Multilevel Flexible Specification of the Production Function in Health Economics

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Abstract

Previous studies on efficiency of hospitals often refer to quite restrictive functional forms for the technology. In this paper, referring to a study about some hospitals in Lombardy, we propose a statistical model based on the translogarithmic function - the most widely used flexible functional form (Christensen, Jorgenson and Lau, 1973). More specifically, in order to take into consideration the hierarchical structure of the data (as in Gori, Grassetti and Rossi, 2004), we propose a multilevel model, ignoring for the moment the one-side error specification, typical of stochastic frontier analysis. Given this simplification, however, we are easily able to take into account some typical econometric problems as, for example, heteroscedasticity.

The estimated production function can be used to identify the technical inefficiency of hospitals (as already seen in previous works), but also to draw some economic considerations about scale elasticity, scale efficiency and optimal resource allocation of the productive units. We will show, in fact, that for the translogarithmic specification it is possible to obtain the elasticity of the output (regarding an input) at hospital level as a weighted sum of elasticities at ward level. Analogous results can be achieved for scale elasticity, which measures how output changes in response to simultaneous inputs variation. In addition, referring to scale efficiency and to optimal resource allocation, we will consider the results of Ray (1998) in our context.

The interpretation of the results is surely an interesting administrative instrument for decision makers in order to analyze the productive conditions of each hospital and its single wards and also to decide the preferable interventions.

Keywords: Elasticity, Multilevel Models, Production Function Analysis, Scale Efficiency, Scale Elasticity, Translog Function.

1 Introduction

Previous studies on hospitals efficiency often refer to quite restrictive functional forms for the technology. In this paper, referring to a study about some hospitals in Lombardy, we formulate convenient correctives to a statistical model based on the translogarithmic function, which is the most widely used flexible functional form for economic functions (Christensen, Jorgenson and Lau, 1973; Kim, 1992; Grant, 1993; Ryan and Wales, 2000). This model allows to obtain also closed form measures of scale elasticity and scale efficiency, readily computable from the fitted model. The aim of this work is, indeed, to provide an administrative instrument, based on stochastic production function analysis, which is able both to identify non standard productive conditions and propose convenient correctives, in the sense of inputs re-allocation.

The translogarithmic specification is the second order Taylor approximation of a generic production function. In the simple case of one output (y) and two input variables (x_1, x_2), it is equal to:

$$\ln(y) = \alpha_0 + \alpha_1 \ln(x_1) + \alpha_2 \ln(x_2) + \frac{\beta_{11}}{2}[\ln(x_1)]^2 + \beta_{12} \ln(x_1) \ln(x_2) + \frac{\beta_{22}}{2}[\ln(x_2)]^2. \quad (1)$$

The success of this functional form in many econometric applications is due to its flexibility (for example, the elasticity of the output with respect to an input is not constant, as for the Cobb-Douglas, but it is a function of the inputs). This flexibility allows a large adaptability of the model, but, at the same time, increases the multicollinearity.

In the present case of study the choice of the translogarithmic specification is mainly connected to the non-linearity of the relation between beds number (an input variable) and the hospitalizations number (the output variable). In addition, it allows to make some economic considerations about scale elasticity, scale efficiency and optimal resource's allocation of the productive units.

In order to take into consideration the hierarchical structure of the data, typical of health context (Leyland and Goldstein, 2001), we propose a multilevel model, ignoring for the moment the one-side error specification, typical of stochastic frontier analysis. Given this simplification, however, we are able to take into account some typical econometric problems as, for example, heteroscedasticity.

The data involved in the study are characterized by the presence of two levels of data collection. While the first one regards the single wards in each hospital, the second is identified by the hospitals.

The paper is organized as follows. Section 2 describes the data involved in the study. Section 3 illustrates the model proposed. Section 4 presents the results of the analysis. Section 5 proposes some economic considerations on the results. Section 6 gives some conclusions.

2 The Data

As just said the data involved in the study are characterized by a two level hierarchical structure. More, in depth, the considered dataset refers to 1478 first level units (wards) observed in 178 second level units (hospitals) of Italian region Lombardy in 1997. This dataset, described in Gori, Grassetti and Rossi (2004), has been obtained from a deterministic linkage of three distinct archives: the discharge database, the beds allocation database and the staff database. These three archives are collected at different levels of detail. The discharge files are available for each hospitalization and, among the other variables, includes the ward and the hospital identifiers. This information allows to calculate the total number of hospitalizations for each ward in a hospital (i.e. the total number of cases treated in the observed year), which is the output variable (indicated by *NHosp*). The beds number (indicated by *Beds*) is available for each ward in a hospital. The staff variable (*Staff*) is available only at hospital level, without the possibility of distinguishing among substructures, and corresponds to the total number of doctors, nurses and administrative staff.

Then, some other informations are taken into consideration. First of all, in order to distinguish the hospitalizations by degree of complexity, there is the case-mix index, which represents the relative level of case complexity in each ward, given the regional mean. In particular, we use a standardized version of the logarithm of this index (indicated by $std(\ln(CMix))$). Then, in order to take into account the type of hospitalizations disease, an addictive linear component is introduced in the production function model, consisting of four variables. These four variables are two transformations of the Disease Related Groups (DRG) weight variable, calculated at hospital and ward level. The DRG code derive from an international clusterization of the hospitalization cases; the DRG weight provides a "proxy" measure of the complexity of every treated case. The first transformation is the mean of DRG weights (indicated by $\mu_{Weig,ij}$ and $\mu_{Weig,i}$) and represents the mean complexity of observed cases at ward and hospital level. The second one is the standard deviation of DRG weights ($\sigma_{Weig,ij}$ and $\sigma_{Weig,i}$) and allows to control the variability of the complexity in treated cases at the two levels considered. These factors are introduced to affect only the level of production and not the production process itself.

In Table 1 are summarized the descriptive statistics of the variables involved in the model.

Variable	Min	Median	Max	Mean	Stand. Deviation
$\ln(NHosp)$	0.000	6.903	9.806	6.589	1.947
$\ln(Beds)$	0.000	3.296	6.234	3.166	0.926
$\ln(Staff)$	2.833	6.497	8.322	6.442	1.147
mean DRG Weight (Ward lev.)	0.160	0.850	8.320	1.076	0.776
s.d. DRG Weight (Ward lev.)	0.000	0.420	6.870	0.682	0.937
mean DRG Weight (Hospital lev.)	0.630	0.850	2.130	0.886	0.030
s.d. DRG Weight (Hospital lev.)	0.120	0.670	1.950	0.695	0.093
$std(\ln(CMix))$	-10.350	-0.023	2.935	0.000	1

Table 1: Descriptive Statistics

The pair-wise scatter plot in Figure 1 shows the relation between the output variable and the input variables and in particular highlights the strong dependence between hospitalizations number and beds number. From in-depth studies this relationship can not be considered linear.

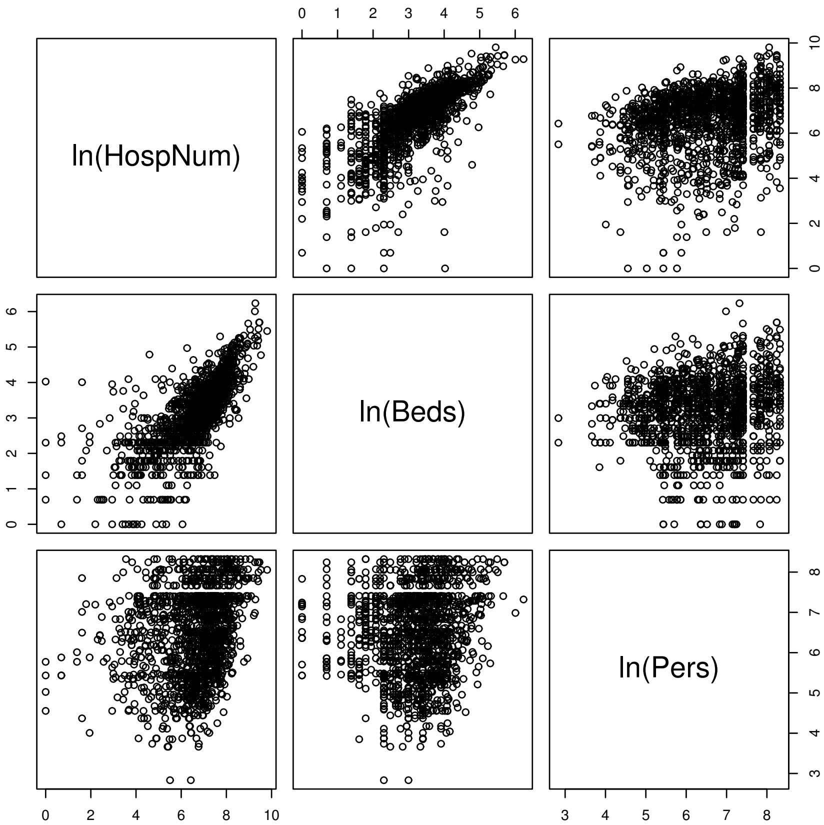

Figure 1: Pair-wise Scatter Plot among output variable and input variables

3 The Model

In order to model the hospitalization phenomenon we use a modified translog specification. In particular, the complete translog model for the hospitalizations number, function of the beds number and the staff of observed units, is modified by the addition of two other components. The first one is a multiplicative term that linearly depends from case-mix index (indicated by $std(\ln(CMix_{ij})))$. The second one is the additive linear component illustrated in Section 2 (indicated in the following by f(Weights)).

Given the hierarchical structure of the data, we propose to adopt a multilevel model, with two levels and random intercept, which can be formulated as follows:

$$
\begin{aligned}
\ln(NHosp_{ij}) &= \alpha(CMix_{ij}) + \beta(CMix_{ij}, Beds_{ij}) + \\
&+ \gamma(CMix_{ij}, Staff_i) + \delta(CMix_{ij}, Beds_{ij}, Staff_i) \\
&+ f(Weights) + u_i + \epsilon_{ij},
\end{aligned} \tag{2}
$$

where

$$
\begin{aligned}
\alpha(CMix_{ij}) &= \alpha_0 + \alpha_1 std(\ln(CMix_{ij})) \\
\beta(CMix_{ij}, Beds_{ij}) &= [\beta_0 + \beta_1 std(\ln(CMix_{ij}))] \cdot \left[\ln(Beds_{ij}) + \tfrac{\beta_2}{2} \ln(Beds_{ij})^2\right] \\
\gamma(CMix_{ij}, Staff_i) &= [\gamma_0 + \gamma_1 std(\ln(CMix_{ij}))] \cdot \left[\ln(Staff_i) + \tfrac{\gamma_2}{2} \ln(Staff_i)^2\right] \\
\delta(CMix_{ij}, Beds_{ij}, Staff_i) &= [\delta_0 + \delta_1 std(\ln(CMix_{ij}))] \cdot [\ln(Beds_{ij}) \cdot \ln(Staff_i)] \\
f(Weights) &= \lambda_1 \mu_{Weig,ij} + \lambda_2 \sigma_{Weig,ij} + \lambda_3 \mu_{Weig,i} + \lambda_4 \sigma_{Weig,i},
\end{aligned}
$$

$u_i \sim N(0, \sigma_u^2)$ are the residuals at hospital level, i=1,...,N,

$\epsilon_{ij} \sim N(0, \sigma_\epsilon^2)$ are the residuals at ward level, j=1,...,M, with $u_i \perp \epsilon_{ij}$.

Using a classical notation for the coefficients of mixed models, equation (2) can be re-written as:

$$
\begin{aligned}
&\ln(NHosp_{ij}) = \alpha_{0i} + \alpha_1 \ln(Beds_{ij}) + \tfrac{\alpha_2}{2}(\ln(Beds_{ij})^2) + \alpha_3 \ln(Staff_i) + \\
&+ \tfrac{\alpha_4}{2}(\ln(Staff_i)^2) + \alpha_5(\ln(Beds_{ij}) \cdot \ln(Staff_i)) + \alpha_6(\ln(Beds_{ij}) \cdot std(\ln(CMix_{ij}))) + \\
&+ \alpha_7(\ln(Staff_i) \cdot std(\ln(CMix_{ij}))) + \alpha_8(\ln(Beds_{ij}) \cdot \ln(Staff_i) \cdot std(\ln(CMix_{ij})))) + \\
&+ \alpha_9(\ln(Beds_{ij})^2 \cdot std(\ln(CMix_{ij})))) + \alpha_{10}(\ln(Staff_i)^2 \cdot std(\ln(CMix_{ij})))) + \\
&\alpha_{11} std(\ln(CMix_{ij}))) + \lambda_1 \mu_{Weig,ij} + \lambda_2 \sigma_{Weig,ij} + \lambda_3 \mu_{Weig,i} + \lambda_4 \sigma_{Weig,i} + \epsilon_{ij},
\end{aligned} \tag{3}
$$

where $\alpha_{0i} = \alpha_0 + u_i$, $\alpha_1 = \beta_0$, $\alpha_2 = \beta_0\beta_2$, etc.
Note that the model defined in (3) can be seen as an unconstrained version of model (2). Equation (3) is obtained by substitution of $\alpha(CMix_{ij})$, $\beta(CMix_{ij})$, $\gamma(CMix_{ij})$, $\delta(CMix_{ij})$ and $f(Weights)$ in (2). This operation leads to a linear form with some restriction on parameter values. For example, coefficients connected to $\ln(Beds)$ correspond to β_0, β_1, $\frac{\beta_0\beta_2}{2}$ and $\frac{\beta_1\beta_2}{2}$, that are only three different coefficients in model (2) and, respectively, α_1, α_6, α_9 and $\frac{\alpha_2}{2}$ in model (3). In order to justify the re-parametrization in (3), we have performed a hypothesis test about non-linear restrictions (Godfrey, 1988). The Wald test statistic presents a p-value of 0.0699, which allows to accept the null hypothesis. Given this result, the following analysis is based on model (3), without any constraints on the interaction parameters. The choice of formulation (3) is due not only to the ease in estimation by means of conventional statistical software, but also to the immediate interpretability of the estimated coefficients.

Then, regarding the errors at first level, ϵ_{ij}, usually called residuals, the estimation results, performed with the statistical software `R` (see Cribari-Neto and Zarkos, 1999; Ihaka and Gentleman, 1996), show a high level of heteroscedasticity (see Figure 2-A). In order to include this information in the model, we have defined a dependence of ϵ from one of the most significant inputs (the beds number), by means of an exponential multiplicative variance function. Then, the corresponding model assumes that the within-group errors are heteroscedastic, with variance function equal to:

$$Var(\epsilon_{ij}) = \sigma_\epsilon^2 |Beds_{ij}|^{2\theta}. \tag{4}$$

The plot of the standardized residuals (estimated by considering the inclusion of heteroscedasticity in the model) in Figure 2-B, shows that the problem of heteroscedasticity has been reduced.

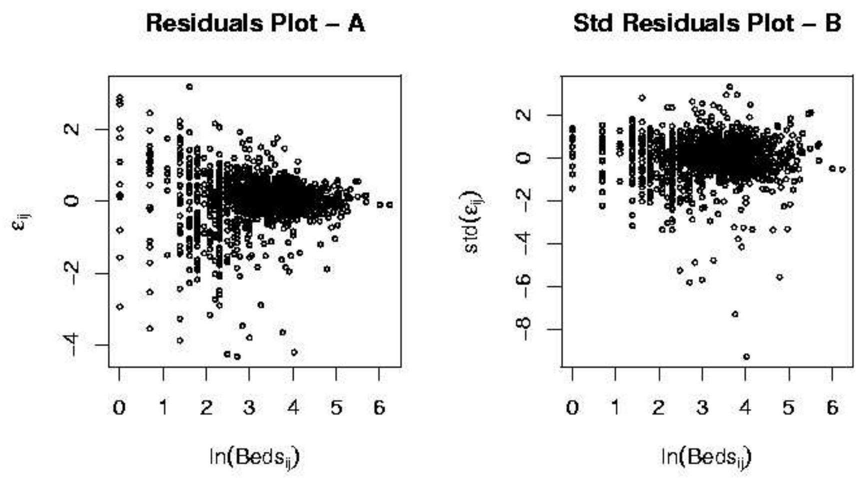

Figure 2: Effect on the residuals of inclusion of heteroscedasticity in the model

Finally, the 2^{nd} level errors, u_i, can be interpreted as efficiency indicators, as illustrated for example in Gori, Grassetti and Rossi (2004), and can be estimated by "Empirical Bayes" (EB) approach as described in Pinheiro and Bates (2000) and Verbeke and Molenberghs (2000).

4 The Results

In Table 2 we summarize the estimated coefficients for fixed and random effects of the model introduced in equation (3), which takes also into consideration the heteroscedasticity. For variance components, as Wald statistics based on the asymptotic standard error are not reliable, we provide the 95% confidence intervals. For the parameters in the linear mixed-effects model approximate confidence intervals are obtained by using the normal approximation of the distribution of the maximum likelihood estimators.

The prior aim of this paper is to focus on the economic interpretation of the estimated model. The obtained coefficients show that quadratic forms of original variables have negative effects on number of hospitalization as well as the various complexity indexes ($std(\ln(CMix_{ij}))$, $\mu_{Weig,ij}$ $\sigma_{Weig,ij}$, $\mu_{Weig,i}$ and $\sigma_{Weig,i}$). A positive effect is, indeed, connected to the original input variables, also taking into consideration their interation with the standardized case-mix index.

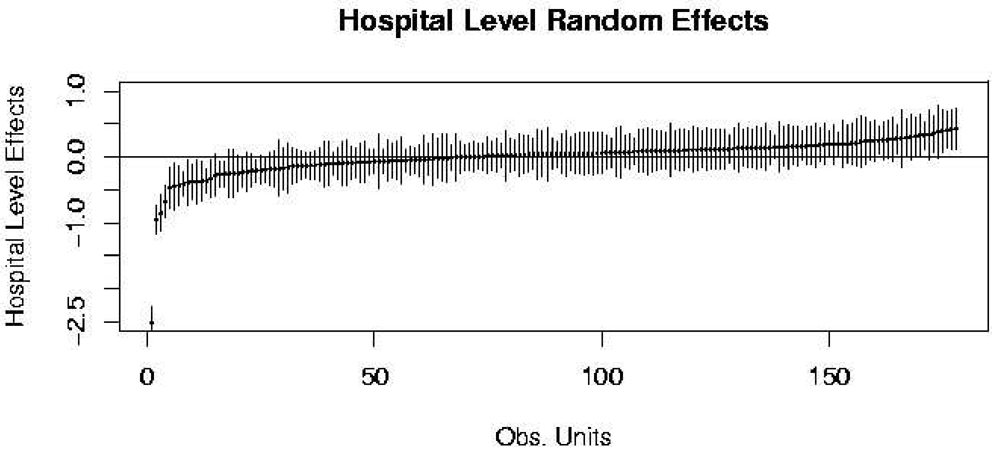

Figure 3: Confidence intervals for estimated BLUP random effects (u_i) at hospital level

In Figure 3 we have summarized the estimated BLUP error components. Confidence intervals have been calculated on the basis of pairwise comparison theory given in Goldstein and Healy (1995). The estimated BLUP error components and their standard errors have been computed following chapter 7 of Pinheiro and Bates (2000). The point es-

Fixed Effects Coefficients			
	Value	Std.Error	p-value
α_0	0.7941	1.0367	0.4434
α_1	1.2328	0.1541	0.0000
α_2	-0.0165	0.0163	0.3101
α_3	0.9683	0.3033	0.0017
α_4	-0.0630	0.0251	0.0129
α_5	-0.0114	0.0236	0.6275
α_6	0.3265	0.1756	0.0632
α_7	0.9390	0.2315	0.0001
α_8	0.0185	0.0232	0.4252
α_9	-0.0443	0.0195	0.0232
α_{10}	-0.0774	0.0183	0.0000
α_{11}	-3.2004	0.8323	0.0001
λ_1	-0.4798	0.0493	0.0000
λ_2	-0.0591	0.0493	0.2305
λ_3	-1.2266	0.3833	0.0016
λ_4	0.6379	0.2689	0.0188
Random Effects Coefficients			
	Value	Lower Bound	Upper Bound
θ	-0.3773	-0.4170	-0.3375
σ_u	0.3502	0.3026	0.4053
σ_ϵ	2.0625	1.8123	2.3474

Number of Observations: **1478** - Number of Groups: **178**

Table 2: Estimated coefficients of the mixed model with heteroscedasticity

timates and their confidence intervals (C.I.) are easily obtained from fixed effects and variance maximum likelihood (ML) estimates.

5 Some Economic Considerations

As just pointed out, the aim of this article is to investigate the elasticity of the hospitalizations number with respect to the inputs of the model, in particular *Staff* and *Beds*, which are, indeed, the only variables directly under hospital control. As mentioned previously, the case-mix index is treated as an exogenous variable, in order to distinguish among different types of hospitalizations. Looking at our analysis results, in fact, it results that elasticity is strongly affected by this variable; we will show in the following how elasticity varies for its different levels.

The *elasticity* of the output y for an input x_r, defined as the marginal productivity of x_r divided by the average productivity of x_r, i.e.

$$e(x_r) = \frac{\partial y}{\partial x_r} / \frac{y}{x_r} = \frac{\partial \ln(y)}{\partial \ln(x_r)}, \tag{5}$$

is the percentage change in output associated with a unitary percent change in the r-th input, holding all other inputs constant, and represents a unit-free measure of the marginal productivity (Chambers, 1988).

From this formulation we obtain directly the elasticity at ward level, denoted by e^W. For *Beds* we have:

$$\begin{aligned} e^W(Beds)_{ij} &= \tfrac{\partial \ln(NHosp_{ij})}{\partial \ln(Beds_{ij})} = \\ &= [\beta_0 + \beta_1 std(\ln(CMix_{ij}))] \cdot [1 + 2\beta_2 \ln(Beds_{ij})] + \\ &+ [\delta_0 + \delta_1 std(\ln(CMix_{ij}))] \cdot \ln(Staff_i). \end{aligned} \tag{6}$$

Analogously, for *Staff* we have:

$$\begin{aligned} e^W(Staff)_{ij} &= \tfrac{\partial \ln(NHosp_{ij})}{\partial \ln(Staff_i)} = \\ &= [\gamma_0 + \gamma_1 std(\ln(CMix_{ij}))] \cdot [1 + 2\gamma_2 \ln(Staff_i)] + \\ &+ [\delta_0 + \delta_1 std(\ln(CMix_{ij}))] \cdot \ln(Beds_{ij}). \end{aligned} \tag{7}$$

In both cases, the elasticity depends on the three explicative variables of the model. As one can see in Appendix A (see Table 6), it is mainly affected by the case-mix value. In fact, given a positive index, the estimated elasticity of the hospitalizations number with respect to *Beds* decreases with *Beds*, when *Staff* is fixed, and increases with *Staff*, when *Beds* are fixed. On the contrary, given a negative case-mix value, specular patterns can be observed. Finally, given a null value of the index, both elasticities are almost constant.

Tables 4 and 5, in Appendix A, summarize the estimated elasticity for *Beds* and *Staff* calculated for every observed macro units (i.e. at hospital level). Equation (5) can be, in fact, reformulated at hospital level by writing output y as the sum of outputs of single micro unit, reducing itself to the weighted sum of elasticities at ward level:

$$\begin{aligned} e^H(Staff)_i &= \tfrac{\partial \sum_j y_{ij}}{\partial Staff_i} \tfrac{Staff_i}{\sum_k y_{ik}} = \textstyle\sum_j \left(\tfrac{\partial y_{ij}}{\partial Staff_i} \tfrac{Staff_i}{\sum_k y_{ik}} \tfrac{y_{ij}}{y_{ij}} \right) \\ &= \textstyle\sum_j \left(\tfrac{\partial y_{ij}}{\partial Staff_i} \tfrac{Staff_i}{y_{ij}} \tfrac{y_{ij}}{\sum_k y_{ik}} \right) = \sum_j \left(\tfrac{\partial y_{ij}}{\partial Staff_i} \tfrac{Staff_i}{y_{ij}} w_{ij} \right) \\ &= \textstyle\sum_j \left(\tfrac{\partial \ln y_{ij}}{\partial \ln Staff_i} w_{ij} \right) = \sum_j e^W(Staff)_{ij} w_{ij}, \end{aligned} \tag{8}$$

where e^H indicates the elasticity at hospital level and w_{ij} are to be intended as weights of the elasticities at ward level.

It can be simply demonstrated, by means of the limit of the difference quotient, that analogous results can be achieved for *Beds*, obtaining:

$$e^H(Beds)_i = \textstyle\sum_j \left(\tfrac{\partial \ln y_{ij}}{\partial \ln Beds_{ij}} w_{ij} \right) = \sum_j e^W(Beds)_{ij} w_{ij}. \tag{9}$$

The estimates of elasticity at hospital level are summarized in Figure 4.

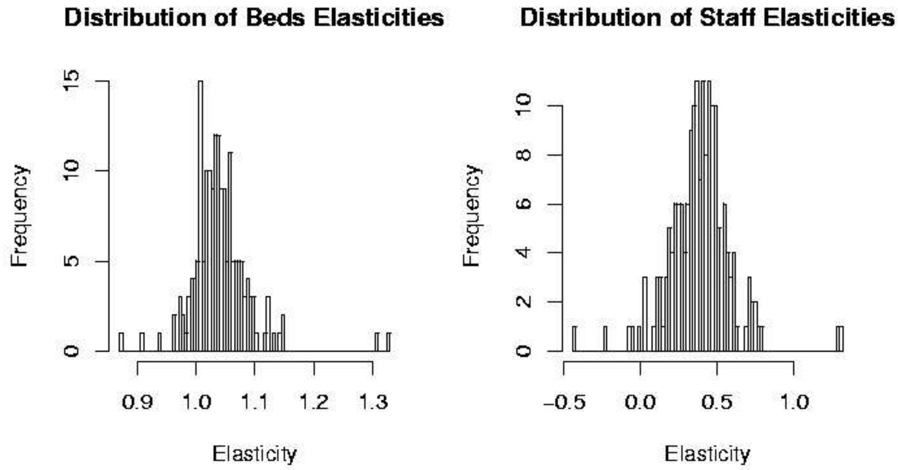

Figure 4: Histograms of hospital elasticities for *Beds* and *Staff*

From this figure one can notice that, for *Beds*, the elasticity is concentrated around the unity. For *Staff*, however, the plot shows that quite all units work in over-dimensioned conditions, and a few have even reached a congested estate (the negative values). From a re-allocative point of view we can then identify situations where elasticity values can justify an input increase (elasticities greater than one) and cases for which an additional input brings up no proportional output improvement (elasticities lower than one).

Considering the sum of the elasticities for *Beds* and *Staff* we obtain a slightly interpretation of elasticity results as the local returns to scale (so called *elasticity of scale*)

$$e(x) = e(Beds)_i + e(Staff)_i, \tag{10}$$

where x indicates the input vector (*Beds*, *Staff*). The elasticity of scale is a scalar-valued measure of how output changes in response to simultaneous input variation (Chambers, 1988). Here simultaneous input variation is restricted to variations that do not change relative input utilization; that is, the ratios (x_r/x_s) are constant for all r and s.

"By geometrical point of view, the elasticity of scale is interpretable as measuring how accurately the distance between isoquants in input space reflects the distance in output space. In particular, there are three possible characterizations of production functions. If

$e(x) = 1$, the production function exhibits constant returns to scale, and the isoquants are evenly spaced. If $e(x) < 1$, the production function exhibits decreasing returns to scale, and the distance between isoquants in input space overestimates the distance in output space. Finally, if $e(x) > 1$, the distance in input space underestimates the distance in output space, and the production function exhibits increasing returns to scale; isoquants, therefore, tend to be more crowded together as one moves along a ray from the origin" (Chambers, 1988).

The preceding concepts have some economical interpretations. "Assume that, by a given endowment of inputs, the goal is producing as much output as possible and that we can decide whether or not it would be better to split up the resource endowment equally into m separate operations or to produce everything in one large operation. For convenience, suppose also that both alternatives are equally costly. If the available technology is characterized by decreasing returns to scale, there is no incentive to centralize the operation, and it is better to split up the operation; exactly analogous arguments show that when $e(x) = 1$, centralization and decentralization are indifferent, and when $e(x) > 1$, centralization is preferable" (Chambers, 1988). These situations can be interpreted as the estimated productive conditions of each hospital and each ward.

Focusing our analysis on hospitals level results, an estimated scale elasticity under the unity identifies hospitals presenting a congested condition, where investments are not useful and it is better to reduce the dimension; on the contrary, hospitals that have an higher return to scale are the ones with unused productive capacity, which could increase their dimension. This interpretation could be very useful by the administrative point of view. Tables 4 and 5, in Appendix A, shows also the estimated scale elasticity for each hospital. It can be noted, however, that the differences among hospitals are due quite all to *Staff* elasticity. *Staff* and *Beds* values are also given in these tables in order to allow a straightforward interpretation of the estimated values.

Another interesting measure, related to the returns to scale of a technology, is the *scale efficiency*, which measures the ray average productivity at the observed input scale on the production frontier relative to the maximum ray average productivity attainable at the *most productive scale size* (Banker, 1984), which is an input bundle x characterized by $e(x) = 1$. "It needs to be emphasized that scale efficiency is lower than one whenever the observed input mix is not scale-optimal, i.e. where locally constant returns to scale does not hold. Scale elasticity, on the other hand, can be either greater than or less than unity. Only at the *most productive scale size* both measures equal unity and are, therefore, equal to one another. Elsewhere, they differ and the magnitude of scale elasticity does not directly reveal anything about the level of scale efficiency" (Ray, 1988). Ray developed an input-oriented measure of scale efficiency, directly obtainable from an empirically estimated single output multiple input translog production function. For example, regarding the translog model

$$\begin{aligned} \ln(NHosp) &= \alpha_0 + \alpha_1 \ln(Beds) + \alpha_2 \ln(Staff) + \\ &+ \tfrac{\beta_{11}}{2}[\ln(Beds)]^2 + \beta_{12} \ln(Beds)\ln(Staff) + \tfrac{\beta_{22}}{2}[\ln(Staff)]^2 \end{aligned} \tag{11}$$

and in the absence of technical inefficiency, the scale efficiency is equal to:

$$SE(x) = exp\left\{\frac{(1 - e(x))^2}{2\beta}\right\}, \tag{12}$$

where $\beta = \sum_{r=1}^{2}\sum_{s=1}^{2}(\beta_{rs})$.

In the more general case involving technical inefficiency, it is equal to:

$$SE(x) = exp\left\{\frac{[1 - \sqrt{e(x)^2 - 2\beta\theta}]^2}{2\beta}\right\}, \tag{13}$$

where θ is the technical inefficiency (in our case we can obtain it from a MOLS or COLS transformation - Greene, 1993 - of the second level error component). This measure can be computed for each hospital from the fitted translog production function.

As previously seen, a practical implication of returns to scale analysis is that any input characterized by increasing returns to scale should be expanded, while one with diminishing returns should be scaled down in order to attain full scale efficiency. Supposing it is possible to change relative input utilization, an interesting question would be: what level of *Staff* combined with the given quantity of *Beds* would result in a scale-optimal input-mix?

Ray (1998) developed an index, in the two input case, which measures the extent to which the observed quantity of *Staff* differs from what would be the optimal level in light of the size of its actual *Beds*.

Let B_0 be the exogenously fixed quantity of *Beds*, S_0 the observed quantity of *Staff* and (S_0^*) the optimal quantity of *Staff*. Ray (1998) defines

$$\sigma = \frac{S_0^*}{B_0} / \frac{S_0}{B_0} = \frac{S_0^*}{B_0} \cdot \frac{B_0}{S_0} = \frac{S_0^*}{S_0}. \tag{14}$$

Clearly, $\sigma = 1$ if and only if the observed pair (S_0, B_0) is itself scale-optimal. Otherwise, $\sigma > 1$ implies that the actual *Beds*-*Staff* ratio is higher than the optimal. Similarly, $\sigma < 1$ implies excessive number of *Staff* relative to the optimal level.

For the translog case the index is equal to:

$$\sigma = exp\left\{\frac{1 - e(S_0, B_0)}{\beta_{22} + \beta_{12}}\right\}, \tag{15}$$

that is it depends on both the observed scale elasticity, $e(S_0, B_0)$, and the estimated values of the parameters in the denominator. Thus, the mere fact that increasing returns to scale hold at any given input bundle does not by itself imply that the observed quantity of *Staff* is too low. "It should be noted, however, that if $\beta_{22} + \beta_{12} = 0$, there will not exist a finite S_0^* for the given B_0" (Ray, 1998).

6 Conclusions

The decision to measure efficiency of health services by the given model is due to both interpretability and flexibility of the functional form. The model proposed is certainly a simplified version of the complete econometric model specification (some other variables, in fact, can affect the analyzed phenomenon) but, also at this preliminary stage, some of the obtained results are really closed to the desirable hypotheses. Then one can conclude that the application of this methodology provides useful and reliable results. Some more attention should be focused, however, on typical econometric problems, like for example outlier detection; this is, in fact, still an unsolved problem in the multilevel framework, because it is not easy to identify at which level outliers should be searched (see Langford and Toby, 1998 and Barnett and Toby, 1994).

By means of deterministic COLS and MOLS approaches (Greene, 1993), decision makers can interpret the random effect BLUP estimates ($\hat{u}_i$) as indicators of structure efficiency. The larger is the effect, the better is the productive process. Then, the interpretation of estimated elasticities provides some information about the productive conditions of observed units.

Our results show that, as beds elasticities are mainly concentrated around unity, the interest of decision makers should be focused on estimated staff elasticities. Both staff elasticity and scale elasticity highlight the presence of over- and under-dimensioned units, i.e. situations where a re-allocation of staff is necessary.

The sample used for the analysis included different kinds of hospital structures. As reported in Table 3, while structures classified as “Hospitals” and “Classified Hospitals” are almost homogenous in terms of staff elasticity, “Private and Public Clinics” present a large variability. “Research structures” are instead characterized by a low average staff elasticity and, consequently, worse productive conditions, maybe due to the different goal of these structures. Here staff is quite completely devoted to research and the health service is considered only as a secondary aim.

Beds Elasticities **Structure Type**	$y_{0.01}$	$y_{0.25}$	$y_{0.5}$	$y_{0.75}$	$y_{0.99}$	**Mean**	**Std. Dev.**	**Obs. Number**
Hospitals	0.971	1.015	1.032	1.051	1.088	1.033	0.025	118
Research Institutes	0.913	0.992	1.013	1.036	1.086	1.010	0.042	18
Classified Hospitals	1.010	1.022	1.025	1.042	1.072	1.035	0.022	5
Private and Public Clinics	0.919	1.028	1.068	1.099	1.317	1.067	0.074	51
Staff Elasticities **Structure Type**	$y_{0.01}$	$y_{0.25}$	$y_{0.5}$	$y_{0.75}$	$y_{0.99}$	**Mean**	**Std. Dev.**	**Obs. Number**
Hospitals	0.003	0.302	0.375	0.449	0.600	0.369	0.123	118
Research Institutes	-0.045	0.187	0.265	0.354	0.703	0.282	0.183	18
Classified Hospitals	0.269	0.321	0.353	0.426	0.563	0.387	0.104	5
Private and Public Clinics	-0.326	0.253	0.478	0.604	1.297	0.453	0.303	51

Table 3: Elasticity summary statistics of hospitalization structures clusters

In our analysis, the *Case-Mix* is treated as an exogenous variable, which does not interfere with hospital politics. This is correct in the case that we consider as fixed the service demand for each structure. For a few diseases with a lower case-mix, however, the demand can be affected by single hospital marketing strategies. As one can see in Table 6 of Appendix A, substantial changes in case-mix index cause the raise of different elasticity patterns.

Future developments will consider the generalization of Ray's results (Ray, 1998) to our model specification and the consequent estimation of scale efficiency and σ-index for scale optimality of input mix.

In conclusion, we think that the presented methodology and relative results can be considered really interesting for the decision making processes. In fact, both random and fixed effect estimates are easily interpretable. From an administrative point of view they can be used to identify the different productive observed conditions (both technical and scale inefficiency) and, in case, to decide the preferable interventions.

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Appendix A: Elasticity Results

HospID	Beds Elasticity	Staff Elasticity	Scale Elasticity	Beds	Staff		HospID	Beds Elasticity	Staff Elasticity	Scale Elasticity	Beds	Staff
1	1.010	0.330	1.339	170	351		51	1.068	0.518	1.586	271	485
2	1.031	0.367	1.397	853	1327		52	1.062	0.472	1.534	329	676
3	1.015	0.311	1.326	622	1187		53	1.021	0.113	1.135	58	55
4	0.997	0.238	1.235	630	1042		54	1.025	0.353	1.378	351	559
5	1.001	0.185	1.186	151	217		55	1.122	0.697	1.819	168	219
6	1.039	0.401	1.440	293	579		56	1.125	0.710	1.835	110	168
7	1.007	0.206	1.213	268	285		57	1.068	0.478	1.546	80	48
8	1.060	0.475	1.535	123	228		58	1.048	0.150	1.198	282	309
9	1.054	0.411	1.465	400	95		59	1.133	0.719	1.852	130	134
10	1.141	0.765	1.907	126	143		60	1.146	0.713	1.860	300	365
11	1.061	0.509	1.570	214	559		61	0.872	-0.430	0.442	160	170
12	1.044	0.363	1.406	77	154		62	1.023	0.245	1.268	71	65
13	1.050	0.444	1.494	368	702		63	1.084	0.564	1.648	60	55
14	1.046	0.443	1.489	140	280		64	1.328	1.303	2.631	136	232
15	1.042	0.422	1.464	102	234		65	0.995	0.261	1.256	260	350
16	1.042	0.426	1.469	258	408		66	1.039	0.397	1.436	182	240
17	1.073	0.569	1.642	270	932		67	1.039	0.378	1.417	91	98
18	1.022	0.321	1.343	201	321		68	1.061	0.464	1.525	190	185
19	1.074	0.520	1.594	180	182		69	1.092	0.601	1.692	198	358
20	0.989	0.193	1.182	106	126		70	1.116	0.758	1.874	380	681
21	1.068	0.471	1.538	100	100		71	1.086	0.743	1.830	60	70
22	1.043	0.244	1.287	170	183		72	1.016	0.226	1.243	144	96
23	0.965	-0.223	0.742	60	42		73	1.025	0.307	1.332	105	242
24	1.028	0.296	1.324	32	91		74	1.045	0.445	1.490	495	943
25	1.036	0.432	1.468	525	926		75	1.084	0.557	1.640	129	264
26	1.049	0.439	1.488	130	267		76	1.030	0.366	1.396	212	377
27	1.059	0.494	1.553	140	331		77	1.008	0.263	1.271	157	240
28	1.027	0.288	1.315	92	180		78	0.963	0.179	1.142	163	226
29	1.026	0.315	1.340	342	538		79	1.033	0.352	1.384	140	244
30	1.056	0.482	1.538	172	461		80	1.028	0.342	1.371	186	232
31	1.006	0.233	1.239	260	474		81	1.063	0.443	1.506	170	321
32	1.089	0.606	1.695	349	663		82	1.064	0.487	1.551	68	165
33	1.088	0.581	1.669	72	240		83	1.074	0.563	1.637	279	585
34	0.998	0.240	1.238	587	977		84	0.986	0.122	1.108	78	182
35	1.044	0.446	1.491	549	1270		85	1.306	1.290	2.596	130	229
36	1.053	0.482	1.534	391	857		86	1.100	0.570	1.670	148	107
37	1.089	0.592	1.681	188	434		87	0.979	0.172	1.151	142	152
38	1.033	0.374	1.408	258	469		88	0.982	0.175	1.156	135	174
39	1.033	0.374	1.407	493	1016		89	1.032	0.387	1.418	319	415
40	1.036	0.413	1.449	511	1347		90	1.047	0.449	1.496	420	551
41	1.015	0.354	1.368	575	1109		91	0.976	0.022	0.998	92	128
42	1.010	0.198	1.209	223	333		92	0.974	-0.056	0.918	108	184
43	1.047	0.414	1.461	141	262		93	1.035	0.357	1.392	35	47
44	1.010	0.321	1.331	504	1085		94	1.060	0.493	1.553	236	616
45	1.022	0.334	1.357	238	493		95	1.052	0.482	1.534	366	667
46	1.006	0.238	1.244	128	246		96	1.037	0.378	1.414	167	368
47	1.049	0.445	1.494	162	322		97	1.016	0.319	1.335	197	292
48	1.016	0.240	1.256	75	202		98	1.036	0.415	1.451	155	272
49	1.031	0.374	1.405	165	303		99	1.020	0.345	1.366	203	378
50	1.006	0.133	1.138	50	102		100	1.031	0.383	1.414	168	300

Table 4: Estimated elasticities computed with respect to each input variable and scale elasticity

HospID	Beds Elasticity	Staff Elasticity	Scale Elasticity	Beds	Staff	HospID	Beds Elasticity	Staff Elasticity	Scale Elasticity	Beds	Staff
101	1.008	0.284	1.292	90	211	140	1.054	0.451	1.505	269	541
102	1.083	0.601	1.684	118	391	141	1.037	0.404	1.441	405	789
103	1.029	0.247	1.276	100	92	142	1.079	0.503	1.582	180	79
104	1.009	0.267	1.276	378	624	143	1.123	0.794	1.917	744	210
105	1.095	0.554	1.649	275	430	144	1.017	0.358	1.375	1021	1644
106	1.066	0.484	1.550	105	107	145	1.004	0.263	1.267	481	732
107	1.073	0.492	1.564	145	133	146	1.057	0.489	1.546	230	487
108	1.078	0.552	1.630	319	421	147	1.042	0.460	1.502	917	2135
109	0.996	0.024	1.019	86	65	148	1.030	0.411	1.440	895	1653
110	1.052	0.445	1.497	320	325	149	1.035	0.400	1.436	724	1407
111	1.047	0.402	1.449	120	113	150	1.015	0.327	1.342	706	1461
112	1.018	0.288	1.306	122	146	151	1.018	0.390	1.408	1574	3198
113	1.023	0.251	1.274	86	123	152	0.997	0.301	1.298	2155	3802
114	1.028	0.368	1.396	102	192	153	1.029	0.415	1.445	747	1558
115	1.035	0.335	1.370	140	171	154	1.012	0.337	1.348	941	1630
116	1.057	0.419	1.476	125	205	155	1.010	0.360	1.370	1298	2570
117	1.071	0.548	1.619	345	661	156	1.032	0.426	1.458	644	1462
118	1.047	0.465	1.512	452	745	157	0.971	0.087	1.058	494	1081
119	1.104	0.621	1.724	100	89	158	1.015	0.399	1.414	746	2369
120	0.974	0.027	1.001	76	61	159	1.007	0.370	1.377	1608	4113
121	1.098	0.608	1.706	160	135	160	1.009	0.348	1.358	564	1529
122	1.100	0.559	1.659	30	17	161	1.008	0.280	1.289	766	1642
123	1.003	0.182	1.185	57	39	162	1.024	0.384	1.408	613	1401
124	1.076	0.508	1.584	147	142	163	1.025	0.215	1.240	80	103
125	1.059	0.435	1.494	162	229	164	0.991	0.306	1.297	750	2520
126	1.079	0.590	1.668	147	210	165	0.936	0.108	1.044	510	1511
127	1.040	0.450	1.490	472	902	166	1.036	0.407	1.443	207	525
128	1.020	0.246	1.266	95	124	167	1.032	0.428	1.460	1394	2965
129	0.991	-0.001	0.990	97	115	168	0.990	0.103	1.093	53	192
130	1.023	0.345	1.368	120	127	169	1.004	0.206	1.210	173	253
131	1.041	0.373	1.413	137	143	170	1.016	0.273	1.289	130	194
132	1.077	0.485	1.562	145	113	171	1.037	0.259	1.296	64	57
133	1.056	0.467	1.524	164	320	172	1.091	0.731	1.823	189	410
134	1.019	0.199	1.218	90	142	173	1.056	0.542	1.597	993	3150
135	1.057	0.480	1.537	135	327	174	0.993	0.137	1.130	154	321
136	1.066	0.476	1.541	174	211	175	0.908	-0.071	0.837	110	747
137	1.057	0.461	1.518	95	181	176	1.010	0.337	1.347	242	487
138	1.042	0.423	1.465	201	534	177	1.038	0.325	1.363	159	247
139	1.146	0.733	1.879	94	85	178	1.001	0.203	1.205	287	390

Table 5: Estimated elasticities computed with respect to each input variable and scale elasticity

Elasticity of *Nhosp calculated with respect to Staff given different input mix* and different levels of case-mix (*ln(std(CMix))*)

Case-mix = 2	Beds						
Staff	10	20	30	40	50	100	500
10	1.902	1.920	1.930	1.938	1.943	1.961	2.002
50	1.201	1.219	1.229	1.236	1.242	1.260	1.301
200	0.597	0.615	0.625	0.633	0.638	0.656	0.697
500	0.198	0.216	0.226	0.233	0.239	0.257	0.298
Case-mix = 1	Beds						
Staff	10	20	30	40	50	100	500
10	1.277	1.282	1.285	1.287	1.288	1.293	1.305
50	0.825	0.830	0.833	0.835	0.837	0.841	0.853
200	0.436	0.441	0.444	0.446	0.447	0.452	0.464
500	0.179	0.183	0.186	0.188	0.190	0.195	0.206
Case-mix = 0	Beds						
Staff	10	20	30	40	50	100	500
10	0.652	0.644	0.639	0.636	0.634	0.626	0.607
50	0.449	0.441	0.437	0.433	0.431	0.423	0.405
200	0.275	0.267	0.262	0.259	0.256	0.248	0.230
500	0.159	0.151	0.147	0.143	0.141	0.133	0.114
Case-mix = -1	Beds						
Staff	10	20	30	40	50	100	500
10	0.027	0.006	-0.006	-0.015	-0.021	-0.042	-0.090
50	0.073	0.053	0.041	0.032	0.025	0.004	-0.044
200	0.113	0.093	0.081	0.072	0.065	0.044	-0.004
500	0.140	0.119	0.107	0.098	0.092	0.071	0.023
Case-mix = -2	Beds						
Staff	10	20	30	40	50	100	500
10	-0.598	-0.632	-0.652	-0.665	-0.676	-0.710	-0.788
50	-0.303	-0.336	-0.356	-0.370	-0.381	-0.414	-0.492
200	-0.048	-0.081	-0.101	-0.115	-0.126	-0.159	-0.237
500	0.121	0.087	0.067	0.053	0.043	0.009	-0.069

Elasticity of *Nhosp calculated with respect to Staff given different input mix* and different levels of case-mix (*ln(std(CMix))*)

Case-mix = 2	Beds						
Staff	10	20	30	40	50	100	500
10	1.460	1.315	1.229	1.169	1.122	0.976	0.637
50	1.502	1.356	1.271	1.210	1.163	1.017	0.679
200	1.537	1.391	1.306	1.246	1.199	1.053	0.714
500	1.561	1.415	1.329	1.269	1.222	1.076	0.738
Case-mix = 1	Beds						
Staff	10	20	30	40	50	100	500
10	1.295	1.211	1.162	1.127	1.100	1.015	0.819
50	1.307	1.223	1.173	1.138	1.111	1.027	0.831
200	1.317	1.232	1.183	1.148	1.121	1.036	0.841
500	1.323	1.239	1.190	1.154	1.127	1.043	0.847
Case-mix = 0	Beds						
Staff	10	20	30	40	50	100	500
10	1.131	1.108	1.094	1.085	1.077	1.055	1.001
50	1.112	1.089	1.076	1.066	1.059	1.036	0.983
200	1.096	1.073	1.060	1.050	1.043	1.020	0.967
500	1.086	1.063	1.050	1.040	1.033	1.010	0.957
Case-mix = -1	Beds						
Staff	10	20	30	40	50	100	500
10	0.966	1.004	1.027	1.043	1.055	1.094	1.183
50	0.917	0.956	0.978	0.995	1.007	1.046	1.135
200	0.876	0.914	0.937	0.953	0.965	1.004	1.094
500	0.848	0.887	0.910	0.926	0.938	0.977	1.066
Case-mix = -2	Beds						
Staff	10	20	30	40	50	100	500
10	0.801	0.901	0.959	1.001	1.033	1.133	1.365
50	0.723	0.823	0.881	0.923	0.955	1.055	1.287
200	0.655	0.755	0.814	0.855	0.888	0.988	1.220
500	0.611	0.711	0.770	0.811	0.843	0.943	1.176

Table 6: Estimated elasticities computed with respect to each input by given values of case-mix index. Negative values of case-mix index corresponds to easier treatments which need more Beds than Staff. In a specular way positive case-mix identifies the treatments that need more Staff than Beds.