Fully Funded Social Security and Allocation of Resources: The case for an Environmental Motive in a two countries overlapping Generations Model.

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Abstract

The aims of the model discussed in this paper are twofold. In the …rst strand we show the e¤ect of an environment motive in the saving decision, and its impact on capital accumulation. In the second strand we extend the analysis of the issues by showing the e¤ect of the free movement of factors of production and retired people on capital accumulation, in a two country overlapping generations model.

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1 Introduction

Social Security systems have played an important role in the economic life of European countries. They not only provide security for the elderly, but can also ful…ll a role of automatic stabilizers. They provide a means of income redistribution, as well as impacting upon capital accumulation, and labour supply.

The essential thrust of economic integration in its various forms, from trade liberalization to enhanced labour and capital mobility, can be summarized thus: economic integration, while generally welcomed, usually has adverse consequences for relatively ine¢cient producers. The interaction between social security and economic integration then becomes particularly interesting whenever it induces some gain or losses between countries. The aim of my thesis is to provide a theoretical investigation in relation to speci…c aspects of social security systems, using various overlapping generation models. The main dimensions of my analysis relates to the e¢ciency of public pensions, in particular thir impact upon economic growth in the light of new challenges arising from European economic integration.

In this article I am going to consider a simple model of resource allocation using a two-country overlapping generation model with free migration of workers, and the retired. We assume that the only role played by the government is to provide Social Security by a capital reserve system. The model de…nes households characterized by utility functions with the usual formal properties. One new element introduced will be that agents, in making their saving decisions (for pernsions) will also consider an environmental incentive. We will look at the determinants of migration choices for workers or pensioners, and examine the e¤ects of wages (workers) or di¤erent level of environment (pensioners) on these choices. Di¤erent wage conditions and di¤erent environment levels, between the two countries will impact upon migration. We will produce a model illustratimg the implications of both being with or without trade. Some preliminary results from this model are discussed in the next sections of this paper.

After a literature review on Social Security, conducted in section 2, we will use sections 3, 4 and 5 to present the basic model of our analysis. The aim of this model is to set the basis for future analysis considering a two-county, in…nite overlapping generations model with di¤erent saving preferences. We are going to consider a fully-funded pension system in an overlapping generation

model à la Diamond[1]. As we mentioned before, a new element introduced will be that agents in making their saving decisions may consider an environmental incentive. When they make a saving decision, agents will also consider the value of the environment over their retirement period. The value of the environment in the next period (retirement) is one argument within the consumers' utility function. We assume that the value of the environment is a function of an original exogenous endowment of the environment, which is assumed to have an identical value across periods in present value terms. The qualitiy value of the environment is assumed to be negatively related to the presence of negative externalities and hence positively related to the presence of positive externalities. We also assume that the value of the environment is a negative function of the expected level of capital deployed during the next period. Implicit in this assumption is the view that capital investment undermines the sustainability of the environment. We assume that individuals ignore the impact of individual decision making on the overall level of capital accumulation. They simply maximize their utility function subject to a budget constraint, disregarding the impact on the overall level of capital accumulation. Because the individual can by himself have only a minuscule impact upon the overall level of capital accumulation, it is assumed that such an impact is disregarded when consumers make their decisions. In e¤ect an individual consumers' optimum may lead to a sub-optimal level of capital accumulation at a social level. [Therefore we are assuming that dynamics implicit in learning-by-doing models do not operate.]

In sections 3 and 4 of this paper we are going to show the e¤ect on saving decisions (and also capital accumulation) of the presence of an environmental motive in a closed economy. We will show how saving decisions change if agents consider also the e¤ect of their decisions on the environment when they are retired.

In section 5 we are going to consider resource allocation in a model with two countries. The model is an overlapping generation model with free migration, which is otherwise symmetric except for the assumption of di¤erent endowments of environmental resources between the two countries, and the impact of the di¤erent levels of development in the two countries. We will allow individuals (workers and the retired) and capital, to migrate freely across national borders.

In section 6 we summarise the conclusions of the model.

[1] Diamon (1965)

2 Literature review on Social Security

A social security scheme either may rely on pay-as-you-go principles or take the form of a capital reserve system. With a pay-as-you-go system (PAYG), the younger households' contributions are immediately distributed to the old; with a capital reserve system (CR), a capital stock, which is built up for each young generation, will be consumed when this generation becomes old. Most Western European pension systems are funded on a Pay-as-you-go basis.

When pension provision is introduced, there is often little consideration and knowledge of the e¤ects on macroeconomic variables, and little thought about long term sustainability. After the second world war this PAYG approach to providing for old age was adopted in most Western European Countries.

Nowadays, many economists would raise severe concerns about the introduction of an old-age security system based upon a Pay-As-You-Go approach. The wide interest in economic theory relating to public pension systems in Europe, came as a consequence of three facts:

1. In no other part of the world is provision for old-age so predominantly provided by the state.

2. Most Western European Countries have lower fertility rates than the rest of the world. Without immigration, their populations will already have substantially decreased.

3. The introduction of the Single Market (1992) and the Euro (2002), have increased the degree of economic integration and mobility of factors of production between European Countries.

Following the seminal work of Samuelson (1975), a theoretical literature has grown examining the macroeconomic relationships between social security systems, aggregate savings and the allocation of resources within an overlapping generations framework[2]. An important aspect of the analysis of the social security system involves analysing of the relationship between social security, and dynamic e¢ciency in terms of an optimal capital-labour ratio. This analysis is generally conducted within a simple two-sector, two-period overlapping generations (closed) economy model with emphasis being placed upon the steady state. In the literature, the term e¢ciency has also been related to the "Golden Rule" concept, which broadly de…nes the optimal level of capital for a dynamic economy as that which maximizes the

[2] The theoretical literature is reviewed by Felderer B. (1993)

steady state lifetime utility for each generation.

The theory of ine¢ciency of intergenerational transfers provided by Pay-as-you-go schemes is based on two di¤erent sets of models. The …rst set of models assumes a "small open economy". In this case the economies are so small in relation to the size of the world economy that the market rate of interest is assumed to be given. A number of important papers [P.A. Samuelson (1958), H.Aaron (1966), K.Spremann (1984), St.Homburg (1990)] use these kind of models. The second set of models assumes a "closed economy". The models are based upon a life-cycle theory and assume factor prices are determined endogenously. [P.A. Diamond (1965) P.A. Samuelson (1975), F.Breyer/M.Straub (1991)]

It is interesting to note that both groups of hypotheses lead to the same conclusions in terms of the welfare implications of Pay-as-you-go systems (given that an in…nite sequence of overlapping generations are assumed): Steady state welfare is improved by introducing a system of transfers from younger to older generations, so long as the interest rate is smaller than the growth rate of the population. That is because the rate of return in social security is assumed to be the growth rate of the population, rather than the interest rate as in a capital reserve system.

Samuelson (1975) employs a simple two-sector (private and government) two period overlapping generations model, to characterize the optimal steady state social security programme. In his work he establishes that a less than fully funded state pension scheme can be used to manipulate the level of aggregate saving and hence capital accumulation, in the economy, thereby securing attainment of the Golden Rule capital-labor ratio, where it would otherwise not prevail. This result arises because the introduction of a social security programme reduces the need for individuals to save for their retirement. It can therefore be used to reduce aggregate saving to the Golden Rule level. This Pareto improvement can only be achieved if the initial state is one of dynamic ine¢ciency with too much (and not too little) capital.

The Samuelson (1975) paper has provided the foundations for much subsequent work, with later authors investigating the impact of relaxing various assumptions. Among the more noteworthy of these are:

1. Presence or absence of "altruism" that can motivate intergenerational transfers such as gifts and bequests[3].

[3]Barro (1975), Bernheim at al (1984, 1985), Feldstain (1988), Abel (1987), Kimball (1987) and Weil (1987).

2. Uncertainty with respect to output [4]and length of lifetimes[5].

3. Myopia in the saving decision[6].

4. Inelastic or exogenous supply of labour.

5. Retirement behavior[7].

6. Public choice voting systems [8].

From these works it is possible to extract some themes of particular relevance:

=) The presence of some of these factors, such a bequest, uncertainty and myopia, may themselves justify the introduction of a compulsory social security scheme.

=) Social Security with a Pay as You Go scheme is a substitute for private saving; consequently its availability reduces the need to save for retirement.

=) Social Security reduces the e¤ective cost of retirement, and may therefore encourage earlier retirement. This in turn may induce additional saving to provide for the longer expected periods of retirement.

=) Under the assumption of a single majority voting system, the contributions to the public pension systems are too large, and such a level generally does not converge to the one which maximizes individual utility in the long-run equilibrium.

=) Empirically the quantitative e¤ect of the saving rate is still a controversial point, because of the impact of several variables.

=) The introduction of uncertainty seems to con…rm the ine¢ciency of a pay-as-you-go scheme. However, research on this type of model is still being carried out.

=) There is a concentrated discussion on the programs of reform in Western Europe. Due to demographic factor, all the Western European countries are facing the prospect of higher and higher contribution rates to retain the same level of pension-income[9]. The possible increase in women's employment will o¤er only partial relief. Simulation exercises indicate that in addition to a rise in the retirement age, further measures will be necessary.

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[4]Sheshinski and Weiss(1981) consider uncertainty with respect to mortality.

[5]W. Richter and W. Enders (1982), and H.E Lapan. As a staring point in both papers it is assumed that there is uncertainty with respect to labour productivity.

[6]Feldestain (1985)

[7]Danziger (1981), Aaron (1982), Lazear (1986), Hu(1979)

[8]Browning (1975), P.G.C Townley (1981)

[9]See European Commission 2001.

3 The basic model

We consider an overlapping-generations economy in which individuals live for two periods, so that at any point in time, the economy is composed of two cohorts, or generations, the young (Y) and the old (O). At any one time individuals of di¤erent generations are alive and may engage in trading with one another, each generation trades with di¤erent generations in di¤erent periods of their life. Individuals work only when young , he/she can transfer consumption between periods t, and t + 1, by saving an amount s_t,and receiving an interest rate r_{t+1}. The economy has a …xed endownment of enviromental resources $\overline{e}$. The only role played by the government is to provide Social Security by a capital reserve system in which in period t the government raises d_t in contributions from the young, and invests the contributions in capital and returns in period t + 1, to the old generation. The savings of the young in period t and the pension contributions collected by the government in period t generate the capital stock that is used to produce output in period t + 1, in combination with the labor supplied by the younger generation during period t + 1.

3.1 The Decentralized Equilibrium

The economy is composed of individuals, government and …rms. We assume that the total population of the economy P_t, grows at constant relative rate n and is composed of L_t young workers and R_t retired people. Each person born at t lives for two periods and is capable of providing one unit of labour in the …rst period L_t = (1 + n)R_t. Let the life time utility $U(t)$ of a representative individual born at t be dependent upon his consumption in both periods, and the value of the environmental variable in the second period:

$$U_t = U[c_t^Y, c_{t+1}^O, e_{t+1}] \tag{1}$$

where c_t^Y denotes his/her consumption when he/she is young, $c_{t+1}^O$is consumption when retired , e_{t+1} is the value at time t of the environment function at time t + 1. It is assumed that the life time utility function is increasing and quasi concave with respect to all its arguments.

$$U'(¢) > 0, U''(¢) < 0$$

3.2 The Environment function

In our model the environment variable, broadly interpreted, refers to the standard of living that a country can o¤er to retired people i.e. natural resources. The key idea in the model is that elderly people consider this environment variable to be a substitute for consumption over the retirement time. The higher the level of this environment variable in a speci…c country, smaller is the need for saving when people are young. The environment variable enters into the overall utility function (U_t). In making a consumption decision consumers also consider the value of e_{t+1} in the next period i.e. when they are old. We assume in (2) that the value of the environment, is a function of the environmental endowment ($\bar{e}$) i.e. environmental capital stock, minus the sum of the congestion coe¢cient (ϖ) multiplied by the term in brackets adjacent to ϖ that represents the value of the congestion in period $t+1$. The numerator of this expression contains the term R_{t+1} (retired people) plus M_{t+1} which relates to migration of retired people in the country. Whilst the denominator contains only the variable R_{t+1}. The idea about the congestion coe¢cient is that the higher the number of retired people resident in one country the smaller is the value of the environment variable for a single retired person (i.e. a congestion e¤ect). We can think here of a attractive country with a lot of sun and natural resources and perhaps an asthetically pleasing environment. The presence of many residents in that country can reduce the value of its natural resources.

The …nal expression ($\beta(k_t)$) involves the coe¢cient β multiplied by the expected value of (k_t), which is the ratio of capital to labour. We assume that a capital intensive production produces more pollution that a labour intensive one. The more capital intensive is production, the more the productive sector in the economy is environmental unfriendly. The pollution produced by highly capital intensive production, reduces the value of the environment variable by the coe¢cient β.

$$e_{t+1} = \bar{e} - \varpi \left[\frac{R_{t+1} + M_{t+1}}{R_{t+1}} \right] - \beta(k_t) \qquad (2)$$

Note that the value for a single individual of the enviroment function at time $t+1$ is a function of the capital at time t. That is because we assume that young individuals observe the level of the capital-labour ratio at period t, and they ignore the impact of the individuals decision making, upon the overall level capital accumulation. Thus for single individuals:

$$\frac{dk_{t+1}}{ds_t} = 0 \tag{3}$$

In doing so young individual will think that $k_{t+1} = k_t$, and they will evaluate the value of e_{t+1} as a function of the observed value of the capital k_t.

3.3 The Role of the Government

Without further complications we assume that the only role played by the government is to provide Social Security by a capital reserve system which in period t raises d_t in contributions from the young, invests the contributions as capital, and pays back b_{t+1} to the same generation when they are old.

$$b_{t+1} = (1 + r_{t+1})d_t \tag{4}$$

The returns to the contributions to public pensions are the same as returns to private savings. The role of a Social Security system of this kind can be interpreted as involving compulsory savings imposed by the government on the young i.e. to prevent people living miserably in old age because they do not realize when they are young the future consequences of their private saving habits. The government using public pension contributions faciltates capital formation for the next period. We assume that the government does not consider the impact upon the environment variable given in (2) by his saving decision.

3.4 The Household Sector

At this stage in order to develop intuition from the model, we need to specify a utility function that will allow us to obtain full and analytical results. We assume that the typical household has a log-linear utility function over consumption level (c_t^Y, c_{t+1}^O) and over the level of the environment (e_{t+1}).

$$U = \ln(c_t^Y) + \alpha \ln[c_{t+1}^O + e_{t+1}] \tag{5}$$

The lifetime allocation problem for the typical household is that of choosing $c_1$and c_2 subject to the lifetime income constraint (6) and the value of the environmental function (7):

$$\begin{aligned} c_t^Y &= w_t - s_t - b_t \\ c_{t+1}^O &= (s_t + b_t)(1 + r_{t+1}) \end{aligned} \tag{6}$$

$$e_{t+1} = \overline{e} - \varpi \left(\frac{R_{t+1} - M_{t+1}}{R_{t+1}} \right) - \beta(k_t) \tag{7}$$

where w_t is the wage received in period t and r_{t+1} is the interest rate paid on private savings (s_t) and public pension contribution from period t to period $t + 1$. Note that having been determined in the period $t - 1$ the value of k_t is exogenous in the period t. The value of the environment variable e_{t+1} depends on the value of k_t (exogenous within the period t), because as we mentioned before a single individual will ignore the impact of individual decision making upon the overall level of capital accumulation. By maximizing (5) subject to (6) and (7), we have :

$$\frac{1 + r_{t+1}}{\alpha} = \frac{c_{t+1}^O + e_{t+1}}{c_t^Y} \tag{8}$$

Equation (8) asserts that the marginal rate of substitution of present consumption for future consumption, plus the environmental value, must equal the interest rates discounted by the time preference parameter α.

3.5 The Production Sector

When refering to the production sector, in order to develop some intuition we use a speci…c production function that allows us to obtain a full and analytical solution. We consider a competitive economy in which …rms act competitively and assume log-linear constant returns to technology:

$$Y_t = L_t a \ln(k_t + 1), \ a > 0 \tag{9}$$

where Y_t is the level of output in period t, L_t is the labour input (the number of young people in period t.), $k_t \acute{} K_t/L_t$ the capital-labour ratio and a is an exogenous technology coe¢cient. We assume that (5) is a net production function, with depreciation already accounted for. Output for worker $y_t = Y_t/L_t$, and is given by the production function[10]:

$$y_t = a \ln(k_t + 1) \tag{10}$$

Firms maximize pro…ts, taking wage rate w_t, and the rental rate on capital, r_t as given. They act competitively, hiring labour from the young cohorts, and renting capital from the government or the private sector. Hence, the equilibrium wage rate and the rate of return on capital, are equal to the respective marginal products of labour and capital:

$$\begin{aligned} r_t &= f'(k_t) = \frac{a}{k_t + 1} \\ w_t &= f(k_t) - k_t f'(k_t) = a \ln(k_t + 1) - \frac{k_t}{k_t + 1} \end{aligned} \tag{11}$$

It is helpful to note that y, k, r and w are uniquely and monotonically related by the production function (10) and the marginal productivity conditions (11). The market clearing condition will require that total investment be equal to total savings including the public pension contribution, so:

$$K_{t+1} - K_t = L_t(s_t + d_t) - K_t$$

Where the left-hand side is the net investment and the right-hand side is the net savings (saving of the young - dissaving of the old). Eliminating K_t from both sides and with $L_{t+1} = (1 + n)L_t$ we can determine the capital accumulation equation in terms of the capital-labour ratio:

$$k_{t+1} = \frac{(s_t + d_t)}{(1 + n)} \tag{12}$$

From (12) we can observe that the capital-labour ratio in period $t + 1$ is determined by public and private savings in period t. Equation (12) represents the solution to the model in terms of dynamic behavior of the capital

[10] Note that for $k_t > 0$, the production function respect the usual concavity assumption: $f'(k_t) = \frac{a}{k_t+1} > 0$ and $f''(k_t) = -(\frac{a}{(k_t+1)^2}) < 0$.

stock k_t. In the next section we will derive from (12) the capital accumulation equation $k_{t+1} = f(k_t)$ for a closed economy. The capital accumulation equation is the relationship between k_{t+1} and k_t that describes the path of the capital stock. In the second stage we will analyze the effect of the environment variable on capital accumulation. Finally in order to discuss a full and analytical solution, we will present some numerical evaluation of the capital accumulation equation.

4 Dynamics and Steady State in a close economy

We define as a closed economy an economy where there is no trade between the rest of the world and that economy. The capital stock is generated internally and no capital flows are allowed to enter or exit the economy. Also the labour force is generated internally and there is no migration of workers or retired people between countries. This will imply $M_t = 0$ all the time so we can simplify (7) as:

$$e_{t+1} = \bar{e} - \varpi - \beta(k_t) \tag{7'}$$

Note that the value of k_t is exogenous within the period because it has been predetermined by the savings in period $t - 1$. This implies also that the value of the enviroment variable e_{t+1} is exogenous within the period.

4.1 Saving decisions

The lifetime allocation problem for the young person in a closed economy will be to choose the total amount of saving s_t, so as to maximize the lifetime utility function (5) subject to the income constraint (6), the government budget constraint (4), and the value on the environment given by (7'). The savings as a function of current income is:

$$s_t = \frac{\alpha}{1+\alpha} w_t - \frac{\bar{e} - \varpi - \beta k_t}{(1+r_{t+1})(1+\alpha)} - d_t \tag{13}$$

$1 > s^{0}_{w_t} > 0,\ s^{0}_{r_{t+1}}$ **7** 0(for $\hat{e}^{1}$ **7** $\varpi + \beta k_t$), $s^{0}_{\hat{e}} < 0,\ s^{0}_{d_t}$ = ¡ 1, $s^{0}_{k} > 0, s^{0}_{\varpi} > 0$

Using (13) we can easily see that s_t is increasing with respect to the current wage, and with respect to the depletion of the environment given by βk_t and ϖ. It is also decreasing with respect to the environmental endowment. Note that in the case of a fully funded pension system private savings are a perfect substitute for the contribution d_t. Any increase in social security saving, d_t, is exactly o¤set by a decrease in private savings in such a way that the total $s_t + d_t$ is constant.

$$s_t + d_t = \frac{\alpha}{1+\alpha} w_t \text{ ¡ } \frac{\hat{e}^{1} \text{ ¡ } \varpi \text{ ¡ } \beta k_t}{(1 + r_{t+1})(1+\alpha)} \tag{14}$$

The sum $s_t + d_t$ represents the total savings of the private agents. It is increasing to note that if the government imposes a public pension contribution d_t **1** $\frac{\alpha}{1+\alpha} w_t$ ¡ $\frac{\hat{e} \text{ ¡ } \varpi \text{ ¡ } \beta k_t}{(1+r_{t+1})(1+\alpha)}$, equation (14) does not hold and s_t = 0[11]. The economic intuition behind this is straightforward: social security contributions provide a rate of return equal to the private saving ones; this means that the consumer is, therefore, indi¤erent to who does the saving, they care only about the rate of return; any change in the public pension contributions will be fully balanced out by an equivalent increase/decrease in private savings. If the government imposes a public pension contribution d_t above the optimal level de…ned in (14), than agents will avoid any kind of private saving. In our analysis we assume that governments set each period $d_t < \frac{\alpha}{1+\alpha} w_t$ ¡ $\frac{\hat{e} \text{ ¡ } \varpi \text{ ¡ } \beta k_t}{(1+r_{t+1})(1+\alpha)}$. This means that social security contributions do not exceed the amount of savings that the economy would otherwise have incurred.

4.2 Capital accumulation

We assume the supply of capital in period t is determined by the sum of total saving decisions of the young, and the government in period t ¡ 1. We also assume labour is supplied inelastically. The equilibrium in the capital/labour markets is obtained when wage and interest rates are such that …rms use the

[11] That is the unusual case where only government provide public saving. The capital accumulation equation in this case is simply determinate exogenously by the government. $k_{t+1} = \frac{(d_t)}{(1+n)}$.

total amount of capital and labour disposable on the markets. The factor market equilibrium is therefore given by equation (11). If we combine this equilibrium together with the capital accumulation equation (12), and the optimal saving decision equation of (13) we can derive the implicit function of the dynamic behavior of capital-labour ratio for a closed economy:

$$k_{t+1} = \frac{\alpha}{(1+\alpha)(1+n)} a \left[\ln(k_t + 1) - \frac{k_t}{k_t + 1} \right] - \frac{\left[\bar{e} - \varpi - \beta k_t \right]}{1 + \frac{a}{k_{t+1}} \quad (1+\alpha)(1+n)} \tag{15}$$

Equation (15) states the relationship between k_{t+1} and k_t that de…nes our curve of capital accumulation for the economy. One important property of this function depends on the derivative:

$$\frac{dk_{t+1}}{dk_t} = \frac{\frac{\alpha k_t a}{(k_t+1)^2}\left[(1 + \frac{a}{k_{t+1}})^2 + \beta(1 + \frac{a}{k_{t+1}})\right]}{(1+\alpha)(1+n)(1 + \frac{a}{k_{t+1}})^2 + (\bar{e} - \varpi - \beta k_t)(\frac{a}{k_{t+1}^2})} \tag{16}$$

The numerator of equation (16) is positive for every $k_t > 0$; it is possible to show also that the denominator is positive even when $\bar{e} < \varpi + \beta k_t$. This means that for $k_t > 0$, $\frac{dk_{t+1}}{dk_t} > 0$ so the capital accumulation equation given by (15) is always increasing. At any point at which the capital accumulation (15) crosses the 45-degree line at which $k_{t+1} = k_t$ there are steady state points. To de…ne the nature of the steady state (stable/unstable) we need to evaluate (16) at the point of steady state; for $\left| \frac{dk_{t+1}}{dk_t} \right|_{k_{t+1} = \bar{k}} < 1$, we will have a stable steady state solution otherwise the steady state will be unstable.

Since analytical results in terms of stability of the steady state are hard to obtain given the complexity of (15) and (16), the future analysis will be carried out using numerical evaluation.

4.3 A numerical evaluation

Since our model is a very simpli…ed it is not our aim to provide realistic quantitative estimates; but to derive more realistic results from the overlapping generations model, we need to adopt empirically signi…cant parameter estimates for the simulation analysis.

4.3.1 The basic case

Using the famous empirical work of Romer, Mankiw and Weil (1992) we set the technology coe¢cient of our production function (10) to a = 350[12] . We assume a population growth n = 0.025[13]. We also assume the rate of intertemporal substitution α = 0.95. Setting $\overline{e}$ = 0, ϖ = 0, β = 0 (14) we get:

$$k_{t+1} = \frac{\alpha}{(1+\alpha)(1+n)} a \left[\ln(k_t + 1) - \frac{k_t}{k_t + 1} \right] \qquad (14')$$

equation (14') represents the dynamic solution for the basic model. In this simple case since the environment variable is e_{t+1} = 0 the environment enters with a zero value in the utility function (5). The numerical evaluation of (14') brings us also to a unique equilibrium in steady state with a positive capital stock. This solution is represented graphically in …gure 1 where the ss represents the 45 degree steady state line. The path to the steady state level depends upon the initial level of k_o. The economy starts out at k_o and gradually moves towards the steady state capital stock. For example in the case of k_0 in …gure 1, the per-capita level of capital will gradually increase to k^{α}, the opposite applies to a starting point like $k_0^{'}$.

[12] In their work Romer, Mankiw and Weil using a sample of 22 OECD over the period 1960-80 estimated a value of the logoutput per working age income of ln(y) = 7.8. In our model a = 350 is the value that allows our production function (10) to reach a steady state at the same per capita income level.

[13] That is the value estimate in terms of Birth rate-Death rate by the Population Reference Bureau (http://www.prb.org) during the last 20 years in the more developed counties.

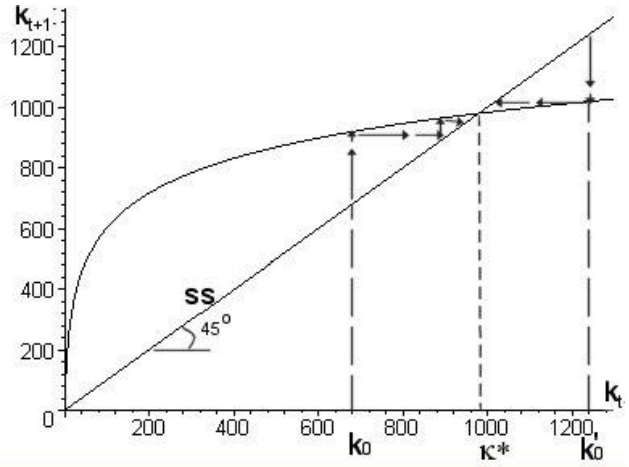

Figure 1 Dynamic adjustment for the simplest case $\overline{e}$=0,ϖ=0,β=0

We next want to examine how the level of the steady state changes if we specify speci…c values for $\overline{e}$, and β.

4.3.2 The environmental endowment $\overline{e}$

In …gure 2 we show the indications of (14) with three positive levels of endowment of the environment $\overline{e}$, and keeping $\varpi = 0, \beta = 0$. Note that in this case the value of the environment function ($e_{t+1} = \overline{e} - \varpi - \beta(k_t)$) is exogenous in the model and is simple given by $e_{t+1} = \overline{e}$. Even this numerical solution gives us a unique stable path for the capital accumulation equation (14), with a positive level of capital in steady state. These di¤erent levels are represented by the dotted lines. The continuous line is the same one examined in section 4.3.1 with $\overline{e} = 0$, $\varpi = 0$, $\beta = 0$. It is immediately possible to observe that the larger is the level of the endowment level of environment taken in account in the saving decision, the lower is the level of the Steady State for the economy. The economic intuition behind this is that: the environmental resources are a substitute for the savings of retired people. The higher is the environmental level, the lower is the incentive for each younger generation to save. This obviously is re‡ected in a lower level of capital in Steady State.

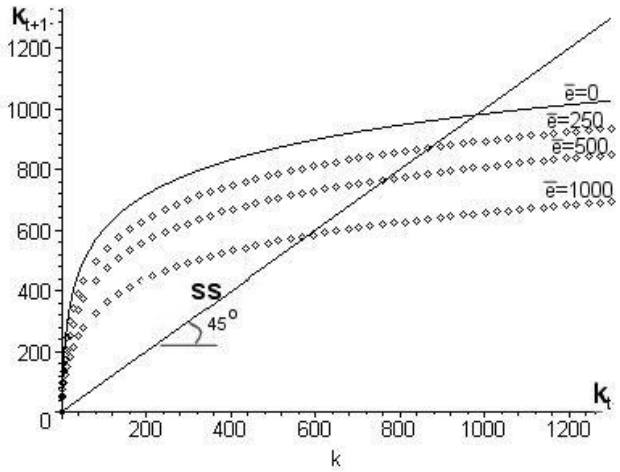

Figure 2 Dynamic adjustment for the cases $\bar{e}$=0, $\bar{e}$=250, $\bar{e}$=500, $\bar{e}$=1000

4.3.3 The environment depreciation coefficient β

In figure 3 we repeat the same experiment conducted for $\bar{e}$ testing the effect of positive level of β on the steady state solution. Even in this experiment we find that a positive level of β does not compromise the stability of the capital accumulation equation. We tested three different levels of β and we observed that the higher is the level of the environmental depreciation coefficient, then the higher is the level of capital in steady state.

The economic reason for this arises because of coordination failure arising when the younger generation make their own saving decisions. As we have discussed in section 3.2, agents evaluate the variable e_{t+1} as a function of k_t because they think that the level of k_{t+1} will not be affected by their own saving decision. Also the government with a pension system fully funded and with $d_t < \frac{\alpha}{1+\alpha} w_t - \frac{\bar{e} + \varpi + \beta k_t}{(1+r_{t+1})(1+\alpha)}$ is not able to affect the overall saving $(d_t + s_t)$[14].

If agents evaluate the variable e_{t+1} as a function of k_t, the value of e_{t+1} is exogenous with respect to the period t ($e_{t+1} = \bar{e} - \varpi - \beta(k_t)$). We need to remember from (12) that the value of the capital k_t is determined by total savings in period $t - 1$. This implies that k_t is exogenous with respect to the period t.

Agents think that they cannot affect the overall level of capital in the next period, and consequently the value of the environmental variable (7').

[14] See section 4.1

Therfore to compensate for the loss arising from the lower level of the environment, the typical agents will save more when young. In doing so they will compensate for the loss in the environment with more consumption when old. These factors will obviously result in a higher level of Steady State capital as shown in …gure 3.

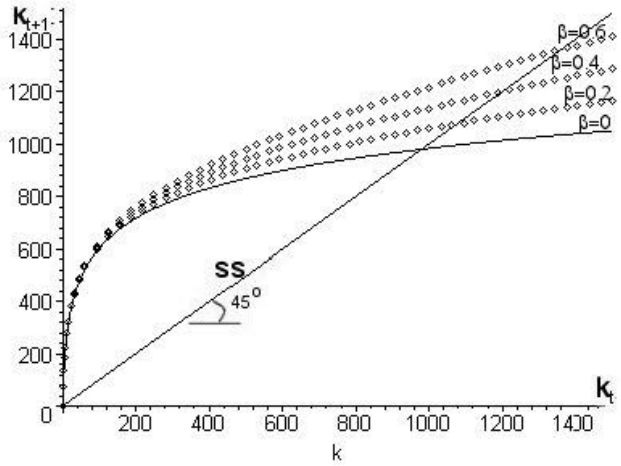

Figure 3 Dynamic adjustment for the cases β=0, β=250, β=500, β=1000

4.3.4 Some preliminary results

This section of the article has extended the analysis of macroeconomic allocation of resources in a dynamic economy introducing an environmental motive into the utility function. We can draw some stylized results from the case observed.

=) The introduction of an environmental motive can shift down the capital accumulation curve bringing about a lower level of capital stock in Steady State.

=) Two countries with access to the same technology and same utility functions, but with di¤erent levels of endowment of environment ($\bar{e}$) have two di¤erent capital accumulation curves, and two di¤erent Steady States.

=) The fact that individuals discount any impact that they individually can have upon capital accumulation can result in a higher level of overall saving. This is in order to compensate for the loss in utility arising from lower environmental levels.

=) If we assume that agents are able to predict correctly the level of capital in the next period, the …ndings in 3.3.3 can be reversed.[15]

[15] Due to space constraints we do not discuss this result. If we assume $e_{t+1} = \bar{e}$ ¡

5 A two country global-equilibrium model

In this Section we consider resource allocation in a two-country [Home and Foreign (*)] overlapping generations model with free mobility of factors, and free migration of labour and retired people[16].

5.1 The two economies

We assume that these two countries at time t = 0 form a Community comprising only of two countries. The community as a whole is characterized in the following way: First, we have free trading of goods and services. Second, when integration takes place the two economies converge to a fully integrated capital market with a unique interest rate r_t and unique wage rate w_t. Thirdly, householders can freely migrate within the community.

Each country has access to the same production technology given in equation (10) but may be at a di¤erent stage of development. This means that at the time of the integration, the two countries can have two di¤erent levels of capital stock.

$$k_0 \neq k_0^{\ast} \tag{17}$$

We also assume that residents of the two countries have identical logarithmic preferences given by eq (5), and identical population growth rates $n = n^{\ast}$. Regarding the environment function we assume that the countries have the same environment function, but di¤erent endownments of environment. For this speci…c case we will assume that the foreign country has a higher endownment.

$$\bar{e}^{\ast} < \bar{e} \quad \varpi^{\ast} = \varpi \ \beta^{\ast} = \beta \tag{18}$$

If there were no integration between the two countries we would have two di¤erent capital accumulation curves, and also two di¤erent levels of capital stock in steady state, $k_{SS} > k_{SS}^{\ast}$[17]. This is shown in Figure 5.

$\varpi \equiv \frac{R_{t+1} - M_{t+1}}{R_{t+1}}$ ¡ $\beta(k_{t+1})$ it is possible to show that for higher levels of β the capital accumulation curve is shifted down to lower a level of capital stock in Steady State.

[16] See Obstfeld and Rogo¤ (1998) chapter 3.

[17] This is the case discussed in section 3.3.2 where we show that the higher the endowment $\bar{e}$ the lower the Steady State k_{SS}.

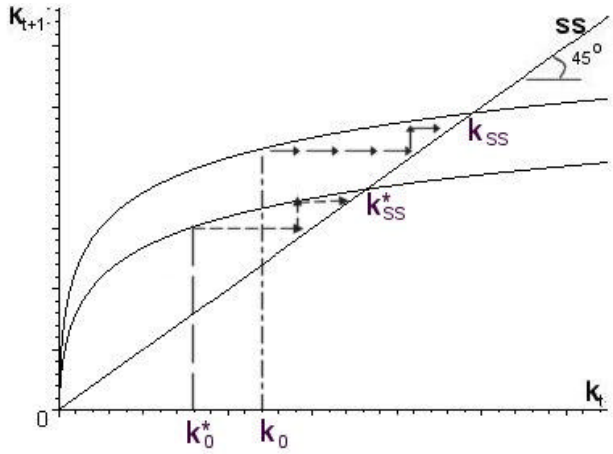

Figure 5 Dynamic adjustment for the two counties without integration

5.2 The world competitive equilibrium

In this world comprising of two countries [Home and Foreign (*)], a competitive equilibrium requires that each household resides in one of the two countries, that markets clear, no household can make itself better o¤ by selecting a di¤erent consumption bundle, or by locating in another jurisdiction, and no …rms can make higher pro…ts.

5.2.1 Capital and labour market

With the Home and Foreign markets integrated by a world capital and labour market, if labour and capital move in opposite directions this will equate at time $t = 0$ the interest rate ($r_1^W = r_1^¤ = r_1$) and the wage ($w_0^W = w_0^¤ = w_0$) in both countries, so that $k_0^W = k_0 = k_0^¤$.

$$r_1^W = r_1^¤ = r_1 = \frac{a}{k_0^W + 1} = \frac{a}{k_0 + 1} = \frac{a}{k_0^¤ + 1} \tag{19}$$

$$w_0^W = w_0^¤ = w_0 = a\left[\ln(k_0^W + 1) - \frac{k_0^W}{k_0^W + 1}\right] = a\left[\ln(k_0^¤ + 1) - \frac{k_0^¤}{k_0^¤ + 1}\right] \tag{20}$$
$$= a\left[\ln(k_0 + 1) - \frac{k_0}{k_0 + 1}\right]$$

k_0^W is the unique common capital-labour ratio across the two economies after the integration has taken place. This implies identical production technologies, and is given by the endowment of the labour force and capital in the two economies:

$$k_0^W = k_0 = k_0^* = \frac{K_0 + K_0^*}{L_0 + L_0^*} \tag{21}$$

5.2.2 The environment function

In the context of open economies retired people can migrate across national borders looking for better living conditions. We assume that governments allow retired people to collect their full pension even if they move abroad. According to our model economic integration implies that young agents will also consider the possibility of moving abroad for the retirement period. This possibility is implied in equation (2) when $M \neq 0$. If integration takes place in period 0 and $\bar{e}^* > \bar{e}$ a competitive equilibrium requires that in period 1 we observe migration of retired people from home to abroad up to the point where:

$$e_1^* = e_1 = e_1^W = \bar{e} - \varpi \left(\frac{R_1 - M_1}{R_1} \right) - \beta(k_0^W) = \bar{e}^* - \varpi \left(\frac{R_1^* + M_1}{R_1^*} \right) - \beta(k_0^W) \tag{22}$$

Solving (21) with respect to M_1 we can calculate the value of the migration flow between the two countries:

$$M_1 = \frac{\bar{e}^* - \bar{e}}{R_1^* + R_1} \frac{R_1 R_1^*}{\varpi} \tag{23}$$

Substituting (22) in (21) we can calculate the value of the global environmental function that is given by:

$$e_1^W = \bar{e}^* \frac{R_t^*}{R_1 + R_t^*} + \bar{e} \frac{R_t}{R_t + R_t^*} - \varpi - \beta(k_0^W) \tag{24}$$

Note that in the new world equilibrium the congestion coefficient ϖ does not change. The new world equilibrium is a function of the population size and the endownment of the environment of the two economies. At this moment we can define the endowment of environment of the world economy such that:

$$\overline{e}^W = \overline{e}^{¤}\frac{R_t^{¤}}{R_1 + R_t^{¤}} + \overline{e}\frac{R_t}{R_t + R_t^{¤}} \quad (25)$$

for $\overline{e}^{¤} > \overline{e}$ =) $\overline{e}^{¤} > \overline{e}^W > \overline{e}$

We assume now that at time 0, when economic integration takes place, young people will have perfect expectations about future migration of retired people, and will make saving decisions considering the new world level of environment given in (23). At this point it is possible to de…ne the World Capital accumulation function, that is represented in …gure 6. Since $\overline{e}^{¤} > \overline{e}^W > \overline{e}$ the new accumulation function lies between the home and the overseas function.

5.2.3 The Dynamics of equilibrium and Steady State in the global economy

We are now in a position to characterize world equilibrium. In period 0 when the integration takes place movement of capital from home to abroad and/or an opposite movement of labour will take place in order to balance the capital-labour ratio according to equation (21)[18], that is represented in Figure 6. The home economy will move from point C up to B, and the overseas one will move from A to B. Note that the bigger the gap between the two economies in terms of the capital-labor ratio, the bigger the jump that the two economies will make in period 0 to reach the world capital accumulation curve. After period 0, the two economies will move across the common World Capital accumulation curve up to the new Steady State k_{SS}^W.

The world capital accumulation equation will be given by:

$$k_{t+1}^W = \frac{\alpha}{(1+\alpha)(1+n)} a \left[\ln(k_t^W + 1) - \frac{k_t^W}{k_t^W + 1} \right] - \frac{\overline{e}^{¤}\frac{R_t^{¤}}{R_1 + R_t^{¤}} + \overline{e}\frac{R_t}{R_t + R_t^{¤}} - \varpi - \beta k_t^W}{1 + \frac{a}{k_{t+1}^W}(1+\alpha)(1+n)} \quad (26)$$

It is interesting to note that the new equilibrium in terms of the capital-labour ratio will be smaller for the home economy and bigger for the foreign one. Note also that the world equilibrium k_{SS}^W does not depend upon the starting points of the two economies. We would have the same result even

[18] No assumptions are made to specify how much capital, and how much labour needs to move in order to reach the equilibrium.

if $k_0 < k_0^{¤}$, but it would depend upon the endowment of the environment $\dot{e}$ that a¤ects the saving decisions of the agents. The path of the capital-labour ratio is also represented in the time-capital axis in …gure 7.

One criticism that can be made of this kind of solution is that at the time of the integration (t = 0) the model implies a big shock in terms of the capital stock for the two economies. Many people may think that even with full mobility of capital and labour, it is unrealistic in the short run to have that kind of shock. Actually the basic idea that we need to keep in mind is that in a overlapping generation fashion, each period corresponds to the half life of an individual, so one period in our model corresponds to many years in the life of individuals.

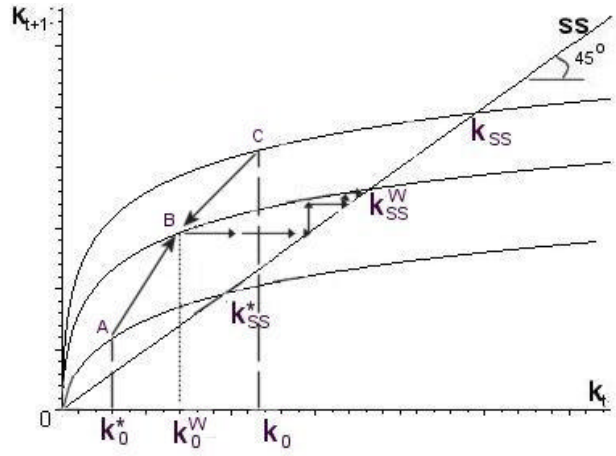

Figure 6 Dynamic and Steady State of the world capital-ratio.

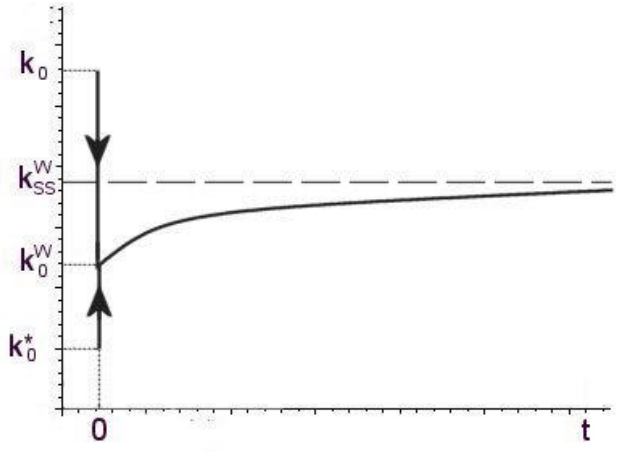

Figure 7 Dynamic path of the world capital-ratio.

6 Conclusions

In this article we have analyzed the allocation e¤ects of an environmental motive in a two country overlapping generation model. Our principal …ndings have been the following:

1) Using a logarithmic production function and a log linear utility function we …nd a unique stable path for the capital accumulation equation. Given the shape of the capital accumulation curve, we also have a unique and stable equilibrium for positive levels of the capital-labour ratio in Steady State.

2) An environmental motive in the saving decision shifts the capital accumulation curve, and also the steady state capital level down.

3) If young people think that they are no longer able to a¤ect the capital level in the next period, they will save more in order to balance with consumption the loss in environment when they are old. This will raise the Steady State capital level. If the young are conscious of the e¤ect of their own saving decisions on capital, then the previous result can be reversed.

4) In a two-country integrated model di¤erent endownments of environmental resources will activate ‡ows of migration between the two countries such that the value of the environment function is the same in both countries. This phenomenon will also modify the capital accumulation curve (and the Steady State), for both countries.

5) The bigger the di¤erence in terms of the stage of development between the two countries that are going to integrate, the higher the shock that economic integration will cause to the economies.

We now want to point out in an informal manner some generalizations as well as limitations of our analysis. In the above idealized model, households respond quickly to wage di¤erentials and immediately move to the country o¤ering the highest salary. A natural objection to this would be that in reality many people are immobile, i.e. they will stay at their original location irrespective of wage di¤erentials. We admit, of course that a Spanish worker is unlikely to move to Germany at once if he …nds wages to be in…nitesimally higher there[19]. Yet, as the 21st century will probably become a "century of migration", we also feel that this point should not be pushed too far. For our results to hold qualitatively, it su¢ces if there are always some people who consider moving location due to higher wages. It seems to be clear that the

[19] This is not least because there are transaction costs associated with moving to another location.

development of the Common Market, the single currency, and the formation of a political union, will make EC residents even more mobile.

References

[1] Aaron, H. J. (1996) "The social Insurance Paradox" Canadian Journal of Economics and Political Science 32

[2] Aaron, H. J. (1982) "Economic e¤ects of social security" Washington

[3] Abel A. (1987) "Operative gift and bequest motives" NBER Working Paper n.2331 Cambridge MA.

[4] Altig D. and S.J. Davis (1991)"Borrowing constraints and two sided altruism with an application to social security" NBER Working Paper n.3913 Cambridge MA.

[5] Aylott G. (1996), "Social security the Golden Rule and the optimal allocation of resources: The case of endogenous retirement and a strategic bequest motive", Warwick University.

[6] Barro, (1974), "Are Government Bonds Net Wealth", Journal of Political Economy 82 1095-1117.

[7] Bernheim D., A.Shleifer and L.Summers (1984) "Bequest as a means of payment", NBER Working Paper n. 1303 Cambridge MA.

[8] Bertola G. Dolado J., Bentolila S. Bean C "Social Europe: One for All?" CEPR Monitoring European Integration 8.

[9] Bernheim D., A.Shleifer and L.Summers (1985) "The strategic bequest motive". Journal of Public Economics 93: 1045-76

[10] Blanchard O.J and S. Fischer (1989), Lectures on Macroeconomics. Cambridge Mass MIT Press.

[11] Bloom D.E and R.B Freeman (1992) "The fall of Private Pension Coverage in US (1979) American Economic Review 82 146-158

[12] Breyer, F. (1989) "On the Intergenerational E¢ciency of Pay-as-you-go Financed Pension Systems". Journal of Institutional and Theoretical Economics 145:643-658

[13] Brey F. and Straub, M (1991) "Welfare E¤ects of Unfunded Pension systems when Labour Supply is Endogenous" forthcoming in: Journal of Public Economics.

[14] Browning, E. K.(1975): "Why the social insurance budget is too large in a democracy" Economic Inquiry 13:373-388

[15] Brunner, J. K. Wickstrom, B.A. "Politically Stable Pay As you Go" Pension System: When the social insurance Budget is too small in a Democracy" in B. Felderer ed Public Pension Economics. Zeitschrift fur Nationalokonomie, Supplementum 7.

[16] Diamon, P.A. (1965): "National Debt in a Neoclassical Growth Model" American Economic Review 55: 1126-1150

[17] Danziger S., R.Haverman and R.Plotnick (1981)" How income transfer programmes a¤ect work, savings, and income distribution: a critical review", Journal of Economic Literature 19:975-1028

[18] Enders W. and H.E. Lapan (1982) "Social Security taxation and intergenerational risk sharing" International Economic Review 23: 647-58

[19] Enders W. and H.E. Lapan (1982) "A model of …rst and second-best social security programs", in B. Felderer ed Public Pension Economics. Zeitschrift fur Nationalokonomie, Supplementum 7.

[20] Felder, B (1992): "Does a public pension System Reduce the Saving rate and birth rate?" Journal of Institutional and theoretical Economics 148: 314-325

[21] Feldstein, Martin S. (1974) "Social security, Induced Retirement and Aggregate Capital Accumulation and e¢cient Growth: A complete Characterization" Journal of Economic Theory 4

[22] Homburg, S. (1990a) "The e¢ciency of Unfunded Pension Schemes" Journal of Institutional and theoretical Economics 146: 450-459

[23] Hu S.C. (1979) "Social security, the supply of labour, and capital accumulation" American Economic Review 69: 274-283

[24] Kieran Mc Morrow, Werner Roeger (2002) "EU Pension reform. An overview of the debate and an empirical assessment of the main policy reform options" Economi Paper n.162 European Commission.

[25] Obstfeld M. Rogo¤ Kenneth, Fundations of internationalt Macroeconomics, Cambridge Mass MIT Press.

[26] Rankin N. Ro¤a B. "Maximum sustainable Government Debt in the Overlapping Generations Model. Warwick Univesity.

[27] Romer D., Mankiw G.N.,Weil, N.D. "A Contribution to the Empirics of Economics Growth. The Quarterly Journal of Economics, Volume 107 Isue 2 407-437.

[28] Samuelson P.A. (1958), "An Exact Consumption-Loan Model of Interest with or without the social contract of Money" Journal of Political Economy 66: 467-482

[29] Samuelson P.A. (1975),"Optimum social security in a life-cycle growth model" International Economic Review 69: 539-544

[30] Sayan S. and Uyar A. "Directions of trade ‡ows and labour movement between high and low population Growth countries: An overlapping generations general eqilibrium analysis"

[31] Sinn H.W. (1999),"The crisis of German's Pensions insurance system and how it can be resolved" C.E.P.R 2175

[32] Xavier Sala-I-Martini (1999),"Social Security in Theory and Practice Fact and Political Theories.

[33] Weil P. (1987) "Permanent budget de…cits and in‡ation" Journal of Monetary Economics20 :393-410.