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INCENTIVES, MORAL HAZARD AND ADVERSE SELECTION

Silvia Platoni

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INCENTIVES, MORAL HAZARD AND ADVERSE SELECTION†

SILVIA PLATONI[‡]

Università Cattolica del Sacro Cuore

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ABSTRACT

This paper proposes a model which analyses not only the provision of incentives (see, e.g., Gershkov et. al 2006 and Huck et al. 2001) and the *moral hazard* problem (see, e.g., Holmström 1982), but also the *adverse selection* problem (i.e. the workers are *heterogeneous*). Moreover, unlike the previous works, the paper introduces also the *time* dimension: we consider a firm with an infinite time horizon and individuals whose working life is split into two phases, the young phase and old phase. By comparing the results of the classical incentives scheme with those of a rewarding incentives scheme, we can conclude that this last scheme allows a higher production level.

1. INTRODUCTION

As Gibbons (1996) points out, a no-economist might be surprised to learn that modern labour economics has little to say about activities inside the firms.

While several research areas focus on what happens before an employment relationship begins (i.e. the interest for the unemployment duration and for the labour-force participation or the fact that labour demand theory considers how many workers should be hired rather than what the firm should do with them), other research areas reduce the employment relationship to the wage or, at most, to the wage profile (i.e. the models which analyse the job search, the labour supply and the human capital). Moreover, on the one hand the research on the return to seniority more often focuses on econometric issues than on what actually happens during an employment relationship, on the other the research on training focuses on pre-employment government-sponsored programs rather than on skill development inside firms.

In the field of the literature on incentives, Holmström (1982) stresses the relevance of the *moral hazard* problem in case of labour *team*; if the firm and the workers are two different subjects with different goals (the profit for the firm and the utility for the workers), the incentives scheme developed by the profit maximising firm may be not budget balanced if the effort of the workers is not observable.

Gershkov et. al (2006) analyses the incentives problem when the output is a function of the total effort of *homogeneous* workers and the firm maximises the collective wellbeing (i.e. the sum of the

‡ Dipartimento di Scienze Economiche e Sociali, Università Cattolica del Sacro Cuore, via Emilia Parmense 84, 29100 Piacenza, Italy (silvia.platoni@unicatt.it).

workers utilities); if the workers are paid according to their effort, Gershkov et. al (2006) find that the workers choose the first-best effort.

Huck et al. (2001) underline how the workers behaviour is governed not only by the economic incentives, but also by the *social norms*. In their analysis they assume the worker utility negatively depends on the embarrassment of exerting less effort than the other workers' average effort.

In this work we present a model which analyses not only the provision of incentives (see, e.g., Gershkov et. al 2006 and Huck et al. 2001) and the *moral hazard* problem (see, e.g., Holmström 1982), but also the *adverse selection* problem. Moreover, unlike previous works, we introduce also the *time* dimension: we consider a firm with an infinite time horizon and individuals whose working life is split into two phases (in the first phase the workers are young and in the second phase they are old).

In both phases of his working life each individual applies a certain effort e and the joined effort of all the individuals determines the output level y (see, e.g., Gershkov et al. 2006).

If the aims of firm and workers were disjointed and the firm who decided to set up an incentives scheme could not observe the worker effort, then the *moral hazard* problem would arise: the worker would attempt to work less than what he could (see, e.g., Huck et al. 2001). The *classical incentives scheme* solves this problem by relating the wage to the production y in a contest where the firm maximises the workers' wellbeing and not the profit.

Furthermore, we assume there are two individual types, i.e. we assume the individuals are *heterogeneous*: the type P worker (who can become *partner*) and type E worker (who will remain *employed*). The cost function of the type P worker is lower than the cost function of the type E worker for any given effort level $\overline{e}$: $c_P\left(\overline{e}\right) < c_E\left(\overline{e}\right)$. Therefore, type P individuals are inclined to work with more effort than type E individuals.

If the firm decides to reward type P individuals for their higher effort by making them partners in the second phase of their working life (that is if the firm sets up an incentives scheme such that the type P wage in the second phase depends not only on the production level y, but also on the profit π), then the labour relationship would be affected also by the *adverse selection* problem. In fact if the firm could not distinguish the two individual types (i.e. the firm could not observe the cost functions $c_P\left(\bullet\right)$ and $c_E\left(\bullet\right)$), then in the first phase of their working life the type E individuals would "mimic" the type P individuals to become partners in the second phase. In this case if the firm didn't disincentive the type E individuals (*self-selection constraint*), then these individuals would be better off to the detriment of the type P individuals.

The remainder of the paper proceeds as follows. In Section 2 we describe the setup of the model. In Section 3 we consider a model without incentives: if the firm doesn't set up any incentives scheme, then the workers take *hidden actions*. In Section 4 we present the model with the classical incentives scheme; this scheme solves the *moral hazard* problem (the problem of individuals who attempt to work less than what they can). In Section 5 we propose a model of incentives scheme which awards the type P individuals; in this case the firm has to face also the *adverse selection* problem since the type E individuals are tempted to mimic the type P individuals. Finally Section

6 draws some conclusions, i.e. we outline the benefits of the rewarding incentives scheme with respect to those of the classical incentives scheme.

2. THE MODEL

The working life of the individual i is split into two phases $\tau = 1, 2$. Therefore the worker's utility is

$$U^i = u_1^i + \beta \cdot u_2^i \tag{1}$$

where u_1^i is the period utility in the phase $\tau = 1$ of the worker i , u_2^i is the period utility in the phase $\tau = 2$ of the worker i and $0 \le \beta \le 1$ is the rate of time preference.

In every period t the firm employs N_t young workers and N_{t-1} old workers (hired in $t-1$); if we consider a constant employment rate n , then we can write $N_t = (1+n) \cdot N_{t-1}$.

Moreover in every period t the individual i during the working phase τ makes the effort $e_{\tau,t}^i \in [\underline{e}, 1]$, where $\underline{e}$ is the minimum effort required by the firm.

As anticipated, the effort e entails for the individual $i = P, E$ the cost $c_i(e)$. We assume the cost function $c_i(\bullet)$ satisfies $c_i' > 0$ and $c_i'' > 0$ (in the following we will consider the functional form $c(e) = (e)^{exp}$ with $\underline{e} \le e \le 1$).

In the previous section we have already assumed there are two individual types (P and E) characterized by $c_P(\overline{e}) < c_E(\overline{e})$ for any given effort $\overline{e}$; this implies the effort e entails a lower cost for the individual type P than for the individual type E . The figure 1 represents the cost functions $c_P(e) = (e)^4$ and $c_E(e) = (e)^3$ with $\underline{e} = 0.2$ and then with $0.2 \le e \le 1$.

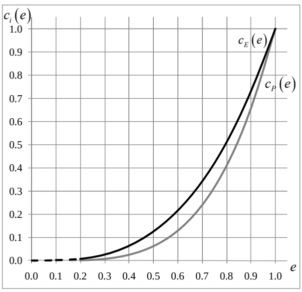

Figure 1: The Effort Cost

In the figure 1 we can see how the cost for the individual type P is always lower than the cost for the individual type E .

The probability a worker belongs to the group $i = P, E$ is p^i , where $p^P + p^E = 1$.

In this simplified analysis we assume the labour is the only production factor. The production technology is linear: in the period t the output y_t is given by the sum of the efforts made by the workers (see, e.g., Gershkov et al. 2006). Thus, in the first period $t = 0$, when the firm employs only the N_0 young workers (in the working phase $\tau = 1$), we have

$$y_0 = y(\boldsymbol{e_0}) = N_0 \cdot \left(\sum_{i=P,E} p^i \cdot e_{1,0}^i \right) \tag{2}$$

where $\boldsymbol{e_0} = \left(e_{1,0}^P, e_{1,0}^E\right)$, with $e_{1,0}^P$ the effort made by the $N_0 \cdot p^P$ individuals belonging to group P and $e_{1,0}^E$ the effort made by the $N_0 \cdot p^E$ individuals belonging to group E. In every following period $t > 0$ we have

$$\begin{aligned} y_t &= y(\boldsymbol{e_t}) = N_t \cdot \sum_{i=P,E} p^i \cdot e_{1,t}^i + N_{t-1} \cdot \sum_{i=P,E} p^i \cdot e_{2,t}^i = \\ &= N_0 \cdot (1+n)^t \cdot \left(\sum_{i=P,E} p^i \cdot e_{1,t}^i + \frac{1}{1+n} \cdot \sum_{i=P,E} p^i \cdot e_{2,t}^i \right) \end{aligned} \tag{3}$$

where $\boldsymbol{e_t} = \left(e_{1,t}^P, e_{1,t}^E, e_{2,t}^P, e_{2,t}^E\right)$, with $e_{1,t}^P$ the effort made by the $N_0 \cdot (1+n)^t \cdot p^P$ young workers belonging to group P, $e_{1,t}^E$ the effort made by the $N_0 \cdot (1+n)^t \cdot p^E$ young workers belonging to group E, $e_{2,t}^P$ the effort made by the $N_0 \cdot (1+n)^{t-1} \cdot p^P$ old workers belonging to group P and $e_{2,t}^E$ the effort made by the $N_0 \cdot (1+n)^{t-1} \cdot p^E$ old workers belonging to group E.

If the firm doesn't set up any incentives scheme, the wage w doesn't depend on the workers' efforts and thus the utility in (1) is function only of the cost c the effort e implies

$$U_t^i = \left(w - c_i\left(e_{1,t}^i\right)\right) + \beta \cdot \left(w - c_i\left(e_{2,t+1}^i\right)\right) \tag{4}$$

On the contrary if the firm sets up a classical incentives scheme (the wage w is related to the production level y, and then to the efforts made by all the workers), in $t = 0$ (when the firm employs only individuals in the first phase of their working life $\tau = 1$) the wage w of any individual type $i = P, E$ is

$$w_0 = w(\boldsymbol{e_0}) = b \cdot \frac{y(\boldsymbol{e_0})}{N_0} = b \cdot \frac{N_0 \cdot \left(\sum_{i=P,E} p^i \cdot e_{1,0}^i \right)}{N_0} = b \cdot \sum_{i=P,E} p^i \cdot e_{1,0}^i \tag{5}$$

where b is the share of the production y assigned to cover the labour cost. In $t > 0$ (when the firm employs individuals in both phases of the working life $\tau = 1, 2$) the wage for each worker $i = P, E$ is

$$w_t = w(\boldsymbol{e_t}) = b \cdot \frac{y(\boldsymbol{e_t})}{N_t + N_{t-1}} = b \cdot \frac{1+n}{2+n} \cdot \left(\sum_{i=P,E} p^i \cdot e_{1,t}^i + \frac{1}{1+n} \cdot \sum_{i=P,E} p^i \cdot e_{2,t}^i \right) \tag{6}$$

Therefore, in $t = 0$ the utility in (1) can be rewritten as a function of the efforts e made by the workers, that is as a function not only of the cost c the effort e implies, but also of the production level y the efforts e determine

$$
\begin{aligned}
U_0^i &= u_{1,0}^i + \beta \cdot u_{2,1}^i = \\
&= \left(w(\boldsymbol{e_0}) - c_i\left(e_{1,0}^i\right)\right) + \beta \cdot \left(w(\boldsymbol{e_1}) - c_i\left(e_{2,1}^i\right)\right) = \\
&= \left(b \cdot \frac{y(\boldsymbol{e_0})}{N_0} - c_i\left(e_{1,0}^i\right)\right) + \beta \cdot \left(b \cdot \frac{1+n}{2+n} \cdot \frac{y(\boldsymbol{e_1})}{N_1} - c_i\left(e_{2,1}^i\right)\right) = \\
&= \left(b \cdot \sum_{i=S,E} p^i \cdot e_{1,0}^i - c_i\left(e_{1,0}^i\right)\right) + \beta \cdot \left[b \cdot \frac{1+n}{2+n} \cdot \left(\sum_{i=P,E} p^i \cdot e_{1,1}^i + \frac{1}{1+n} \cdot \sum_{i=P,E} p^i \cdot e_{2,1}^i \right) - c_i\left(e_{2,1}^i\right)\right]
\end{aligned}
\tag{7}
$$

where $\boldsymbol{e_0} = \left(e_{1,0}^P, e_{1,0}^E\right)$ - effort made in $t=0$ by the young individuals (in the phase $\tau=1$) belonging to the group P and to the group E - and $\boldsymbol{e_1} = \left(e_{1,1}^P, e_{1,1}^E, e_{2,1}^P, e_{2,1}^E\right)$ - effort made in $t=1$ by the young individuals (in the phase $\tau=1$) belonging to the group P and to the group E and by the old individuals (in the phase $\tau=2$) belonging to the group P and to the group E.

In every following period $t>0$ the utility in (1) rewritten as a function of the efforts e becomes

$$
\begin{aligned}
U_t^i &= u_{1,t}^i + \beta \cdot u_{2,t+1}^i = \\
&= \left(w(\boldsymbol{e_t}) - c_i\left(e_{1,t}^i\right)\right) + \beta \cdot \left(w(\boldsymbol{e_{t+1}}) - c_i\left(e_{2,t+1}^i\right)\right) = \\
&= \left(b \cdot \frac{1+n}{2+n} \cdot \frac{y(\boldsymbol{e_t})}{N_t} - c_i\left(e_{1,t}^i\right)\right) + \beta \cdot \left(b \cdot \frac{1+n}{2+n} \cdot \frac{y(\boldsymbol{e_{t+1}})}{N_{t+1}} - c_i\left(e_{2,t+1}^i\right)\right) = \\
&= \left[b \cdot \frac{1+n}{2+n} \cdot \left(\sum_{i=P,E} p^i \cdot e_{1,t}^i + \frac{1}{1+n} \cdot \sum_{i=P,E} p^i \cdot e_{2,t}^i \right) - c_i\left(e_{1,t}^i\right)\right] + \beta \cdot \left[b \cdot \frac{1+n}{2+n} \cdot \left(\sum_{i=P,E} p^i \cdot e_{1,t+1}^i + \frac{1}{1+n} \cdot \sum_{i=P,E} p^i \cdot e_{2,t+1}^i \right) - c_i\left(e_{2,t+1}^i\right)\right]
\end{aligned}
\tag{8}
$$

where $\boldsymbol{e_t} = \left(e_{1,t}^P, e_{1,t}^E, e_{2,t}^P, e_{2,t}^E\right)$ - effort made in t by the young individuals (in the phase $\tau=1$) belonging to the group P and to the group E and by the old individuals (in the phase $\tau=2$) belonging to the group P and to the group E - and $\boldsymbol{e_{t+1}} = \left(e_{1,t+1}^P, e_{1,t+1}^E, e_{2,t+1}^P, e_{2,t+1}^E\right)$ - effort made in $t+1$ by the young individuals (in the phase $\tau=1$) belonging to the group P and to the group E and by the old individuals (in the phase $\tau=2$) belonging to the group P and to the group E.

If the firm decides to reward the type P individuals, whose effort e implies a lower cost c, by promoting them partners, then these workers in the second phase of their working life will receive also a share of the profit π. Since the rewarding incentives scheme consists in the assignment of a share of the profit to the old individuals (individuals in the second phase of their working life $\tau=2$), the period $t=0$ - when the firm employs only young individuals - is not considered. In each period $t>0$ the profit is the difference between the production level y and the labour cost

$$
\begin{aligned}
\pi_t = \pi(\boldsymbol{e_t}) &= y(\boldsymbol{e_t}) - \left(N_t + N_{t-1}\right) \cdot w(\boldsymbol{e_t}) = y(\boldsymbol{e_t}) - \frac{2+n}{1+n} \cdot N_t \cdot w(\boldsymbol{e_t}) = \\
&= y(\boldsymbol{e_t}) - \frac{2+n}{1+n} \cdot N_t \cdot \left(b \cdot \frac{1+n}{2+n} \cdot \frac{y(\boldsymbol{e_t})}{N_t}\right) = (1-b) \cdot y(\boldsymbol{e_t})
\end{aligned}
\tag{9}
$$

Thus while b is the share of the production y assigned to cover the labour cost, the share $1-b$ is the profit π.

In the phase $\tau=2$ the wage of the individual $i=P$ is given not by the equation (6), but by the following equation which includes both the labour income and the capital income

$$
\begin{aligned}
w_{2,t}^P \quad \equiv w_2^P\left(\boldsymbol{e_t}\right) \quad &= \frac{b\cdot y\left(\boldsymbol{e_t}\right)}{N_t+N_{t-1}}+\frac{\left(1-b\right)\cdot y\left(\boldsymbol{e}_t\right)}{p^P\cdot N_{t-1}}= \\
&=\left(b+\frac{\left(1-b\right)\cdot\left(2+n\right)}{p^P}\right)\cdot\frac{1+n}{2+n}\cdot\frac{y\left(\boldsymbol{e_t}\right)}{N_t}= \\
&=\left(b+\frac{\left(1-b\right)\cdot\left(2+n\right)}{p^P}\right)\cdot\frac{1+n}{2+n}\cdot\left(\sum_{i=P,E}p^i\cdot e_{1,t}^i+\frac{1}{1+n}\cdot\sum_{i=P,E}p^i\cdot e_{2,t}^i\right)
\end{aligned}
\tag{10}
$$

where the grey element is related to the capital income (the distinction between the equation (10) and the equation (6)).

Hence in $t=0$ the utility of the individuals $i=P$ becomes

$$
\begin{aligned}
U_0^P \quad &= u_{1,0}^P+\beta\cdot u_{2,1}^P= \\
&=\left(w\left(\boldsymbol{e_0}\right)-c_P\left(e_{1,0}^P\right)\right)+\beta\cdot\left(w_2^P\left(\boldsymbol{e_1}\right)-c_P\left(e_{2,1}^P\right)\right)= \\
&=\left(b\cdot\frac{y\left(\boldsymbol{e_0}\right)}{N_0}-c_P\left(e_{1,0}^P\right)\right)+\beta\cdot\left[\left(b+\frac{\left(1-b\right)\cdot\left(2+n\right)}{p^P}\right)\cdot\frac{1+n}{2+n}\cdot\frac{y\left(\boldsymbol{e_1}\right)}{N_1}-c_P\left(e_{2,1}^P\right)\right]= \\
&=\left(b\cdot\sum_{i=P,E}p^i\cdot e_{1,0}^i-c_P\left(e_{1,0}^P\right)\right)+\beta\cdot\left[\left(b+\frac{\left(1-b\right)\cdot\left(2+n\right)}{p^P}\right)\cdot\frac{1+n}{2+n}\cdot\left(\sum_{i=P,E}p^i\cdot e_{1,1}^i+\frac{1}{1+n}\cdot\sum_{i=P,E}p^i\cdot e_{2,1}^i\right)-c_P\left(e_{2,1}^P\right)\right]
\end{aligned}
\tag{11}
$$

where $\boldsymbol{e_0}=\left(e_{1,0}^P,e_{1,0}^E\right)$ and $\boldsymbol{e_1}=\left(e_{1,1}^P,e_{1,1}^E,e_{2,1}^P,e_{2,1}^E\right)$. In $t>0$ the utility of type P worker becomes

$$
\begin{aligned}
U_t^P \quad &= u_{1,t}^P+\beta\cdot u_{2,t+1}^P= \\
&=\left(w\left(\boldsymbol{e}_t\right)-c_P\left(e_{1,t}^P\right)\right)+\beta\cdot\left(w_2^P\left(\boldsymbol{e_{t+1}}\right)-c_P\left(e_{2,t+1}^P\right)\right)= \\
&=\left(b\cdot\frac{1+n}{2+n}\cdot\frac{y\left(\boldsymbol{e}_t\right)}{N_t}-c_P\left(e_{1,t}^P\right)\right)+\beta\cdot\left[\left(b+\frac{\left(1-b\right)\cdot\left(2+n\right)}{p^P}\right)\cdot\frac{1+n}{2+n}\cdot\frac{y\left(\boldsymbol{e_{t+1}}\right)}{N_{t+1}}-c_P\left(e_{2,t+1}^P\right)\right]= \\
&=\left[b\cdot\frac{1+n}{2+n}\cdot\left(\sum_{i=P,E}p^i\cdot e_{1,t}^i+\frac{1}{1+n}\cdot\sum_{i=P,E}p^i\cdot e_{2,t}^i\right)-c_P\left(e_{1,t}^P\right)\right]+\beta\cdot\left[\left(b+\frac{\left(1-b\right)\cdot\left(2+n\right)}{p^P}\right)\cdot\frac{1+n}{2+n}\cdot\left(\sum_{i=P,E}p^i\cdot e_{1,t+1}^i+\frac{1}{1+n}\cdot\sum_{i=P,E}p^i\cdot e_{2,t+1}^i\right)-c_P\left(e_{2,t+1}^P\right)\right]
\end{aligned}
\tag{12}
$$

where $\boldsymbol{e_t}=\left(e_{1,t}^P,e_{1,t}^E,e_{2,t}^P,e_{2,t}^E\right)$ and $\boldsymbol{e_{t+1}}=\left(e_{1,t+1}^P,e_{1,t+1}^E,e_{2,t+1}^P,e_{2,t+1}^E\right)$. Also in the equations (11) and (12) the grey elements are related to the capital income, which distinguishes these two equations from the equations (7) and (8).

3. NO INCENTIVES MODEL

In this section we consider a firm which doesn't sep up any incentives scheme.

If the firm is profit maximising, then the effort e demanded to each individual would be determined by

$$
\max_{\boldsymbol{e_0},\boldsymbol{e_t}} \Pi:=\sum_{t=0}^{\infty}\pi_t=\sum_{t=0}^{\infty}\left(1-b\right)\cdot y\left(\boldsymbol{e_t}\right)
\tag{13}
$$

If the effort e was not observable, then the workers would attempt to make less effort e than what pre-determined. Therefore the firm should maximise "her" profit subject to the constraint the workers maximise "their" utility (*incentive constraint*)

$$\begin{aligned}&\max_{\boldsymbol{e}_t}\ \Pi &&:= \sum_{t=0}^{\infty}(1-b)\cdot y(\boldsymbol{e}_t)\\ &\text{s.t.} && \boldsymbol{e}_t^i \text{ risolve } \max_{e_t^i}\ U_t^i := \left(w(\boldsymbol{e}_t)-c_i\left(e_{1,t}^i\right)\right)+\beta\cdot\left(w(\boldsymbol{e}_t)-c_i\left(e_{2,t+1}^i\right)\right)\ \left(\mu_t^i\cdot p^i\cdot N_t\right)\end{aligned} \tag{14}$$

However the *moral hazard* problem can be solved upstream. In fact the firm can be considered as an economic subject where ownership and workers cooperates by combining and then by realizing different (if not opposite) interests. In this contest, the firm maximises the wellbeing of the current and the future workers

$$\begin{aligned}\max_{\boldsymbol{e}_0,\boldsymbol{e}_t}\ W &:= \sum_{t=0}^{\infty} N_t\cdot\sum_{i=P,E} p^i\cdot U_t^i\\ &:= \sum_{t=0}^{\infty} N_0\cdot(1+n)^t\cdot\sum_{i=P,E} p^i\cdot\left(w-c_i\left(e_{1,t}^i\right)\right)+\beta\cdot\left(w-c_i\left(e_{2,t+1}^i\right)\right)\end{aligned} \tag{15}$$

Since the wage w is given, the utility of each worker (young or old, belonging to type P or to type E) is not a function of his effort e: the worker is not interested in the effect his effort e has on the production level y. Therefore the effort e made by all the workers (P and E) in both phases of their working life ($\tau=1$ and $\tau=2$) is the minimum effort $\underline{e}$

$$e_{1,t}^i = e_{2,t+1}^i = \underline{e} \Rightarrow e_{1,t}^P = e_{2,t+1}^E = 0.2 \tag{16}$$

The table 1 displays the productions levels y by considering $N_0=10$ and $N_{100}=2000$; thus with $n=\left(N_T/N_0\right)^{1/T}-1$ the employment rate is $n=0.0544$.

Table 1: Production Levels without Incentives

	0	25	50	75	100
N_t	10	38	141	532	2000
y_t	4	15	55	207	779

Even if all the workers make the minimum effort $\underline{e}=0.2$, the production level y increases because of $n=0.0544$ (with $n=0$ the production level y would be constant, i.e. $y_t=4$ for every t).

4. Classical Incentives Scheme

In this section we consider the wage w as a function of the production y and thus as a function of the effort e made by all the workers (see equations (5) and (6)).

Since in $t=0$ there are only young workers and in $t>0$ there are both young and old workers, the equation which describes the production in $t=0$ is different from the equation which describes the production in $t>0$ (compare the equations (2) and (3)). Hence with the classical incentives scheme also the utility function in $t=0$ is different from the utility function in $t>0$ (compare the equations (7) and (8)) and the maximisation problem is

$$
\begin{aligned}
\max_{e_0, e_t} W \quad &\coloneqq N_0 \cdot \sum_{i=P,E} p^i \cdot U_0^i + \sum_{t=1}^{\infty} N_t \cdot \sum_{i=P,E} p^i \cdot U_t^i \\
&\coloneqq N_0 \cdot \sum_{i=P,E} p^i \cdot U_0^i + \sum_{t=1}^{\infty} N_0 \cdot (1+n)^t \cdot \sum_{i=P,E} p^i \cdot U_t^i
\end{aligned} \tag{17}
$$

where we sum up the utilities of the workers hired in $t=0$ and the utilities of the workers hired in the following periods $t>0$, with $t=1\ldots\infty$.

Let us consider the wellbeing of the individuals hired in $t=0$. In $t=0$ these workers are young (in the working phase $\tau=1$) and they make the effort $e_{1,0}^i$; the *FOCs* are

$$
\begin{aligned}
\frac{\partial W}{\delta e_{1,0}^P} &= N_0 \cdot p^P \cdot \frac{\partial U_0^P}{\delta e_{1,0}^P} + N_0 \cdot p^E \cdot \frac{\partial U_0^E}{\delta e_{1,0}^P} = N_0 \cdot \left(b \cdot p^P\right) - N_0 \cdot p^P \cdot c_P'\left(e_{1,0}^P\right) = 0 \\
\frac{\partial W}{\delta e_{1,0}^E} &= N_0 \cdot p^E \cdot \frac{\partial U_0^E}{\delta e_{1,0}^E} + N_0 \cdot p^P \cdot \frac{\partial U_0^P}{\delta e_{1,0}^E} = N_0 \cdot \left(b \cdot p^E\right) - N_0 \cdot p^E \cdot c_E'\left(e_{1,0}^E\right) = 0
\end{aligned} \tag{18}
$$

In $t=1$ the workers hired in $t=0$ are old (in the working phase $\tau=2$) and they make the effort $e_{2,1}^i$. However, the firm has to account the effort $e_{2,1}^i$ affects also the utility U_1^i of the young workers (in the working phase $\tau=1$) hired in $t=1$. Therefore the *FOCs* are

$$
\begin{aligned}
\frac{\partial W}{\delta e_{2,1}^P} &= N_0 \cdot \left(p^P \cdot \frac{\partial U_0^P}{\delta e_{2,1}^P} + p^E \cdot \frac{\partial U_0^E}{\delta e_{2,1}^P} \right) + N_0 \cdot (1+n) \cdot \left(p^P \cdot \frac{\partial U_1^P}{\delta e_{2,1}^P} + p^E \cdot \frac{\partial U_1^E}{\delta e_{2,1}^P} \right) = \\
&= N_0 \cdot \left\{ \left[\beta + (1+n)\right] \cdot \left(b \cdot \frac{1}{2+n} \cdot p^P \right) - \beta \cdot p^P \cdot c_P'\left(e_{2,1}^P\right) \right\} = 0 \\
\frac{\partial W}{\delta e_{2,1}^E} &= N_0 \cdot \left(p^E \cdot \frac{\partial U_0^E}{\delta e_{2,1}^E} + p^P \cdot \frac{\partial U_0^P}{\delta e_{2,1}^E} \right) + N_0 \cdot (1+n) \cdot \left(p^E \cdot \frac{\partial U_1^E}{\delta e_{2,1}^E} + p^P \cdot \frac{\partial U_1^P}{\delta e_{2,1}^E} \right) = \\
&= N_0 \cdot \left\{ \left[\beta + (1+n)\right] \cdot \left(b \cdot \frac{1}{2+n} \cdot p^E \right) - \beta \cdot p^E \cdot c_E'\left(e_{2,1}^E\right) \right\} = 0
\end{aligned} \tag{19}
$$

The individuals hired in t when they are young (in the working phase $\tau=1$) make the effort $e_{1,t}^i$; the effort $e_{1,t}^i$ affects not only their utility U_t^i, but also the utility U_{t-1}^i of the individuals hired in $t-1$ who are old (in the working phase $\tau=2$) in t. Hence the *FOCs* are

$$
\begin{aligned}
\frac{\partial W}{\delta e_{1,t}^P} &= N_0 \cdot (1+n)^{t-1} \cdot \left(p^P \cdot \frac{\partial U_{t-1}^P}{\delta e_{1,t}^P} + p^E \cdot \frac{\partial U_{t-1}^E}{\delta e_{1,t}^P} \right) + N_0 \cdot (1+n)^t \cdot \left(p^P \cdot \frac{\partial U_t^P}{\delta e_{1,t}^P} + p^E \cdot \frac{\partial U_t^E}{\delta e_{1,t}^P} \right) = \\
&= N_0 \cdot (1+n)^t \cdot \left\{ \left[\beta + (1+n)\right] \cdot \left(b \cdot \frac{1}{2+n} \cdot p^P \right) - p^P \cdot c_P'\left(e_{1,t}^P\right) \right\} = 0 \\
\frac{\partial W}{\delta e_{1,t}^E} &= N_0 \cdot (1+n)^{t-1} \cdot \left(p^E \cdot \frac{\partial U_{t-1}^E}{\delta e_{1,t}^E} + p^P \cdot \frac{\partial U_{t-1}^P}{\delta e_{1,t}^E} \right) + N_0 \cdot (1+n)^t \cdot \left(p^E \cdot \frac{\partial U_t^E}{\delta e_{1,t}^E} + p^P \cdot \frac{\partial U_t^P}{\delta e_{1,t}^E} \right) = \\
&= N_0 \cdot (1+n)^t \cdot \left\{ \left[\beta + (1+n)\right] \cdot \left(b \cdot \frac{1}{2+n} \cdot p^E \right) - p^E \cdot c_E'\left(e_{1,t}^E\right) \right\} = 0
\end{aligned} \tag{20}
$$

The workers hired in t are old (in the working phase $\tau = 2$) in the following period $t+1$ when they make the effort $e^i_{2,t+1}$; since the effort $e^i_{2,t+1}$ affects also the utility U^i_{t+1} of the young workers (in the working phase $\tau = 1$) hired in $t+1$, the *FOCs* are

$$\begin{aligned}
\frac{\partial W}{\delta e^P_{2,t+1}} &= N_0 \cdot (1+n)^t \cdot \left(p^P \cdot \frac{\partial U^P_t}{\delta e^P_{2,t+1}} + p^E \cdot \frac{\partial U^E_t}{\delta e^P_{2,t+1}} \right) + N_0 \cdot (1+n)^{t+1} \cdot \left(p^P \cdot \frac{\partial U^P_{t+1}}{\delta e^P_{2,t+1}} + p^E \cdot \frac{\partial U^E_{t+1}}{\delta e^P_{2,t+1}} \right) = \\
&= N_0 \cdot (1+n)^t \cdot \left\{ \left[\beta + (1+n) \right] \cdot \left(b \cdot \frac{1}{2+n} \cdot p^P \right) - p^P \cdot \beta \cdot c'_P \left(e^P_{2,t+1} \right) \right\} = 0 \\
\frac{\partial W}{\delta e^E_{2,t+1}} &= N_0 \cdot (1+n)^t \cdot \left(p^E \cdot \frac{\partial U^E_t}{\delta e^E_{2,t+1}} + p^P \cdot \frac{\partial U^P_t}{\delta e^E_{2,t+1}} \right) + N_0 \cdot (1+n)^{t+1} \cdot \left(p^E \cdot \frac{\partial U^E_{t+1}}{\delta e^E_{2,t+1}} + p^P \cdot \frac{\partial U^P_{t+1}}{\delta e^E_{2,t+1}} \right) = \\
&= N_0 \cdot (1+n)^t \cdot \left\{ \left[\beta + (1+n) \right] \cdot \left(b \cdot \frac{1}{2+n} \cdot p^E \right) - p^E \cdot \beta \cdot c'_E \left(e^E_{2,t+1} \right) \right\} = 0
\end{aligned} \tag{21}$$

Thus the effort $e^i_{1,0}$ of the young workers (in the working phase $\tau = 1$) in $t = 0$ satisfies

$$\left.\begin{aligned} b - c'_P\left(e^P_{1,0}\right) = 0 \\ b - c'_E\left(e^E_{1,0}\right) = 0 \end{aligned}\right. \Rightarrow c'_i\left(e^i_{1,0}\right) = b \tag{22}$$

and, if we assume the cost function $c_i\left(e^i_{\tau,t}\right) = \left(e^i_{\tau,t}\right)^{exp_i}$, we can write the optimal level of the effort $e^i_{1,0}$ the firm demands in $t = 0$ to the young workers (in the working phase $\tau = 1$) for different values of the share b of the production y assigned to cover the labour cost

$$\begin{aligned} 4 \cdot \left(e^P_{1,0}\right)^3 = b \Rightarrow e^P_{1,0} = \sqrt[3]{\frac{b}{4}} \\ 3 \cdot \left(e^E_{1,0}\right)^2 = b \Rightarrow e^E_{1,0} = \sqrt[2]{\frac{b}{3}} \end{aligned} \tag{23}$$

The figure 2 displays the values of $e^P_{1,0}$ and $e^E_{1,0}$ for $0 \le b \le 1$; obviously we obtain $e^P_{1,0} > e^E_{1,0}$ for every value of b.

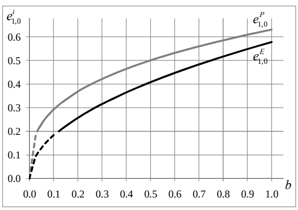

Figure 2: The Young Workers Effort in $t = 0$

From the figure 2 it is evident the type P workers reach the minimum effort level $\underline{e} = 0.2$ for a value b lower than the type E workers ($b = 0.032$ and not $b = 0.120$).

The effort $e^i_{1,t}$ of the young workers (in the working phase $\tau = 1$) in $t > 0$ satisfies

$$\begin{array}{l}\left[\beta+(1+n)\right]\cdot\left(b\cdot\dfrac{1}{2+n}\right)-c_P'\left(e_{1,t}^P\right)=0\\ \left[\beta+(1+n)\right]\cdot\left(b\cdot\dfrac{1}{2+n}\right)-c_E'\left(e_{1,t}^E\right)=0\end{array}\Rightarrow c_i'\left(e_{1,t}^i\right)=\left[\beta+(1+n)\right]\cdot\left(b\cdot\frac{1}{2+n}\right) \tag{24}$$

and with the cost function $c_i\left(e_{\tau,t}^i\right)=\left(e_{\tau,t}^i\right)^{exp_i}$ the optimal effort demanded to these workers is

$$\begin{array}{l}4\cdot\left(e_{1,t}^P\right)^3=\left[\beta+(1+n)\right]\cdot\left(b\cdot\dfrac{1}{2+n}\right)\Rightarrow e_{1,t}^P=\sqrt[3]{\dfrac{\left[\beta+(1+n)\right]\cdot\left(b\cdot\dfrac{1}{2+n}\right)}{4}}\\ 3\cdot\left(e_{1,t}^E\right)^2=\left[\beta+(1+n)\right]\cdot\left(b\cdot\dfrac{1}{2+n}\right)\Rightarrow e_{1,t}^E=\sqrt[2]{\dfrac{\left[\beta+(1+n)\right]\cdot\left(b\cdot\dfrac{1}{2+n}\right)}{3}}\end{array} \tag{25}$$

With $\beta=0.8$ and $n=0.0544$, the figure 3 displays the values of $e_{1,t}^P$ and $e_{1,t}^E$ for $0\le b\le 1$; obviously $e_{1,t}^P>e_{1,t}^E$ for every value of b.

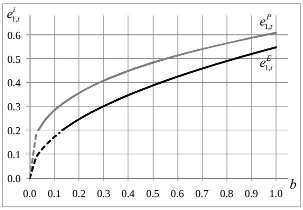

Figure 3: The Young Workers Effort in $t>0$

Also in the figure 3 it is evident how the type P workers make the minimum effort $\underline{e}=0.2$ with a share b of the production y assigned to cover the labour cost lower than the type E workers ($b=0.035$ and not $b=0.133$).

The effort of the old workers (in the working phase $\tau=2$) satisfies the same equation both if they've been hired in $t=0$ ($e_{2,1}^i$) and if they have been hired in $t>0$ ($e_{2,t+1}^i$); in fact by considering $t\ge 0$ from the equations (19) and (21) we have

$$\begin{array}{l}\left[\beta+(1+n)\right]\cdot\left(b\cdot\dfrac{1}{2+n}\right)-\beta\cdot c_P'\left(e_{2,t+1}^P\right)=0\\ \left[\beta+(1+n)\right]\cdot\left(b\cdot\dfrac{1}{2+n}\right)-\beta\cdot c_E'\left(e_{2,t+1}^E\right)=0\end{array}\Rightarrow c_i'\left(e_{2,t+1}^i\right)=\frac{\left[\beta+(1+n)\right]\cdot\left(b\cdot\dfrac{1}{2+n}\right)}{\beta} \tag{26}$$

and thus, by assuming the cost function $c_i\left(e_{\tau,t}^i\right)=\left(e_{\tau,t}^i\right)^{exp_i}$, we can derive the effort demanded to the old workers in every period $t\ge 0$

$$
\begin{aligned}
&4\cdot\left(e_{2,t+1}^{P}\right)^{3}=\frac{\left[\beta+\left(1+n\right)\right]\cdot\left(b\cdot\frac{1}{2+n}\right)}{\beta}\Rightarrow e_{2,t+1}^{P}=\sqrt[3]{\frac{\left[\beta+\left(1+n\right)\right]\cdot\left(b\cdot\frac{1}{2+n}\right)}{\beta\cdot 4}}\\
&3\cdot\left(e_{2,t+1}^{E}\right)^{2}=\frac{\left[\beta+\left(1+n\right)\right]\cdot\left(b\cdot\frac{1}{2+n}\right)}{\beta}\Rightarrow e_{2,t+1}^{E}=\sqrt[2]{\frac{\left[\beta+\left(1+n\right)\right]\cdot\left(b\cdot\frac{1}{2+n}\right)}{\beta\cdot 3}}
\end{aligned}
\tag{27}
$$

In the figure 4 the values of $e_{2,t+1}^{P}$ and $e_{2,t+1}^{E}$ for $0\le b\le 1$ are displayed and also in this case we obtain $e_{2,t+1}^{P}>e_{2,t+1}^{E}$ for every value of b.

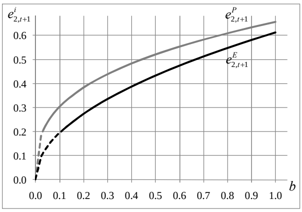

Figure 4: The Old Workers Effort in $t+1$ with $t\ge 0$

As in the two first cases (the figures 2 and 3), from the figure 4 we can see the type P workers make the minimum effort $\underline{e}=0.2$ with a value b lower than the type E workers ($b=0.028$ and not $b=0.106$).

While the effort of the young individuals in the equation (22) doesn't depend on the rate of time preference β, the effort of young and old individuals in the equations (24) and (26) depends on β, and thus on the weight these individuals give to the future with respect to the present.

In the figures 5 and 6 we compare the production levels y obtained without incentives scheme (when the workers make the minimum effort $\underline{e}=0.2$) and the production levels y obtained with the classical incentives scheme.

As previously said, in $t=0$ the workers make at least the minimum effort $\underline{e}=0{,}2$ when the share of the production y assigned to cover the labour cost is $b=0.120$ ($b=0.120$ for the type E workers and $b=0.032$ for the type P workers).

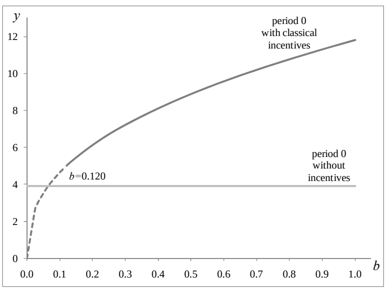

Figure 5: The Production y in Period $t=0$

In the figure 5 it is shown how the production obtained when the wage doesn't depend on the production level (light grey line) is always lower than the production obtained when the firm sets up a scheme to motivate the workers to make more effort (dark grey line).

From the previous analysis we know that in $t > 0$ the workers make the minimum effort $\underline{e} = 0.2$ if the share of the production y assigned to cover the labour cost is at least $b = 0.133$ ($b = 0.133$ for the type E young workers, $b = 0.035$ for the type P young workers, $b = 0.106$ for the type E old workers and $b = 0.028$ for the type P old workers).

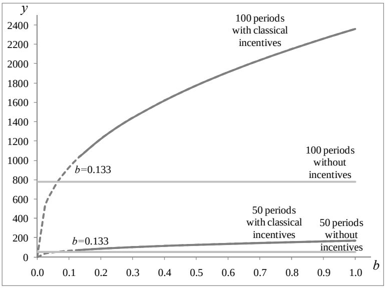

Figure 6: The Production y in Period $t > 0$

Also from the figure 6 it is evident the production obtained without incentives (light grey line) is lower than the production obtained if the firm motivates the workers (dark grey line).

5. Rewarding Incentives Scheme

The results obtained in the previous section suggest the firm could obtain an higher production level y by setting up an incentives scheme which rewards the individuals willing to work with an higher effort e. This scheme consists in the assignment of a share of the profit to the type P individuals in the second phase of their working life ($\tau = 2$): the type P individual becomes a partner. As said in the second section, if the incentives system is structured such that to reward the workers able to become partners, the type E individuals are inclined to mimic the type P individuals.

If the firm can identify the two individual types (the cost functions are observable) the maximisation problem is the (17) where U_0^E and U_t^E are the equations (7) and (8) - the wage of type E workers depends only on the production y - and U_0^P and U_t^P are the equations (11) and (12) - the wage of type P workers depends also on the profit π.

Let us consider the wellbeing of individuals hired in $t = 0$. These workers in $t = 0$ are young (in the working phase $\tau = 1$) and they make the effort $e_{1,0}^i$: the *FOCs* are the conditions (18) obtained with the classical incentives scheme.

In $t = 1$ the workers hired in $t = 0$ are old (in the working phase $\tau = 2$) and they make the effort $e_{2,1}^i$; the *FOCs* are

$$
\begin{aligned}
\frac{\partial W}{\delta e_{2,1}^{P}} &= N_0\cdot\left(p^P\cdot\frac{\partial U_0^P}{\delta e_{2,1}^P}+p^E\cdot\frac{\partial U_0^E}{\delta e_{2,1}^P}\right)+N_0\cdot(1+n)\cdot\left(p^P\cdot\frac{\partial U_1^P}{\delta e_{2,1}^P}+p^E\cdot\frac{\partial U_1^E}{\delta e_{2,1}^P}\right)=\\
&= N_0\cdot\left\{\left[\beta+(1+n)\right]\cdot\left(b\cdot\frac{1}{2+n}\cdot p^P\right)-\beta\cdot p^P\cdot c_P'\left(e_{2,1}^P\right)+\beta\cdot p^P\cdot(1-b)\right\}=0\\
\frac{\partial W}{\delta e_{2,1}^{E}} &= N_0\cdot\left(p^E\cdot\frac{\partial U_0^E}{\delta e_{2,1}^E}+p^P\cdot\frac{\partial U_0^P}{\delta e_{2,1}^E}\right)+N_0\cdot(1+n)\cdot\left(p^E\cdot\frac{\partial U_1^E}{\delta e_{2,1}^E}+p^P\cdot\frac{\partial U_1^P}{\delta e_{2,1}^E}\right)=\\
&= N_0\cdot\left\{\left[\beta+(1+n)\right]\cdot\left(b\cdot\frac{1}{2+n}\cdot p^E\right)-\beta\cdot p^E\cdot c_E'\left(e_{2,1}^E\right)+\beta\cdot p^E\cdot(1-b)\right\}=0
\end{aligned}
\tag{28}
$$

where the grey elements differentiate these conditions from the conditions (19).

The individuals hired in t in the working phase $\tau=1$ (when they are young) make the effort $e_{1,t}^{i}$ and the *FOCs* are

$$
\begin{aligned}
\frac{\partial W}{\delta e_{1,t}^{P}} &= N_0\cdot(1+n)^{t-1}\cdot\left(p^P\cdot\frac{\partial U_{t-1}^P}{\delta e_{1,t}^P}+p^E\cdot\frac{\partial U_{t-1}^E}{\delta e_{1,t}^P}\right)+N_0\cdot(1+n)^{t}\cdot\left(p^P\cdot\frac{\partial U_t^P}{\delta e_{1,t}^P}+p^E\cdot\frac{\partial U_t^E}{\delta e_{1,t}^P}\right)=\\
&= N_0\cdot(1+n)^{t}\cdot\left\{\left[\beta+(1+n)\right]\cdot\left(b\cdot\frac{1}{2+n}\right)-c_P'\left(e_{1,t}^P\right)+\beta\cdot(1-b)\right\}=0\\
\frac{\partial W}{\delta e_{1,t}^{E}} &= N_0\cdot(1+n)^{t-1}\cdot\left(p^E\cdot\frac{\partial U_{t-1}^E}{\delta e_{1,t}^E}+p^S\cdot\frac{\partial U_{t-1}^P}{\delta e_{1,t}^E}\right)+N_0\cdot(1+n)^{t}\cdot\left(p^E\cdot\frac{\partial U_t^E}{\delta e_{1,t}^E}+p^P\cdot\frac{\partial U_t^P}{\delta e_{1,t}^E}\right)=\\
&= N_0\cdot(1+n)^{t}\cdot\left\{\left[\beta+(1+n)\right]\cdot\left(b\cdot\frac{1}{2+n}\right)-c_E'\left(e_{1,t}^E\right)+\beta\cdot(1-b)\right\}=0
\end{aligned}
\tag{29}
$$

where the grey elements differentiate these conditions from the conditions (20).

The workers hired in t in the following period $t+1$ are old (in the working phase $\tau=2$) and they make the effort $e_{2,t+1}^{i}$; the *FOCs* are

$$
\begin{aligned}
\frac{\partial W}{\delta e_{2,t+1}^{P}} &= N_0\cdot(1+n)^{t}\cdot\left(p^P\cdot\frac{\partial U_{t}^P}{\delta e_{2,t+1}^P}+p^E\cdot\frac{\partial U_{t}^E}{\delta e_{2,t+1}^P}\right)+N_0\cdot(1+n)^{t+1}\cdot\left(p^P\cdot\frac{\partial U_{t+1}^P}{\delta e_{2,t+1}^P}+p^E\cdot\frac{\partial U_{t+1}^E}{\delta e_{2,t+1}^P}\right)=\\
&= N_0\cdot(1+n)^{t}\cdot\left\{\left[\beta+(1+n)\right]\cdot\left(b\cdot\frac{1}{2+n}\right)-\beta\cdot c_P'\left(e_{2,t+1}^P\right)+\beta\cdot(1-b)\right\}=0\\
\frac{\partial W}{\delta e_{2,t+1}^{E}} &= N_0\cdot(1+n)^{t}\cdot\left(p^E\cdot\frac{\partial U_{t}^E}{\delta e_{2,t+1}^E}+p^P\cdot\frac{\partial U_{t}^P}{\delta e_{2,t+1}^E}\right)+N_0\cdot(1+n)^{t+1}\cdot\left(p^E\cdot\frac{\partial U_{t+1}^E}{\delta e_{2,t+1}^E}+p^P\cdot\frac{\partial U_{t+1}^P}{\delta e_{2,t+1}^E}\right)=\\
&= N_0\cdot(1+n)^{t}\cdot\left\{\left[\beta+(1+n)\right]\cdot\left(b\cdot\frac{1}{2+n}\right)-\beta\cdot c_E'\left(e_{2,t+1}^E\right)+\beta\cdot(1-b)\right\}=0
\end{aligned}
\tag{30}
$$

where, as in the two previous cases, the grey elements differentiate these conditions from the conditions (21).

As with the classical incentives scheme, in $t=0$ the effort of the young individuals satisfies the (22) and, if we assume the cost function $c_i\left(e_{\tau,t}^{i}\right)=\left(e_{\tau,t}^{i}\right)^{exp_i}$, the optimal value of $e_{1,0}^{i}$ demanded to young workers in $t=0$ for different values of the share b of the production y assigned to cover the labour cost is the (23). Obviously the values of $e_{1,0}^{P}$ and $e_{1,0}^{E}$ for $0 \le b \le 1$ are the same values displayed in the figure 2 and we obtain $e_{1,0}^{P} > e_{1,0}^{E}$ for every value of b.

Differently the effort of the young individuals for $t>0$ satisfies

$$\begin{rcases} \left\{\left[\beta+(1+n)\right]\cdot\left(b\cdot\frac{1}{2+n}\right)+\beta\cdot(1-b)-c_P'\left(e_{1,t}^{P}\right)\right\}=0 \\ \left\{\left[\beta+(1+n)\right]\cdot\left(b\cdot\frac{1}{2+n}\right)+\beta\cdot(1-b)-c_E'\left(e_{1,t}^{E}\right)\right\}=0 \end{rcases} \Rightarrow c_i\left(e_{1,t}^{i}\right)=\left[\beta+(1+n)\right]\cdot\left(b\cdot\frac{1}{2+n}\right)+\beta\cdot(1-b) \quad (31)$$

and with the cost function $c_i\left(e_{\tau,t}^{i}\right)=\left(e_{\tau,t}^{i}\right)^{exp_i}$ the optimal effort demanded to these workers is

$$\begin{aligned} 4\cdot\left(e_{1,t}^{P}\right)^{3}&=\left[\beta+(1+n)\right]\cdot\left(b\cdot\frac{1}{2+n}\right)+\beta\cdot(1-b)\Rightarrow e_{1,t}^{P}=\sqrt[3]{\frac{\left[\beta+(1+n)\right]\cdot\left(b\cdot\frac{1}{2+n}\right)+\beta\cdot(1-b)}{4}} \\ 3\cdot\left(e_{1,t}^{E}\right)^{2}&=\left[\beta+(1+n)\right]\cdot\left(b\cdot\frac{1}{2+n}\right)+\beta\cdot(1-b)\Rightarrow e_{1,t}^{E}=\sqrt[2]{\frac{\left[\beta+(1+n)\right]\cdot\left(b\cdot\frac{1}{2+n}\right)+\beta\cdot(1-b)}{3}} \end{aligned} \quad (32)$$

With $\beta=0.8$ and $n=0.0544$, the figure 7 displays the values of $e_{1,t}^{P}$ and $e_{1,t}^{E}$ for $0 \le b \le 1$; obviously also in this case we obtain $e_{1,t}^{P} > e_{1,t}^{E}$ for every value of b.

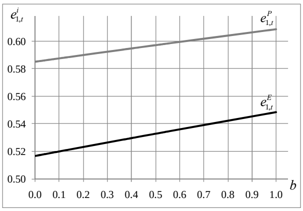

Figure 7: The Effort of Young Workers in $t>0$

From the figure 7 it is evident both the type P and the type E workers are demanded an effort level higher than the minimum level $\underline{e}=0.2$ for every share b of the production y assigned to cover the labour cost.

The effort of the old workers (in the working phase $\tau=2$) satisfies the same equation both if they've been hired in $t=0$ ($e_{2,1}^{i}$) and if they have been hired in $t>0$ ($e_{2,t+1}^{i}$); in fact by considering $t \ge 0$ from the equations (28) and (30) we have

$$\begin{rcases} \left[\beta+(1+n)\right]\cdot\left(b\cdot\frac{1}{2+n}\right)-\beta\cdot c_P'\left(e_{2,t+1}^{P}\right)+\beta\cdot(1-b)=0 \\ \left[\beta+(1+n)\right]\cdot\left(b\cdot\frac{1}{2+n}\right)-\beta\cdot c_E'\left(e_{2,t+1}^{E}\right)+\beta\cdot(1-b)=0 \end{rcases} \Rightarrow c_i'\left(e_{2,t+1}^{i}\right)=\frac{\left[\beta+(1+n)\right]\cdot\left(b\cdot\frac{1}{2+n}\right)+\beta\cdot(1-b)}{\beta} \quad (33)$$

and thus, by assuming $c_i\left(e_{\tau,t}^i\right)=\left(e_{\tau,t}^i\right)^{exp_i}$, we can derive the effort demanded to the old individuals in every period $t \geq 0$

$$\begin{aligned}
4\cdot\left(e_{2,t+1}^P\right)^3 &= \frac{\left[\beta+(1+n)\right]\cdot\left(b\cdot\dfrac{1}{2+n}\right)+\beta\cdot(1-b)}{\beta} \Rightarrow e_{2,t+1}^P=\sqrt[3]{\frac{\left[\beta+(1+n)\right]\cdot\left(b\cdot\dfrac{1}{2+n}\right)+\beta\cdot(1-b)}{\beta\cdot 4}} \\
3\cdot\left(e_{2,t+1}^E\right)^2 &= \frac{\left[\beta+(1+n)\right]\cdot\left(b\cdot\dfrac{1}{2+n}\right)+\beta\cdot(1-b)}{\beta} \Rightarrow e_{2,t+1}^E=\sqrt[2]{\frac{\left[\beta+(1+n)\right]\cdot\left(b\cdot\dfrac{1}{2+n}\right)+\beta\cdot(1-b)}{\beta\cdot 3}}
\end{aligned} \tag{34}$$

In the figure 8 the efforts $e_{2,t+1}^P$ and $e_{2,t+1}^E$ for $0 \leq b \leq 1$ are displayed; also in this case we obtain $e_{2,t+1}^P > e_{2,t+1}^E$ for every value of b.

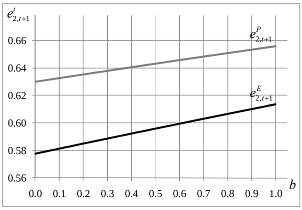

Figure 8: The Old Workers Effort in $t+1$ with $t \geq 0$

In the figure 9 we compare the production level y for different values of b (share of the production y assigned to cover the labour cost) if the firm decides to reward the type P workers and the production level y if the firm sets up the classical incentives scheme.

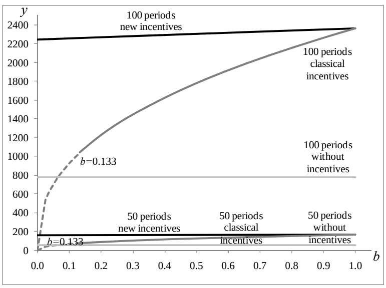

Figure 9: The Production y in the Period $t > 0$

From the figure 9 it is evident how the rewarding incentives scheme (black line) allows to obtain a production level y higher than the production level y obtainable with the classical incentives scheme (dark grey line) for every value of the share b of the production y assigned to cover the labour cost.

If the firm cannot distinguish the two individual types, the firm has to maximise the utility of all the workers taking into account the *adverse selection* problem

$$\begin{aligned}\max_{\boldsymbol{e}_0,\boldsymbol{e}_t} W \quad &:= N_0\cdot\sum_{i=P,E} p^i\cdot U_0^i+\sum_{t=1}^{\infty}N_t\cdot\sum_{i=P,E}p^i\cdot U_t^i = N_0\cdot\sum_{i=P,E}p^i\cdot U_0^i+\sum_{t=1}^{\infty}N_0\cdot(1+n)^t\cdot\sum_{i=P,E}p^i\cdot U_t^i \\ s.t. \quad & \hat{U}_t^P\left(e_{1,t}^E\right)\geq U_t^P\left(e_{1,t}^P\right)\end{aligned} \tag{35}$$

The *self-selection constraint* is specified as follows

$$\left(w(\boldsymbol{e_t})-c_E\left(e_{1,t}^E\right)\right)+\beta\cdot\left(w(\boldsymbol{e_{t+1}})-c_E\left(e_{2,t+1}^E\right)\right)\geq\left(w(\boldsymbol{e_t})-c_E\left(e_{1,t}^P\right)\right)+\beta\cdot\left(w_2^P(\boldsymbol{e_{t+1}})-c_E\left(e_{2,t+1}^E\right)\right) \tag{36}$$

The utility guaranteed by the firm to type E individuals has to be higher or equal than the utility these individuals could obtain if they would mimic the type P individuals.

The figure 10 displays the utility of the type P individuals (grey line), the utility of the type E individuals (dark line) and the utility of type E individuals who mimic the type P individuals in the working phase $\tau=1$ (dashed black line).

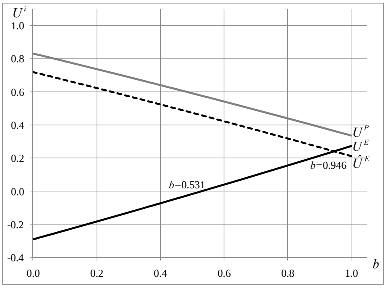

Figure 10: The Wellbeing of Partner and Employed

The type E individuals obtain the same utility if they mimic and if they don't when $b=0.946$ (where the two utility lines cross). Therefore, we can conclude for sufficiently high values of b the type E workers don't mimic the type P workers.

6. Conclusions

The paper analyses different incentives schemes when the workers are heterogeneous: there are workers who are willing to make more effort than the others.

Thus the analysis compares two incentives schemes: with the classical incentives scheme the wage of all the workers depends on the production obtained given their joined effort and with the rewarding incentives scheme the more worth workers receive also a share of the profit.

The rewarding incentives scheme allows to obtain a higher production level for every share of the production assigned to cover the labour cost and this scheme is *self-selection* compatible if the wage received by all the workers is appropriate (i.e. if the share of the production assigned to cover the labour cost is sufficiently high).

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