relates to the estimation of the rating migration risk for the banks' economic capital: to be effective, internal rating models should be designed coherently not only with the actual borrowers' standing, but also with their expected assessment pattern. As a matter of fact, companies are characterized by different propensities to change and some of them could be defined as stayers, as opposed to movers. A good transition estimate should be able to differentiate the two clusters and recognize the trajectories of movers. An inconsistent rating transition model inevitably affects the robustness of capital absorption. Finally, we question whether the credit risk approximation due to the migration risk could increase the cyclicality and consequent probability of crisis.

To address these issues in depth, we focus our analysis on transition matrices applied to credit risk which show the pattern of changes for different borrowers over time from one rating notch to another. Every row of the matrix can be defined as a set of probabilities describing the likelihood of credit quality staying unchanged or moving to any of the other rating classes over a given time horizon, conditioned to the starting state. In most of applications, matrices are based on a Markov transition probability model.

Nevertheless, the pure Markov approach is unable to model the increasing probability for companies to stay within a rating class once they have been rated for a long period in that notch (Lando and Skødeberg, 2002; Altman, 1998 and Figlewski et al., 2006). Moreover, pure Markov models are not influenced by business cycles, while Nickel et al. (2000) show that rating changes can be affected differently. Again, if individuals are heterogeneous, migration probabilities may depend on their individual characteristics (Gomez-Gonzalez and Kiefer, 2009). Finally, transition matrices computed within a pure Markovian approach, are affected by the presence of an absorbing state that dictates that sooner or later, rated companies will be attracted by the "black hole" of the default state (Kremer and Weissbach, 2013).

Our paper helps overcome all of these drawbacks, as it applies the mover-stayers model to the estimation of rating migration matrices. Nevertheless the mover-stayer model generalizes the Markov chain model, and it is based upon two types of agents: (a) the "stayers" who are assumed to remain in the same category during the period; (b) the "movers" which are expected to move from a rating class to another with a probability described by a pure Markov chain. The empirical evidence of the existence of stayers allows us to solve the issue of an absorbing default state for borrowers.

The economic issue about the estimations of transition matrices pertains to the structural impact of financial crisis on the estimations, and the implications for regulators and credit policy makers. Since the crisis has potentially changed the patterns of defaults experienced before, all the matrices were biased by different estimates. Moreover, the regulatory response, particularly the introduction of the

Incremental Risk Charge for the trading book, was based on the Markov way of estimating the migration risk.
One of the most important goals behind the current debate aimed at revising the Basel Capital Accord is to replace the existing risk weights with a system which more clearly recognizes the differences in risk of various instruments. It is likely that rating systems will play a larger role in quantifying these differences. This is particularly relevant for small and medium enterprises (SMEs), without an external rating, that are generally rated by banks' internal rating models. To capture this factor we apply a statistical model calibrated to assess SMEs ratings as used by financial institutions.
Our prediction is that when banks accept to lend money incoherently with the borrowers' trajectory, this inevitably contributes to worsen asset quality which leads to a hyper-speculative position and, within an origination-to-distribute model with a large credit risk transfer via securitization, also to a highly distressed system. Moreover, the application of incorrect transition matrices causes over-estimated credit risk, a consequent capital misallocation and an inefficient increase, with the perspective that the loan process will be affected by growing transaction costs and bounded rationality. Our research question is to prove that this scenario particularly affects small and medium enterprises search for credit.
The remainder of this paper is organized as follows. Section 2 describes the database. Section 3 explains how we construct the Movers Stayers model compared with pure Markov chains that are applied to the transition matrices. In Section 4 we present the results. Section 5 concludes.

2. Data description

Our study has been designed to compare the credit transition approach based upon the pure Markovian model against the Movers-Stayers approach in order to verify whether commercial banks could optimize their credit risk portfolio models when lending to small and medium enterprises. We study the Italian loan market as it is peculiar for three reasons: only a small number of very large companies are rated by rating agencies; Italian GDP is strongly dependent on the

output of SMEs; finally, the high bank dependent liability rate of Italian firms, which enforces credit institutions to improve their internal rating models.
The database is composed of the balance sheets and income statements recorded for a large set of Italian corporate firms collected from Aida-Bureau van Dijk. The sample contains 44192 firms over the 11-year period from 1999 to 2010. We do not incorporate new companies into the sample that are not built present at the starting period: in this way we do not allow the cross-section to vary over time. One of the original contributions of our research is that for the first time, to our knowledge, the issue of pure Markovian weaknesses has been addressed specifically with regard to small and medium sized companies that are without an external rating and any bond issuance. This allows us to compare our sample to the credit portfolios of retail and commercial banks with an internal rating based model. Consequently, we run an internal rating approach to all the companies of the sample for every year of our time series. Ratings are grouped into six rating notches from A, characterized by the lowest level of default probability, to F. The default state is added and quantitative data are collected at the end of the year.
Another key point of our approach is that a sample of small/medium companies rated with an internal model allows us to control for the issue of movements to the not-rated category. Some researchers (Carty, 1997; Nickell et al., 2000) suggest that issuers who experience a transition to the not-rated category should be excluded from samples to calibrate the transition matrices. With our approach theses event do not occur and we can manage a complete credit analysis, keeping the number of rated companies constant over time.
The design of an internal rating system is an original component of our study because, in contrast to other research, it offers the opportunity to endogenously assign the probability of default. We used the data from the accounting reports of Italian companies collected from AIDA-Bureau van Dijk, a large financial and balance sheets information provider.
Through an extraction of sound firms with the same characteristics (base upon industry, size, accounting years availability and

geographical area) of default companies, we generated 50 matched samples of firms. Finally, we randomly split each of the 50 samples into equally sized sub samples: a *learning sample*, from which we derive the classification model, and a *control sample*, used to select the best model (out of sample forecast). Once we have obtained the final database, we compute a set of financial ratios to cover the most relevant areas of a firm's activity such as leverage, profitability, and solvency. Moreover, we proceeded to add Altman et al. (1977) ratios.
The logistic regression (LR) methodology was used to develop the model which not only allows us to derive models at different points in time before failure to estimate the chances of a firm going bankrupt, but it also enables the detection of the probability of default as it approaches. To select the best model, we ran a logistic regression on the selected ratios for each of the 50 *learning samples.* Finally, the model with the best classification in the respective *control sample* was selected.
The main characteristics of the model can be summarized as follows: (i) the discriminatory power of the model in the control sample to classify the bankrupt firms was around 80; (ii) the error of classifying an insolvent firm as financially sound (I type) was less than the error of classifying a sound firm as insolvent; and (iii) the accuracy ratio, measured with an ROC (receiver operating characteristic) curve, was more than 85.
Table 1 shows the absolute and percentage distribution for the entire sample of firms, respectively, by year and rating class.
Most of the companies have been rated within the A, B, or C rating notches whereas a lower percentage falls within the speculative or non-investment grades (i.e., D or below). This fact depends on the logic applied to fit the rating model. First, the default probability estimated for firms is drawn by a model calibrated on this sample and does not come from rating agencies or large banking institutions, as shown in previous studies[1]. Moreover, in our study we refer to not

[1]In Fei, Fuertes and Kalotychou (2011) data are derived from the S&P CreditPro 7.7 database. In Dietsch and Petey (2004), relating to the empirical studies on asset correlation, data were collected from the internal rating system of Coface for France and Creditform for Germany. Similarly, in Bandyopadhyay et al. (2007), results

listed companies which contrast to firms issuing publicly traded debt that are usually rated by the large international rating agencies.

We grouped ratings into a granular rating system with six rating notches by mapping the S&P scale. Mapping is done looking at the ranges of expected default frequencies estimated with an internal model and assigning each of them to one of the classes of rating agencies. Doing a mechanical comparison between the S&P scale and the one derived from our analysis, we observe that even the best firms within of the data set will receive only a BBB+ rating.

As a result of the mapping process, the default probabilities of class A do not match with that of the international rating agencies. Indeed, the lowest probability of default drawn from our model (0.26 per cent) corresponds, approximately, to a BBB+/BBB notch. Similarly, the B risk class includes more than one notch where the highest probability to default is close to 1 per cent, corresponding to a BB/BB- notch. This fact explains why more than 50 of firms fall within the best two classes. It is worth highlighting that the distribution of our sample is in line with the real distribution of Italian small and medium enterprises.

Table 1: Companies' distribution. Absolute and relative values (percentages) by year (1999 - 2010) and by rating classes.

	A	*B*	*C*	*D*	*E*	*F*	*Default*
1999	19265	7804	8313	7268	975	567	0
	43.59%	17.66%	18.81%	16.45%	2.21%	1.28%	0.00%
2000	17950	7906	8715	7551	966	468	636
	40.62%	17.89%	19.72%	17.09%	2.19%	1.06%	1.44%
2001	12433	9457	10679	9188	1062	147	1226
	28.13%	21.40%	24.17%	20.79%	2.40%	0.33%	2.77%
2002	13136	9346	9930	8579	1168	270	1763
	29.72%	21.15%	22.47%	19.41%	2.64%	0.61%	3.99%
2003	13194	9275	9571	8471	1135	290	2256
	29.86%	20.99%	21.66%	19.17%	2.57%	0.66%	5.10%
2004	13298	9208	9508	8129	973	250	2826

were based on CRISIL's (a lending credit rating agency in India) annual ratings of long-term bonds. For more details on the process we applied to compute PD, see Gabbi and Vozzella, 2013.

	30.09%	20.84%	21.52%	18.39%	2.20%	0.57%	6.39%
2005	13367	9152	9490	7764	950	263	3206
	30.25%	20.71%	21.47%	17.57%	2.15%	0.60%	7.25%
2006	13136	8940	9500	7804	956	281	3575
	29.72%	20.23%	21.50%	17.66%	2.16%	0.64%	8.09%
2007	13565	8709	9212	7373	1012	201	4120
	30.70%	19.71%	20.85%	16.68%	2.29%	0.45%	9.32%
2008	16860	8581	7431	5482	965	317	4556
	38.15%	19.42%	16.82%	12.40%	2.18%	0.72%	10.31%
2009	17297	7954	6896	5506	1220	303	5016
	39.14%	18.00%	15.60%	12.46%	2.76%	0.69%	11.35%
2010	16892	8044	6984	5601	1186	414	5071
	38.22%	18.20%	15.80%	12.67%	2.68%	0.94%	11.47%

Rating classes range from A (lowest probability of default, PDs) to F (highest PDs). To maintain the total number of firms equal each year we cumulate defaults which appear to increase over time.

Table 2 reports the evolution of firms leaving their risk rating states each year, those never moving from the starting state, and defaults over the sample period 2000-2010. Table 2 Panel 1 confirms the expected monotonic pattern of defaults by rating notches. The dynamic is recorded at year 11 (2010) for the bankruptcy procedures length, even though corporate failures significantly decrease. This problem does not corrupt the purpose of our analysis, that is unconditioned by the probability of default. Panel 2 shows the share of firms that at time $t+1$ recorded a change (upgrading or downgrading) compared to time t. Unsurprisingly, the percentage of shifts across rating notches is not only high throughout the observed period but it increases as credit quality deteriorates. Finally, Panel 3 shows the percentage of firms for every state that have never changed their rating over the sample period (potential stayers). The introduction of this category of firms suggests that, in contrast to the pure Markov chains that implicitly assume the default as an absorbing state, a non-pure Markovian behavior in ratings migrations occurs. Modeling the existence of this kind of companies allows us to manage most of the pure Markovian assumptions implied in the transition matrices usually computed within external and internal credit or bond portfolio models.

Table 2: Structure of the firms' states evolution (percentages on the number of firms starting from every notch).

	A	*B*	*C*	*D*	*E*	*F*
	Panel 1: Failed firms over time by rating class					
2000	0.55	0.83	1.23	3.59	6.15	7.41
2001	0.45	0.72	1.14	3.35	6.21	8.55
2002	0.20	0.27	1.24	2.90	5.93	17.01
2003	0.13	0.31	0.88	2.77	6.34	17.78
2004	0.15	0.49	1.01	2.90	7.93	24.83
2005	0.11	0.25	0.74	2.01	5.86	21.20
2006	0.04	0.27	0.60	2.22	6.74	17.11
2007	0.14	0.44	1.07	2.37	8.68	41.99
2008	0.10	0.32	1.00	3.08	4.45	14.93
2009	0.08	0.44	1.00	2.75	7.36	35.65
2010	0.02	0.05	0.13	0.47	0.82	0.99
	Panel 2: Firms leaving their state (up and downgrading)					
2000	21.52	42.86	41.09	34.92	61.33	79.01
2001	40.67	40.29	36.73	30.57	56.83	88.89
2002	12.96	39.09	40.54	33.02	59.32	67.35
2003	14.17	37.79	38.91	31.06	61.73	72.22
2004	12.26	35.30	36.20	30.20	65.64	70.69
2005	11.92	34.81	35.45	29.66	59.92	61.20
2006	12.60	35.13	34.03	27.41	55.89	57.79
2007	10.18	35.62	35.58	29.61	55.54	79.00
2008	7.66	48.09	50.85	46.40	60.67	59.70
2009	8.84	38.85	41.08	33.35	55.23	73.50
2010	10.00	33.43	35.19	29.84	57.54	58.09
	Panel 3: Firms never moving from their starting state					
	30.09	1.60	2.06	5.27	3.08	0.00

Panel 1 shows the pattern of failed firms over time. Panel 2 contains the incidence of companies moving from one notch to another. Panel 3 shows the incidence of potential stayers, that is companies whose rating has never changed over the 11 years.

3. Markov chains and the Movers Stayers model

In the classical Markov Chain (MC), single names are supposed to move from state *i* to state *j* with a given probability $p_{ij} = \Pr(X_t = j | X_{t-1} = i)$, depending only on *i* and *j*. Individuals are then homogeneous since they move according to only with their starting state *i*, and not with some idiosyncratic features.

The Mover-Stayer model (MS) arises from the mixture of two different Markov chains, one of which degenerates with transition probabilities $p_{ij} = \delta_{ij}$. Then individuals are actually divided in two subsets: the set of Movers (M), following a classical MC with transition probabilities m_{ij}, and the set of Stayers (S), constantly living in their initial state. This model is introduced in Blumen, Kogan and McCharty (1955) with the aim of including heterogeneity among individuals. Furthermore, MS avoids some problems related to the simple MC, for example the tendency to underestimate the probability p_{ii} to remain in the same state.

For any individual, we consider the probability s_i to be a Stayer, given the starting state *i*. Note that this probability depends again only on *i*, and not on individual factors. The global transition probability is then given by:

$$p_{ij} = s_i \cdot \delta_{ij} + (1 - s_i) \cdot m_{ij}, \qquad [1]$$

where δ_{ij} is the Kronoecker's Delta.

The main difference with the MC model relates to the computation of the transition probabilities after *s* steps: if $p_{ij}{}^{(s)} = \Pr(X_t = j | X_{t-s} = i)$ and $P^{(s)} = \{p_{ij}{}^{(s)}\}$ is the corresponding matrix, then:

$$P^{(s)} = P^s \qquad [2]$$

in the classical MC, and

$$P^{(s)} = S + (I - S) * M^s \qquad [3]$$

in the MS model (with $S = diag\{s_1, \dots, s_k\}$).

When banks monitor their borrowers, they collect data which are characterized by a continuous stream over the lending period. Measuring credit transitions in the continuous time has a number of advantages: (i) it allows a rigorous formulation and testing of assumptions 'rating drift' and other non-Markov type behavior (Altman and Kao, 1992a,b, Lucas and Lonski, 1992, and Carty and Fons, 1993); (ii) it permits us to formulate and test the dependence on external covariates, and quantify changes in regimes either due to business cycles (Nickell et al., 2000), or changes in rating policies (Blume et al., 1998); (iii) it leads to better estimates of rating-based term structure modeling by different rating classes, especially when their slopes are unorthodox (Jarrow et al., 1997, Lando, 1998, and Das and Tufano, 1996).

In the aforementioned models, transitions are allowed to happen only at equi-spaced instants of time *t*, *t+1, t+2, ..., t+n*. The "continuous-in-time" version of MC (CTMC) instead permits transitions at any point of time *t* in *R*. It means that p_{ij} is actually a continuous function of t:

$$p_{ij}(t) = \Pr(X_t = j | X_0 = i) \qquad [4]$$

for every *t* in *R*.

The continuous version of MS (CTMS herein) is defined in the same way: Movers may move at any instant *t*, and the global transition probabilities are:

$$p_{ij} = s_i \cdot \delta_{ij} + (1 - s_i) \cdot m_{ij}(t). \qquad [5]$$

The continuous time models are characterized by the existence of a matrix *Q* describing the rate of transitions among the states. *Q* is this generating matrix and satisfies the following properties:

1. $q_{ij} \geq 0 \; \forall i \neq j$ and $q_{ii} < 0$;
2. $\sum_{j=1}^{k} q_{ij} = 0 \; \forall i$;
3. $M(t) = e^{tQ}$ for all *t>0*, where $e^{(.)}$ stands for the exponential matrix function (Golub and Van Loan, 1996).

The MS version of this property will be:

$$P(t) = S + (I - S) * e^{tQ} \qquad [6]$$

As we will see in the subsequent sections, Q is beneficial to obtain additional information about the persistence of borrowers (single names) in every state.

In our research we focus on the continuous version of MC and MS. In this sense it is worth noting two different problems affecting such models: embeddability and aliasing (Fougere and Kamionka, 2003). Indeed, from the estimated one-year transition matrix $\hat{M}$ (for the Movers) we obtain the estimated $\hat{Q}$ as a solution of $M = e^{Q}$. Two cases may happen:

1. Embeddability: there is no solution $\hat{Q}$ which also satisfies the aforementioned properties of a generating matrix. This drawback arises because not every discrete time Markov chain can be realized as a discretized continuous-time chain (Israel et al., 2001). Hence, it may be impossible from a one-year transition matrix (for example, if it contains zeros in some of the non-default rows) to structure a continuous-time chain which has the one-year transition matrix as its "marginal".
2. Aliasing: more than one continuous time chain exists, from which the discrete process arises. This drawback is caused by the fact that the equation $M = e^{Q}$ may have different solutions.

If one of the aforementioned problems occurs, we cannot find the right estimates for MS parameters: in the first case the underlying continuous-in-time model does not exist, and in the second case we have to choose among several models, all of them fitting the observed data.

In Fougere and Kamionka (2003) a method is proposed to check if embeddability or aliasing arise from our data. In particular to estimate the parameters s, M and Q they proposed a Bayesian framework, later re-elaborated in Cipollini et al. (2013), which is based on the Gibbs Sampling algorithm. Shortly, given the starting values M_0, S_0, and defining Z as the random variable describing the number of Stayers in every state, at the *l-th* iteration the algorithm is based on the following steps (more details are in Cipollini et al., 2013):

1. it randomly draws Z_l from a binomial distribution depending on S_{l-1}, M_{l-1};
2. it updates S_{l-1}, M_{l-1} to S_l, M_l randomly drawing from a probability density function which is a sort of multivariate Dirichlet distribution, with parameters depending on Z_l and on the observed starting distribution n_0 and the total observed number n_{ij} of transition from *i* to *j*, for every couple *i* and *j*.

The algorithm allows us to estimate also the standard error for $\hat{S}$, $\hat{M}$, and the matrix $\hat{Q}$. The same procedure permits us to estimate the probability that an underlying continuous model exists (if it is <1 then embeddability may occur) and also the number of existing models (if it is >1 then aliasing occurs). We will see in the following section that neither embeddability nor aliasing affect our estimates.

4. Results

The application of the MS approach to Italian data allows the estimation of model parameters along with the transition matrix. It is therefore possible to compare the results from both the pure Markovian chain and Movers-Stayers models. The latter shows its performance in terms of equilibrium distribution, persistence and mobility measures.

4.1. Estimated parameters

We apply the Gibbs sampling algorithm to our data with 50000 iterations. The convergence is reached by cutting away a burn-in period of 10000 iterations. The fundamental parameters to be shown for the MS models are S and Q, since from Q we can obtain the transition matrix $M(t)$ for any time t. The probability of embeddability results to be 1 for every iteration, as for the number of possible models (Table 3).

Table 3: Estimated generating matrix for the MC model (panel 1) and estimated generating matrix and probability to be a 'Stayer' for the MS model (panel 2)

	A	*B*	*C*	*D*	*E*	*F*	*Default*
	Panel 1: Estimated generating matrix for MC						
A	-0.1853	0.1376	0.0195	0.0213	0.0022	0.0037	0.0010
se	*0.0012*	*0.0012*	*0.0007*	*0.0005*	*0.0002*	*0.0002*	*0.0001*
B	0.2532	-0.5433	0.2776	0.0000	0.0063	0.0031	0.0030
se	*0.0020*	*0.0031*	*0.0024*	*0.0000*	*0.0005*	*0.0004*	*0.0003*
C	0.0081	0.2934	-0.5676	0.2461	0.0080	0.0048	0.0072
se	*0.0009*	*0.0025*	*0.0032*	*0.0022*	*0.0008*	*0.0005*	*0.0004*
D	0.0182	0.0066	0.2893	-0.4541	0.1010	0.0124	0.0267
se	*0.0007*	*0.0012*	*0.0026*	*0.0030*	*0.0018*	*0.0009*	*0.0007*
E	0.0276	0.0360	0.0272	0.6555	-0.9634	0.1525	0.0646
se	*0.0030*	*0.0038*	*0.0061*	*0.0117*	*0.0129*	*0.0072*	*0.0037*
F	0.1762	0.0955	0.1402	0.2190	0.3870	-1.3138	0.2959
se	*0.0113*	*0.0111*	*0.0135*	*0.0179*	*0.0211*	*0.0301*	*0.0131*
Default	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001	0.0000
se	*3.6E-05*	*4.2E-05*	*4.3E-05*	*3.8E-05*	*5.0E-05*	*5.7E-05*	*9.0E-05*
Prob. of embeddability = 1, nr. of possible models (aliasing) = 1							
	Panel 2: Estimated generating matrix and prob. to be a Stayer for MS						
A	-0.2978	0.2223	0.0300	0.0346	0.0034	0.0060	0.0016
se	*0.0023*	*0.0021*	*0.0011*	*0.0008*	*0.0004*	*0.0004*	*0.0002*
B	0.2721	-0.5661	0.2818	0.0000	0.0065	0.0027	0.0030
se	*0.0022*	*0.0033*	*0.0024*	*0.0000*	*0.0005*	*0.0004*	*0.0003*
C	0.0071	0.3015	-0.5828	0.2550	0.0072	0.0048	0.0072
se	*0.0010*	*0.0026*	*0.0033*	*0.0023*	*0.0009*	*0.0005*	*0.0004*
D	0.0203	0.0047	0.3075	-0.4824	0.1093	0.0125	0.0280
se	*0.0008*	*0.0013*	*0.0029*	*0.0034*	*0.0020*	*0.0009*	*0.0008*
E	0.0297	0.0372	0.0209	0.7006	-1.0165	0.1611	0.0670
se	*0.0034*	*0.0041*	*0.0063*	*0.0131*	*0.0141*	*0.0075*	*0.0039*

F	0.1876	0.0855	0.1392	0.2122	0.3971	-1.3175	0.2959
se	*0.0122*	*0.0110*	*0.0138*	*0.0190*	*0.0221*	*0.0305*	*0.0135*
Default	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001	0.0000
se	*3.7E-05*	*4.3E-05*	*4.3E-05*	*4.1E-05*	*4.7E-05*	*6.0E-05*	*9.2E-05*
Stayers	0.2625	0.0115	0.0167	0.0424	0.0318	0.0018	0.5015
se	*0.0036*	*0.0014*	*0.0016*	*0.0027*	*0.0056*	*0.0019*	*0.2884*
Prob. of embeddability = 1, nr. of possible models (aliasing) = 1							

Panel 1 shows the estimated generating matrix for the Markov Chain model. Panel 2 shows the estimated generating matrix and the probability to be a 'Stayer' according to the Movers-Stayers model.

4.2. The estimated annual transition matrix

The second step of the analysis is the estimation of the annual transition matrix given the existence of the continuous-time Markov chain (Table 4, panel 1) and the annual global matrix obtained from the Movers-Stayers model (Table 4, panel 2).

Table 4: Estimated annual transition matrices $\hat{\mathbf{P}}$ (percentages).

	A	*B*	*C*	*D*	*E*	*F*	*Default*
	Panel 1: Estimated annual transition matrix for MC						
A	84.46	10.02	2.89	1.94	0.27	0.22	0.19
B	18.13	61.68	16.67	2.43	0.49	0.22	0.40
C	3.24	17.52	61.28	15.53	1.15	0.35	0.93
D	1.92	3.24	18.23	67.70	5.36	0.91	2.64
E	3.02	3.12	7.22	34.67	40.79	5.22	5.96
F	10.14	6.17	8.93	15.97	13.32	27.92	17.54
Default	0.00	0.00	0.00	0.00	0.00	0.00	100.00
	Panel 2: Estimated annual transition matrix for MS						
A	82.62	11.19	3.22	2.19	0.30	0.25	0.22
B	18.04	61.63	16.62	2.61	0.49	0.22	0.40
C	3.23	17.48	61.33	15.54	1.15	0.35	0.93
D	1.92	3.24	18.19	67.76	5.36	0.91	2.63
E	3.01	3.11	7.21	34.56	40.98	5.20	5.93
F	10.11	6.16	8.92	15.95	13.29	28.06	17.51
Default	0.00	0.00	0.00	0.00	0.00	0.00	100.00

As expected, both credit migration matrices are diagonally dominant implying a relatively large ratings stability over a one-year horizon. However, the diagonal entries are smaller for speculative grade ratings than for investment grade ones, confirming that low ratings are more volatile. Unsurprisingly, the probability of default increases monotonically as credit quality deteriorates. Also in the case of SMEs, our estimates confirm the stylized row monotonicity in rating migrations, as frequently observed for ratings assigned by rating agencies (Nickell et al., 2000, Bangia et al., 2002, Lando and Skodeberg, 2002, and Fuertes and Kalotychou, 2007). However, contrary to the values generally observed in the transition matrices of international rating agencies for listed companies, within both our estimated matrices, with the exception of the F notch the immediate off-diagonal elements are generally larger for upgrades than downgrades. Moreover, both estimated annual transition matrices MC and MS, suggest a non-zero probability to default for “A” firms however this is approximately 0.20 per cent higher than of that commonly found by rating agencies. This result is consistent with the fact that our notch “A” is PD-equivalent to BBB in S&Ps scale where the transition probability to default is substantially similar.

4.3. Comparison between the Markov chain and Movers Stayers models

Using $\hat{Q}$ and $\hat{S}$ we are allowed to estimate the rating distribution for firms over the period 1999-2010. The comparison between estimated and observed distributions (Table 5) allows us to choose the suitable model to adjust the pure Markovian limits. The estimated distribution for MC and MS is respectively given by:

$$
\begin{aligned}
MC &: \hat{d}_t = d_{1999} \cdot \hat{P}(t)_{MC}, \\
MS &: \hat{d}_t = d_{1999} \cdot (\hat{S} - (I - \hat{S}) \cdot \hat{M}(t)_{MS}),
\end{aligned}
\qquad [7]
$$

where $\hat{d}_t$ and d_{1999} are the estimated distribution for any time t and the observed distribution in the year 1999 respectively (which corresponds to the starting distribution, being 1999 the first observed year).

Table 5: Estimated distribution for the MC and MS models (percentages).

	A	*B*	*C*	*D*	*E*	*F*	*Default*
	Panel 1: Estimated distributions for MC						
2000	41.14	19.24	19.00	16.30	2.37	0.82	1.12
2001	39.32	19.97	19.26	16.21	2.38	0.70	2.17
2002	37.91	20.27	19.45	16.15	2.36	0.67	3.19
2003	36.77	20.35	19.57	16.11	2.34	0.65	4.21
2004	35.82	20.30	19.61	16.07	2.33	0.65	5.22
2005	35.01	20.18	19.59	16.03	2.32	0.64	6.23
2006	34.31	20.02	19.53	15.97	2.31	0.64	7.23
2007	33.68	19.83	19.43	15.90	2.30	0.63	8.23
2008	33.11	19.64	19.30	15.82	2.29	0.63	9.22
2009	32.58	19.43	19.16	15.72	2.27	0.62	10.21
2010	32.10	19.23	19.01	15.61	2.26	0.62	11.18
	Panel 2: Estimated distributions for MS						
2000	40.32	19.73	19.14	16.45	2.39	0.84	1.13
2001	37.97	20.67	19.56	16.48	2.41	0.72	2.19
2002	36.19	21.06	19.89	16.52	2.41	0.68	3.24
2003	34.81	21.15	20.11	16.57	2.40	0.67	4.29
2004	33.68	21.10	20.22	16.60	2.40	0.67	5.33
2005	32.75	20.95	20.25	16.62	2.40	0.66	6.37
2006	31.95	20.77	20.21	16.60	2.40	0.66	7.41
2007	31.26	20.56	20.13	16.57	2.39	0.66	8.44
2008	30.64	20.33	20.02	16.50	2.38	0.65	9.47
2009	30.09	20.10	19.88	16.42	2.37	0.65	10.49
2010	29.58	19.87	19.72	16.32	2.36	0.64	11.51

Re-proposing (and re-elaborating) the method introduced in Frydman et al. (1985), we use the estimated distributions to choose between MC and MS. Indeed we are able to evaluate the error affecting our estimates by means of the following formula:

$$err_t = \frac{\left\| \hat{d}_t - d_t^{obs} \right\|}{\left\| d_t^{obs} \right\|} \qquad [8]$$

which represents a sort of percentage error caused by using the estimated distribution at time t ($\hat{d}_t$) instead the observed distribution at the same time ($\hat{d}_t^{obs}$).

Table 6 shows the estimation errors for the transition matrices, computed with the Markov chain assumption and with the Movers-Stayers model.

Table 6: Errors estimated applying the equation [9] for Markov Chain (MC) and Movers-Stayers (MS) models (percentages).

	'00	*'01*	*'02*	*'03*	*'04*	*'05*	*'06*	*'07*	*'08*	*'09*	*'10*
MC	3.6	25.5	18.2	15.4	12.8	10.6	10.4	6.9	12.9	16.2	14.7
MS	4.1	23.1	15.6	12.8	10.4	8.57	8.59	5.5	14.0	16.9	15.2

The mean error is 13.39 for MC and 12.29 for MS. The latter's dominance deteriorates after the beginning of the crisis (from 2007 to 2010), when MS errors are 0.255 larger than MC. This is due to the increased instability of the stayers' structure. Nevertheless, we observe that the error gap was particularly high in 2008 (1.13) and that this reduces progressively when the MS model appears to become more reliable. Since MS, like all the Markov models, suffers when there is a huge shock in credit markets and a turning point in credit cycles given the lack of temporal homogeneity, their application in these cases could deviate from picturing the transition risk as is.

4.4. The equilibrium distribution

When time t tends to infinity, individuals reach the equilibrium distribution among the states, given that there exists the limit for $t \to \infty$ in equation [6]. Table 7 enlightens the implications of the absorbing state, which in the case of MC matrices attracts all the existing firms to a default state.

Table 7: Estimated equilibrium distributions (percentages).

	A	*B*	*C*	*D*	*E*	*F*	*Default*
MC	0.00	0.00	0.00	0.00	0.00	0.00	100.00
MS	11.78	0.52	0.64	0.97	0.11	0.02	85.96

The equilibrium distribution highlights an important drawback of the MC model: since "default" is by definition an absorbing state (at least for companies), estimating transition matrices with the pure MC model, any firm will default sooner or later. On the other hand, the MS model facilitates the appraisal of a cluster of firms (the Stayers) which are unlikely to fail. The same fact is supported by Table 8, containing the last column of the transition matrix for different values of *t*. The distribution of stayers is determined by the credit mapping by rating notch, which is highly concentrated within the A-class. Since Table 8 is directly generated by equation [5] when $t \to \infty$, this means that the real credit state as observed over the period is coherently represented in the long run.

When the equilibrium distribution is run for all the rating classes, with *t* ranging from 1 to 1000 periods (Table 8), we can appreciate how the deterioration process, particularly for the best quality borrowers, is differently designed. The adoption of MC models implies that lenders should be very careful to maintain long-term investments, and mortgages (say between 20 and 50 years) are scarcely "rational", even when supplied to very good firms.

Table 8: Estimated transition probabilities to default. Comparison of MC and MS models with $1 < t < 1000$ (percentages).

	A	*B*	*C*	*D*	*E*	*F*
t	*Panel 1: Default probabilities for MC*					
1	0.19	0.40	0.93	2.63	5.96	17.53
2	0.53	0.96	2.11	5.08	10.30	23.77
3	0.98	1.66	3.40	7.27	13.36	26.65
4	1.53	2.47	4.71	9.20	15.64	28.42
5	2.15	3.34	6.01	10.93	17.47	29.75
10	6.10	8.15	11.91	17.60	23.88	34.54
20	15.33	17.69	21.62	27.07	32.65	41.80
50	39.28	41.03	43.91	47.86	51.85	58.36
100	65.18	66.19	67.84	70.10	72.39	76.12
500	99.59	99.60	99.62	99.65	99.68	99.72
1000	100.00	100.00	100.00	100.00	100.00	100.00
	Panel 2: Default probabilities for MS					
1	0.22	0.40	0.93	2.63	5.93	17.51
2	0.58	0.99	2.13	5.05	10.20	23.79
3	1.05	1.75	3.44	7.20	13.19	26.73

4	1.61	2.63	4.79	9.10	15.43	28.55
5	2.23	3.60	6.13	10.79	17.23	29.94
10	5.94	9.01	12.35	17.44	23.67	35.07
20	13.94	19.80	23.01	27.39	32.99	43.15
50	33.24	45.33	47.36	49.51	53.63	61.47
100	52.61	70.92	71.73	71.63	74.28	79.81
500	73.64	98.70	98.19	95.63	96.68	99.71
1000	73.75	98.85	98.33	95.77	96.81	99.82

4.5. Time Persistence

The analysis is based on the notion of persistence, defined as the random variable describing the time that individuals spend in every state. It is possible to prove that a single company following a pure MC process and being in *i* at time *t*, will leave such state after a random time *T*, which is distributed as an exponential r.v. with parameter $-q_{ii}$ (Grimmett and Stirzaker, 1992).This means that the probability to leave *i* after a time *T*>*s* is given by $exp(q_{ii} \cdot s)$. From the properties of the exponential distribution, the mean persistence time in *i* is given by $-\frac{1}{q_{ii}}$.

The same result can be applied to the group of Movers which follow a classical MC, whereas Stayers have necessarily an infinite persistence time.

Table 9: Mean persistence time for rating notches. Time is measured in years.

	A	*B*	*C*	*D*	*E*	*F*	*Default*
E(T)	3.3581	1.7660	1.7161	2.0738	0.9837	0.7595	Inf

We note that the first notch A has the maximal mean persistence time (3 years and 4 months), joined with the largest amount of Stayers (26.25%, as reported in the last row of Table 3) . This is a good results highlighting the major stability of firms with the highest grade with respect to the others.

4.6. Upgrading probability as function of t

After having analyzed the predicted stability of firms in the rating notches, we aim to evaluate their capacity to improve the current condition through the inspection of the upgrading probability. In other terms, we estimate the probability for firms starting from ratings lower than A to change their credit quality after t years and to become classified in better rating notches (Figure 1).

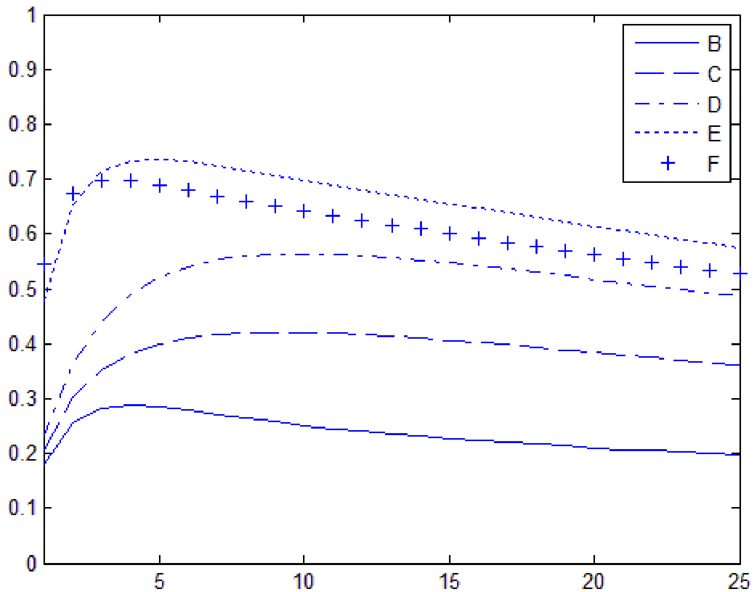

Figure 1: Upgrading probability by time. The vertical axis shows the probability, the horizontal axis shows time in years. The five curves compare the upgrade path probability to a better rating notch.

Unsurprisingly, the upgrading probability is much higher the lower the start rating is. The figure suggests that, starting from rating E or F, the probability of a firm to reach a better state is around 70 and 75 per cent after five years respectively, upon which it tends to be smooth. This is a non-trivial finding. It means that firms have a higher probability to improve their credit quality in the first phases of their life. In particular, companies rated E and F are expected to improve with a high probability (around 70) within their 4-5 first years. Afterwards, the probability declines. This results underlines the issue about the pure Markovian hypothesis, which is adequately solved applying the MS approach.

4.7. Mobility measurements

In the previous sections we showed that the way ratings change over time is not adequately captured by the pure MC model. In order to measure this more fittingly, first the wavering of single firms and, second the prevailing trajectory towards an upgrade or a downgrade rating notch, we introduce mobility indices, defined as functions assigning to every transition matrix P a real value $I(P)$. This addresses the rating dynamics which transition matrices should exhibit. The aim is to obtain a univariate measure allowing us to compare two different matrices. In this sense, the lower is $I(P)$, the lower the mobility of individuals is which move according to P.
We apply two mobility indices: the trace index, proposed in Shorrocks (1978) divided by $\frac{k-1}{k}$ to obtain values in [0,1], defined in equation [9]:

$$I_{tr}(P) = \frac{1}{k}\left(k - \sum_{i=1}^{k} p_{ii}\right) \qquad [9]$$

and the directional index, introduced in Ferretti and Ganugi (2013) as in equation [10]:

$$I_{dir}(P) = \frac{1}{Z}\sum_{i=1}^{k} \omega_i \sum_{j=1}^{k} p_{ij} \cdot sign(j-i) \cdot v(|j-i|) \qquad [10]$$

where $(\omega_1, \dots, \omega_k)$ is a vector of weights to be attributed to the states (labeled with $1,\dots,k$), $sign(x)$ is the sign function, equal to -1 if $x<0$, +1 if $x>0$ and 0 if $x=0$, and ν is a function to measure the

magnitude of the jumps from the i-th to the j-th state, Z is a normalizing constant to have values among -1 and +1. Since the directional index is defined supposing that states are ordered from the worst to the best ones, while ratings are in the opposite order, we have changed the sign of the index.
The trace index and the directional index measure two different features of mobility: the former one assumes values in [0,1] and describes the turbulence of individuals, whereas the latter also measures the prevailing direction towards an upgrade or a downgrade of firm ratings.
Another important feature of both the aforementioned indices is that they can be decomposed as a sum:

$$I(P) = \sum_{i=1}^{k} \omega_i I_i(P) \qquad [11]$$

where $I_i(P)$ measures the mobility of individuals starting from the i-th state (and $\omega_i = 1/k$ for the trace index), providing then additional information about the whole mobility.
In Table 9 we show the 1-year (panel 1) and 10-year (panel 2) transition matrices for both the Movers and the whole set of firms estimated with the MS model. We also exhibit the results of the global mobility indices. Firstly we evaluate the mobility for every state, and the same indices on the whole matrices (setting $(\omega_1, \dots, \omega_k) = d_{1999}$ and $v(|j-i|) = |j-i|$ for the directional index).

Table 10: Mobility indices for Movers (*I(M)*) and for the whole sample (*I(P)*), evaluated by starting rating and for the whole matrices (percentages).

	Panel 1: Mobility indices on the 1-year estimated matrices.			
Starting Notch	*I(M)*		*I(P)*	
	directional	trace	directional	trace
A	-6.32	23.56	-4.66	17.38
B	-1.65	38.82	-1.63	38.37
C	0.68	39.33	0.67	38.68
D	5.35	33.66	5.12	32.24

E	13.76	60.96	13.32	59.03
F	25.98	72.07	25.93	71.94
Default	0.00	0.00	0.00	0.00
total	-2.54	38.34	-1.67	36.80
	Panel 1: Mobility indices on the 10-years estimated matrices.			
Starting Notch	*I(M)*		*I(P)*	
	directional	trace	directional	trace
A	-29.36	72.20	-21.65	53.25
B	-17.42	77.06	-17.21	76.17
C	-3.11	77.56	-3.06	76.28
D	18.43	80.74	17.65	77.32
E	31.89	97.32	30.88	94.24
F	37.23	99.42	37.17	99.25
Default	0.00	0.00	0.00	0.00
total	-16.71	72.04	-12.56	68.07

The mobility of Movers is always higher (in absolute value) than the whole mobility (since Stayers tend to slow down the movements). We also note that the turbulence in the dynamics of firms is quite high, but the global directional mobility tends to be negative. This means that, on average, companies are more likely to be downgraded. This is true not only for A-rated companies, but also for firms that were originally B- and C-rated.

5. Conclusion

Through conducting the research in this paper we reach two relevant conclusions. First, we find that the rating trajectory cannot be estimated with a pure Markov chain without incurring the risk of an absorbing state which stands for a bankruptcy. Therefore, banks are over-estimating their credit risk resulting in excessive regulatory capital. This may have important macroeconomic implications since holding a large capital buffer is costly for banks and this in turn influences their ability to lend. This conclusion is particularly true during economic downturns with the consequence of exacerbating the cyclicality in risk capital which therefore further aggravates economic conditions. These implications are confirmed also when we compared the pure Markov chain with the Mover-Stayer model.

Our analysis shows that not only is the default not an absorbing state, as assumed in the pure Markov chain model, but also that the equilibrium distribution could be more adequately estimated through the MS approach. Our analysis supports the idea that the MS approach, on average, is more efficient because it provides a lower error than the Markov chain method. The results suggest that separate transition matrices should be applied for small and medium enterprises, and that credit risk is statistically and economically overestimated by the pure MC approach relative to the MS one. This implies that, if employed for SMEs, the capital charges prescribed by the Markov chain approach are higher than that drawn from the MS model, suggesting an increase of cyclicality especially during downturns. Promoting the estimation of transition matrices through the MS method should be encouraged, in order to adjust the procyclicality induced by the use of point-in-time ratings.

Second, we show that the immediate off-diagonal elements of the transition matrices for SMEs confirm the ineffectiveness of the naïve transition matrix to estimate credit migration when we refer to small and medium firms. Indeed, upgrades are generally larger than downgrades, suggesting greater capital requirements in a one-year horizon when naïve transition matrices are applied. The mobility indices outcomes show that in the case of SMEs, the trace index increases along with the probability of default, both when we take into consideration only movers and when we add stayers. These findings confirm that asset correlations increase in the worst rating notches (Gabbi and Vozzella, 2013). When designed by regulators, the supervisory formula for concentration risk aimed to combat pro-cyclicality with a negative link between asset correlations and probabilities of default. Our paper shows that this miscalibration could increase the pro-cyclical impact of the use of the wrong transition matrices.

Our findings also explain part of the misevaluation of borrowers and the actual relevant weight of non-performing loans within banking portfolios: the approximation of pure-Markov transition matrices increases the transaction costs and information costs involved with the lending activity. Transaction costs include imperfect foresights, bounded rationality and their economic consequences. Neoclassical approaches and prescriptions cannot be considered as effective as expected by regulators who have designed the “new” regulation in response to the most recent crisis. The Mover-Stayers approach helps to reduce the rough calculation of historical movements of borrowers’ rating and, consequently, the efficacy of their allocative process and the expected industry stability.

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