Interactions between Nominal and Real Rigidities with Microfoundations, and Monetary Policy

Marco Arnone and Diego Scalise
Catholic University and CeMaFiR (Milan, Italy)

Abstract

This paper persents a dynamic stochastic general equilibrium model with imperfections in the labor market. We introduce hiring costs for firms together with a bargaining process for the real wage in what is usually known as New Keynesian model. We analyze both the flexible price equilibrium and the Calvo staggered-price adjustment mechanism. In this context in the flexible price allocation there is persistent and equilibrium unemployment, the GDP shows inertia and regulation in the labor, and product market can influence the level of GDP. In the staggered-price equilibrium we obtain a revised version of the New Keynesian Phillips curve that helps to overcome some of its shortcomings, and a linkage between the degree of imperfection in the labor market and the effectiveness of monetary policy.

1 Introduction[1]

As pointed out by Blanchard and Gali (2005) the New Neoclassical Synthesis equlibrium is characterized by the lack of unvolountary unemployment; both in the flexible and sticky prices allocation workers are always on their labor supply curve. In this class of models the labor market is simple: as the classical theory predicts, it is a decentralized

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market where all buyers and sellers of labor meet and trade at a single, equilibrium price.

Search theory has provided a rigorous yet tractable framework to understand how trading frictions work and change macroeconomic outcomes. As noted by Gali (1998) accounting for unemployment in a general equilibrium framework requires introducing heterogeneity of workers and/or jobs or a cost of transition between states, and, as a result, search. Introducing such a labor market into a dynamic stochastic general equilibrium model (DSGE) is challenging and appealing because it allows us to derive a "Keynesian" result within this class of models and evaluate the consequences of the introduction of such an hypothesis on monetary policy.

Within this framework it is possible to study in a dynamic, microfounded setting the depth interactions between imperfections in the labour market, real and nominal rigidites, monetary policy and macroeconomic outcomes.

In this paper we introduce in a DSGE model with money an imperfect labor market, à la Howitt (1988), in which firms face a hiring cost and the wage is determined after a privately efficient Nash bargaining; we introduce imperfect competition in goods market to make it possible some form of nominal rigity and more easily compare our results with those of the standard model. The demand side of the model is reppresented by the standard optimizing IS-LM model. We analyze both the equilibrium with flexible and sticky prices evaluating the different policy implications of the model.

The paper is organized as follows: in section 2 we present the model, in section 3 the flexible price equilibrium and in section 4 the sticky price equilibrium.

2 The model

2.1 Consumers

The representative consumer seeks to maximize the objective function:

$$E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, \frac{M_t}{P_t}, N_t) \tag{1}$$

where E_0 is the conditional expectation in $t = 0$, β is the subjective discount factor[2], $\frac{M_t}{P_t}$ are real money balances, N_t are the number of hours supplied in the labor market and:

[2] $\beta = \frac{1}{1+\rho}$ where ρ is the subjective discount rate.

$$C_t \equiv \left[\sum_{i=1}^{z} C_t(i)^{1-\frac{1}{\epsilon}} \right]^{\frac{\epsilon}{\epsilon-1}} \tag{2}$$

$$P_t \equiv \left[\sum_{i=1}^{z} P_t(i)^{1-\epsilon} \right]^{\frac{1}{1-\epsilon}} \tag{3}$$

subject to a sequence of budget constraints given by:

$$\sum_{i=1}^{z} P_t(i)C_t(i) + M_t + R_t^{-1}B_t \leq M_{t-1} + B_{t-1} + W_tN_t - T_t \tag{4}$$

where $R = (1 + i)$, where i reppresents the nominal interest rate, B the nominal amount of public bonds held by the consumer and T_t a lump-sum tax. Also, a solvency constraint must be imposed in order to avoid Ponzi games; this can be written as :

$$\lim_{T\to\infty} \left(\prod_{j=t}^{T} R_j^{-1} \right) R_T \geq 0 \tag{5}$$

where R_T reppresents total wealth ($B + M$ at time T).

Defining Z_t as $\sum_{i=1}^{z} P_t(i)C_t(i)$ it can be shown that (see mathematical appendix xx):

$$C_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\epsilon} \frac{Z_t}{P_t} \tag{6}$$

for all $i \in [0, z]$. Also, the budget constraint can be rewritten as (see mathematical appendi xx):

$$P_tC_t + M_t + R_t^{-1}B_t \leq M_{t-1} + B_{t-1} + W_tN_t - T_t \tag{7}$$

which holds for all t.

Let us define $X_t \equiv \left[C_t, \frac{M_t}{P_t}, N_t \right]$ the vector of choice variables that maximizes (1). The F.O.C. of consumer's problem can be written as:

$$\frac{U_m(X_t)}{U_c(X_t)} = (1 - R_t^{-1}) \tag{8}$$

$$-\frac{U_n(X_t)}{U_c(X_t)} = \frac{W_t}{P_t} \tag{9}$$

$$U_c(X_t) = \beta R_t E_t \left[U_c(X_t) \left(\frac{P_t}{P_{t+1}} \right) \right] \tag{10}$$

The conditions represents the intratemporal optimality condition setting thr marginal rate of substitution between money and consumption equal to the opportunity cost of holding money, the intratemporal optimality condition setting the marginal rate of substitution between leisure and consumption equal to the real wage and the Euler condition for the optimal intertemporal allocation of consumption.

Assuming the period utility:

$$U(C_t, \frac{M_t}{P_t}, N_t) = \frac{C_t^{1-\sigma}}{1-\sigma} + \frac{(M_t/P_t)^{1-\nu}}{1-\nu} - \frac{N_t^{1+\varphi}}{1+\varphi}$$

the consumer's optimality conditions become:

$$\frac{M_t}{P_t} = \left(\frac{C_t^{\sigma}}{1 - R_t^{-1}} \right)^{\frac{1}{\nu}} \tag{11}$$

$$\frac{W_t}{P_t} = C_t^{\sigma} N_t^{\sigma} \tag{12}$$

$$1 = \beta R_t E_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \left(\frac{P_t}{P_{t+1}} \right) \right] \tag{13}$$

2.2 Firms

The market of goods is characterized by imperfect competition; we assume a set of z firms indexed by $i \in [1, z]$. Each firm produces a differentiated good, with the same technology:

$$Y_t(i) = A_t N_t(i) \tag{14}$$

where $N_t(i)$ is the quantity of labor employed by firm i and A_t represents the level of technology, evolving exogenously according to a stationary MA process:

$$A_t = \bar{A} + \epsilon_t \tag{15}$$

with:

$$\epsilon_t \sim N(0, \sigma^2)$$

Let us define the (net) markup charged by firm i (in log) as

$$\mu_t(i) = p_t(i) - mc_t^n(i)$$

$$= (p_t(i) - p_t) - mc_t(i)$$

where $mc_t^n(i)$ is the (log) nominal marginal cost faced by firm i .

The aggregate price level in (3) can be log-linearized (see appendix xx) about a symmetric steady state to yeld:

$$p_t = \sum_{i=1}^{z} p_t(i)$$

Defining the average price markup in the economy as $\mu_t = \sum_{i=1}^{z} \mu_t(i)$ it follows that:

$$\mu_t = -mc_t \tag{16}$$

where $mc_t = \sum_{i=1}^{z} mc_t(i)$ is the (log) average real marginal cost in the economy. This very important equivalence will turn to be very useful when describing the sticky price allocation.

In addition it is possible to show (see appendix xx) that the assumption of common technology implies that:

$$Y_t(i) = A_t N_t(i)$$

at aggregate level.

2.3 Labor market

Workers are identical. In each period a fraction δ (exogenous) of the employed is laid off and joins the unemployment pool. Firms hire workers from the unemployment pool since it is not possible to hire from other firms. Also, firms face a cost for hiring h_t workers in period t given by:

$$C(h_t) = \left(\frac{G}{2}\right)\left(\frac{h_t^2}{U_{t-1}}\right)$$

where G is a constant (we can immagine that is fixed by the government or by the degree of imperfection in the recruitment process due to burocracy, for instance), U_{t-1} is the unemployment rate at the end of $t-1$, and h_t is the number of people hired by the firm in t. The cost is increasing in the rate of hiring: this captures the idea that a high rate of hiring may force firms to increase their sarch intensity. Also, this cost is a decreasing function of aggregate unemployment: this captures the idea that a high rate of unemployment makes it easier and cheaper for the firm to find willing and competent workers. Employment evolves following the law of motion:

$$N_t = (1-\delta)N_{t-1} + h_t \tag{17}$$

Real wage is not determined by the classical decentralized labor market; on the contrary it derives from a process of bargaining between firms and workers. The decisional process of hiring for firms implies equalization between the marginal cost of hiring and the marginal benefit of doing so:

$$\frac{Gh_t}{U_{t-1}} = (1-s){q_t}^3$$

where s is the contactual strength of workers and q_t is defined as:

$$q_t = E_t\left[\sum_{i=0}^{\infty}\beta^t(1-\delta)^t A_{t+i}\right]$$

q_t reppresents the discounted future surplus of the job divided between workers (given their contactual strength, s) and firms.

Given (15) q_t can be rewritten as:

$$q_t = \frac{1}{1-\beta(1-\delta)}\bar{A}$$

3 Flexible price equilibrium

Each period the typical firm solves the problem:

$$\max\ P_t(i)Y_t(i) - TC_t(Y_t(i))$$

subject to (6). The F.O.C. is given by:

$$P_t(i) = \left(\frac{\epsilon}{\epsilon-1}\right)MC_t^n(i) \tag{18}$$

thus implying that $\mu_t = \log\left(\frac{\epsilon}{\epsilon-1}\right) = \mu$ for all t that is optimal price setting implies a *constant markup over marginal cost*. The assumption of common technology implies that:

$$P_t(i) = P_t \text{ for all } i$$

[3] $(1-s)q_t$ is the net- of the wage paid to workers i.e. sq_t- benefit of the job to the firm.

and:

$$MC_t = \left(\frac{\epsilon - 1}{\epsilon}\right)^4 \tag{19}$$

In this model the total cost function for firms is:

$$TC = \frac{W_t}{P_t} N_t + \frac{h_t^2}{U_{t-1}} \frac{G}{2} \tag{20}$$

and the marginal cost can be shown to have the following form:

$$\frac{sq_t}{A_t} + \frac{1}{A_t}\left[\frac{Y_t}{A_t} - \frac{Y_{t-1}}{A_{t-1}}\right] \frac{1}{(l - z\frac{Y_{t-1}}{A_{t-1}})} G \tag{21}$$

where l is the labor force.
The optimal pricing rule implies that:

$$\frac{sq_t}{A_t} + \frac{1}{A_t}\left[\frac{Y_t}{A_t} - \frac{Y_{t-1}}{A_{t-1}}\right] \frac{1}{(l - z\frac{Y_{t-1}}{A_{t-1}})} G - \frac{\epsilon - 1}{\epsilon} = 0 \tag{22}$$

Equation (16) reppresents an aggregate supply relation. Equations (11), (12), (13), (15) and (16) together with a description of how monetary policy is conducted and the market clearing condition:

$$Y_t = C_t \tag{23}$$

complete the model.

As it can be seen the model displays both neutrality and superneutrality, since all real variables (GDP, employment, real interest rate and real wage) are determined indipendently of prices and inflation.

Also, it is possible to show, using the implicit function theorem that (see appendix xx):

$$\frac{dY_t}{dG} < 0, \quad \frac{dY_t}{dY_{t-1}} > 0, \quad \frac{dY_t}{d\epsilon} > 0, \quad \frac{dY_t}{dA_t} > 0, \quad \frac{dY_t}{dA_{t-1}} > 0$$

The equilibrium is a not a Pareto optimum[5]*:* both imperfections in the labor and goods market make GDP lower than the perfect competition level. Hence in the long run policy of deregulation in goods (higher ϵ) and labor market (lower G) are beneficial and welafare improving.

GDP desplays inertia: the link is reppresented by the law of motion of employment (17). A lot of empirical works confirm this fact. Also,

[4] That is the explicit, no logaritmic form of the equivalence (16).

[5] It is possible to note that conditions to apply the second welfare theorem do not hold in this setting.

it is interesting to observe that the model implies that the current level of GDP is influenced by the previous period level of technology, despite the description of the preocess for technology. Again the channel is reppresented by the law of motion for employment.

4 Sticky price equilibrium

Following the formalism in Calvo (1983), we assume that each firm may reset its price with probability $(1-\Theta)$ each period, independently of the time elapsed since the last adjustment. Thus each period, a measure $(1-\Theta)$ of producers reset their prices, while a fraction Θ keep their prices unchanged. Aggregate prices follow the law of motion:

$$p_t = \Theta p_{t-1} + (1-\Theta)p_t^* \tag{24}$$

where p_t^* denote the (log) price set by firms adjusting their prices in t.

Equation (23) implies that:

$$\pi_t = (1-\Theta)(p_t^* - p_{t-1}) \tag{25}$$

where π_t is the inflation rate at time t. If there were no constraint on the adjustment of prices the typical firm would set:

$$p_t^*(i) = \mu + mc_t^n(i) \quad \forall t$$

Under the Calvo (1983) price setting structure it turns out that $p_{t+k}(i) = p_t^*(i)$ with probability Θ^k; hence, a forward looking firm, resetting its price in t chooses $P_t^*(i)$ in order to maximize:

$$\max_{p_t^*} \sum_{k=0}^{\infty} \Theta^k E_t \left[Q_{t,t+k} \left(P_t^* Y_{t+k}(i) - TC(Y_{t+k}(i)) \right] \right. \tag{26}$$

where $Q_{t,t+k} = \beta^k \left[\frac{C_{t+k}}{C_t}\right]^{-\sigma} \frac{P_t}{P_{t+k}}$ is the relevant discount factor for nominal pay offs; the expectation is conditional on P_t remaining effective and the maximization is subject to the sequence of budget constraints:

$$Y_{t+k}(i) = \left(\frac{P_t^*(i)}{P_{t+k}}\right)^{-\epsilon} C_{t+k} \equiv Y_{t+k}^D((P_t^*(i))) \tag{27}$$

The F.O.C. associated with the firm's problem can be written as:

$$\sum_{k=0}^{\infty} \Theta^k E_t \left[Q_{t,t+k} Y_{t+k}^D(P_t^*(i)) \left[\frac{P_t^*(i)}{P_{t-1}} - \left(\frac{\epsilon}{\epsilon - 1}\right) \pi_{t-1,t+k} MC_{t+k} \right] \right] \tag{28}$$

where $\pi_{t-1,t+k} \equiv \left(\frac{P_{t+k}}{P_{t-1}}\right)$. The optimal price setting rule can be log-linearized in a neighborood of a zero steady state inflation equilibrium:

$$p_t^*(i) = \mu + (1-\beta\Theta)\sum_{k=0}^{\infty}(\beta\Theta)^k E_t\left[mc_{t+k}^n(i)\right] \tag{29}$$

Thus firms will set a price equal to a markup μ over a weighted average of expected future nominal marginal costs, with the weights associated with each horizon k proportional to the probability that the chosen price remain effective k periods ahead[6]. Using the fact that all firms resetting prices in period t will choose the same price p_t^*, equation (28) can be rewritten as :

$$p_t^* - p_{t-1} = (1-\beta\Theta)\sum_{k=0}^{\infty}(\beta\Theta)^k E_t\left[\hat{m}c_{t+k}\right] + \sum_{k=0}^{\infty}(\beta\Theta)^k E_t\left(\pi_{t+k}\right) \tag{30}$$

where $\hat{m}c_t = mc_t - mc$, and $mc = -\mu$; $\hat{m}c_t$ is the deviation of marginal cost from its flexible price level. More compactly:

$$p_t^* - p_{t-1} = \beta\Theta E_t\left[p_{t+1}^* - p_t\right] + (1-\beta\Theta)\hat{m}c_t + \pi_t \tag{31}$$

combined with (24), yelds the inflation equation:

$$\pi_t = \beta E_t(\pi_{t+1}) + \lambda\hat{m}c_t \tag{32}$$

with:

$$\lambda = \frac{(1-\Theta)(1-\beta\Theta)}{\Theta}$$

Considering the definition of the marginal cost given in (21) it turns out (see appendix xx) that:

$$\begin{aligned}\hat{m}c_t &= mc_t - mc\\ &= \gamma_1\left(y_t - \bar{y}_t\right) + \gamma_2\left(y_{t-1} - \bar{y}_{t-1}\right)\end{aligned}$$

or more compatcly:

$$\pi_t = \beta E_t(\pi_{t+1}) + \alpha_1\tilde{y}_t + \alpha_2\tilde{y}_{t-1} \tag{33}$$

where $\tilde{y}_t \equiv (y_t - \bar{y}_t)$ and $\tilde{y}_{t-1} \equiv (y_{t-1} - \bar{y}_{t-1})$ reppresent current and previous period "output gap" and:

[6] A rigorous derivation of the optimal price setting rule can be found in Yun (1996) or Woodford (1996).

$\alpha_1 \equiv \frac{(1-\Theta)(1-\beta\Theta)}{\Theta}\gamma_1$ and $\alpha_2 \equiv \frac{(1-\Theta)(1-\beta\Theta)}{\Theta}\gamma_2$

$\gamma_1 \equiv G\frac{Y_{SS}}{A_{ss}^2\left(l-z\frac{Y_{SS}}{A_{SS}}\right)MC_{ss}}$ and

$\gamma_2 \equiv \left[\frac{Y_{SS}}{A_{SS}^2} + \frac{z}{A_{SS}}\left(l - z\frac{Y_{SS}}{A_{SS}}\right)^{-2} - \frac{l}{\left(l-z\frac{Y_{SS}}{A_{SS}}\right)^2}\right]\frac{GY_{SS}}{MC_{SS}}.$

where the subscript SS indicates zero inflation steady state values of the different variables.

Equation (33) reppresents a form of the New Keynesian Phillips Curve (NKPC) linking at the same time current inflation, expected inflation, current output gap and past output gap. This new form of the curve, obtained combining nominal and real rigidities provides a richer description of inflation dynamics, and their linkage with real activity. Recall that the NKPC[7], derived assuming perfect labor market was:

$$\pi_t = \beta E_t(\pi_{t+1}) + k\tilde{y}_t$$

with $k = \frac{(1-\Theta)(1-\beta\Theta)}{\Theta}(\sigma + \varphi)$.

Our new derivation includes in the inflation equation both previous output gap ($\tilde{y}_{t-1}$) and the indicator of labor market imperfection, namely the cost of hiring G. In particular G enters the coefficient of present and past output gap: higher values of G make the trade off between inflation and output gap worse. This link captures the strategic incentive for firms: the higher G, the more expensive is hiring, the stronger the incentive for firms to adjust prices and not quantities, following any monetary action. This is the core of the interaction between nominal and real rigidities in the model. As we have shown, reduction of G is beneficial also in the flexible price allocation. In the context of sticky price equilibrium policies reducing G (i.e. policies aiming to make labor market more flexible) make monetary policy more effective when pursuing an intervention on real activity. From our model emerges a new link between fiscal policies of liberalization in the labor market and monetary policy.

In addition the introduction of real rigidities allow to overcome some empirical shortcuts of the standard NKPC; first a trade off between output gap and inflation emerges. As pointed out by Blanchard and Gali (2005) in the standard new keynesian model no trade off between

[7]For a detailed discussion of the NKPC see Fuhrer and Moore (1995), Gali and Gertler (1999), Gali, Gertler and Lopez Salido (2003) and Sbordone (2002).

the two exists, due to the presence of the so called "divine coincidence"[8]. On the contrary, in our model stabilizing inflation does not necessarly imply stabilization of output gap due to the extra term $\alpha_2\tilde{y}_{t-1}$.

Furthemore our framework predicts some sort of inflation inertia[9]; current inflation can be written as:

$$\pi_t = \beta E_t(\pi_{t+1}) + \alpha_1\tilde{y}_t + \frac{\alpha_2}{\alpha_1}\left(\pi_{t-1} - \alpha_2\tilde{y}_{t-2} - \beta E_{t-1}(\pi_t)\right)$$

Also, this model predicts a persistent and unvoluntary unemployment rate. Its existence is due to imperfections in the labor market, namely real rigidities, and it can be reduced both by fiscal policy and monetary policy.

4.1 General equilibrium

We now have all the components of a simple general equilibrium model that is consistent with optimizing behavior of the part of households and firms. Equations (11), (12) and (13) can be log-linearized and written as follows (because consumption is equal to output in this model):

:

$$m_t - p_t = y_t - \eta i_t{}^{10} \tag{34}$$

$$w_t - p_t = \sigma y_t + \varphi n_t \tag{35}$$

$$y_t = E_t\left[y_{t+1}\right] - \frac{1}{\sigma}\left[i_t - E_t\left[\pi_{t+1}\right] - \rho\right] \tag{36}$$

Equation (36), known as the new IS, linking current output, real interest rate and subjective discount rate can be rewritten in terms of the output gap:

$$\tilde{y}_t = E_t\left[\tilde{y}_{t+1}\right] - \frac{1}{\sigma}\left[i_t - E_t\left[\pi_{t+1}\right] - \bar{r}r_t\right] \tag{37}$$

where $\bar{r}r_t = \rho + \sigma E_t\left[\Delta\bar{y}_{t+1}\right]$ is the *natural rate of interest* that is *the one that would obtain under flexible prices.*

Equation (34) is the money market equilibrium condition: it can be rewritten in terms of the output gap:

$$\tilde{y}_t - \eta i_t = m_t - p_t - \bar{y}_t \equiv mpy_t \tag{38}$$

[8] See Blanchard and Gali (2005).

[9] About the lack of inflation inertia in the new keynesian model see, for example, Gali and Gertler (1999).

[10] Assuming a unit income elasticity ($\sigma = \nu$).

Equation (35) represents labor supply: in this model of imperfect labor market it can be used to calculate unvoluntary unemployment.

We assume that monetary policy is represented by a rule for setting the nominal rate of interest; the nominal quantity of money is then endogenously determined to achieve the desired nominal interest rate[11]. On the other hand the new keynesian Phillips curve describes the optimal price setting behavior for firms.

Hence, model's dynamics are determined by the following two log-linearized equations plus an interest rate rule:

$$\tilde{y}_t = E_t\left[\tilde{y}_{t+1}\right] - \frac{1}{\sigma}\left[i_t - E_t\left[\pi_{t+1}\right] - \bar{r}r_t\right]$$

$$\pi_t = \beta E_t(\pi_{t+1}) + \alpha_1 \tilde{y}_t + \alpha_2 \tilde{y}_{t-1}$$

We specify monetary policy assuming that the central bank responds to both inflation and the output gap according to a standard *Taylor rule*[12]:

$$i_t = \bar{r}r_t + \delta_\pi \pi_t + \delta_{\tilde{y}_t} \tilde{y}_t \tag{39}$$

Equations (33), (37) and (39) close the model. The resulting system can be written, following Blanchard and Kahn (1980), as:

$$\begin{bmatrix} 1 & -q_1 & -q_3 \\ 0 & \beta & 0 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} X_{2t+1} \\ E_t(\pi_{t+1}) \\ X_{1t} \end{bmatrix} = \begin{bmatrix} -q_2 & 0 & 0 \\ -\alpha_2 & 1 & -\alpha_1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X_{2t} \\ \pi_t \\ X_{1t-1} \end{bmatrix} \tag{40}$$

where: $q_1 = \frac{(1-\delta_\pi\beta)}{\left(1+\frac{\delta_\pi\alpha_1}{\sigma}+\frac{\delta_{\tilde{y}_t}}{\sigma}\right)}$, $q_2 = \frac{\left(\frac{\delta_\pi\alpha_2}{\sigma}\right)}{\left(1+\frac{\delta_\pi\alpha_1}{\sigma}+\frac{\delta_{\tilde{y}_t}}{\sigma}\right)}$, $q_3 = \left(1 + \frac{\delta_\pi\alpha_1}{\sigma} + \frac{\delta_{\tilde{y}_t}}{\sigma}\right)^{-1}$ and $X_{1t} = \tilde{y}_{t+1}$, $X_{2t} = \tilde{y}_{t-1}$, $X_{2t+1} = \tilde{y}_t = X_{1t-1}$ and α_1, α_2 have been defined above. Determinant of matrix A is βq_3; hence the matrix is invertible and the system can be rewritten as:

$$\begin{bmatrix} X_{2t+1} \\ E_t(\pi_{t+1}) \\ X_{1t} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ -\frac{1}{\beta}\alpha_2 & \frac{1}{\beta} & -\frac{1}{\beta}\alpha_1 \\ \frac{q_2}{q_3} + \frac{1}{\beta}\alpha_2\frac{q_1}{q_3} & -\frac{1}{\beta}\frac{q_1}{q_3} & \frac{1}{q_3} + \frac{1}{\beta}\alpha_1\frac{q_1}{q_3} \end{bmatrix} \begin{bmatrix} X_{2t} \\ \pi_t \\ X_{1t-1} \end{bmatrix} \tag{41}$$

[11] Most central banks today use a short-term nominal interest rate as their instrument for implementing monetary policy. There are important issues involved in choosing between money supply policy procedures and interest rate procedures: for detailed discussions see Walsh (2003) and Woodford (2003).

[12] This type of policy rule has been shown to provide a reasonable empirical description of the policy behavior of many central banks (Clarida, Gali and Gertler, 2000).

X_{2t} is predetermined, while π_t and X_{1t-1} are not. Hence, following Blanchard and Kahn (1980), stability and uniqueness of the solution requires two eigenvalues outside the unit circle and one inside. Considering that α_1 and α_2 depend on G (namely the cost of hiring, the indicator of the imperfection in the labor market), also determinacy conditions on policy parameters δ_π and $\delta_{\tilde{y}_t}$ will depend on G (analytical conditions are derived in appendix xx): as a result, when designing monetary policy the central bank have to look at the labor market, in order to guarantee stable economic outcomes. In this model the presence of a real distortion in the labor market create an additional claim for coordination between monetary and liberalization policies.

5 Conclusions

The introduction of an imperfect labor market, characterized by efficient bargaining in the determination of the real wage and by the presence of a hiring cost for firms in a standard New Keynesian model, delivers non trivial results.

In the flexible price equilibrium, GDP shows inertia and is affected by the past level of technology; moreover, it is influenced both by regulation in the labor market - i.e. policies aiming to reduce G - and by pro-competitive policies in goods market - i.e. policies aiming to reduce ϵ.

In the short run, we derived a NKPC linking current inflation with expected inflation, current and *past output gap*: the last additional term, with respect to the traditional NKPC, generates (1) a policy trade-off between output gap and inflation stabilization, absent in the standard New Keynesian model, in line with the insights in Blanchard and Gali (2005), and (2) inflation inertia, consistent with a substantial body of empirical findings. The degree of labor market imperfection (namely, the magnitude of G) influences the trade-off: the higher G, the less effective is monetary policy in affecting real activity. Also, in the choice of monetary policy parameters (δ_π and $\delta_{\tilde{y}_t}$ in the Taylor rule, i.e. the magnitude of responses to inflation and output gap) the central bank has to take into account the regulation in the labour market to obtain stable and determinate macroeconomic outcomes.

Finally, from the theoretical point of view, it is important to underline that in this DSGE model we obtain persistent, involuntary, and equlibrium unemployment both in the flexible and sticky price allocation.

To deepen the understanding of the linkages between real and nominal rigidities, the framework built in this paper can be extended in several ways. For example the introduction of an imperfect capital market in the model would allow us to study the interactions between labor, goods and financial markets liberalization and monetary policy. Also, in this framework it would be interesting to investigate more deeply conditions for determinacy of different monetary policy rules. In addition, the introduction of different shocks would make it possible to address the problem of optimal monetary policy and the credibility of the central banker.

On the empirical side, it would be interesting to estimate the slope of our NKPC for similar countries with different levels of regulation in the labor market (in particular regarding costs of hiring for firms) and to study responses to different types of shocks.

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