Mandatory Social Security with Social Planner and with Majority Rule

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Abstract

Several authors have argued that a mandatory social security program undertaken by Social Planner can have positive effects on welfare when individuals possess hidden information about their longevity. Davies and Kuhn [3] have considered the related problem of the effects of a mandatory social security program undertaken by Social Planner when individuals can take hidden actions to affect their longevity and they have shown that social security never raises welfare in a pure moral hazard economy. Anderberg [1] has considered voting over a mandatory social security program when there is an annuity market characterized by adverse selection and he has shown that a majority voting can be either the median type's ideal policy or an ends-against-the-middle equilibrium. In this work I will consider the case in which the annuity market is characterized by both adverse selection and moral hazard and I will analyse the effect of a mandatory social security program undertaken by Social Planner and by individuals through a Majority Rule: a mandatory social security program can have positive effects on welfare.

1 Introduction

According to many authors a large scale of interventions in the programs of health insurance, unemployment insurance, disability insurance and public pensions (insurance against longevity) in Western democracies must be considered not remedies for any of the classical market failures,

not efficient devices for redistribution but consequences of asymmetric information [1].

Several authors ([10], [12], [8], [9] and [4]) have argued that a mandatory social security program undertaken by Social Planner can have positive effects on welfare when individuals possess hidden information about their longevity, that is when there is adverse selection in the annuity market and a group of individuals (low risk individuals) is affected by negative externalities. Davies and Kuhn [3] have considered the related problem of the effects of a mandatory social security program undertaken by Social Planner when individuals can take hidden actions to affect their longevity by consuming "health-related goods", that is when there is moral hazard in the annuity market and all individuals are affected by negative esternalities; they have shown that social security never raises welfare in a pure moral hazard economy.

Anderberg [1] has considered voting over a mandatory social security program when there is an annuity market characterized by adverse selection and he has shown that a majority voting can be either the median type's ideal policy or an ends-against-the-middle equilibrium.

In this work I will consider the case in which the annuity market is characterized by both adverse selection (individuals possess hidden information about their longevity) and moral hazard (individuals can take hidden actions to affect their longevity).

When the annuity market is characterized by private information annuity supplying firms offer the utility-maximizing actuarially fair annuity contracts (either separating contracts or pooling contract) subject to the constraint that individuals cannot take hidden actions and subject to the constraint that individuals cannot exploit hidden information. Then all individuals, and in particular low-risk individuals, suffer from negative esternalities.

Given these negative esternalities the investigation of possible Pareto improving policies which either Social Planner or individuals through a Majority Rule can undertake is interesting. Then I will analyse the effect of a mandatory social security program undertaken by Social Planner and by individuals through a Majority Rule.

I will show that a mandatory social security program can have positive effects on welfare and I will compare the improvement of welfare when the size of the public provided annuity is decided by Social Planner and when the size of the public provided annuity is decided by individuals through a Majority Rule.

2 The Basic Structure

The economy to be studied is a variant of Samuelson's pure-exchange overlapping generations model [11]. At each period t $(t \geq 1)$ the population consists of old members of generation $t - 1$ who all die at the end of that period and young members of generation t.

Generations are of equal size (the population growth rate is $n = 0$) and there is no altruism; thus if there were no uncertainty each individual would leave a bequest of zero.

In this economy there is a single non-storable and non-producible consumption good c. Each young agent is endowed at birth with w units of the consumption good.

All individuals live for a maximum of two periods $(t = 1, 2)$. All members of a given generation are alive for certain in the first period, and survive with some probability p into the second period.

If we consider the health care as "capabilities and mechanism of defense that protect an organism from external stress" [2], an individual can increase his survival probability into the second period by investing in health care in the first period: an individual survives with probability $p(h)$ into the second period, where h represents the investment level in health care in the first period [3].

Since investment in health care in the first period decreases the probability of death at the beginning of the second period, we have $\dfrac{\delta p(h)}{\delta h} > 0$. Moreover I suppose $\dfrac{\delta^2 p(h)}{\delta h \delta h} = 0$. With h^{max} the maximum feasible investment in h, an individual survives with probability $p(h)$ into the second period with $0 \leq p(h) \leq 1$, $p(h) \longrightarrow 0$ as $h \longrightarrow 0$ and $p(h) \longrightarrow 1$ as $h \longrightarrow h^{\mathrm{max}}$.

The cost of investing in a level of health care h is given by the function $c(h)$, with $c(0) = 0$ and $\dfrac{\delta c(h)}{\delta h} > 0$. Moreover I suppose $\dfrac{\delta^2 c(h)}{\delta h \delta h} = 0$.

Then the representative individual's expected lifetime utility is given by

$$U = u(c_1) + p(h) \cdot u(c_2)$$

with $u_c > 0$, $u_{cc} < 0$, $u_c \longrightarrow \infty$ as $c \longrightarrow 0$ and $u_c \longrightarrow 0$ as $c \longrightarrow \infty$ and the individuals' consumption sets are bounded by

$$0 \leq h \leq h^{\mathrm{max}}$$

2.1 Myopia

I suppose that in this economy individuals are myopic. According to traditional definition of myopia [6] consumers can fail to appreciate their

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later needs, either discounting the future completely or placing a lower weight upon it than would capture their true preferences, or they may make mistakes in their planning, have lack information or simply be irrational. Although the behavioural foundations of myopia differ significantly from those of consistent utility maximization, the formulation of myopic behaviour of Feldstein [5] can be incorporated into the traditional analysis. If we define μ the index which captures the degree of myopia (with $0 \leq \mu \leq 1$), the representative individual's expected lifetime utility becomes

$$U = u(c_1) + \mu \cdot p(h) \cdot u(c_2)$$

2.2 Annuity Contract

Agents allocate consumption intertemporally by purchasing annuities which are supplied by competitive firms that specialize in holding the safe asset. An annuity bond at period t is a claim to a certain quantity of the consumption good at period $t+1$ which is payable only if the original purchaser of the annuity is alive. Normalizing the purchasing price of a period t annuity to one unit of the good at t, the annuity's rate of return represents the intertemporal terms of trade faced by its buyer.

Then we define an annuity contract as a two dimensional vector (s, R) (with s quantity and R rate of return) so that if a young agent purchase this contract his consumption vector (c_1, c_2) becomes $(w - c(h) - s, s \cdot R)$ if he lives two periods and $(w - c(h) - s, 0)$ if he lives only one period.

Firms decide the values of quantity s and rate of return R by maximizing the expected utility of young agents

$$\underset{c_1, c_2, h}{Max} u(c_1) + \mu \cdot p(h) \cdot u(c_2)$$

$$\begin{aligned} s.t. \quad & c_1 = w - c(h) - s \\ & c_2 = s \cdot R \\ & 0 \leq h \leq h^{\max} \\ & s \geq 0 \end{aligned}$$

This maximization problem may be rewritten using the indirect utility form in the simpler form

$$\underset{s,h}{Max}\, \nu(w - c(h) - s) + \mu \cdot p(h) \cdot \nu(s \cdot R)$$

$$\begin{aligned} s.t. \quad & 0 \leq h \leq h^{\max} \\ & s \geq 0 \end{aligned}$$

The constraint $s \geq 0$ marks that the consumer cannot sell off the future income for most first period income.

2.3   Equilibrium Concepts

In the following analysis I will consider a population in which each generation is partitioned into three distinct groups according to the degree of myopia (and then according to the survival probability) of agents. We suppose that the relative size of these groups is fixed. Given this heterogeneity of the population, we can think of two kinds of annuity equilibria:

1. a *separating equilibrium* in which agents with different survival probabilities purchase annuities with different rate of return,

2. a *pooling equilibrium* in which the same annuity is purchased by members of different groups.

Since relative size of groups is fixed, there is absence of aggregate uncertainty regarding the number of deaths in each group and then absence of uncertainty regarding the profits of the annuity-supplying firms. Therefore in either a pooling or a separating equilibrium real profits must be equal to zero.

Rothschild and Stiglitz [10] consider firms that accomplish a *screening* strategy and they define an equilibrium as a set of contracts such that when agents choose contract to maximize their expected utility

1. no contract in the equilibrium set makes negative expected profits,

2. there is no contract outside the equilibrium set that, if offered, will make a non-negative profit.

Given these basic features of the equilibrium, in a Rothschild-Stiglitz equilibrium each firm assumes that the contract its competitors offer are independent of its own actions (Nash-Cournot type equilibrium) [10], [4]. Then each contract offered in equilibrium earns zero profits: positive profits on any single contract are eliminating by *undercutting* among the firms and *cross-subsidization* among different contracts offered by any given firm can be rule out by noting that firms will withdraw contracts that persistently yield negative profits.

However a Rothschild-Stiglitz equilibrium has problems of non-existence:

1. there cannot be a Rothschild-Stiglitz pooling equilibrium,

2. for a relatively small number of high-risk agents (agents with higher survival probability), there does not exist a Rothschild-Stiglitz equilibrium.

Wilson solves the problems of non-existence of a Rothschild-Stiglitz equilibrium in a contest in which firms carry on accomplishing a *screening* strategy: each firm will correctly anticipate which of those policies that are offered by other firms will become unprofitable as a consequence of any change in its own policies [12], [4]. Then a firm offers a new policy only if it makes non-negative profits after all the other firms have made the expected adjustment in their policy offers[1]. Then in the following analysis I will consider a Wilson equilibrium.

2.4 Parameterized Example

I will clarify the results by presenting a computational model. I assume that preferences are given by a logarithmic function. Thus representative individual's expected lifetime utility is given by

$$U = \ln(c_1) + \mu \cdot p(h) \cdot \ln(c_2)$$

With q_h the price of a unit of investment in health care h, I will consider the cost function $c(h) = q_h \cdot h$. Moreover $h^{\max} = \dfrac{w}{q_h}$ and then $p(h) = \dfrac{h}{h^{\max}} = \dfrac{h \cdot q_h}{w}$. In the numerical examples the endowed income is $w = 1000$ and the price of a unit of investment in health care is $q_h = 1.25$. The simulations are made with *GAUSS*.

3 The Model

Each generation t is partitioned into three distinct groups, L, M and H, whose relative size is fixed for all t, so that for each agent of type L there are γ_M agents of type M and γ_H agents of type H, with $\gamma_M, \gamma_H > 0$. Individuals of three groups have a different degree of myopia μ^i (with $0 \leq \mu^i \leq 1$): members of group L are more myopic than members of group M and members of group M are more myopic than members of group H ($\mu^L < \mu^M < \mu^H$).

The absence of aggregate uncertainty regarding the number of deaths in each group (and then the absence of uncertainty regarding the prof-

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[1]Riley solves the problems of non-existence of a Rothschild-Stiglitz equilibrium in a context in which firms adopt a *signalling* strategy [9, Riley (1979b)]. In this model the signal should be the investment in health care in the first period, but low-risk individuals invest less in health care than high-risk individuals. Since high-risk individuals have incentive to declare the lower investment in health care in the first period, firms cannot adopt a signalling strategy.

its of the annuity-supplying firms) implies that a stationary allocation (c_1^i, c_2^i, h^i) $(i = L, M, H)$ is feasible if it satisfies

$$\left[\left(c_1^L + c\left(h^L \right) \right) + \gamma_M \cdot \left(c_1^M + c\left(h^M \right) \right) + \gamma_H \cdot \left(c_1^H + c\left(h^H \right) \right) \right]$$
$$+ \left[p\left(h^L \right) \cdot c_2^L + \gamma_M \cdot p\left(h^M \right) \cdot c_2^M + \gamma_H \cdot p\left(h^H \right) \cdot c_2^H \right] = w \cdot (1 + \gamma_M + \gamma_H)$$

A feasible stationary allocation $\left(\bar{c}_1^i, \bar{c}_2^i, \bar{h}^i \right)$ $(i = L, M, H)$ is optimal if there does not exist another feasible stationary allocation (c_1^i, c_2^i, h^i) $(i = L, M, H)$ such that

$$U^i \left(c_1^i, c_2^i, h^i \right) \geq U^i \left(\bar{c}_1^i, \bar{c}_2^i, \bar{h}^i \right)$$

with strict inequality for some i. Moreover an interior allocation (c_1^i, c_2^i, h^i) is optimal if for some $\beta^i > 0$ $(i = L, M, H)$ it solves the problem

$$Max \; \beta^L \cdot U^L \left(c_1^L, c_2^L, h^L \right) + \beta^M \cdot U^M \left(c_1^M, c_2^M, h^M \right) + \beta^H \cdot U^H \left(c_1^H, c_2^H, h^H \right)$$

$$s.t. \quad \left[\left(c_1^L + c\left(h^L \right) \right) + \gamma_M \cdot \left(c_1^M + c\left(h^M \right) \right) + \gamma_H \cdot \left(c_1^H + c\left(h^H \right) \right) \right] +$$
$$+ \left[p\left(h^L \right) \cdot c_2^L + \gamma_M \cdot p\left(h^M \right) \cdot c_2^M + \gamma_H \cdot p\left(h^H \right) \cdot c_2^H \right] =$$
$$w \cdot (1 + \gamma_M + \gamma_H)$$

$$c_1^L, c_1^M, c_1^H \geq 0$$
$$c_2^L, c_2^M, c_2^H \geq 0$$
$$0 \leq h^L, h^M, h^H \leq h^{\max}$$

Ignoring the inequality constraints, if we maximize with respect to the consumption goods in the two periods we obtain

$$\frac{u_{c_1}^i}{u_{c_2}^i} = \mu^i \tag{1}$$

Then we can write

$$\frac{u_{c_1}^L}{\mu^L \cdot u_{c_2}^L} = \frac{u_{c_1}^M}{\mu^M \cdot u_{c_2}^M} = \frac{u_{c_1}^H}{\mu^H \cdot u_{c_2}^H} = 1 \tag{2}$$

defined by Eckestein, Eichenbaum and Peled [4] as the *necessary and sufficient condition for an interior allocation to be optimal.*

Ignoring the inequality constraints and maximizing also with respect to the investment in health care we obtain

$$u^i \left(c_2^i \right) - c_2^i \cdot u_{c_2}^i = \frac{\dfrac{\delta c\left(h^i \right)}{\delta h}}{\dfrac{\delta p\left(h^i \right)}{\delta h}} \cdot \frac{u_{c_1}^i}{\mu^i} = \frac{\dfrac{\delta c\left(h^i \right)}{\delta h}}{\dfrac{\delta p\left(h^i \right)}{\delta h}} \cdot u_{c_2}^i \tag{3}$$

From the last equation we can argue that

	$\mu^L = 0.4$	$\mu^M = 0.7$	$\mu^H = 1.0$
c_1^i	565.44	323.11	226.18
c_2^i	**226.18**	**226.18**	**226.18**
h^i	283.52	441.63	504.87
p^i	35.44%	55.20%	63.11%
U^i	7.1061	7.8729	8.8426
	$\mu^L = 0.5$	$\mu^M = 0.7$	$\mu^H = 0.9$
c_1^i	452.35	323.11	251.31
c_2^i	**226.18**	**226.18**	**226.18**
h^i	357.30	441.63	488.47
p^i	44.66%	55.20%	61.06%
U^i	7.3251	7.8729	8.5059
	$\mu^L = 0.6$	$\mu^M = 0.7$	$\mu^H = 0.8$
c_1^i	376.96	323.11	282.72
c_2^i	**226.18**	**226.18**	**226.18**
h^i	406.49	441.63	467.98
p^i	50.81%	55.20%	58.50%
U^i	7.5849	7.8729	8.1815

Table 1: Optimal Allocations

- if $h^i > 0$ and then if $p(h^i) > 0$ (with $i = L, M, H$), members of three groups choose the same level of consumption in the second period: $c_2^L = c_2^M = c_2^H > 0$;

- if $h^i = 0$ and then if $p(h^i) = 0$ (with $i = L, M, H$), the utility function becomes $U^i = u(c_1^i)$.

In the following analysis I will consider only the case $h^i > 0$ (with $i = L, M, H$). The results of simulation are summarized in table 1.

In the first period members of group L consume more and invest less in health care than members of group M and members of group M consume more and invest less in health care than members of group H (with a consumption in the second period equal for members of three groups).

Given heterogeneity with respect to degree of myopia ($\mu^L < \mu^M < \mu^H$), optimal allocations have the property that ex ante marginal rates of substitution are not equalized across members of different groups

$$\frac{1}{p(h^L)} \cdot \frac{u_{c_1}^L}{\mu^L \cdot u_{c_2}^L} \neq \frac{1}{p(h^M)} \cdot \frac{u_{c_1}^M}{\mu^M \cdot u_{c_2}^M} \neq \frac{1}{p(h^H)} \cdot \frac{u_{c_1}^H}{\mu^H \cdot u_{c_2}^H} \quad (4)$$

In a competitive equilibrium agents equate their expected intertemporal marginal rate of substitution to the rate of return on saving that they face:

$$\frac{1}{p\left(h^i\right)} \cdot \frac{u^i_{c_1}}{\mu^i \cdot u^i_{c_2}} = R^i \ \ \text{for } i = L, M, H$$

Consequently, the competitive equilibrium will be full information Pareto optimal if and only if all agents face actuarially fair rates of return $\dfrac{1}{p\left(h^i\right)}$ for agents of type i (with $i = L, M, H$). When

$$R^i = \frac{1}{p\left(h^i\right)} \ \ \text{for } i = L, M, H$$

we obtain

$$\frac{u^i_{c_1}}{\mu^i \cdot u^i_{c_2}} = 1 \ \ \text{for } i = L, M, H$$

and the necessary and sufficient condition for an interior allocation to be optimal is satisfied.

4 Private Information

In the case of private information regarding the degree of myopia (and then the investment in health care in the first period h and the survival probability $p\left(h\right)$) we have the following two situations.

Individuals can take *hidden actions* to affect their longevity (in annuity markets there is *moral hazard)* becouse individuals choose the optimal level of h in response to contract $\left(s^i, R\left(p\left(h^i\right)\right)\right)$ offered by annuity-supplying firms [3]: for individuals the rate of return $R\left(p\left(h^i\right)\right)$ is given.

No agent (individuals, firms and government) knows whether any particular individual belongs to group L, M or H. In a previous work [7] I showed that the contract $\left(s^L, R\left(p\left(h^L\right)\right)\right)$ (*low-risk contract*), the contract preferred by members of group L, the contract $\left(s^M, R\left(p\left(h^M\right)\right)\right)$ (*medium-risk contract*), the contract preferred by members of group M, and the contract $\left(s^H, R\left(p\left(h^H\right)\right)\right)$ (*high-risk contract*), the contract preferred by members of group H, are characterized by $s^H > s^M > s^L$, $R\left(p\left(h^H\right)\right) < R\left(p\left(h^M\right)\right) < R\left(p\left(h^L\right)\right)$ and $s^H \cdot R\left(p\left(h^H\right)\right) = s^M \cdot R\left(p\left(h^M\right)\right) = s^L \cdot R\left(p\left(h^L\right)\right)$. Then both members of group M and members of group H would prefer contract $\left(s^L, R\left(p\left(h^L\right)\right)\right)$ to contracts $\left(s^M, R\left(p\left(h^M\right)\right)\right)$ and $\left(s^H, R\left(p\left(h^H\right)\right)\right)$: from the point of view of members of groups M and H contract $\left(s^L, R\left(p\left(h^L\right)\right)\right)$ dominates contracts $\left(s^M, R\left(p\left(h^M\right)\right)\right)$ and $\left(s^H, R\left(p\left(h^H\right)\right)\right)$. Hence the medium-risk individuals (members of group M) and the high-risk individuals (members of group H) have *hidden information* about their longevity (in annuity markets there is *adverse selection*).

4.1 Separating Equilibrium

Because individuals can take hidden actions to affect their longevity, competitive firms offer the utility-maximizing actuarially fair annuity contracts $\left(\tilde{s}^i, R\left(p\left(\tilde{h}^i\right)\right)\right)$ subject to the constraint that individuals choose the optimal level of $\tilde{h}^i$ in response to these contracts (*incentive constraint*) [3].

Since contract $\left(\tilde{s}^L, R\left(p\left(\tilde{h}^L\right)\right)\right)$ is actuarially fair for members of group L only, if members of groups M and H purchased it profits of firms would necessarily be negative. Thus from the point of view of firms contract of *separating equilibrium* for group L must not be more attractive to members of group M than contract $\left(\tilde{s}^M, R\left(p\left(\tilde{h}^M\right)\right)\right)$ and contracts of *separating equilibrium* for groups L and M must not be more attractive to members of group H than contract $\left(\tilde{s}^H, R\left(p\left(\tilde{h}^H\right)\right)\right)$ (*incentive-compatibility* or *self-selection constraints*) [4].

Thus competitive firms offer to members of groups L and M the utility-maximizing actuarially fair annuity contracts $\left(\check{s}^L, R\left(p\left(\check{h}^L\right)\right)\right)$ and $\left(\check{s}^M, R\left(p\left(\check{h}^M\right)\right)\right)$ subject to the constraints that individuals choose the optimal level of $\check{h}^L$ and $\check{h}^M$ in response to these contracts (*incentive constraints*) and subject to the constraints that contract of *separating equilibrium* for group L must not be more attractive to members of group M than contract $\left(\check{s}^M, R\left(p\left(\check{h}^M\right)\right)\right)$ and that contract of *separating equilibrium* for group M must not be more attractive to members of group H than contract $\left(\tilde{s}^H, R\left(p\left(\tilde{h}^H\right)\right)\right)$ (*self-selection constraints*).

Hence in the case of separating equilibrium there are two kinds of negative externalities [7].

1. The fact that firms consider the possibility that individuals modify the investment in health care in the first period in response to separating contracts offered to them is a first negative externality: $V^L\left(\tilde{s}^L, \tilde{h}^L\right) < V^L\left(s^L, h^L\right)$, $V^M\left(\tilde{s}^M, \tilde{h}^M\right) < V^M\left(s^M, h^M\right)$ and $V^H\left(\tilde{s}^H, \tilde{h}^H\right) < V^H\left(s^H, h^H\right)$.

2. The presence of high-risk individuals (group H) exerts a second negative externality on agents of group M and the presence of medium-risk individuals (group M) and high-risk individuals (group H) exerts a second negative externality on agents of group L: $V^M\left(\check{s}^M, \check{h}^M\right) < V^M\left(\tilde{s}^M, \tilde{h}^M\right) < V^M\left(s^M, h^M\right)$ and $V^L\left(\check{s}^L, \check{h}^L\right) < V^L\left(\tilde{s}^L, \tilde{h}^L\right) < V^L\left(s^L, h_1^L\right)$.

These negative externalities are purely destructive because members of group L, M and H are worse off than they would be in the absence of private information.

4.2   Pooling Equilibrium

With private information firms don't know whether any particular individual belongs to group L, M or H, then in the case of pooling equilibrium firms offer to the members of three groups not only the same rate of return, but also the same quantity of annuity: a pooling contract is characterized not only by $R_t^L = R_t^M = R_t^H = \bar{R}_t$, but also by $s_t^L = s_t^M = s_t^H = \bar{s}_t$ and $s_{t+1}^L = s_{t+1}^M = s_{t+1}^H = \bar{s}_{t+1}$. Then the condition of zero profits is given by

$$\bar{s}_{t+1} + \gamma_M \cdot \bar{s}_{t+1} + \gamma_H \cdot \bar{s}_{t+1} + $$
$$-\bar{R}_t \cdot \left[p_t \left(\bar{h}^L \right) \cdot \bar{s}_t + \gamma_M \cdot p_t \left(\bar{h}^M \right) \cdot \bar{s}_t + \gamma_H \cdot p_t \left(\bar{h}^H \right) \cdot \bar{s}_t \right] = 0$$

In a stationary equilibrium $\bar{R}_t = \bar{R}$ and $\bar{s}_t = \bar{s}$ for all t and then the rate of return of the pooling contract is

$$\bar{R} = \frac{1 + \gamma_M + \gamma_H}{p\left(\bar{h}^L \right) + \gamma_M \cdot p\left(\bar{h}^M \right) + \gamma_H \cdot p\left(\bar{h}^H \right)} \tag{5}$$

Suppose that annuity-supplying firms offer a pooling contract characterized by a quantity of annuity $\bar{s}$ that doesn't maximize the utility of low-risk individuals. Given this case, if a firm offered a pooling contract characterized by a quantity of annuity $\bar{s}$ that maximizes the utility of low-risk individuals, individuals of group L would purchase this second contract and the profits of firms that offer the first contract would become negative . Thus the quantity of annuity of a pooling contract which assures non-negative profits is

$$\bar{s} = \arg \ \underset{s}{Max} \ \ v^L \left(w - c \left(\bar{h}^L \right) - s \right) + \mu^L \cdot p \left(\bar{h}^L \right) \cdot v^L \left(s \cdot \bar{R} \right)$$

Because individuals can take hidden actions to affect their longevity, competitive firms offer the utility-maximizing actuarially fair annuity contracts $\left(\bar{s}, \bar{R} \right)$ subject to the constraint (*incentive constraint*) that individuals choose the optimal level of h^i in response to this contract [3].

Hence in the case of pooling equilibrium there are two kinds of negative externalities [7].

1. The fact that firms consider the possibility that members of all groups modify the investment in health care in the first period in response to the pooling contract offered to them is a first negative externality. However when a pooling equilibrium is offered by the annuity supplying firms, for members of H this negative externality is completely compensated by $\bar{R} > R \left(p \left(h^H \right) \right)$.

$\gamma_M = 0.50$ and $\gamma_H = 0.50$				
	0.4			
	μ^L	$\check{\mu}^L$	$\bar{\mu}^L$	μ_{SP}^L
$\bar{s}$			84.76	78.55
s^L	80.16	41.37		3.23
$\bar{R}$			1.83	1.85
R^L	2.82	2.41		2.37
V^L	7.1061	**7.0621**	7.0523	7.0564
	0.7			
	μ^M	$\check{\mu}^M$	$\bar{\mu}^M$	μ_{SP}^M
s^M	124.86	88.39		11.16
$\bar{R}$				
R^M	1.81	1.60		1.59
V^M	7.8729	7.8257	7.8774	7.8830
	1.0			
	μ^H	$\tilde{\mu}^H$	$\bar{\mu}^H$	μ_{SP}^H
s^H	142.74	144.06		50.41
$\bar{R}$				
R^H	1.58	1.49		1.46
V^H	8.8426	8.8240	8.9054	8.9179

Table 2: Public provided annuity and Social Planner with gamma M 0.50 and gamma H 0.50 - Case 0.4, 0.7 and 1.0

2. The presence of high-risk individuals (group H) exerts a second negative externality on other agents (group L and M) and the presence of medium-risk individuals (group M) exerts a second negative externality on low-risk agents (group L) because $p\left(h^H\right) > p\left(h^M\right) > p\left(h^L\right)$.

These negative externalities are not purely destructive because while members of group L and members of group M are worse off than they would be in the absence of private information, members of group H are better off.

5 Improving Role of Public Provided Annuity

In the previous sections I have reminded that both in separating equilibrium and in pooling equilibrium there are negative externalities. Given

$\gamma_M = 1.00$ and $\gamma_H = 1.00$				
	0.6			
	μ^L	$\check{\mu}^L$	$\bar{\mu}^L$	μ^L_{SP}
$\bar{s}$			118.07	109.60
s^L	114.92	79.27		0
$\bar{R}$			1.65	1.64
R^L	1.97	1.72		1.76
V^L	7.5849	7.5415	7.5453	**7.5459**
	0.7			
	μ^M	$\check{\mu}^M$	$\bar{\mu}^M$	μ^M_{SP}
s^M	124.86	98.41		0
$\bar{R}$				
R^M	1.81	1.61		1.63
V^M	7.8729	7.8398	7.8560	7.8542
	0.8			
	μ^H	$\tilde{\mu}^H$	$\bar{\mu}^H$	μ^H_{SP}
s^H	132.31	133.92		15.10
$\bar{R}$				
R^H	1.71	1.58		1.56
V^H	8.1815	8.1632	8.1873	8.1831

Table 3: Public provided annuity and Social Planner with gamma M 1.00 and gamma H 1.00 - Case 0.6, 0.7 and 0.8

these negative externalities the investigation of possible Pareto improving policies which either Social Planner or individuals through a Majority Rule can undertake is interesting.

The search for allocations which dominate those obtained as competitive equilibria requires that

- all agents purchase a given amount s_P of public provided annuity that pay a rate of return R_P;

- agents can satisfy the residual demand for annuities in the private market where competitive firms offer separating contracts $\left(s^i, R\left(p\left(h^i\right)\right)\right)$.

When the same quantity of annuity at the same rate of return is

$\gamma_M = 0.50$ and $\gamma_H = 0.50$			
0.6			
μ^L	$\check{\mu}^L$	$\bar{\mu}^L$	μ^L_{SP}
$\bar{s}$		117.80	117.80
s^L 114.92	79.27		0
$\bar{R}$		1.67	1.67
R^L 1.97	1.72		1.76
V^L 7.5849	7.5415	**7.5507**	**7.5507**
0.7			
μ^M	$\check{\mu}^M$	$\bar{\mu}^M$	μ^M_{SP}
s^M 124.86	98.41		0
$\bar{R}$			
R^M 1.81	1.61		1.63
V^M 7.8729	7.8398	7.8627	7.8627
0.8			
μ^H	$\tilde{\mu}^H$	$\bar{\mu}^H$	μ^H_{SP}
s^H 132.31	133.92		0
$\bar{R}$			
R^H 1.71	1.58		1.55
V^H 8.1815	8.1632	8.1952	8.1952

Table 4: Public provided annuity and Social Planner with gamma M 0.50 and gamma H 0.50 - Case 0.6, 0.7 and 0.8

offered to members of all groups the condition of zero profits is given by

$$s_{P_{t+1}} + \gamma_M \cdot s_{P_{t+1}} + \gamma_H \cdot s_{P_{t+1}} + \tag{6}$$
$$-R_{P_t} \cdot \left[p_t \left(\check{h}^L \right) \cdot s_{P_t} + \gamma_M \cdot p_t \left(\check{h}^M \right) \cdot s_{P_t} + \gamma_H \cdot p_t \left(\tilde{h}^H \right) \cdot s_{P_t} \right] = 0$$

In a stationary equilibrium $R_{P_t} = R_P$ and $s_{P_t} = s_P$ for all t and then the rate of return of the pooling contract is

$$R_P = \frac{1 + \gamma_M + \gamma_H}{p \left(\check{h}^L \right) + \gamma_M \cdot p \left(\check{h}^M \right) + \gamma_H \cdot p \left(\tilde{h}^H \right)} \tag{7}$$

5.1 Social Planner

Since both in a separating equilibrium and in a pooling equilibrium negative externalities suffered by low-risk individuals are larger than negative externalities suffered by medium-risk individuals and high-risk

$\gamma_M = 0.50$ and $\gamma_H = 0.50$				
	0.5			
	μ^L	$\check{\mu}^L$	$\bar{\mu}^L$	μ^L_{SP}
$\bar{s}$			104.98	98.17
s^L	101.02	59.87		0.77
$\bar{R}$			1.72	1.73
R^L	2.24	1.93		1.95
V^L	7.3251	7.2751	7.2787	**7.2804**
	0.7			
	μ^M	$\check{\mu}^M$	$\bar{\mu}^M$	μ^M_{SP}
s^M	124.86	91.96		2.25
$\bar{R}$				
R^M	1.81	1.61		1.61
V^M	7.8729	7.8314	7.8726	7.8707
	0.9			
	μ^H	$\tilde{\mu}^H$	$\bar{\mu}^H$	μ^H_{SP}
s^H	138.10	139.56		27.60
$\bar{R}$				
R^H	1.64	1.53		1.50
V^H	8.5059	8.4874	8.5517	8.5469

Table 5: Public provided annuity and Social Planner with gamma M 0.50 and gamma H 0.50 - Case 0.5, 0.7 and 0.9

individuals [7], Social Planner choose the amount s_{SP} of public provided annuity by weighing up only the expected utility of members of group L.

Social Planner decides the level of s_P which maximizes the utility of individuals of group L independently from the existence of the private market: he maximizes the utility of individuals of group L who don't satisfy the residual demand for annuity in the private market. However Social Planner has to consider that individuals can satisfy the residual demand for annuities in the private market: Social Planner maximizes the expected utility of members of group L subject to both *incentive constraints* and *self-selection constraints*. Then Social Planner chooses the utility-maximizing actuarially fair annuity contract (s_{SP}, R_{SP}) by maximizing

$$\underset{s_{SP}}{Max} v^L \left(w - c \left(\check{h}^L \right) - s_{SP} \right) + \mu^L \cdot p \left(\check{h}^L \right) \cdot v^L \left(s_{SP} \cdot R_{SP} \right) \qquad (8)$$

$$s.t. \; \check{h}^L \; \text{solves} \underset{\check{h}^L}{Max} \nu^L \left(w - c \left(\check{h}^L \right) - s_{SP} - \check{s}^L \right) +$$

$$+ \mu^L \cdot p \left(\check{h}^L \right) \cdot \nu^L \left(s_{SP} \cdot R_{SP} + \check{s}^L \cdot R \left(p \left(\check{h}^L \right) \right) \right) \quad \left[\lambda_{mh}^L \right]$$

$$s.t. \; \check{h}^M \; \text{solves} \underset{\check{h}^M}{Max} \nu^M \left(w - c \left(\check{h}^M \right) - s_{SP} - \check{s}^M \right) +$$

$$+ \mu^M \cdot p \left(\check{h}^M \right) \cdot \nu^M \left(s_{SP} \cdot R_{SP} + \check{s}^M \cdot R \left(p \left(\check{h}^M \right) \right) \right) \quad \left[\lambda_{mh}^M \right]$$

$$s.t. \; \tilde{h}^H \; \text{solves} \underset{\tilde{h}^H}{Max} \nu^H \left(w - c \left(\check{h}^H \right) - s_{SP} - \tilde{s}^H \right) +$$

$$+ \mu^H \cdot p \left(\check{h}^H \right) \cdot \nu^H \left(s_{SP} \cdot R_{SP} + \tilde{s}^H \cdot R \left(p \left(\tilde{h}^H \right) \right) \right) \quad \left[\lambda_{mh}^H \right]$$

$$\nu^M \left(w - c \left(\check{h}^M \right) - s_{SP} - \check{s}^M \right) +$$
$$+ \mu^M \cdot p \left(\check{h}^M \right) \cdot \nu^M \left(s_{SP} \cdot R_{SP} + \check{s}^M \cdot R \left(p \left(\check{h}^M \right) \right) \right)$$
$$= \hat{\nu}^M \left(w - c \left(\hat{h}^M \right) - s_{SP} - \check{s}^L \right) +$$
$$+ \mu^M \cdot p \left(\hat{h}^M \right) \cdot \hat{\nu}^M \left(s_{SP} \cdot R_{SP} + \check{s}^L \cdot R \left(p \left(\check{h}^L \right) \right) \right) \quad \left[\lambda_{as}^L \right]$$

$$\nu^H \left(w - c \left(\tilde{h}^H \right) - s_{SP} - \tilde{s}^H \right) +$$
$$+ \mu^H \cdot p \left(\tilde{h}^H \right) \cdot \nu^H \left(s_{SP} \cdot R_{SP} + \tilde{s}^H \cdot R \left(p \left(\tilde{h}^H \right) \right) \right)$$
$$= \hat{\nu}^H \left(w - c \left(\hat{h}^H \right) - s_{SP} - \check{s}^M \right) +$$
$$+ \mu^H \cdot p \left(\hat{h}^H \right) \cdot \hat{\nu}^H \left(s_{SP} \cdot R_{SP} + \check{s}^M \cdot R \left(p \left(\check{h}^M \right) \right) \right) \quad \left[\lambda_{as}^M \right]$$

$$0 \leq \; \check{h}^L, \check{h}^M, \tilde{h}^H \leq h^{\max}$$

$$s_{SP}, \check{s}^L, \check{s}^M, \tilde{s}^H \geq 0$$

Given the public provided annuity (s_{SP}, R_{SP}), competitive firms offer separating contracts $(s^i, R(p(h^i)))$ to satisfy the residual demand for annuities. Since the existence of medium-risk individuals M and high-risk individuals H affects the expected utility of low-risk individuals L through the rate of return of the public provided annuity R_{SP}, the maximization of the expected utilities of three individual types cannot be independent. Then in the private market competitive firms offer the utility-maximizing actuarially fair annuity contracts $(s^i, R(p(h^i)))$ by weighing up only the expected utility of members of group L. Then given the value of s_{SP} decided by Social Planner the annuity supplying firms offer separating contracts such that the expected utility of individuals of group L is maximized.

$$\underset{\check{s}^L, \check{h}^L, \check{s}^M, \check{h}^M, \tilde{s}^H, \tilde{h}^H}{Max} v^L\left(w - c\left(\check{h}^L\right) - s_{SP} - \check{s}^L\right) +$$
$$+ \mu^L \cdot p\left(\check{h}^L\right) \cdot v^L\left(s_{SP} \cdot R_{SP} + \check{s}^L \cdot R\left(p\left(\check{h}^L\right)\right)\right)$$

$$s.t.\ \check{h}^L\ \text{solves} \underset{\check{h}^L}{Max} \nu^L\left(w - c\left(\check{h}^L\right) - s_{SP} - \check{s}^L\right) +$$
$$+ \mu^L \cdot p\left(\check{h}^L\right) \cdot \nu^L\left(s_{SP} \cdot R_{SP} + \check{s}^L \cdot R\left(p\left(\check{h}^L\right)\right)\right)\ \ \left[\lambda_{mh}^L\right]$$

$$s.t.\ \check{h}^M\ \text{solves} \underset{\check{h}^M}{Max} \nu^M\left(w - c\left(\check{h}^M\right) - s_{SP} - \check{s}^M\right) +$$
$$+ \mu^M \cdot p\left(\check{h}^M\right) \cdot \nu^M\left(s_{SP} \cdot s_{SP} + \check{s}^M \cdot R\left(p\left(\check{h}^M\right)\right)\right)\ \ \left[\lambda_{mh}^M\right]$$

$$s.t.\ \tilde{h}^H\ \text{solves} \underset{\tilde{h}^H}{Max} \nu^H\left(w - c\left(\check{h}^H\right) - s_{SP} - \tilde{s}^H\right) +$$
$$+ \mu^H \cdot p\left(\check{h}^H\right) \cdot \nu^M\left(s_{SP} \cdot R_{SP} + \tilde{s}^H \cdot R\left(p\left(\tilde{h}^H\right)\right)\right)\ \ \left[\lambda_{mh}^H\right]$$

$$\nu^M\left(w - c\left(\check{h}^M\right) - s_{SP} - \check{s}^M\right) +$$
$$+ \mu^M \cdot p\left(\check{h}^M\right) \cdot \nu^M\left(s_{SP} \cdot R_{SP} + \check{s}^M \cdot R\left(p\left(\check{h}^M\right)\right)\right)$$
$$= \hat{\nu}^M\left(w - c\left(\hat{h}^M\right) - s_{SP} - \check{s}^L\right) +$$
$$+ \mu^M \cdot p\left(\hat{h}^M\right) \cdot \hat{\nu}^M\left(s_{SP} \cdot R_{SP} + \check{s}^L \cdot R\left(p\left(\check{h}^L\right)\right)\right)\ \ \left[\lambda_{as}^L\right]$$

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$\gamma_M = 2.00$ and $\gamma_H = 2.00$				
	0.6			
	μ^L	$\check{\mu}^L$	$\bar{\mu}^L$	μ^L_{SP}
$\bar{s}$			118.28	108.45
s^L	114.92	79.27		0.51
$\bar{R}$			1.63	1.63
R^L	1.97	1.72		1.76
V^L	7.5849	7.5415	7.5411	**7.5423**
	0.7			
	μ^M	$\check{\mu}^M$	$\bar{\mu}^M$	μ^M_{SP}
s^M	124.86	98.41		1.03
$\bar{R}$				
R^M	1.81	1.61		1.63
V^M	7.8729	7.8398	7.8507	7.8495
	0.8			
	μ^H	$\tilde{\mu}^H$	$\bar{\mu}^H$	μ^H_{SP}
s^H	132.31	133.92		18.80
$\bar{R}$				
R^H	1.71	1.58		1.57
V^H	8.1815	8.1632	8.1809	8.1774

Table 6: Public provided annuity and Social Planner with gamma M 2.00 and gamma H 2.00 - Case 0.6, 0.7 and 0.8

$$\nu^H \left(w - c\left(\tilde{h}^H\right) - s_{SP} - \tilde{s}^H \right) +$$
$$+\mu^H \cdot p\left(\tilde{h}^H\right) \cdot \nu^H \left(s_{SP} \cdot R_{SP} + \tilde{s}^H \cdot R\left(p\left(\tilde{h}^H\right)\right) \right)$$
$$= \hat{\nu}^H \left(w - c\left(\hat{h}^H\right) - s_{SP} - \check{s}^M \right) +$$
$$+\mu^H \cdot p\left(\hat{h}^H\right) \cdot \hat{\nu}^H \left(s_{SP} \cdot R_{SP} + \check{s}^M \cdot R\left(p\left(\check{h}^M\right)\right) \right) \quad \left[\lambda^M_{as}\right]$$

$$0 \leq \check{h}^L, \check{h}^M, \tilde{h}^H \leq h^{\max}$$
$$s_{SP}, \check{s}^L, \check{s}^M, \tilde{s}^H \geq 0$$

The results of simulation are summarized in tables 2, 4, 3, 5 and 6.

In the case $\gamma_M = 0.50$ and $\gamma_H = 0.50$ with $\mu^L = 0.4$, $\mu^M = 0.7$ and $\mu^H = 1.0$ the separating equilibrium dominates the pooling equilibrium ($7.0621 > 7.0523$). In this case the public program is not Pareto

$\gamma_M = 0.50$ and $\gamma_H = 0.50$							
0.6	μ^L	$\check{\mu}^L$	$\bar{\mu}^L$	μ^L_{SP}	$\mu^L_{MR(L)}$	$\mu^L_{MR(M)}$	$\mu^L_{MR(H)}$
s_P			117.80	117.80	**117.80**		
s^L	114.92	79.27		0	0	0	0
R			1.67	1.67	1.67		
R^L	1.97	1.72		1.76	1.76	1.78	1.79
V^L	7.5849	7.5415	**7.5507**	**7.5507**	**7.5507**	7.5497	7.5477
0.7	μ^M	$\check{\mu}^M$	$\bar{\mu}^M$	μ^M_{SP}	$\mu^M_{MR(L)}$	$\mu^M_{MR(M)}$	$\mu^M_{MR(H)}$
s_P						**126.26**	
s^M	124.86	98.41		0	0	0	0
R						1.69	
R^M	1.81	1.61		1.63	1.63	1.65	1.66
V^M	7.8729	7.8398	7.8627	7.8627	7.8627	**7.8639**	7.8633
0.8	μ^H	$\tilde{\mu}^H$	$\bar{\mu}^H$	μ^H_{SP}	$\mu^H_{MR(L)}$	$\mu^H_{MR(M)}$	$\mu^H_{MR(H)}$
s_P							**132.48**
s^H	132.31	133.92		0	0	0	0
R							1.70
R^H	1.71	1.58		1.55	1.55	1.56	1.57
V^H	8.1815	8.1632	8.1952	8.1952	8.1952	8.1987	**8.1994**

Table 7: Ideal amounts of public provided annuity with gamma M 0.50 and gamma H 0.50 - Case 0.6, 0.7 and 0.8

improving: the separating equilibrium dominates also the equilibrium with mandatory annuity program ($7.0621 > 7.0564$) and the reason is the large difference among the degrees of myopia μ^i of three groups.

In the case $\gamma_M = 0.50$ and $\gamma_H = 0.50$ with $\mu^L = 0.6$, $\mu^M = 0.7$ and $\mu^H = 0.8$ the pooling equilibrium dominates the separating equilibrium ($7.5507 > 7.5415$). In this case the public program is not Pareto improving: the equilibrium with mandatory annuity program doesn't dominate the pooling equilibrium ($7.5507 = 7.5507$) and the reasons are both the small difference among the degrees of myopia μ^i of three groups and the relative small number of medium-risk individuals and high-risk individuals with respect to low-risk individuals.

In the case $\gamma_M = 1.00$ and $\gamma_H = 1.00$ with $\mu^L = 0.6$, $\mu^M = 0.7$ and $\mu^H = 0.8$ the pooling equilibrium dominates the separating equilibrium ($7.5453 > 7.5415$). In this case the public program is Pareto improving: the equilibrium with mandatory annuity program dominates the pooling equilibrium ($7.5459 > 7.5453$) and the reason is the relative high number

$\gamma_M = 0.50$ and $\gamma_H = 0.50$							
0.4	μ^L	$\check\mu^L$	$\bar\mu^L$	μ^L_{SP}	$\mu^L_{MR(L)}$	$\mu^L_{MR(M)}$	$\mu^L_{MR(H)}$
s_P			84.76	78.56	**0**		
s^L	80.16	41.37		3.23	41.37	0	0
R			1.83	1.85	1.88		
R^L	2.82	2.41		2.37	2.41	2.40	2.42
V^L	7.1061	**7.0621**	7.0523	7.0564	**7.0621**	7.0372	7.0257
0.7	μ^M	$\check\mu^M$	$\bar\mu^M$	μ^M_{SP}	$\mu^M_{MR(L)}$	$\mu^M_{MR(M)}$	$\mu^M_{MR(H)}$
s_P						**124.24**	
s^M	124.86	88.39		11.16	88.39	0	0
R						1.87	
R^M	1.81	1.60		1.59	1.60	1.63	1.65
V^M	7.8729	7.8257	7.8774	7.8830	7.8257	**7.9090**	7.9057
1.0	μ^H	$\tilde\mu^H$	$\bar\mu^H$	μ^H_{SP}	$\mu^H_{MR(L)}$	$\mu^H_{MR(M)}$	$\mu^H_{MR(H)}$
s_P							**138.79**
s^H	142.74	144.06		50.41	144.06	0	0
R							1.89
R^H	1.58	1.49		1.46	1.49	1.44	1.47
V^H	8.8426	8.8240	8.9054	8.9179	8.8240	8.9836	**8.9885**

Table 8: Ideal amounts of public provided annuity with gamma M 0.50 and gamma H 0.50 - Case 0.4, 0.7 and 1.0

of medium-risk individuals and high-risk individuals with respect to low-risk individuals.

In the case $\gamma_M = 0.50$ and $\gamma_H = 0.50$ with $\mu^L = 0.5$, $\mu^M = 0.7$ and $\mu^H = 0.9$ the pooling equilibrium dominates the separating equilibrium $(7.2787 > 7.2751)$. In this case the public program is Pareto improving: the equilibrium with mandatory annuity program dominates the pooling equilibrium $(7.2804 > 7.2787)$ and the reason is the relative large difference among the degrees of myopia μ^i of three groups.

In the case $\gamma_M = 2.00$ and $\gamma_H = 2.00$ with $\mu^L = 0.6$, $\mu^M = 0.7$ and $\mu^H = 0.8$ the separating equilibrium dominates the pooling equilibrium $(7.5415 > 7.5411)$. In this case the public program is Pareto improving: the equilibrium with mandatory annuity program dominates the separating equilibrium $(7.5423 > 7.5415)$ and the reason is the small difference among the degrees of myopia μ^i of three groups.

Then we obtain that a pooling equilibrium can always be Pareto dominated by a public program, while a separating equilibrium dominates or is dominated by a public program.

5.2   Majority Rule

The ideal amount of the public provided annuity for agents of the group L (s_{MR}^L) that pay a rate of return R_{MR} is obtained by weighing up only the expected utility of members of the group L

$$\underset{s_{MR}^L,\check{s}^L,\check{h}^L,\check{s}^M,\check{h}^M,\tilde{s}^H,\tilde{h}^H}{Max} v^L\left(w - c\left(\check{h}^L\right) - s_{MR}^L - \check{s}^L\right) + \tag{9}$$

$$+ \mu^L \cdot p\left(\check{h}^L\right) \cdot v^L\left(s_{MR}^L \cdot R_{MR} + \check{s}^L \cdot R\left(p\left(\check{h}^L\right)\right)\right)$$

$$s.t. \; \check{h}^L \; \text{solves} \underset{\check{h}^L}{Max} \nu^L\left(w - c\left(\check{h}^L\right) - s_{MR}^L - \check{s}^L\right) +$$

$$+ \mu^L \cdot p\left(\check{h}^L\right) \cdot \nu^L\left(s_{MR}^L \cdot R_{MR} + \check{s}^L \cdot R\left(p\left(\check{h}^L\right)\right)\right) \quad \left[\lambda_{mh}^L\right]$$

$$s.t. \; \check{h}^M \; \text{solves} \underset{\check{h}^M}{Max} \nu^M\left(w - c\left(\check{h}^M\right) - s_{MR}^L - \check{s}^M\right) +$$

$$+ \mu^M \cdot p\left(\check{h}^M\right) \cdot \nu^M\left(s_{MR}^L \cdot R_{MR} + \check{s}^M \cdot R\left(p\left(\check{h}^M\right)\right)\right) \quad \left[\lambda_{mh}^M\right]$$

$$s.t. \; \tilde{h}^H \; \text{solves} \underset{\tilde{h}^H}{Max} \nu^H\left(w - c\left(\check{h}^H\right) - s_{MR}^L - \tilde{s}^H\right) +$$

$$+ \mu^H \cdot p\left(\check{h}^H\right) \cdot \nu^M\left(s_{MR}^L \cdot R_{MR} + \tilde{s}^H \cdot R\left(p\left(\tilde{h}^H\right)\right)\right) \quad \left[\lambda_{mh}^H\right]$$

$$\nu^M\left(w - c\left(\check{h}^M\right) - s_{MR}^L - \check{s}^M\right) +$$

$$+ \mu^M \cdot p\left(\check{h}^M\right) \cdot \nu^M\left(s_{MR}^L \cdot R_{MR} + \check{s}^M \cdot R\left(p\left(\check{h}^M\right)\right)\right)$$

$$= \hat{\nu}^M\left(w - c\left(\hat{h}^M\right) - s_{MR}^L - \check{s}^L\right) +$$

$$+ \mu^M \cdot p\left(\hat{h}^M\right) \cdot \hat{\nu}^M\left(s_{MR}^L \cdot R_{MR} + \check{s}^L \cdot R\left(p\left(\check{h}^L\right)\right)\right) \quad \left[\lambda_{as}^L\right]$$

$$\nu^H\left(w - c\left(\tilde{h}^H\right) - s_{MR}^L - \tilde{s}^H\right) +$$

$$+ \mu^H \cdot p\left(\tilde{h}^H\right) \cdot \nu^H\left(s_{MR}^L \cdot R_{MR} + \tilde{s}^H \cdot R\left(p\left(\tilde{h}^H\right)\right)\right)$$

$$= \hat{\nu}^H\left(w - c\left(\hat{h}^H\right) - s_{MR}^L - \check{s}^M\right) +$$

$$+ \mu^H \cdot p\left(\hat{h}^H\right) \cdot \hat{\nu}^H\left(s_{MR}^L \cdot R_{MR} + \check{s}^M \cdot R\left(p\left(\check{h}^M\right)\right)\right) \quad \left[\lambda_{as}^M\right]$$

$$0 \leq \; \check{h}^L, \check{h}^M, \tilde{h}^H \leq h^{\max}$$

$$s_{MR}^L, \check{s}^L, \check{s}^M, \tilde{s}^H \geq 0$$

The ideal amount of the public provided annuity for agents of the group M (s_{MR}^M) that pay a rate of return R_{MR} is obtained by weighing up only the expected utility of members of the group M

$$\underset{s_{MR}^M, \check{s}^L, \check{h}^L, \check{s}^M, \check{h}^M, \tilde{s}^H, \tilde{h}^H}{Max} v^M \left(w - c\left(\check{h}^M\right) - s_{MR}^M - \check{s}^M\right) + \tag{10}$$

$$+ \mu^M \cdot p\left(\check{h}^M\right) \cdot v^M \left(s_{MR}^M \cdot R_{MR} + \check{s}^M \cdot R\left(p\left(\check{h}^M\right)\right)\right)$$

$$s.t. \ \check{h}^L \ \text{solves} \underset{\check{h}^L}{Max} \nu^L \left(w - c\left(\check{h}^L\right) - s_{MR}^M - \check{s}^L\right) +$$

$$+ \mu^L \cdot p\left(\check{h}^L\right) \cdot \nu^L \left(s_{MR}^M \cdot R_{MR} + \check{s}^L \cdot R\left(p\left(\check{h}^L\right)\right)\right) \ \left[\lambda_{mh}^L\right]$$

$$s.t. \ \check{h}^M \ \text{solves} \underset{\check{h}^M}{Max} \nu^M \left(w - c\left(\check{h}^M\right) - s_{MR}^M - \check{s}^M\right) +$$

$$+ \mu^M \cdot p\left(\check{h}^M\right) \cdot \nu^M \left(s_{MR}^M \cdot R_{MR} + \check{s}^M \cdot R\left(p\left(\check{h}^M\right)\right)\right) \ \left[\lambda_{mh}^M\right]$$

$$s.t. \ \tilde{h}^H \ \text{solves} \underset{\tilde{h}^H}{Max} \nu^H \left(w - c\left(\check{h}^H\right) - s_{MR}^M - \tilde{s}^H\right) +$$

$$+ \mu^H \cdot p\left(\check{h}^H\right) \cdot \nu^M \left(s_{MR}^M \cdot R_{MR} + \tilde{s}^H \cdot R\left(p\left(\tilde{h}^H\right)\right)\right) \ \left[\lambda_{mh}^H\right]$$

$$\nu^M \left(w - c\left(\check{h}^M\right) - s_{MR}^M - \check{s}^M\right) +$$
$$+ \mu^M \cdot p\left(\check{h}^M\right) \cdot \nu^M \left(s_{MR}^M \cdot R_{MR} + \check{s}^M \cdot R\left(p\left(\check{h}^M\right)\right)\right)$$
$$= \hat{\nu}^M \left(w - c\left(\hat{h}^M\right) - s_{MR}^M - \check{s}^L\right) +$$
$$+ \mu^M \cdot p\left(\hat{h}^M\right) \cdot \hat{\nu}^M \left(s_{MR}^M \cdot R_{MR} + \check{s}^L \cdot R\left(p\left(\check{h}^L\right)\right)\right) \ \left[\lambda_{as}^L\right]$$

$$\nu^H \left(w - c\left(\tilde{h}^H\right) - s_{MR}^M - \tilde{s}^H\right) +$$
$$+ \mu^H \cdot p\left(\tilde{h}^H\right) \cdot \nu^H \left(s_{MR}^M \cdot R_{MR} + \tilde{s}^H \cdot R\left(p\left(\tilde{h}^H\right)\right)\right)$$
$$= \hat{\nu}^H \left(w - c\left(\hat{h}^H\right) - s_{MR}^M - \check{s}^M\right) +$$
$$+ \mu^H \cdot p\left(\hat{h}^H\right) \cdot \hat{\nu}^H \left(s_{MR}^M \cdot R_{MR} + \check{s}^M \cdot R\left(p\left(\check{h}^M\right)\right)\right) \ \left[\lambda_{as}^M\right]$$

$$0 \leq \ \check{h}^L, \check{h}^M, \tilde{h}^H \leq h^{\max}$$
$$s_{MR}^M, \check{s}^L, \check{s}^M, \tilde{s}^H \geq 0$$

The ideal amount of the public provided annuity for agents of the group H (s_{MR}^H) that pay a rate of return R_{MR} is obtained by weighing up only the expected utility of members of the group H

$$\underset{s_{MR}^H, \check{s}^L, \check{h}^L, \check{s}^M, \check{h}^M, \tilde{s}^H, \tilde{h}^H}{Max} \quad v^H\left(w - c\left(\tilde{h}^H\right) - s_{MR}^H - \tilde{s}^H\right) + \qquad (11)$$

$$+ \mu^H \cdot p\left(\tilde{h}^H\right) \cdot v^H\left(s_{MR}^H \cdot R_{MR} + \tilde{s}^H \cdot R\left(p\left(\tilde{h}^H\right)\right)\right)$$

$s.t.$ $\check{h}^L$ solves $\underset{\check{h}^L}{Max} \, \nu^L\left(w - c\left(\check{h}^L\right) - s_{MR}^H - \check{s}^L\right) +$

$$+ \mu^L \cdot p\left(\check{h}^L\right) \cdot \nu^L\left(s_{MR}^H \cdot R_{MR} + \check{s}^L \cdot R\left(p\left(\check{h}^L\right)\right)\right) \quad \left[\lambda_{mh}^L\right]$$

$s.t.$ $\check{h}^M$ solves $\underset{\check{h}^M}{Max} \, \nu^M\left(w - c\left(\check{h}^M\right) - s_{MR}^H - \check{s}^M\right) +$

$$+ \mu^M \cdot p\left(\check{h}^M\right) \cdot \nu^M\left(s_{MR}^H \cdot R_{MR} + \check{s}^M \cdot R\left(p\left(\check{h}^M\right)\right)\right) \quad \left[\lambda_{mh}^M\right]$$

$s.t.$ $\tilde{h}^H$ solves $\underset{\tilde{h}^H}{Max} \, \nu^H\left(w - c\left(\check{h}^H\right) - s_{MR}^H - \tilde{s}^H\right) +$

$$+ \mu^H \cdot p\left(\check{h}^H\right) \cdot \nu^M\left(s_{MR}^H \cdot R_{MR} + \tilde{s}^H \cdot R\left(p\left(\tilde{h}^H\right)\right)\right) \quad \left[\lambda_{mh}^H\right]$$

$$\nu^M\left(w - c\left(\check{h}^M\right) - s_{MR}^H - \check{s}^M\right) +$$
$$+ \mu^M \cdot p\left(\check{h}^M\right) \cdot \nu^M\left(s_{MR}^H \cdot R_{MR} + \check{s}^M \cdot R\left(p\left(\check{h}^M\right)\right)\right)$$
$$= \hat{\nu}^M\left(w - c\left(\hat{h}^M\right) - s_{MR}^H - \check{s}^L\right) +$$
$$+ \mu^M \cdot p\left(\hat{h}^M\right) \cdot \hat{\nu}^M\left(s_{MR}^H \cdot R_{MR} + \check{s}^L \cdot R\left(p\left(\check{h}^L\right)\right)\right) \quad \left[\lambda_{as}^L\right]$$

$$\nu^H\left(w - c\left(\tilde{h}^H\right) - s_{MR}^H - \tilde{s}^H\right) +$$
$$+ \mu^H \cdot p\left(\tilde{h}^H\right) \cdot \nu^H\left(s_{MR}^H \cdot R_{MR} + \tilde{s}^H \cdot R\left(p\left(\tilde{h}^H\right)\right)\right)$$
$$= \hat{\nu}^H\left(w - c\left(\hat{h}^H\right) - s_{MR}^H - \check{s}^M\right) +$$
$$+ \mu^H \cdot p\left(\hat{h}^H\right) \cdot \hat{\nu}^H\left(s_{MR}^H \cdot R_{MR} + \check{s}^M \cdot R\left(p\left(\check{h}^M\right)\right)\right) \quad \left[\lambda_{as}^M\right]$$

$$0 \leq \check{h}^L, \check{h}^M, \tilde{h}^H \leq h^{\max}$$
$$s_{MR}^H, \check{s}^L, \check{s}^M, \tilde{s}^H \geq 0$$

$\gamma_M = 1.00$ and $\gamma_H = 1.00$							
0.6	μ^L	$\check{\mu}^L$	$\bar{\mu}^L$	μ^L_{SP}	$\mu^L_{MR(L)}$	$\mu^L_{MR(M)}$	$\mu^L_{MR(H)}$
s_P			118.07	109.60	**109.60**		
s^L	114.92	79.27		0	0	0	0
R			1.65	1.64	1.64		
R^L	1.97	1.72		1.76	1.76	1.78	1.79
V^L	7.5849	7.5415	7.5453	**7.5459**	**7.5459**	7.5443	7.5423
0.7	μ^M	$\check{\mu}^M$	$\bar{\mu}^M$	μ^M_{SP}	$\mu^M_{MR(L)}$	$\mu^M_{MR(M)}$	$\mu^M_{MR(H)}$
s_P						**126.56**	
s^M	124.86	98.41		0	0	0	0
R						1.66	
R^M	1.81	1.61		1.63	1.63	1.65	1.66
V^M	7.8729	7.8398	7.8560	7.8542	7.8542	**7.8572**	7.8566
0.8	μ^H	$\tilde{\mu}^H$	$\bar{\mu}^H$	μ^H_{SP}	$\mu^H_{MR(L)}$	$\mu^H_{MR(M)}$	$\mu^H_{MR(H)}$
s_P							**132.80**
s^H	132.31	133.92		15.10	15.10	0	0
R							1.67
R^H	1.71	1.58		1.56	1.56	1.56	1.57
V^H	8.1815	8.1632	8.1873	8.1831	8.1831	8.1907	**8.1915**

Table 9: Ideal amounts of public provided annuity with gamma M 1.00 and gamma H 1.00 - Case 0.6, 0.7 and 0.8

The results of simulation are summarized in tables 8, 7, 9, 10 and 11. In all cases obviously we have $s^L_{MR} < s^M_{MR} < s^H_{MR}$: the ideal amount of the public provided annuity of individual L is lower than this one of individual M which is lower than this one of individual H.

In the case $\gamma_M = 0.50$ and $\gamma_H = 0.50$ with $\mu^L = 0.4$, $\mu^M = 0.7$ and $\mu^H = 1.0$ the separating equilibrium dominates the pooling equilibrium $(7.0621 > 7.0523)$. In this case

- the public program decided by Social Planner is not Pareto improving: the separating equilibrium dominates also the equilibrium with mandatory annuity program $(7.0621 > 7.0564)$ and the reason is the large difference among the degrees of myopia μ^i of three groups;

- the ideal amount of the public provided annuity for agents of group L (s^L_{MR}) is equal to **0**.

In the case $\gamma_M = 0.50$ and $\gamma_H = 0.50$ with $\mu^L = 0.6$, $\mu^M = 0.7$ and $\mu^H = 0.8$ the pooling equilibrium dominates the separating equilibrium $(7.5507 > 7.5415)$. In this case

- the public program decided by Social Planner is not Pareto improving: the equilibrium with mandatory annuity program doesn't dominate the pooling equilibrium $(7.5507 = 7.5507)$ and the reasons are both the small difference among the degrees of myopia μ^i of three groups and the relative small number of medium-risk individuals and high-risk individuals with respect to low-risk individuals;

- the ideal amount of the public provided annuity for agents of group L (s_{MR}^L) is equal to the **amount of the pooling contract** (117.80).

In the case $\gamma_M = 1.00$ and $\gamma_H = 1.00$ with $\mu^L = 0.6$, $\mu^M = 0.7$ and $\mu^H = 0.8$ the pooling equilibrium dominates the separating equilibrium $(7.5453 > 7.5415)$. In this case

- the public program decided by Social Planner is Pareto improving: the equilibrium with mandatory annuity program dominates the pooling equilibrium $(7.5459 > 7.5453)$ and the reason is the relative high number of medium-risk individuals and high-risk individuals with respect to low-risk individuals;

- the ideal amount of the public provided annuity for agents of group L (s_{MR}^L) is equal to the **amount of the pooling contract and of the public program decided by Social Planner** (109.60).

In the case $\gamma_M = 0.50$ and $\gamma_H = 0.50$ with $\mu^L = 0.5$, $\mu^M = 0.7$ and $\mu^H = 0.9$ the pooling equilibrium dominates the separating equilibrium $(7.2787 > 7.2751)$. In this case

- the public program decided by Social Planner is Pareto improving: the equilibrium with mandatory annuity program dominates the pooling equilibrium $(7.2804 > 7.2787)$ and the reason is the relative large difference among the degrees of myopia μ^i of three groups;

- the ideal amount of the public provided annuity for agents of group L (s_{MR}^L) is different and **Pareto improving** with respect to the public program decided by Social Planner.

In the case $\gamma_M = 2.00$ and $\gamma_H = 2.00$ with $\mu^L = 0.6$, $\mu^M = 0.7$ and $\mu^H = 0.8$ the separating equilibrium dominates the pooling equilibrium $(7.5415 > 7.5411)$. In this case

$\gamma_M = 0.50$ and $\gamma_H = 0.50$							
0.5	μ^L	$\check{\mu}^L$	$\bar{\mu}^L$	μ^L_{SP}	$\mu^L_{MR(L)}$	$\mu^L_{MR(M)}$	$\mu^L_{MR(H)}$
s_P			104.98	98.17	**87.49**		
s^L	101.02	59.87		0.77	4.83	0	0
R			1.72	1.73	1.73		
R^L	2.24	1.93		1.95	1.95	1.99	2.00
V^L	7.3251	7.2751	7.2787	7.2804	**7.2806**	7.2737	7.2674
0.7	μ^M	$\check{\mu}^M$	$\bar{\mu}^M$	μ^M_{SP}	$\mu^M_{MR(L)}$	$\mu^M_{MR(M)}$	$\mu^M_{MR(H)}$
s_P						**125.52**	
s^M	124.86	91.96		2.25	11.16	0	0
$\bar{R}$						1.75	
R^M	1.81	1.61		1.61	1.61	1.64	1.66
V^M	7.8729	7.8314	7.8726	7.8707	7.8660	**7.8803**	7.8784
0.9	μ^H	$\tilde{\mu}^H$	$\bar{\mu}^H$	μ^H_{SP}	$\mu^H_{MR(L)}$	$\mu^H_{MR(M)}$	$\mu^H_{MR(H)}$
s_P							**136.46**
s^H	138.10	139.56		27.60	38.48	0	0
R							1.77
R^H	1.64	1.53		1.50	1.50	1.50	1.51
V^H	8.5059	8.4874	8.5517	8.5469	8.5394	8.5721	**8.5746**

Table 10: Ideal amounts of public provided annuity with gamma M 0.50 and gamma H 0.50 - Case 0.5, 0.7 and 0.9

- the public program is Pareto improving: the equilibrium with mandatory annuity program dominates the separating equilibrium ($7.5423 > 7.5415$) and the reason is the small difference among the degrees of myopia μ^i of three groups;

- the ideal amount of the public provided annuity for agents of group L (s^L_{MR}) is different and **Pareto improving** with respect to the public program decided by Social Planner.

6   Conclusive Policy Recommendations

Given the public provided annuity (s_P, R_P), competitive firms offer separating contracts $(s^i, R(p(h^i)))$ to satisfy the residual demand for annuities. Since the existence of medium-risk individuals M and high-risk individuals H affects the expected utility of low-risk individuals L through the rate of return of the public provided annuity R_P, the maximization of the expected utilities of three individual types cannot be independent.

$\gamma_M = 2.00$ and $\gamma_H = 2.00$							
0.6	μ^L	$\check{\mu}^L$	$\bar{\mu}^L$	μ^L_{SP}	$\mu^L_{MR(L)}$	$\mu^L_{MR(M)}$	$\mu^L_{MR(H)}$
s_P			118.28	108.45	**89.02**		
s^L	114.92	79.27		0.51	10.97	0	0
R			1.63	1.63	1.62		
R^L	1.97	1.72		1.76	1.75	1.78	1.79
V^L	7.5849	7.5415	7.5411	7.5423	**7.5425**	7.5401	7.5381
0.7	μ^M	$\check{\mu}^M$	$\bar{\mu}^M$	μ^M_{SP}	$\mu^M_{MR(L)}$	$\mu^M_{MR(M)}$	$\mu^M_{MR(H)}$
s_P						**126.79**	
s^M	124.86	98.41		1.03	16.77	0	0
$\bar{R}$						1.64	
R^M	1.81	1.61		1.63	1.62	1.65	1.66
V^M	7.8729	7.8398	7.8507	7.8495	7.8472	**7.8519**	7.8513
0.8	μ^H	$\tilde{\mu}^H$	$\bar{\mu}^H$	μ^H_{SP}	$\mu^H_{MR(L)}$	$\mu^H_{MR(M)}$	$\mu^H_{MR(H)}$
s_P							**133.05**
s^H	132.31	133.92		18.80	39.11	0	0
R							1.65
R^H	1.71	1.58		1.57	1.57	1.56	1.57
V^H	8.1815	8.1632	8.1809	8.1774	8.1738	8.1844	**8.1851**

Table 11: Ideal amounts of public provided annuity with gamma M 2.00 and gamma H 2.00 - Case 0.6, 0.7 and 0.8

Then in the private market competitive firms offer the utility-maximizing actuarially fair annuity contracts $(s^i, R(p(h^i)))$ by weighing up only the expected utility of members of group L.

The results of simulation are summarized in tables 12, 13, 14, 15 and 16 where "V^L" is the utility of individuals of group L who don't satisfy the residual demand for annuity in the private market.

In the case $\gamma_M = 0.50$ and $\gamma_H = 0.50$ with $\mu^L = 0.4$, $\mu^M = 0.7$ and $\mu^H = 1.0$ (table 12) the separating equilibrium dominates the pooling equilibrium $(7.0621 > 7.0523)$. In this case

- the public program decided by Social Planner is not Pareto improving: the separating equilibrium dominates also the equilibrium with mandatory annuity program $(7.0621 > 7.0564)$ and the reason is the large difference among the degrees of myopia μ^i of three groups;

- the ideal amount of public provided annuity for agents of group L (s^L_{MR}) is equal to **0**.

If $s_{MR}^L \leq s_P < s_{SP}$ individuals of groups M and H support an increase of the amount of the public provided annuity and individuals of group L oppose this increase. Since $\gamma_M = 0.50$ and $\gamma_H = 0.50$ (the size of coalition of groups M and H is equal to the size of group L) the amount of the public provided annuity doesn't change.

If $s_{SP} \leq s_P < s_{MR}^M$ individuals of groups M and H support an increase of the amount of the public provided annuity and individuals of group L oppose this increase. Since $\gamma_M = 0.50$ and $\gamma_H = 0.50$ (the size of coalition of groups M and H is equal to the size of group L) the amount of the public provided annuity doesn't change.

If $s_P = s_{MR}^M$ individuals of group H support an increase of the amount of the public provided annuity and individuals of groups L and M oppose this increase. Since $\gamma_M = 0.50$ and $\gamma_H = 0.50$ (the size of coalition of groups L and M is equal to the size of group H) the amount of the public provided annuity doesn't change.

If $s_{MR}^M < s_P \leq s_{MR}^H$ individuals of groups L and M support a decrease of the amount of the public provided annuity and individuals of group H oppose this decrease. Since $\gamma_M = 0.50$ and $\gamma_H = 0.50$ (the size of coalition of groups L and M is larger than the size of group H) the amount of the public provided annuity decreases at $\mathbf{s_{MR}^M}$.

If $s_P > s_{MR}^H$ individuals of groups L, M and H support a decrease of the amount of the public provided annuity and then the amount of the public provided annuity decreases at $\mathbf{s_{MR}^H}$.

Then

- if $0 \leq s_P \leq s_{MR}^M$ the amount of the public provided annuity doesn't change and then individuals of group L can be better off or worse off with respect to the case in which Social Planner decides the amount of the public provided annuity;

- if $s_P > s_{MR}^M$ the amount of the public provided annuity decreases at $\mathbf{s_{MR}^M}$ and then individuals of group L are worse off with respect to the case in which Social Planner decides the amount of the public provided annuity.

In the case $\gamma_M = 0.50$ and $\gamma_H = 0.50$ with $\mu^L = 0.6$, $\mu^M = 0.7$ and $\mu^H = 0.8$ (table 13) the pooling equilibrium dominates the separating equilibrium ($7.5507 > 7.5415$). In this case

- the public program decided by Social Planner is not Pareto improving: the equilibrium with mandatory annuity program doesn't dominate the pooling equilibrium ($7.5507 = 7.5507$) and the reasons are both the small difference among the degrees of myopia

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	$\mu^L = 0.4$			$\mu^M = 0.7$		$\mu^H = 1.0$	
s_P	s^L	V^L	"V^L"	s^M	V^M	s^H	V^H
$s_{MR}^L = 0$	41.36	**7.0621**	6.3711	88.39	7.8257	144.06	8.8240
50	14.78	7.0600	7.0344	38.28	7.8622	82.46	8.8832
$s_{SP} = 79$	3.23	7.0564	**7.0543**	11.16	7.8830	50.41	8.9179
100	0	7.0497	7.0497	0	7.8979	9.79	8.9479
$s_{MR}^M = 124$	0	7.0372	7.0372	0	**7.9090**	0	8.9836
130	0	7.0330	7.0330	0	7.9084	0	8.9868
$s_{MR}^H = 139$	0	7.0257	7.0257	0	7.9057	0	**8.9885**
150	0	7.0152	7.0152	0	7.8991	0	8.9859

<center>$\gamma_M = 0.50$ and $\gamma_H = 0.50$</center>

Table 12: Public provided annuity with gamma M 0.50 and gamma H 0.50 - Case 0.4, 0.7 and 1.0

μ^i of three groups and the relative small number of medium-risk individuals and high-risk individuals with respect to low-risk individuals;

- the ideal amount of the public provided annuity for agents of group L (s_{MR}^L) is equal to the amount of the **pooling contract** (117.80).

If $0 \leq s_P < s_{MR}^L = s_{SP}$ individuals of groups L, M and H support an increase of the amount of the public provided annuity and then the amount of the public provided annuity increases at $\mathbf{s_{MR}^L = s_{SP}}$.

If $s_{MR}^L = s_{SP} \leq s_P < s_{MR}^M$ individuals of groups M and H support an increase of the amount of the public provided annuity and individuals of group L oppose this increase. Since $\gamma_M = 0.50$ and $\gamma_H = 0.50$ (the size of coalition of groups M and H is equal to the size of group L) the amount of the public provided annuity doesn't change.

If $s_P = s_{MR}^M$ individuals of group H support an increase of the amount of the public provided annuity and individuals of groups L and M oppose this increase. Since $\gamma_M = 0.50$ and $\gamma_H = 0.50$ (the size of coalition of groups L and M is equal to the size of group H) the amount of the public provided annuity doesn't change.

If $s_{MR}^M \leq s_P < s_{MR}^H$ individuals of groups L and M support a decrease of the amount of the public provided annuity and individuals of group H oppose this decrease. Since $\gamma_M = 0.50$ and $\gamma_H = 0.50$ (the size of coalition of groups L and M is larger than the size of group H) the amount of the public provided annuity decreases at $\mathbf{s_{MR}^M}$.

If $s_P > s_{MR}^H$ individuals of groups L, M and H support a decrease of the amount of the public provided annuity and then the amount of the public provided annuity decreases at $\mathbf{s_{MR}^H}$.

Then

- if $0 \le s_P < s_{MR}^L = s_{SP}$ the amount of the public provided annuity increases at $\mathbf{s_{MR}^L} = \mathbf{s_{SP}}$ and the well-being of individuals of group L doesn't change with respect to the case in which Social Planner decides the amount of the public provided annuity;

- if $s_{MR}^L = s_{SP} \le s_P \le s_{MR}^M$ the amount of the public provided annuity doesn't change and then individuals of group L can be better off or worse off with respect to the case in which Social Planner decides the amount of the public provided annuity;

- if $s_P > s_{MR}^M$ the amount of the public provided annuity decreases at $\mathbf{s_{MR}^M}$ and then individuals of group L are worse off with respect to the case in which Social Planner decides the amount of the public provided annuity.

In the case $\gamma_M = 1.00$ and $\gamma_H = 1.00$ with $\mu^L = 0.6$, $\mu^M = 0.7$ and $\mu^H = 0.8$ (table 14) the pooling equilibrium dominates the separating equilibrium ($7.5453 > 7.5415$). In this case

- the public program decided by Social Planner is Pareto improving: the equilibrium with mandatory annuity program dominates the pooling equilibrium ($7.5459 > 7.5453$) and the reason is the relative high number of medium-risk individuals and high-risk individuals with respect to low-risk individuals;

- the ideal amount of the public provided annuity for agents of group L (s_{MR}^L) is equal to the **amount of the pooling contract and of the public program decided by Social Planner** (109.60).

If $0 \le s_P < s_{MR}^L = s_{SP}$ individuals of groups L, M and H support an increase of the amount of the public provided annuity and then the amount of the public provided annuity increases at $\mathbf{s_{MR}^L} = \mathbf{s_{SP}}$.

If $s_{MR}^L = s_{SP} \le s_P < s_{MR}^M$ individuals of groups M and H support an increase of the amount of the public provided annuity and individuals of group L oppose this increase. Since $\gamma_M = 1.00$ and $\gamma_H = 1.00$ (the size of coalition of groups M and H is larger then the size of group L) the amount of the public provided annuity increases at $\mathbf{s_{MR}^M}$.

$\gamma_M = 0.50$ and $\gamma_H = 0.50$							
	$\mu^L = 0.6$			$\mu^M = 0.7$		$\mu^H = 0.8$	
s_P	s^L	V^L	"V^L"	s^M	V^M	s^H	V^H
0	79.27	7.5415	6.0364	98.41	7.8398	133.92	8.1632
50	38.04	7.5454	7.4507	50.56	7.8474	78.50	8.1728
75	19.34	7.5476	7.5222	27.74	7.8520	50.49	8.1789
$s^L_{MR} = 118$	0	**7.5507**	**7.5507**	0	7.8627	0	8.1952
120	0	7.5506	7.5506	0	7.8632	0	8.1964
$s^M_{MR} = 126$	0	7.5497	7.5497	0	**7.8639**	0	8.1987
130	0	7.5486	7.5486	0	7.8636	0	8.1993
$s^H_{MR} = 132$	0	7.5477	7.5486	0	7.8633	0	**8.1994**
150	0	7.5373	7.5373	0	7.8555	0	8.1942

Table 13: Public provided annuity with gamma M 0.50 and gamma H 0.50 - Case 0.6, 0.7 and 0.8

If $s_P = s^M_{MR}$ individuals of group H support an increase of the amount of the public provided annuity and individuals of groups L and M oppose this increase. Since $\gamma_M = 1.00$ and $\gamma_H = 1.00$ (the size of coalition of groups L and M is larger then the size of group H) the amount of the public provided annuity doesn't change.

If $s^M_{MR} < s_P \leq s^H_{MR}$ individuals of groups L and M support a decrease of the amount of the public provided annuity and individuals of group H oppose this decrease. Since $\gamma_M = 1.00$ and $\gamma_H = 1.00$ (the size of coalition of groups L and M is larger than the size of group H) the amount of the public provided annuity decreases at $\mathbf{s^M_{MR}}$.

If $s^H_{MR} < s_P$ individuals of groups L, M and H support a decrease of the amount of the public provided annuity and then the amount of the public provided annuity decreases at $\mathbf{s^H_{MR}}$.

Then

- if $0 \leq s_P < s^M_{MR}$ the amount of the public provided annuity increases at $\mathbf{s^M_{MR}}$ and then individuals of group L are worse off with respect to the case in which Social Planner decides the amount of the public provided annuity;

- if $s_P = s^M_{MR}$ the amount of the public provided annuity doesn't change and then individuals of group L are worse off with respect to the case in which Social Planner decides the amount of the public provided annuity;

- if $s_P > s^M_{MR}$ the amount of the public provided annuity decreases at $\mathbf{s^M_{MR}}$ and then individuals of group L are worse off with respect to

$\gamma_M = 1.00$ and $\gamma_H = 1.00$							
	$\mu^L = 0.6$			$\mu^M = 0.7$		$\mu^H = 0.8$	
s_P	s^L	V^L	"V^L"	s^M	V^M	s^H	V^H
0	79.27	7.5415	6.0364	98.41	7.8398	133.92	8.1632
105	1.77	7.5458	7.5452	3.40	7.8533	19.97	8.1817
$s_{MR}^L = 110$	0	**7.5459**	**7.5459**	0	7.8542	15.10	8.1831
120	0	7.5453	7.5453	0	7.8565	0	8.1883
$s_{MR}^M = 127$	0	7.5443	7.5443	0	**7.8572**	0	8.1907
130	0	7.5433	7.5433	0	7.8570	0	8.1913
$s_{MR}^H = 133$	0	7.5423	7.5423	0	7.8566	0	**8.1915**
150	0	6.4507	6.4507	0	7.8490	0	8.1864

Table 14: Public provided annuity with gamma M 1.00 and gamma H 1.00 - Case 0.6, 0.7 and 0.8

the case in which Social Planner decides the amount of the public provided annuity.

In the case $\gamma_M = 0.50$ and $\gamma_H = 0.50$ with $\mu^L = 0.5$, $\mu^M = 0.7$ and $\mu^H = 0.9$ (table 15) the pooling equilibrium dominates the separating equilibrium (7.2787 > 7.2751). In this case

- the public program decided by Social Planner is Pareto improving: the equilibrium with mandatory annuity program dominates the pooling equilibrium (7.2804 > 7.2787) and the reason is the relative large difference among the degrees of myopia μ^i of three groups;

- the ideal amount of the public provided annuity for agents of group L (s_{MR}^L) is different and **Pareto improving** with respect to the public program decided by Social Planner.

If $0 \leq s_P < s_{MR}^L$ individuals of groups L, M and H support an increase of the amount of the public provided annuity and then the amount of the public provided annuity increases at $\mathbf{s_{MR}^L}$.

If $s_{MR}^L \leq s_P < s_{SP}$ individuals of groups M and H support an increase of the amount of the public provided annuity and individuals of group L oppose this increase. Since $\gamma_M = 0.50$ and $\gamma_H = 0.50$ (the size of coalition of groups M and H is equal to the size of group L) the amount of the public provided annuity doesn't change.

If $s_{SP} \leq s_P < s_{MR}^M$ individuals of groups M and H support an increase of the amount of the public provided annuity and individuals

		$\mu^L = 0.5$		$\mu^M = 0.7$		$\mu^H = 0.9$	
				$\gamma_M = 0.50$ and $\gamma_H = 0.50$			
s_P	s^L	V^L	"V^L"	s^M	V^M	s^H	V^H
0	59.87	7.2751	6.1760	91.96	7.8314	139.56	8.4874
$s^L_{MR} = 87$	4.83	**7.2806**	7.2773	11.16	7.8660	38.48	8.5394
90	3.77	7.2806	7.2783	9.07	7.8671	35.79	8.5412
$s_{SP} = 98$	0.77	7.2804	**7.2801**	2.25	7.8707	27.60	8.5469
110	0.22	7.2785	7.2785	0.66	7.8760	25.96	8.5459
$s^M_{MR} = 126$	0	7.2737	7.2737	0	**7.8803**	0	8.5721
130	0	7.2714	7.2714	0	7.8800	0	8.5737
$s^H_{MR} = 136$	0	7.2674	7.2674	0	7.8784	0	**8.5746**
150	0	7.2568	7.2568	0	7.8714	0	8.5711

Table 15: Public provided annuity with gamma M 0.50 and gamma H 0.50 - Case 0.5, 0.7 and 0.9

of group L oppose this increase. Since $\gamma_M = 0.50$ and $\gamma_H = 0.50$ (the size of coalition of groups M and H is equal to the size of group L) the amount of the public provided annuity doesn't change.

If $s_P = s^M_{MR}$ individuals of group H support an increase of the amount of public provided annuity and individuals of groups L and M oppose this increase. Since $\gamma_M = 0.50$ and $\gamma_H = 0.50$ (the size of coalition of groups L and M is equal to the size of group H) the amount of the public provided annuity doesn't change.

If $s^M_{MR} < s_P \leq s^H_{MR}$ individuals of groups L and M support a decrease of the amount of the public provided annuity and individuals of group H oppose this decrease. Since $\gamma_M = 0.50$ and $\gamma_H = 0.50$ (the size of coalition of groups L and M is larger than the size of group H) the amount of the public provided annuity decreases at $\mathbf{s^M_{MR}}$.

If $s_P > s^H_{MR}$ individuals of groups L, M and H support a decrease of the amount of the public provided annuity and then the amount of the public provided annuity decrease at $\mathbf{s^H_{MR}}$.

Then

- if $0 \leq s_P < s^L_{MR}$ the amount of the public provided annuity increases at $\mathbf{s^L_{MR}}$ and then individuals of group L are better off with respect to the case in which Social Planner decides the amount of the public provided annuity;

$\gamma_M = 2.00$ and $\gamma_H = 2.00$							
	$\mu^L = 0.6$			$\mu^M = 0.7$		$\mu^H = 0.8$	
s_P	s^L	V^L	"V^L"	s^M	V^M	s^H	V^H
0	79.27	7.5415	6.0364	98.41	7.8398	133.92	8.1632
$s_{MR}^L = 89$	10.97	**7.5425**	7.5328	16.77	7.8472	39.11	8.1738
100	4.54	7.5425	7.5400	7.70	7.8484	27.50	8.1757
$s_{SP} = 108$	0.51	7.5423	**7.5421**	1.03	7.8495	18.80	8.1774
120	0	7.5411	7.5411	0	7.8511	0	8.1819
$s_{MR}^M = 127$	0	7.5401	7.5401	0	**7.8519**	0	8.1844
130	0	7.5392	7.5392	0	7.8518	0	8.1850
$s_{MR}^H = 133$	0	7.5381	7.5381	0	7.8513	0	**8.1851**
150	0	7.8814	7.8814	0	7.8439	0	8.1803

Table 16: Public provided annuity with gamma M 2.00 and gamma H 2.00 - Case 0.6, 0.7 and 0.8

- if $s_{MR}^L \leq s_P \leq s_{MR}^M$ the amount of the public provided annuity doesn't change and then individuals of group L can be better off or worse off with respect to the case in which Social Planner decides the amount of the public provided annuity;

- if $s_P > s_{MR}^M$ the amount of the public provided annuity decrease at $\mathbf{s_{MR}^M}$ and then individuals of group L are worse off with respect to the case in which Social Planner decides the amount of the public provided annuity.

In the case $\gamma_M = 2.00$ and $\gamma_H = 2.00$ with $\mu^L = 0.6$, $\mu^M = 0.7$ and $\mu^H = 0.8$ (table 16) the separating equilibrium dominates the pooling equilibrium ($7.5415 > 7.5411$). In this case

- the public program is Pareto improving: the equilibrium with mandatory annuity program dominates the separating equilibrium ($7.5423 > 7.5415$) and the reason is the small difference among the degrees of myopia μ^i of three groups;

- the ideal amount of the public provided annuity for agents of group L $\left(s_{MR}^L\right)$ is different and **Pareto improving** with respect to the public program decided by Social Planner.

If $0 \leq s_P < s_{MR}^L$ individuals of groups L, M and H support an increase of the amount of public provided annuity and then the amount of the public provided annuity increases at $\mathbf{s_{MR}^L}$.

If $s_{MR}^L \leq s_P < s_{MR}^M$ individuals of groups M and H support an increase of the amount of the public provided annuity and individuals of group L oppose this increase. Since $\gamma_M = 2.00$ and $\gamma_H = 2.00$ (the size of coalition of groups M and H is larger then the size of group L) the amount of the public provided annuity increases at $\mathbf{s_{MR}^M}$.

If $s_P = s_{MR}^M$ individuals of group H support an increase of the amount of the public provided annuity and individuals of groups L and M oppose this increase. Since $\gamma_M = 2.00$ and $\gamma_H = 2.00$ (the size of coalition of groups L and M is equal to the size of group H) the amount of the public provided annuity doesn't change.

If $s_{MR}^M < s_P \leq s_{MR}^H$ individuals of groups L and M support a decrease of the amount of the public provided annuity and individuals of group H oppose this decrease. Since $\gamma_M = 2.00$ and $\gamma_H = 2.00$ (the size of coalition of groups L and M is larger than the size of group H) the amount of the public provided annuity decreases at $\mathbf{s_{MR}^M}$.

If $s_P > s_{MR}^H$ individuals of groups L, M and H support a decrease of the amount of the public provided annuity and then the amount of the public provided annuity decrease at $\mathbf{s_{MR}^H}$.

Then

- if $0 \leq s_P < s_{MR}^M$ the amount of the public provided annuity increases at $\mathbf{s_{MR}^M}$ and then individuals of group L are worse off with respect to the case in which Social Planner decides the amount of the public provided annuity;

- if $s_P = s_{MR}^M$ the amount of the public provided annuity doesn't change and then individuals of group L are worse off with respect to the case in which Social Planner decides the amount of the public provided annuity;

- if $s_P > s_{MR}^M$ the amount of the public provided annuity decrease at $\mathbf{s_{MR}^M}$ and then individuals of group L are worse off with respect to the case in which Social Planner decides the amount of the public provided annuity.

Social Planner decides the level of the public provided annuity which maximizes the utility of individuals of group L independently from the existence of the private market: he maximizes the utility of individuals of group L who don't satisfy the residual demand for annuity in the private market. For this reason the ideal amount of the public provided annuity for agents of group L is generally different from the amount chosen by Social Planner. However the amount of the public provided annuity decided by individuals through a Majority Rule can be not the

ideal amount of the public provided annuity for agents of group L: the result depends on the relative size of three groups.

References

[1] ANDERBERG, D. Determining the mix of public and private provision of insurance by majority rule. *European Journal of Political Economy 15* (1999), 417–440.

[2] CORREA, H. *A Conceptual Model of Health.* in Salkever D., I. Siralgeldin and A. Sorkin, Jai Press Inc., London, 1983.

[3] DAVIES, J. D., AND KUHN, P. Social security, longevity, and moral hazard. *Journal of Public Economics 49* (1992), 91–106.

[4] ECKESTEIN, Z., EICHENBAUM, M., AND PELED, D. Uncertain lifetimes and the welfare enhancing properties of annuity markets and social security. *Journal of Public Economics 26* (1985), 303–326.

[5] FELDSTEIN, M. S. The optimal level of social security benefits. *Quarterly Journal of Economics 100* (1985), 303–320.

[6] MYLES, G. D. *Public Economics.* Cambridge University Press, Cambridge, 1995.

[7] PLATONI, S. Adverse selection and moral hazard in the annuity markets. *Working Paper* (2004).

[8] RILEY, J. G. Informational equilibrium. *Econometrica 47* (1979a), 331–359.

[9] RILEY, J. G. Noncooperative equilibrium and market signaling. *American Economic Review 69* (1979b), 303–307.

[10] ROTHSCHILD, M., AND STIGLITZ, J. Equilibrium in competitive insurance markets: An essay on the economics of imperfect information. *Quarterly Journal of Economics 90* (1976), 629–649.

[11] SAMUELSON, P. A. An exact consumption-loan model of interest with or without the social contrivance of money. *Journal of Political Economy 66* (1958), 467–482.

[12] WILSON, C. A model of insurance markets with incomplete information. *Journal of Economic Theory 16* (1977), 167–207.