FACOLTÀ DI SCIENZE MATEMATICHE, FISICHE E NATURALI
DIPARTIMENTO DI MATEMATICA E FISICA "NICCOLÒ TARTAGLIA"

Analytic solutions of 2D Navier-Stokes equations with concentrated vorticity

Introduce:
Prof. Alfredo Marzocchi
Università Cattolica del Sacro Cuore

Interviene:
Prof. Marco SAMMARTINO
Università degli Studi di Palermo

Abstract

In this talk we shall consider the 2D Navier-Stokes equations

$$\begin{aligned} \partial_t \omega + \mathbf{u} \cdot \nabla \omega &= \nu \mathbf{\Delta} \omega \\ u &= \nabla^{\perp} \mathbf{\Delta}^{-1} \omega \\ \omega(x, t=0) &= \omega_0(x) \end{aligned}$$

with initial data concentrated on a small set and satisfying the bound

$$\int |\omega_0| dx \leq c.$$

A particularly significant instance of this configuration is when the vorticity $\omega = O(\epsilon^{-1})$ is distributed close to a curve $y = \phi_0(x)$ and decays away from it on a scale $O(\epsilon)$, being $\epsilon > 0$ a small parameter.

In a recent paper the authors considered a similar problem for the analytic solutions of the Euler equations, proving that, for a short time and in the limit $\epsilon \to 0$, the solutions do not develop oscillations or concentrations. The authors proved also that the vorticity remains supported close to a curve $y = \phi(x, t)$ whose dynamics is ruled by the Birkhoff-Rott equation.

The aim of this talk is to take into account the viscosity effects, assuming the diffusive scaling $\epsilon = O(\sqrt{\nu})$. We shall see that, for an almost flat layer, the vorticity follows, to the leading order, the Euler dynamics; and that the viscosity effects, confined to the correction equation, can be described by a controllable weakly nonlinear convection-diffusion equation.

This is joint work with R.Caflisch and M.C.Lombardo.

Seminario

Venerdì 7 luglio 2017
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