Complexity of band structures: Finite Element calculation of complex band structures for one and two dimensional phononic crystals

Interviene
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Abstract:
Calculations of complex band structures of phononic crystals (PCs) have revealed that complex – evanescent – waves are present still in complete stop bands\cite{1,2}. Although, complex band structures of non-periodic systems are well-known, such relations in phononic crystals remained uninvestigated until recently. Evanescent waves describe decaying (spatially varying) fields and are important to fulfill the boundary conditions as transient solutions in non-periodic systems or to predict the spatial-temporal life time of elastic waves within complete stop bands in phononic crystals. The finite element method (FEM) is one of the most popular methods to calculate band structures. The method results in an eigenvalue problem, where the unknown frequencies $\omega$ are calculated as eigenvalues for a given wave number $k$. Here, the assumption of a real $k$ in conjunction with Hermitian coefficient matrices leads to real and positive eigenvalues; hence, to real dispersion diagrams.

In this paper we will apply the finite element method to calculate complex band structures for one and two dimensional PCs. We pay particular attention to surface gratings – as one dimensional surface PCs – since this structure exhibits a wide complete stop band due to the hybridization of two surface modes. The application of the semi-analytical FEM\cite{1} is discussed as a special case of frequency domain calculation using the unit cell approach. Complex band structures are particularly important for such gratings, since the folded surface modes become evanescent beyond the sound cone. This behavior and the complex interconnections between the real propagating modes are well visible for the presented dispersion relation in Fig.1. Moreover, an evanescent mode arises within the complete stop band which describes the spatial life time of the elastic waves in the stop band.

This technique is extended also for 2DPCs. The rearranged governing equations, however, include a wave vector with two components: $kx$,$ky$. The different orientations of the wave vector leads to different eigenvalue problems. Orientations parallel to coordinate axes ($\Gamma$-$X$ and $X$-$M$ directions) lead to quadratic eigenvalue problems; the $kx=ky$ direction ($\Gamma$-$M$ direction for a quadratic lattice) leads to a quartic eigenvalue problem. These problems are solved by linearization.


Seminario

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