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Abstract

We study lobbying behavior by firms in a two-region economy, with either centralized or decentralized provision of profit-enhancing local public goods. Firms compete either in the market, lobbying for public good provision once entered in a market, or for the market, lobbying to gain access to it. When firms compete in the market, we show that lobbying is unambiguously less disruptive for social welfare under decentralization. Moreover, foreign rather than domestic private interests may be more powerful in affecting regional policies. On the contrary, when firms compete for the market, lobbying is mostly effective under decentralization, since local firms always end up forming a local monopoly. However, we show that an institutional setting in which competencies are split between the center and the periphery may dominate either full centralization or full decentralization or both.

Keywords: Fiscal federalism, Lobbying, Private interests.

JEL classification: D70, H23, H77.

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1 Introduction

One of the most interesting recent institutional developments in world economies is a marked and widespread tendency toward decentralization within countries accompanied by an enlargement of international unions among countries. In the nineties about 95% of all countries in the world undertook steps toward a decentralization of functions to local governments. In some cases, this tendency was so strong to bring about the dissolution of previously existing political entities (Bolton et al., 1996, Alesina et al., 2000). At the same time, new international forms of cooperation where established (i.e. the Nafta treaty) and old ones were expanded (i.e. the European Union). These events generated a renewed interest by academic economists on the issue of the optimal organization of governments (e.g. Besley and Coate, 2002, Lockwood, 2002, Bordignon et al., 2001, Alesina et al., 2003). On average, this scrutiny tended to confirm Oates’s (1972) intuition on the existence of potentially important efficiency gains associated with decentralization. However, some policy oriented economists remained highly skeptical. For example, in a very influential policy paper, Prud’homme (1995) severely warned against “the dangers of decentralization”. His main (efficiency) argument against decentralization lies in a (presumed) stronger influence of corruption and lobbying by local interest groups on local governments. Recent empirical studies do not substantiate this hypothesis (e.g. Fisman and Gatti, 2002). Nonetheless, it is quite common to hear Prud’homme’s type of arguments being repeated in political and economic circles as, for example, in the recent debate on whether competition policy should remain in the hands of the European Commission or being partly decentralized to member countries. The issue seems therefore to deserve a more detailed analysis.

Surprisingly enough, while there is a large economic literature on interest groups’ influence on policy (e.g. Grossman and Helpman, 2001), very few studies have concentrated on the specific issue of the relationship between interest groups and decentralization. De Melo et al. (1993) find a positive correlation between decentralization and lobbying, due to the existence of a preference dilution effect. More recently, Redoano (2002) shows that the net effect of decentralization on lobbying is a priori uncertain. However, these studies only focus on the higher heterogeneity...
of preferences under centralization as the main engine for lobbies' formation and influence. Prud'homme's argument, on the other hand, has nothing to do with preferences heterogeneity. It relies instead on a greater “disposition” by local governments to “accept” pressures from local interests, presumably due to the fact that supporting a local interest may generate additional benefits for the local politician than supporting an external one.

To focus on this issue, we build a simple general equilibrium model in which we abstract entirely from heterogeneity of preferences. In our model, there are two regions, one resident firm and a large mass of consumers in each region owning the local firm. The two firms may serve both local markets and in all cases they have an incentive to lobby the governments in charge either to gain access to the local markets or to increase the production of a local public good which is complementary in consumption to the good they sell. We focus on two polar cases, one where all decisions are taken at the central level and the other where all decisions are taken at local level. For simplicity, and also because these effects are already well understood, we abstract entirely from “common pool” effects which may arise out of transfers from the central level to the local one (Persson, 1998), as well as from “fiscal competition” effects which may arise out of the mobility of the tax base (Wilson, 1999) or by “spillover effects” either in local public good production or taxation (Besley and Coate, 2002, Keen and Kotsogiannis, 2002). In our model, nobody moves, there are no spillover effects on either the demand or the supply side of regions, there are no intergovernmental transfers, and each local government finances its supply of local public good out of resident taxation, so as to rule out tax competition effects. The only source of difference we allow between centralization and decentralization is that the central government internalizes as components of social welfare the profits that both firms make in both markets, while under decentralization the local government is only interested to the profits that are made everywhere by its own resident firm (as they increase resident consumers’ income). This captures in the simplest possible way the idea we discussed above that local interests may have a larger weight on local governments’ welfare function.

In this setting, we ask what are the effects of lobbying on economic outcomes and
social welfare in the two cases of decentralization and centralization. We consider two forms of lobbying. In the first one, firms lobby in the market; that is, firms have already gained access to both markets and have an incentive to lobby politicians to increase local public good production. In the second one, firms lobby for the market; that is, they lobby politicians to gain access to local markets.

We get very sharp results. When lobbying is in the market, lobbying behavior under centralization is always at least as bad for social welfare as under decentralization. Under decentralization, when both firms lobby both local politicians, local public goods supply is as distorted as under centralization (and so is social welfare), but lobbies pay higher contributions and so are worse off. However, under decentralization there are also equilibria where each firm lobbies only one politician at the time, while this is not possible under centralization. In this case, contributions are lower and so are the distortions in social welfare. Contrary to common intuition, we show that in many cases it is the foreign firm to lobby local politicians, rather than the home firm. The intuition here is simply that foreign contributions have a larger weight for politicians than contributions from local firms, as the latter contributions also reduce resident consumers’ welfare.

Results are reversed when lobbying is for the market. Under decentralization lobbying always leads the local politicians to give access to the market to the resident firm only, although a duopoly may be better for social welfare. No matter the degree of politicians’ benevolence, in fact, the local firm can always outbid the foreign firm to gain access to the market, because only this firm’s profits matter for the local politicians’ welfare. Under centralization, on the contrary, this effect is absent, which makes the central politician more resilient to lobbying. Finally, we also show that when lobbying is for the market the most effective institutional structure against lobbying distortions may be an intermediate one between centralization and decentralization. Under this structure, which we term “split competencies”, decisions about the number of firms in each market are given to the central government, while decisions about local public good supply are allocated to local governments.

The rest of the paper is organized as follows. In section 2 we set up the model. In section 3 we examine the policy makers’ choices in the benchmark situation of
no lobbying. In section 4 we examine lobbying behavior when both firms compete and lobby in the market. In section 5 we study lobbying for the market. Section 6 concludes. All proofs and further technical details are in the appendix.

2 The model

The economy is composed of two identical regions indexed by $r \in \{a, b\}$. There are four goods: two private consumption goods, $x$ and $z$, a production factor, $y$, and a public investment good, $g$. The latter is purely local, meaning that there is a distinct provision in each region with no spillover effects across regions. In each region live a continuum of identical consumers with a mass of unity, not moving across regions, and there is a firm producing good $x$, indexed by $\rho \in \{\alpha, \beta\}$, where $\alpha$ and $\beta$ are the firms located in regions $a$ and $b$, respectively. In both regions consumers are endowed with a fixed quantity $\bar{y} > 0$ of the production factor and have identical preferences represented by the quasi-linear utility function

$$u(x_r, z_r, g_r) = x_r - \frac{x_r^2}{2g_r} + z_r. \quad (1)$$

We take good $z$ to be the *numeraire* and its (national) market to be perfectly competitive. Technology is linear and units are normalized so that the production of one unit of $z$ requires one unit of input $y$. These assumptions imply that in equilibrium profits in the production of good $z$ are zero and that its supply is perfectly elastic. Moreover, the market price of factor $y$ is equal to one.

Firms $\alpha$ and $\beta$ are entirely owned by consumers living in regions $a$ and $b$, respectively, and their profits are entirely distributed to shareholders.$^{1}$ Hence, consumers’ income is composed of two terms: the market value of the fixed endowment of good $y$, and the distributed firms’ profits, net of contributions to the politicians, if any. Consumer’s income in region $r$ is subject to a proportional income tax at rate $t_r$, $t_r \in [0, 1)$. We let $p_r$ be the price of good $x$ in region $r$, $\Pi_{\rho r}$ be the profits (gross of

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$^{1}$Given quasi-linearity of the utility function, by which all income effects fall on the demand of good $z$, the equilibrium of the economy is independent of the distribution of profits across consumers and across regions.
contributions) earned by firm $\rho$ in region $r$, and $s_{\rho r}$ be the contributions to politicians by firm $\rho$ for public good $g_r$. To simplify the presentation, and without loss of generality (given symmetry between regions), in what follows we focus on region $a$. We denote with $\pi_\alpha = \Pi_{\alpha a} - s_{\alpha a} + \Pi_{\alpha b} - s_{\alpha b}$ the profits distributed by firm $\alpha$. Taking $g_a$ and $\pi_\alpha$ as given, each consumer in region $a$ solves:

$$\max_{x_a, z_a} x_a - \frac{x_a^2}{2g_a} + z_a,$$

s.t. $\ p_a x_a + z_a \leq (1 - t_a)(\bar{y} + \pi_\alpha),$

from which we immediately obtain the inverse demand function for good $x_a$ as

$$p_a(x_a, g_a) = 1 - \frac{x_a}{g_a}. \quad (2)$$

From (2) it is apparent that for any given quantity $x_a > 0$ an increase in $g_a$ increases the marginal willingness to pay for good $x_a$.

2.1 The markets for good $x$

In each region good $x$ is traded in a local duopoly, with one of the firms located within the region and the other one outside it. Firms maximize profits and compete à la Cournot. Good $y$ is the only input into production and technology is linear, so that marginal costs are constant. There is however a source of asymmetry between firms. When a firm supplies to its own regional market (at “home”), the production function is $x = y/c$ (the marginal cost is $c > 0$), while when a firm supplies “abroad” the production function is $x = y/(\delta c)$, $\delta \geq 1$ (the marginal cost is $\delta c$), so that the home firm has a cost advantage over its competitor.$^2$

Denote with $x_{\rho r}$ the quantity sold by firm $\rho$ in region $r$; hence aggregate sales in regions $a$ and $b$ can be written as $x_a = x_{\alpha a} + x_{\beta a}$ and $x_b = x_{\alpha b} + x_{\beta b}$. Using (2),

$^2$For instance, the parameter $\delta$ (strictly speaking, $\delta - 1$) can be interpreted as representing the extra transportation costs needed to transfer one unit of good $x$ across regions. We take the industrial structure as given. In particular, we do not allow for a firm located in one region to open a new plant in the other region so as to avoid paying the extra cost.
firm $\alpha$ solves:

$$\max_{x_{\alpha a}, x_{\alpha b}} \Pi_{\alpha a} + \Pi_{\alpha b} =$$

$$(1 - \frac{x_{\alpha a} + x_{\beta a}}{g_a} - c) x_{\alpha a} + \left(1 - \frac{x_{\alpha b} + x_{\beta b}}{g_b} - \delta c\right) x_{\alpha b}. \quad (3)$$

Solving this problem and the symmetric one for firm $\beta$, we obtain the equilibrium quantities

$$x_{\alpha a}^* = h g_a, \quad x_{\beta b}^* = h g_b, \quad x_{\beta a}^* = f g_a, \quad x_{\alpha b}^* = f g_b,$$

$$x_a^* = (h + f) g_a, \quad x_b^* = (h + f) g_b, \quad (4)$$

and the equilibrium prices

$$p_a^* = p_b^* = p^*, \quad p^* = 1 - (h + f),$$

where

$$h = \frac{1 + \delta c - 2c}{3}, \quad f = \frac{1 + c - 2\delta c}{3}. \quad (5)$$

To ensure that the quantities (and the respective prices) supplied by each firm in both regions are non-negative, we impose the following restrictions on parameters:

**Assumption 1** $0 < c < 1$ and $1 \leq \delta \leq \delta_{\text{max}} = \frac{1 + c}{2c}$.

This framework allows for a wide range of market structures. When $\delta = 1$, then $h = f = (1 - c)/3$, so that there is a symmetric duopoly in each region, since the “home” firm has no cost advantage over its “foreign” rival. At the other extreme, when $\delta = \delta_{\text{max}}$, $h = (1 - c)/2$ and $f = 0$. The cost advantage of the “home” firm is so high that the “foreign” firm does not enter the market, and thus there is a monopoly in each region. A continuum of intermediate cases is obtained for $\delta \in (1, \delta_{\text{max}})$.

Notice that the equilibrium gross profits are linearly increasing in public good provision, so that firms’ managers have an incentive to lobby the policy maker(s) for an expansion in the provision of the public goods:

$$\Pi_a^* = \Pi_{\alpha a}^* + \Pi_{\alpha b}^* = h^2 g_a + f^2 g_b, \quad \Pi_\beta^* = \Pi_{\beta a}^* + \Pi_{\beta b}^* = f^2 g_a + h^2 g_b. \quad (6)$$
2.2 The public sector

We consider two institutional settings. One is a \textit{centralized system}, in which a single policy maker chooses the supply of public goods in both regions. The other is a \textit{decentralized one}, in which each region is characterized by an independent policy maker choosing the local level of the public good. In both cases we assume public goods production to be financed with the residence-based income-tax. Technology for public good production shows decreasing returns, with factor $y$ used as the only input. The corresponding cost function is assumed to be of the form $\phi g^2$, $\phi > 0$. In order to ease the notation, and without any loss of generality, we let $\phi = 1/4$.

Under a centralized system, a single decision maker chooses $g_a$ and $g_b$ and sets a uniform tax rate across regions, $t_a = t_b = t$. The budget constraint is then:

$$\frac{g_a^2 + g_b^2}{4} = t(\pi^*_a + \pi^*_b + 2\bar{y}), \tag{7}$$

where $\pi^*_\rho = \Pi^*_\rho - s_{\rho a} - s_{\rho b}$.

Under a decentralized system, each regional policy maker independently and simultaneously chooses public good provision in her own region, and public expenditure is financed through the local income tax. The regional budget constraints are then:

$$\frac{g_a^2}{4} = t_a(\pi^*_a + \bar{y}), \quad \frac{g_b^2}{4} = t_b(\pi^*_b + \bar{y}). \tag{8}$$

Notice that by Walras’ law the markets for good $z$ and factor $y$ also clear.\footnote{The supply of good $z$ is perfectly elastic and thus its equilibrium quantity is determined by national demand, $z^d$, from consumers. As for factor $y$, national supply from consumers is inelastic, $y^s = 2\bar{y}$. The demand for $y$ comes from three sources: the public sector ($y^d_{PS}$), the firms producing good $z$ ($y^d_Z$), and the firms $\alpha$ and $\beta$ ($y^d_{\alpha+\beta}$). By Walras’ law, given that the centralized public sector’s budget constraint balances, it follows that $y^d_Z + y^d_{PS} + y^d_{\alpha+\beta} = y^s$, where $y^d_{PS} = (g^2_a + g^2_b)/4$, $y^d_Z = z^d = 2\bar{y} + \pi^*_a + \pi^*_b - p^*(x^*_a + x^*_b)$, $y^d_{\alpha+\beta} = c(x^*_{\alpha a} + \delta x^*_{\alpha b} + x^*_{\beta b} + \delta x^*_{\beta a})$. The same holds under decentralization.}

2.3 Social welfare

To compare the two alternative arrangements, we need a normative criteria. Let us then define social welfare as the sum of consumers’ surplus, distributed profits, and
the contributions raised by the government. Substituting the equilibrium values for \( x^*_a, \, \pi^*_a = -p^*x^*_a + (1 - t_a)(\bar{y} + \pi^*_a), \) and \( \pi^*_a \) into the utility function of consumers (1), social welfare in region \( a \) is

\[
W_a = x^*_a - \frac{(x^*_a)^2}{2g_a} - p^*x^*_a + (1 - t_a)(\bar{y} + \Pi^*_{aa} - s_{aa} + \Pi^*_{ab} - s_{ab}) + s_{aa} + s_{\beta a},
\]

which using (4), (6), and (8), can be rewritten as

\[
W_a(g_a, g_b) = W_a(g_a) - s_{ab} + s_{\beta a}, \tag{9}
\]

where

\[
W_a(g_a) = (h + f)^2g_a + 2(h^2g_a + f^2g_b) - \frac{g_a^2}{4} + \bar{y}. \tag{10}
\]

National social welfare, \( W = W_a + W_b, \) is then

\[
W(g_a, g_b) = \frac{(h + f)^2 + 2(h^2 + f^2)}{2}(g_a + g_b) - \frac{g_a^2 + g_b^2}{4} + 2\bar{y}. \tag{11}
\]

Comparing these equations, we notice an important difference. The net effect of lobbyists’ contributions on national social welfare is nil, since they are a pure transfer from lobbyists to politicians. Hence, a fully benevolent social planner under centralization should not take them into account. However, this is not true under decentralization. In this case, a contribution of firm \( \alpha \) to the policy maker of region \( b \) counts as a welfare loss in region \( a \), whereas a contribution of firm \( \beta \) to the policy maker of region \( a \) counts as a welfare gain in region \( a \). Hence, under decentralization, increasing the contributions from foreign firms to home politicians and reducing own firms contributions to foreign politicians count as a net increase in social welfare and as such should be considered by a benevolent planner.

\[4\]Alternatively, we could have defined social welfare as the sum of consumer’s surplus and distributed profits, letting contributions enter the choice function of the government (see eq. 17 below) only as a separate component. Our main results would remain valid under this alternative definition of social welfare.
3 Optimal public good provision without lobbying

Let us begin our analysis by examining policy choices in the benchmark case of no lobbying. Under centralization, the benevolent social planner would choose public goods supply by maximizing (11), giving for both $g_a$ and $g_b$:

$$\hat{g}_C^2 = (h + f)^2 + 2(h^2 + f^2).$$

(12)

Under decentralization, on the other hand, the policy maker of region $a$ would maximize (10) with respect to $g_a$, taking $g_b$ as given (and an analogous problem is solved by the policy maker in region $b$), obtaining the symmetric solution

$$\hat{g}_D^2 = (h + f)^2 + 2h^2.$$ 

(13)

By using (6), (12) and (13), equilibrium profits of each firm under centralization and decentralization are:

$$\hat{\pi}_C^2 = (h^2 + f^2)\hat{g}_C^2,$$

(14)

$$\hat{\pi}_D^2 = (h^2 + f^2)\hat{g}_D^2.$$ 

(15)

It follows:

**Proposition 1** Suppose there is no lobbying. Then if $\delta \in [1, \delta_{\text{max}})$ public good supply, national social welfare and firms’ profits are higher under centralization than under decentralization. In the limiting case $\delta = \delta_{\text{max}}$, the two regimes are equivalent.

**Proof.** The part on public good supply and firms’ profits follows from $f^2 > 0$ if $\delta \in [1, \delta_{\text{max}})$ and $f^2 = 0$ if $\delta = \delta_{\text{max}}$, and by comparison of (12)–(13) and of (14)–(15), respectively. As for aggregate social welfare, since $g_a = g_b = \hat{g}_C^2$ is a global maximum of (11), the latter is not maximized for $g_a = g_b = \hat{g}_D^2 < \hat{g}_C^2$. ■

The intuition is simple. When a regional policy maker considers an increase in local public good supply, she does not internalize as social welfare gains the additional profits made by the non-resident firm. Hence, when both firms sell in both regions, local public good supply is lower under decentralization and so are
profits and national welfare. On the contrary, a centralized policy maker internalizes the entire firms’ profit gains, and hence she has a greater incentive to expand public good supply. These incentives are the same when the resident firm is a monopoly within its own region, and hence \( \hat{g}^C = \hat{g}^D \).

4 Lobbying in the market

We now consider the effect of introducing lobbying into the model. We consider two different cases, lobbying in the market and lobbying for the market. In the first case, firms are already present in the market and have an incentive to lobby politicians to increase public good supply, as this increases their profits. In the second case, firms compete to acquire the right to produce in the market. In both cases, we derive equilibrium contributions and social welfare in the two cases of centralization and decentralization and compare the results.

In this section, we analyze the case of lobbying in the market. In this framework, we study lobbying behavior using the common agency approach developed by Bernheim and Whinston (1986) and popularized by Dixit et al. (1997). Notice however that under decentralization, as there are two principals (firms \( \alpha \) and \( \beta \)) lobbying two agents (policy makers \( a \) and \( b \)), our model combines elements of both the common agency model and the one-principal many-agents model (on the latter, see for instance Mookherjee, 1984, and Ma, 1988). We examine first the case of a centralized system.

4.1 Centralization

A lobby maximizes profits net of contributions to the policy maker, who in turn maximizes a weighted average of social welfare and lobbyists’ contributions. As for the timing, we assume that firms move first, by independently and simultaneously offering the policy maker a contribution schedule defining its monetary contribution as a function of public good provision. Second, upon acceptance of the lobbies contributions, the policy maker chooses public goods supply.

Following Dixit et al. (1997), we focus on truthful equilibria, in which each lobby
offers the policy maker a non-negative compensating contribution schedule, shaped along its iso-profit curve. Firm $\rho$’s compensating contribution schedule is defined as

$$S_\rho(g_a, g_b, \pi_\rho) = \max \{h^2 g_r + f^2 g_r - \pi_\rho, 0\}.$$  

(16)

Using (11) and (16), the policy maker’s objective function is

$$V^C(g_a, g_b, \pi_\alpha, \pi_\beta) = \mu W + (1 - \mu)(S_\alpha + S_\beta).$$  

(17)

The parameter $\mu$, $0 < \mu \leq 1$, captures the degree of “benevolence” of the policy maker. We rule out the unrealistic case that the politician cares about contributions only, i.e. we assume $\mu \neq 0$.

By solving the lobbying game through the maximization of (17), the optimal public good supply, both for $g_a$ and $g_b$, is

$$\tilde{g}^C = \hat{g}^C + 2m(h^2 + f^2),$$  

(18)

where

$$m = \frac{1 - \mu}{\mu}.$$  

(19)

Unsurprisingly, lobbying induces an upward distortion in public good supply, and hence a welfare loss, unless the policy maker is fully benevolent ($\mu = 1$).

Equilibrium net profits and contributions are

$$\tilde{\pi}^C = \hat{\pi}^C + m(h^4 + f^4 + 4h^2 f^2),$$  

(20)

$$\tilde{s}^C = m(h^4 + f^4).$$  

(21)

Eq. (20) shows that profits under lobbying are equal to profits without it, $\hat{\pi}^C$, plus a profit gain from lobbying. As expected, if the policy maker does not care about lobbyists’ contributions, $\tilde{\pi}^C = \hat{\pi}^C$ and $\tilde{s}^C = 0$, since $m = 0$.

The lobbying game in which both firms lobby for both public goods is not the only one conceivable. In fact, each firm has four options — lobby for both public

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5We refer the reader to Appendix A for all the analytical details. Notice that throughout the paper a “hat” denotes the solutions obtained without lobbying, whereas a “tilde” denotes the corresponding solutions under lobbying for public good provision.
goods, lobby for one public good only (either the one produced in its own region or the one produced in the other region), and no lobby. However, we do not need to examine all the corresponding lobbying games, since each firm’s profits are larger if it lobbies for both public goods, no matter what the other firm does. This follows directly from the definition of truthful strategy and the associated compensating contribution function. From proposition 2 in Dixit et al. (1997), a truthful strategy is weakly dominant, and in our setting truthful strategies always involve non-negative contributions by both firms on both public goods.

4.2 Decentralization

Under decentralization, each firm has four possible strategies: lobby both regions (B), lobby only “at home” — inside its region (I), lobby only “abroad” — outside its region (O), and, finally, no lobby (N). This strategy set gives rise to a $4 \times 4$ normal form symmetric game — that we denote as the where-to-lobby game — whose payoffs are the firms’ equilibrium profits at the corresponding truthful equilibrium of the lobbying-game. By symmetry between firms, it is sufficient to consider 9 different lobby games only to construct the where-to-lobby game (in addition to the no-lobby case already examined in section 3). In the following we focus on those lobbying games in which firms play the same strategy, referring the reader to Appendix A for all remaining cases.

Let $S_{\rho}(g_r, \pi_{\rho r})$ be the compensating contribution schedule that firm $\rho$ offers the policy maker of region $r$, where $S_{aa} = \max \{h^2 g_a - \pi_{aa}, 0\}$, $S_{\beta a} = \max \{f^2 g_a - \pi_{\beta a}, 0\}$, $S_{ab} = \max \{f^2 g_b - \pi_{ab}, 0\}$ and $S_{\beta b} = \max \{h^2 g_b - \pi_{\beta b}, 0\}$. When both firms lobby both regions (BB), policy makers maximize\(^6\)

\[
V^D_{BB} = \mu(W_a - S_{ab} + S_{\beta a}) + (1 - \mu)(S_{aa} + S_{\beta a}), \tag{22}
\]

\[
V^D_{BB} = \mu(W_b - S_{\beta a} + S_{ab}) + (1 - \mu)(S_{\beta b} + S_{ab}). \tag{23}
\]

As already noted above, under decentralization, different lobbies’ contributions do not have the same weight into the local politicians’ preferences. One unit of con-

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\(^6\)We assume that the degree of benevolence of regional policy makers is the same as that of the central policy maker.
tribution a firm makes abroad counts as $-\mu$ in the home region but as 1 in the recipient region, while one unit of contribution a firm makes at home counts as $1 - \mu$ in the home region and nothing abroad.

The optimal public good supply in each region is (see Appendix A for details)

$$\tilde{g}^{DBB} = \tilde{g}^D + 2f^2 + 2m(h^2 + f^2),$$ (24)

and total (home plus abroad) net profits of each firm are

$$\tilde{\pi}^{DBB} = \tilde{\pi}^D + mh^4 + 2(1 + 2m)h^2f^2 + (m - m^{-1})f^4.$$ (25)

In a decentralized system lobbies are able to influence public policy even when the social planner is fully benevolent ($\mu = 1$). In fact, even if the regional policy maker does not place any value on contributions \textit{per se}, contributions offered by the foreign firm enter the region social welfare and hence influence her choices, as represented by the second term in (24).

Turning to the case in which both firms lobby their home region only ($II$), the policy makers’ objective functions become

$$V_a^{DII} = \mu W_a + (1 - \mu)S_{aa},$$

$$V_b^{DII} = \mu W_b + (1 - \mu)S_{bb},$$ (27)

and, as it is shown in Appendix A, public good supply and total net profits are, respectively,

$$\tilde{g}^{DII} = \tilde{g}^D + mh^2,$$ (28)

$$\tilde{\pi}^{DII} = \tilde{\pi}^D + mh^4 + 2mh^2f^2.$$ (29)

Finally, when both firms lobby only abroad ($OO$), policy makers maximize

$$V_a^{DOO} = \mu(W_a - S_{ab} + S_{ba}) + (1 - \mu)S_{ba},$$

$$V_b^{DOO} = \mu(W_b - S_{ba} + S_{ab}) + (1 - \mu)S_{ab},$$ (31)

obtaining

$$\tilde{g}^{DOO} = \tilde{g}^D + 2(1 + m)f^2,$$ (32)
and thus total net profits are

\[ \tilde{\pi}^{DOO} = \hat{\pi}^D + 2(1 + m)h^2 f^2 + (1 + m)f^4. \] (33)

Using (25), (29) and (33), as well as the other expressions for equilibrium profits in Table 3, Appendix A, the resulting where-to-lobby game is shown in Table 1. Each cell contains the payoff of the row player, firm \( \alpha \), at the top, and that of the column player, firm \( \beta \), at the bottom. The profit gains from lobbying of the firm playing strategy \( i \) when the opponent is playing \( j \), with \( i, j \in \{B, I, O, N\} \) are denoted with \( \Delta \pi^{ij} = \tilde{\pi}^{Dij} - \hat{\pi}^D \).

As illustrated in Figure 1, the equilibria of the where-to-lobby game depend on the parameter \( \mu \), representing politicians’ preferences for contributions, and on the differential-cost parameter \( \delta \), which influences market structure. The thick curves divide the closed set \( S = (\mu, \delta) \in [0, 1] \times [1, \delta_{max}] \) into three subsets,\(^7\) one in which the unique Nash equilibrium of the where-to-lobby game is \( BB \), one in which it is \( II \), and finally one in which it is \( OO \). Firms lobby both policy makers only if the latter are “greedy” enough, assigning at least as much weight to contributions as to social welfare (i.e. \( \mu \leq \frac{1}{2} \)). On the other hand, if politicians care more about social welfare than contributions, firms lobby at most one politician. Whether it is the home one (equilibrium \( II \)) or the outside one (equilibrium \( OO \)), it depends on the

\(^7\)The meaning of the curves \( \mu^E(\delta; c) \) and \( \mu^S(\delta; c) \), will become apparent in Corollary 1 and Proposition 3, respectively.
Table 1: The where-to-lobby game under decentralization

<table>
<thead>
<tr>
<th></th>
<th>B</th>
<th>I</th>
<th>O</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>f</strong></td>
<td><strong>m</strong></td>
<td><strong>α</strong></td>
<td><strong>BN</strong> = mh^4 + (m - m^{-1})f^4</td>
<td><strong>BI</strong> = mh^4 + 2(1 + 2m)h^2 f^2 + (m - m^{-1})f^4</td>
</tr>
<tr>
<td><strong>i</strong></td>
<td><strong>r</strong></td>
<td><strong>m</strong></td>
<td><strong>BR</strong> = mh^4 + 2(1 + 2m)h^2 f^2 + (m - m^{-1})f^4</td>
<td><strong>IR</strong> = mh^4 + 2mh^2 f^2 + (m - m^{-1})f^4</td>
</tr>
<tr>
<td><strong>m</strong></td>
<td><strong>r</strong></td>
<td><strong>m</strong></td>
<td><strong>BO</strong> = 2(1 + 2m)h^2 f^2 + (1 + m)f^4</td>
<td><strong>IR</strong> = mh^4 + 2(1 + 2m)h^2 f^2 + (m - m^{-1})f^4</td>
</tr>
<tr>
<td><strong>N</strong></td>
<td><strong>m</strong></td>
<td><strong>m</strong></td>
<td><strong>NB</strong> = mh^4 + 2(1 + 2m)h^2 f^2 + (m - m^{-1})f^4</td>
<td><strong>IN</strong> = mh^4 + 2mh^2 f^2 + (1 + m)f^4 = Δπ^{IN}</td>
</tr>
<tr>
<td></td>
<td><strong>m</strong></td>
<td><strong>m</strong></td>
<td><strong>NB</strong> = 2(1 + 2m)h^2 f^2</td>
<td><strong>IN</strong> = 2mh^2 f^2</td>
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</tbody>
</table>
values of $\delta$ and $\mu$. Proposition 2 makes this argument precise.

**Proposition 2** For $\delta \in [1, \tilde{\delta}]$ the unique Nash equilibrium of the where-to-lobby game under decentralization is $BB$ if and only if $\mu \in (0, \mu^{BO}]$ and $OO$ otherwise. For $\delta \in (\tilde{\delta}, \delta_{\text{max}}]$ the equilibrium is $BB$ if and only if $\mu \in (0, \mu^{IO}]$ and $II$ if and only if $\mu \in (\frac{1}{2}, \mu^{IO}]$ and $OO$ otherwise, where $\mu^{BO}(\delta; c) = \frac{1}{2}$, and

$$\mu^{BO}(\delta; c) = 1 - \frac{\left(\sqrt{4h^4 + f^4 - f^2}\right)}{2h^4}, \quad (34)$$

$$\mu^{IO}(\delta; c) = \frac{h^4 - f^4}{h^4}, \quad (35)$$

and

$$\tilde{\delta}(c) = \frac{(2 + \sqrt{2})c + \sqrt{2} - 1}{(1 + 2\sqrt{2})c}. \quad (36)$$

**Proof.** See Appendix A. ■

While under centralization firms always lobby for both public goods, Proposition 2 shows that under decentralization this result does not emerge if politicians are benevolent enough (i.e. $\mu > \frac{1}{2}$). The intuition is that when a firm lobbies abroad, the contribution paid to the politician counts as a welfare loss at home. Hence, in order to successfully lobby at home as well, the firm has to pay a “double” bribe: one to compensate for the welfare loss of lobbying abroad, and one to compensate for the resulting public good distortion at home. Double lobbying turns out to be profitable only if $\mu < \frac{1}{2}$, since it is not necessary, *coeteris paribus*, to pay high contributions to successfully lobby greedy politicians. On the contrary, when firms face politicians who are benevolent enough, it becomes too costly to compensate the home politician for the negative externality caused by lobbying abroad, and hence it becomes profitable to lobby at most in one region.\(^8\) When this is the case,\(^8\)This intuition is evident from the equilibrium contributions shown in Table 4. A firm, say $\alpha$, lobbying both regions pays a contribution $\bar{s}_{\alpha bah}^{BB} = (1 + m)f^4$ to the abroad politician. The contribution paid at home, $\bar{s}_{\alpha ah}^{BB}$, is made up of two terms: $(1 + m^{-1})f^4$ as a compensation for the welfare loss for paying contributions abroad, and $mh^4$ as a compensation for public good distortion at home. Clearly, if $(1 + m^{-1})f^4 > (1 + m)f^4$, which occurs if $\mu > \frac{1}{2}$, it does not pay to lobby both regions.
whether the equilibrium is II or OO hinges upon two contrasting effects. On the one hand, since the weight assigned by the politician to contributions from the home firms is lower than the one assigned to contributions from abroad (1 − µ and 1, respectively), firms have an advantage in lobbying abroad. On the other hand, since a firm is more productive at home, i.e. \( h \geq f \), it makes more profits when lobbying at home. These two contrasting effects — i.e. the fact that the comparative advantage of lobbying abroad is increasing in \( \mu \) whereas that of lobbying at home is increasing in \( \delta \) — explain why the boundary between the equilibria OO and II is given by the increasing function \( \mu^{IO}(\delta; c) \).

It is also worth noting that the equilibria of the where-to-lobby game are not always Pareto efficient in terms of aggregate firms’ net profits. As Corollary 1 shows, when the equilibrium is either BB or OO aggregate firms’ profits are maximized. On the contrary, when the equilibrium is II firms may end up in a prisoner dilemma, in which they both lobby at home while lobbying abroad would be more profitable. As shown in Figure 1, the boundary between the efficient Nash equilibria II and the inefficient ones is given by the curve \( \mu^{E}(\delta; c) \), with the former equilibria lying below the curve.

**Corollary 1** The Nash equilibria BB and OO of the where-to-lobby game are Pareto efficient in terms of aggregate firms’ net profits. The equilibrium II is Pareto efficient if and only if, for \( \delta \in (\bar{\delta}, \delta_{\text{max}}] \), \( \mu \in \left( \frac{1}{2}, \max \{ \mu^{E}, \frac{1}{2} \} \right] \), where \( \mu^{E}(\delta; c) = \frac{h^4 - f^4}{h^4 + 2hf^2} \). Otherwise II is Pareto dominated by the strategy pair OO.

**Proof.** See Appendix A. ■

### 4.3 A comparison

The above results allow for a comparison of lobbying behavior under centralization and decentralization along various dimensions: social welfare and public good supply, firms’ net profits, and contributions to politicians. From Proposition 1 we know that in a world without lobbying aggregate social welfare is higher under centralization than under decentralization, since under the latter regime regional policy makers do not internalize as social welfare the profits of the foreign firm and hence undersupply public goods.
This result is reversed when lobbies influence the policy making process. In a centralized system, since both firms lobby for both public goods and the policy maker internalizes all spillover effects on profits, the resulting upward distortion in public good supply reduces social welfare. When the equilibrium is $BB$ the same distortion, however, occurs under decentralization too, as the supply of public goods is the same under decentralization and centralization. The joint lobbying effort exerted by firms on both regional policy makers induces the latter to implicitly account for the regional profit-spillovers \textit{via} in public good supply. However, when politicians are benevolent enough and firms lobby at most one policy maker (i.e. when the equilibrium is either $II$ or $OO$) lobbying is less effective and the distortion in public good supply and the associated welfare loss are lower under decentralization than under centralization. Moreover, lobbies always prefer a centralized system over a decentralized one, since net profits are higher. This is obvious if the decentralized equilibrium is $BB$, since gross profits are the same under the two regimes whereas contributions are higher under decentralization than under centralization. Firms are also clearly better off under centralization whenever the equilibrium under decentralization is either $II$ or $OO$, since in the latter case gross profits are lower while contributions, though smaller in some cases, do not allow higher net profits compared to centralization.

The following proposition summarizes these results.

\textbf{Proposition 3} In the presence of lobbying, firms’ net profits are higher under centralization than under decentralization. Contributions to politicians are higher under decentralization when the equilibrium is $BB$ and, provided that $\mu > \mu^S(\delta; c) = \frac{h^A}{h + f}$, also when the equilibrium is $OO$; otherwise contributions are higher under centralization. Public good supply and aggregate social welfare are the same under the two regimes when the equilibrium under decentralization is $BB$. When the equilibrium is either $II$ or $OO$, public good supply is lower, whereas aggregate social welfare is higher, under decentralization than under centralization.

\textbf{Proof.} See Appendix A. \hfill \blacksquare
5 Lobbying for the market

We consider now a different political economy framework, one in which firms lobby for acquiring the right to enter the market instead of lobbying for public goods provision. We assume the following time line. In stage 1, the government (central or regional, depending on the case) decides on the number of firms that are allowed to operate in the market for good \( x \). If both firms are allowed to enter, firms have no incentive to pay the politician in stage 2 since we do not allow in this section for lobbying in the market ex post for public good provision (hence the game goes directly to stage 4). Conversely, if the government allows for one entrant only in stage 1, in stage 2 firms competing for the market make a credible commitment to pay politicians a contribution if they are given the monopoly right in the market for good \( x \). In stage 3, the politician, knowing the offer made by the firms in stage 2, assigns the monopoly right to the firm that guarantees her the highest payoff (weighted average of social welfare and lobbies’ contributions) and cashes the contribution. In stage 4, the government chooses public good supply by maximizing social welfare. Finally, in stage 5 market equilibrium is determined along the lines of section 2. The model is solved by backward induction.

The more complex structure of this case allows us to consider three different institutional settings. In the first one, the central government chooses both the number of firms entering each regional market and local public good supplies (full centralization). In the second one, the central government establishes the number of firms that are allowed to operate in each regional market but regional public good supply is decided at the regional level (split competencies). Finally, in the third case, each regional government chooses both the number of firms entering its market and public good supply (full decentralization). The case of split competencies captures the well known fact that in most countries regional and central competencies often overlap (e.g. competition policies), rather than being neatly assigned to one of the two levels of governments. Thus, this allows us to ask if the presence of lobbying may provide a rationale for these arrangements.

To investigate these three cases, we need to compute first market equilibrium and welfare under monopoly (stage 5), thus integrating the duopoly analysis already
provided in section 2. Letting
\[ H = \frac{1 - c}{2} \quad \text{and} \quad F = \frac{1 - \delta c}{2}, \]
(37)
by standard profit maximization, when the regional markets are monopolized the equilibrium quantities are \( x_a^* = H g_a \) and \( x_b^* = H g_b \) if it is the home (foreign) firm that supplies the market. The corresponding equilibrium profits are \( \Pi^*_\alpha = H^2 g_a \) and \( \Pi^*_\beta = H^2 g_b \) (\( \Pi^*_\alpha = F^2 g_a \) and \( \Pi^*_\beta = F^2 g_b \)) when the home (foreign) firm supplies the market.

Focusing again on region \( a \), and depending on which firm operates in each region, social welfare is
\[
W^H_{aH} = \frac{3H^2 g_a}{2} - \frac{g_a^2}{4} + \bar{y}, \quad \text{(38)}
\]
\[
W^F_{aF} = \frac{F^2 g_a + 2F^2 g_b}{2} - \frac{g_b^2}{4} + \bar{y}, \quad \text{(39)}
\]
\[
W^H_{aF} = \frac{3H^2 g_a + 2F^2 g_b}{2} - \frac{g_b^2}{4} + \bar{y}, \quad \text{(40)}
\]
\[
W^F_{aH} = \frac{F^2 g_a}{2} - \frac{g_a^2}{4} + \bar{y}, \quad \text{(41)}
\]
where the apix \( H_aH_b \) (resp. \( F_aF_b \)) denotes that home (resp. foreign) firms are monopolists in both regions, and \( H_aF_b \) (resp. \( F_aH_b \)) that firm \( \alpha \) (resp. \( \beta \)) is a monopolist in both regions. We begin the analysis with the full centralization case.

### 5.1 Full centralization

By symmetry, we only consider the case in which the central government opts in stage 1 for the same policy, one or two firms, in both regions. Suppose first that the government allows for both firms supplying both regional markets. This case has already been studied in section 3, where policy without lobbying was described. Substituting the optimal public good provision given in (12) into (11), the politician’s value function when both firms are allowed to enter the market is then
\[
\hat{V}^{H_f} = \mu \left[ \frac{(h + f)^2}{2} + 2(h^2 + f^2) \right] + 2\mu\bar{y}. \quad \text{(42)}
\]

Consider next the case in which only one firm is allowed to enter the regional markets. The government holds simultaneously an auction for each market, and
firms have now an incentive to compete for it, making contributions to the government. Let \( S^H_\rho \) and \( S^F_\rho \) be the contribution offered by firm \( \rho \) for serving the home and the foreign market, respectively. The following Lemma summarizes the outcome of firms’ competition for the market.

**Lemma 1** Under full centralization, if only one firm is allowed to enter the regional markets, then each firm gets the home market by paying the contribution

\[
\hat{S}^H_\rho = \max \left\{ \hat{T}^H, 0 \right\}, \quad \text{where} \quad \hat{T}^H = - \frac{9\mu(H^4 - F^4)}{4(1 - \mu)} + 3F^4.
\]

The corresponding politician’s value function is

\[
\hat{V}^H = \mu \frac{9H^4}{2} + 2(1 - \mu)\hat{S}^H_\rho + 2\mu\bar{y}.
\]

**Proof.** See Appendix B. ■

The intuition is simple. A local monopoly is always more profitable than a foreign one, since by assumption the home firm has a cost advantage over the foreign one \((H \geq F)\) and the optimal public good supply is higher when the home firm serves the market. Hence each firm wins the home market by outbidding the foreign firm, whose offer \( \hat{S}^F_\rho \) at most equals the profits it would make by serving the foreign market in a monopolistic regime, \( 3F^4 \). Notice however from (43) that the home firm does not need to offer that much, and in some cases it does not even need to make a positive offer to win the market. The reason is that if the foreign firm gets the market, then a welfare loss is observed compared to a home-monopoly. Thus, in order to win the market, the home firm can always offer the politician a lower contribution than the one offered by the foreign firm. Quite intuitively, the higher are \( \mu \) and \( \delta \) the more likely is that the home firm does not need to make a positive offer to win the market.

By comparing (42) and (44), we can finally characterize the central government’s choice in stage 1.

**Proposition 4** Under full centralization, for \( \delta \in [1, \delta_1] \), \( \delta_1(c) = \frac{5 + 17c}{22c} \), there exists a \( \mu_1(\delta; c) \), decreasing in \( \delta \), such that for all \( \mu \leq \mu_1 \) only one firm is allowed to enter each regional market; by Lemma 1, the home firm obtains a monopoly upon
the payment of a contribution. For $\delta \in [1, \delta_1]$ and $\mu > \mu_1$ both firms are allowed into both regional markets. For $\delta \in (\delta_1, \delta_{\max}]$ only one firm is allowed to enter each regional market for all $\mu$ and therefore the home firm gets a monopoly. As for contributions, there exists a $\mu_2(\delta; c)$, decreasing in $\delta$, such that the firm pays a contribution for all $\mu < \mu_2$ and no contribution otherwise.

Proof. See Appendix B.

The results in Proposition 4 are illustrated in Figure 2-a. For $\delta \leq \delta_1$ and $\mu > \mu_1$ the policy maker opts for a duopoly in both markets ($h_f a h_f b$). In all other cases she opts for a monopoly and, given the results in Lemma 1, each firm wins its home market ($H_a H_b$). In this latter case, positive contributions ($\hat{S}_p^H > 0$) are paid if and only if $\mu$ is below a given threshold ($\mu_1$ or $\mu_2$, depending on the value of $\delta$); otherwise the home firm does not need to offer a contribution to gain access to the monopolized market.

To understand the intuition behind these results, suppose first that the politician simply maximizes social welfare (i.e. $\mu = 1$). The proposition then shows that there exists a threshold level of the cost advantage for the home firm, $\delta_1$, such that for $\delta < \delta_1$ ($\delta \geq \delta_1$), social welfare is higher (lower) under a duopoly than under a monopoly. Hence, the fully benevolent politician simply lets both firms enter both markets in the former case and only the home firm in the latter one. If instead $\mu < 1$, the politician faces a trade-off when $\delta < \delta_1$. By creating a monopoly, she gets a contribution from the home firm winning the contest for the market, but at
the cost of the monopoly welfare loss; however, if she lets both firms in, she avoids this welfare cost but does not get any contributions (recall there is no lobbying in the market here). This explains why, for \( \delta < \delta_1 \), a sufficiently benevolent policy maker — one with preferences \( \mu > \mu_1 \) — makes the efficient choice, while a politician who is greedier \( (\mu \leq \mu_1) \) prefers a monopoly by home firms in each regional market. This trade off is absent when \( \delta \geq \delta_1 \), since social welfare is however higher under a home monopoly than under a duopoly. Hence the politician always allows only one firm in each market, no matter her degree of benevolence. The latter only bears on whether contributions are paid to the central politician. If \( \mu > \mu_2 \), i.e. if the politician is sufficiently benevolent, then home firms would not need to bribe the politician in order to win the local monopoly, even though foreign firms made a positive offer. Instead, if the politician is greedy \( (\mu \leq \mu_2) \), the home firm must offer a contribution to outbid the offer made by the foreign firm. Recalling that lobbies’ contributions are pure transfers and that when lobbying is for the market there are no distortions in public goods supply, we can conclude that a loss in social welfare occurs if and only if lobbying induces the central government to opt for local monopolies whenever a benevolent social planner would have opted for local duopolies. Formally:

**Corollary 2** Under full centralization lobbying causes a welfare loss iff \( \delta \in [1, \delta_1) \) and \( \mu \in (0, \mu_1) \).

### 5.2 Split competencies

Consider next the case in which the central government chooses how many firms enter each market, but the regional governments choose public good supply. Since what differentiates split competencies and the fully centralized regimes is only the equilibrium level of public goods supply, we can directly follow the above logic to prove:  

**Proposition 5** Under split competencies, for \( \delta \in [1, \delta_2] \), \( \delta_2(c) < \delta_1(c) \) for all \( c \in (0, 1) \), there exists a \( \mu_3(\delta; c) \) such that for all \( \mu \leq \mu_3 \) only one firm is allowed to enter each regional market, and therefore the home firm obtains a monopoly upon

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\[ ^9 \text{The formal proof is available from the authors upon request.} \]
the payment of a contribution; otherwise both firms are allowed into both regional markets. For \( \delta \in (\delta_2, \delta_{\text{max}}] \) only one firm is allowed to enter each regional market for all \( \mu \), and hence the home firm gets a monopoly. As for contributions, there exists a \( \mu_4(\delta; c) \) such that the firm pays a contribution for all \( \mu < \mu_4 \) and no contribution otherwise.

Split competencies and full centralization are compared in Figure 2-b. Notice that the area in which each firm obtains a monopoly at home upon the payment of a contribution is certainly smaller under split competencies, since the curves \( \mu_3 \) and \( \mu_4 \) for the latter case lie below the respective curves \( \mu_1 \) and \( \mu_2 \) for centralization. Hence, lobbying for the market is less effective under split competencies than under centralization.

However, the comparison in terms of social welfare depends on parameters. As \( \delta_2 < \delta_1 \), there is an area under split competencies — defined by \( \delta \in (\delta_2, \delta_1) \) and \( \mu > \mu_1 \) — in which even a fully benevolent central politician (\( \mu = 1 \)) would opt for a monopoly by the home firm instead of the more efficient duopoly. This is so because under split competencies public good provision is decided at the local level and as shown above (in section 3) local public goods are underprovided by local governments in local duopolies (since local governments do not count profits from foreign firms as social welfare). Hence, allowing for a single home producer by the center is a way to partly counteract this inefficiency at local level. On the other hand, split competencies is more efficient than centralization for \( \delta < \delta_2 \), as the set in which two firms are allowed in both markets (the efficient choice) is larger under split competencies than under centralization, since \( \mu_3 < \mu_1 \). This is again due to the fact that local governments do not consider foreign firms’ profits as part of the (local) social welfare. In fact, in the event of a foreign monopoly, a local government undersupplies the public good compared to a central government. This means that, under split competencies, in order to outbid the foreign competitor home firms need to offer the politician a smaller contribution, which explains why lobbying is more effective under full centralization.
We finally consider the case of full decentralization, in which regional governments (simultaneously) choose first the number of firms that are allowed to enter their market, and then public good supply. The choice on the number of firms gives rise to a $2 \times 2$ normal form game between regional policy makers. Whenever only one firm is allowed to supply a regional market firms compete to gain access to it by bribing the regional policy maker. For any strategy pair, Lemma 2 establishes the outcome of firms’ competition for the market and regional payoffs, shown in Table 2, in terms of the maximum value of politicians’ objective functions.

**Lemma 2** Under full decentralization, whenever a region allows only for one firm to serve its local market, then it is the home firm to gain access to the market, paying the contribution

$$\hat{S}_\rho^H = \max \left\{ -\frac{\mu(9H^4 - F^4)}{4(1 - \mu)} + \frac{F^4}{1 - \mu}, 0 \right\}$$

(45)

to the politician.

Depending on the number of firms allowed into each regional markets, politicians’ value functions are those shown in Table 2.

**Proof.** See Appendix B. ■

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**Table 2:** Politicians’ value functions under full decentralization

<table>
<thead>
<tr>
<th>region b</th>
<th>two firms</th>
<th>one firm</th>
</tr>
</thead>
<tbody>
<tr>
<td>r</td>
<td>two firms</td>
<td>one firm</td>
</tr>
<tr>
<td>e</td>
<td>$V_a = \mu \left( \frac{[(h + f)^2 + 2h^2][2h^2 + 2(h^2 + f^2)]}{4} \right)$</td>
<td>$V_a = \mu \left( \frac{[(h + f)^2 + 2h^2]^2}{4} \right)$</td>
</tr>
<tr>
<td>one firm</td>
<td>$V_a = \mu \left( \frac{[(h + f)^2 + 2h^2]^2}{4} \right)$</td>
<td>$V_a = \mu \left( \frac{[(h + f)^2 + 2h^2]^2}{4} \right)$</td>
</tr>
</tbody>
</table>

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10Since the game is symmetric, the Table shows only the payoffs of region a’s politician. Also, to save space, regional social welfare is net of the endowment $\bar{y}$.
The reasons why, under decentralization, it is always the home firm to gain a monopoly in its market when competing with the foreign firm, are the same already discussed for the other two regimes. From (45) it is immediate to see that $\hat{S}_{H}^H > 0$ if and only if

$$\mu < \mu_5(\delta; c) = \frac{4F^4}{9H^4 - F^4}. \quad (46)$$

Regional politicians choose the number of firms in the market by playing the normal form game given in Table 2. The solution of such a policy game is given in the following proposition:

**Proposition 6** Under full decentralization, it is a dominant strategy for both regional policy makers to admit only one firm in their market for all $\mu \in (0, 1]$ and $\delta \in [1, \delta_{\text{max}}]$. Hence, by Lemma 2 the home firm gets a local monopoly upon the payment of a positive contribution for $\mu < \mu_5(\delta; c)$ and nothing otherwise.

**Proof.** See Appendix B. ■

Proposition 6 shows that lobbying for the market is most effective under full decentralization, with the home firms always gaining a local monopoly in their regional market. When $\delta < \delta_1$, although a duopoly would be the efficient solution in both regions, markets turn out to be fully monopolized no matter the value of $\mu$. This means that in the case of lobbying for the market full decentralization is the less efficient of the three regimes. Moreover, one can show that the Nash equilibrium (one-firm, one-firm) of the game in Table 2 is also Pareto inefficient in terms of politicians’ aggregate value functions for all $\delta < \delta_2$. What makes the difference between full decentralization and split competencies is thus that, while under the former regime regional policy makers end up in a prisoner dilemma, under the latter regime this outcome does not occur, because it is the central policy maker that directly chooses the highest aggregate payoff along the diagonal cells of the game in Table 2.

### 6 Concluding remarks

We began this work by recalling Prud’homme’s argument against the dangers of decentralization due to lobbying effects by local interests. Our analysis made it
clear when this argument is correct and when it is not. If firms lobby for the market, then decentralization is certainly a bad idea. Local governments have a strong incentive to allow only home firms to enter in the market, as their profits only matter for local welfare, which in turn means that a local firm can always easily outbid a foreign one. If firms lobby in the market, on the other hand, lobbying may not be as dangerous under decentralization as it is under centralization. Local governments do not internalize the spillover effects induced by foreign firms’ profits, and while this may be a source of inefficiency for local public goods provision, it has the effect of making local governments more resilient to lobbies’ contributions. This suggests that the best institutional structure as lobbying is concerned is one in which competencies across different levels of government are split, with central government taking care of decisions about the number of firms allowed to operate in the markets and local governments deciding on local public good production.

Our analysis could be extended in several directions. On the one hand, to better focus on the issue at hand, we abstracted from several realistic features of existing federations, such as intergovernmental transfers and firms mobility. Introducing these features may provide a more complete picture of the relationships between decentralization and lobbying. On the other hand, the political side of the model could be expanded, for instance by introducing campaign contributions to political parties and elections, as well as bargaining in legislatures. Allowing for a more complex institutional structure (along the lines, for example, of Persson and Tabellini, 2000, ch. 7, Mitra, 1999, Besley and Coate, 2001, Felli and Merlo, 2001) may highlight other channels of interaction between local interests and local policies.

A Appendix: Lobbying for public good provision

The lobbying-games are solved by extending the logic in proposition 3 in Dixit et al. (1997).
A.1 Centralization

From the first order conditions for maximizing (17),

$$\mu \frac{\partial W}{\partial g_r} + (1 - \mu)(h^2 + f^2) = 0,$$

(47)

we obtain $\tilde{g}^C$ in (18) for both $g_a$ and $g_b$. In deriving the first order condition (47), we ignore the non-negativity constraint on contributions, by letting $S_\rho = h^2 g_r + f^2 g_r - \pi_\rho$ into the objective function, and then checking non-negativity ex post in the computed equilibrium. To compute the equilibrium profits of firm $\beta$ we need first to solve the problem in which firm $\alpha$ is lobbying and firm $\beta$ is not lobbying. Hence, the policy maker maximizes $V_C^\beta = \mu W + (1 - \mu)S_\alpha$. From the corresponding first order conditions:

$$\mu \frac{\partial W}{\partial g_a} + (1 - \mu)h^2 = 0, \quad \mu \frac{\partial W}{\partial g_b} + (1 - \mu)f^2 = 0,$$

we obtain the optimal public good supplies:

$$\tilde{g}^C_{\alpha(-\beta)} = \tilde{g}^C + 2mh^2, \quad \tilde{g}^C_{\beta(-\beta)} = \tilde{g}^C + 2mf^2.$$

Writing the equation $V_C^\beta (\tilde{g}^C_a, \tilde{g}^C_b, \pi_\alpha, \pi_\beta) = V_C^{\beta(-\beta)} (\tilde{g}^C_{\alpha(-\beta)}, \tilde{g}^C_{\beta(-\beta)}, \pi_\alpha)$ and solving for $\pi_\beta$, we obtain the equilibrium profits $\tilde{\pi}^C_\beta$ shown in (20). By symmetry, $\tilde{\pi}^C_\alpha = \tilde{\pi}^C_\beta$. Finally, by substituting (18) and (20) into (16), we check that equilibrium contributions in (21) are non-negative.

A.2 Decentralization: derivation of the where-to-lobby game

We solve the lobby game for each strategy pair occurring under decentralization, ignoring the non-negativity constraint on contributions, letting $S_{\alpha a} = h^2 g_a - \pi_{\alpha a}$, $S_{\beta a} = f^2 g_a - \pi_{\beta a}$, $S_{\alpha b} = f^2 g_b - \pi_{\alpha b}$ and $S_{\beta b} = h^2 g_b - \pi_{\beta b}$. We check ex post that equilibrium contributions are non-negative. $V_r^{Di,j}$ denotes the preferences of policy maker $r$ when firms $\alpha$ and $\beta$ are choosing action $i$ and $j$, respectively, $i, j \in \{B, I, O, N\}$. The results of the analysis are summarized in Table 3 (equilibrium profits) and Table 4 (equilibrium contributions).

Both firms lobbying both regions ($BB$)

When both firms lobby both regions, the policy makers’ objective functions are (22) and (23) in the text. By maximizing (22) with respect to $g_a$ and (23) with respect to $g_b$, we obtain the symmetric solution $\tilde{g}^{D,B,B}$ in (24). To compute the equilibrium profits, assume
Table 3: Firms’ net profits under decentralization

<table>
<thead>
<tr>
<th>Firm</th>
<th>Profit at home ( h^2 g^{D+} )</th>
<th>Profit abroad ( f^2 g^{D+} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \rho )</td>
<td>( mh^4 + 2(1 + m)h^2 f^2 - (1 + m^{-1})f^4 )</td>
<td>( (1 + m)^4 + 2mh^2 f^2 )</td>
</tr>
<tr>
<td>( -\rho )</td>
<td>( mh^4 + 2(1 + m)h^2 f^2 - (1 + m^{-1})f^4 )</td>
<td>( (1 + m)^4 + 2mh^2 f^2 )</td>
</tr>
<tr>
<td>( \rho )</td>
<td>( mh^4 - (1 + m^{-1})f^4 )</td>
<td>( (1 + m)^4 + 2mh^2 f^2 )</td>
</tr>
<tr>
<td>( -\rho )</td>
<td>( mh^4 + 2(1 + m)h^2 f^2 )</td>
<td>( 2mh^2 f^2 )</td>
</tr>
<tr>
<td>( \rho )</td>
<td>( mh^4 + 2(1 + m)h^2 f^2 - (1 + m^{-1})f^4 )</td>
<td>( (1 + m)^4 )</td>
</tr>
<tr>
<td>( -\rho )</td>
<td>( 2(1 + m)h^2 f^2 )</td>
<td>( (1 + m)^4 + 2mh^2 f^2 )</td>
</tr>
<tr>
<td>( \rho )</td>
<td>( mh^4 - (1 + m^{-1})f^4 )</td>
<td>( (1 + m)^4 )</td>
</tr>
<tr>
<td>( -\rho )</td>
<td>( 2(1 + m)h^2 f^2 )</td>
<td>( 2mh^2 f^2 )</td>
</tr>
<tr>
<td>( \rho )</td>
<td>( mh^4 )</td>
<td>( 2mh^2 f^2 )</td>
</tr>
<tr>
<td>( -\rho )</td>
<td>( mh^4 )</td>
<td>( 2mh^2 f^2 )</td>
</tr>
<tr>
<td>( \rho )</td>
<td>( mh^4 + 2(1 + m)h^2 f^2 )</td>
<td>( 0 )</td>
</tr>
<tr>
<td>( -\rho )</td>
<td>( 0 )</td>
<td>( (1 + m)^4 + 2mh^2 f^2 )</td>
</tr>
<tr>
<td>( \rho )</td>
<td>( mh^4 )</td>
<td>( 0 )</td>
</tr>
<tr>
<td>( -\rho )</td>
<td>( 0 )</td>
<td>( 2mh^2 f^2 )</td>
</tr>
<tr>
<td>( \rho )</td>
<td>( 2(1 + m)h^2 f^2 )</td>
<td>( (1 + m)^4 )</td>
</tr>
<tr>
<td>( -\rho )</td>
<td>( 2(1 + m)h^2 f^2 )</td>
<td>( (1 + m)^4 )</td>
</tr>
<tr>
<td>( \rho )</td>
<td>( 0 )</td>
<td>( (1 + m)^4 )</td>
</tr>
<tr>
<td>( -\rho )</td>
<td>( 2(1 + m)h^2 f^2 )</td>
<td>( 0 )</td>
</tr>
<tr>
<td>Contributions at home</td>
<td>Contributions abroad</td>
<td></td>
</tr>
<tr>
<td>-----------------------</td>
<td>----------------------</td>
<td></td>
</tr>
<tr>
<td>firm α B</td>
<td>$s_{\alpha\alpha}^{DBB} = mh^4 + (1 + m^{-1})f^4$</td>
<td>$s_{\alpha\beta}^{DBB} = (1 + m)f^4$</td>
</tr>
<tr>
<td>firm β B</td>
<td>$s_{\beta\beta}^{DBB} = mh^4 + (1 + m^{-1})f^4$</td>
<td>$s_{\beta\alpha}^{DBB} = (1 + m)f^4$</td>
</tr>
<tr>
<td>firm β I</td>
<td>$s_{\beta I}^{DBB} = mh^4$</td>
<td>—</td>
</tr>
<tr>
<td>firm β O</td>
<td>—</td>
<td>$s_{\beta\alpha}^{DOB} = (1 + m)f^4$</td>
</tr>
<tr>
<td>firm β N</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>firm α I</td>
<td>$s_{\alpha\alpha}^{DBI} = mh^4$</td>
<td>—</td>
</tr>
<tr>
<td>firm β I</td>
<td>$s_{\beta I}^{DBB} = mh^4$</td>
<td>—</td>
</tr>
<tr>
<td>firm β O</td>
<td>—</td>
<td>$s_{\beta\alpha}^{DOB} = (1 + m)f^4$</td>
</tr>
<tr>
<td>firm β N</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>firm α O</td>
<td>—</td>
<td>$s_{\alpha\beta}^{DOO} = (1 + m)f^4$</td>
</tr>
<tr>
<td>firm β O</td>
<td>—</td>
<td>$s_{\beta\alpha}^{DOO} = (1 + m)f^4$</td>
</tr>
<tr>
<td>firm β N</td>
<td>—</td>
<td>—</td>
</tr>
</tbody>
</table>

$s_{\rho r}^{Dij}$ denotes the equilibrium contribution made by firm ρ in region r when firm α plays strategy i and firm β plays strategy j, where $i, j \in \{B, I, O, N\}$.

Table 4: Firms’ contributions under decentralization

that firm α lobbies both regions (B) and firm β does not lobby (N). Policy makers maximize:

\[
V_a^{DBN} = \mu(W_a - S_{\alpha b}) + (1 - \mu)S_{\alpha a}, \\
V_b^{DBN} = \mu(W_b + S_{\alpha b}) + (1 - \mu)S_{\alpha b}.
\] (48) (49)

Optimal public goods supplies are:

\[
\tilde{s}_a^{DBN} = \tilde{g}_a^D + 2mh^2, \\
\tilde{s}_b^{DBN} = \tilde{g}_b^D + 2(1 + m)f^2.
\] (50) (51)

Solving

\[
V_a^{DBB} (\tilde{g}_a^{DBB}, \tilde{g}_b^{DBB}, \pi_{\alpha a}, \pi_{\beta a}) = V_a^{DBN} (\tilde{g}_a^{DBN}, \tilde{g}_b^{DBB}, \pi_{\alpha a}, \pi_{\alpha b}), \tag{52}
\]
\[
V_b^{DBB} (\tilde{g}_a^{DBB}, \tilde{g}_b^{DBB}, \pi_{\beta b}, \pi_{\alpha b}, \pi_{\beta a}) = V_b^{DBN} (\tilde{g}_a^{DBB}, \tilde{g}_b^{DBN}, \pi_{ab}), \tag{53}
\]
for $\pi_\beta a$ and $\pi_\beta b$, we get the equilibrium profits shown in Table 3. The reservation utility of the policy maker in region $a$, i.e. the r.h.s. of eq. (52), is defined by assuming that firm $\beta$ does not lobby region $a$ ($g_a = \tilde{g}_a^{DBN}$) while lobbying region $b$ ($g_b = \tilde{g}_b^{DBB}$). By the same token, the reservation utility of the policy maker in region $b$, i.e. the r.h.s. of eq. (53), is defined by assuming that firm $\beta$ does not lobby region $b$ ($g_b = \tilde{g}_b^{DBN}$) while lobbying region $a$ ($g_a = \tilde{g}_a^{DBB}$). The same kind of logic is used below when solving the games $BI$, $BO$ and $BN$, in which one of the firms is lobbying both policy makers.

Finally, equilibrium contributions of the game $BB$, shown in Table 4, are obtained from substitution of optimal public good supplies and profits into the contribution functions, i.e. from $\tilde{s}_\beta^{DBB} = f^2\tilde{g}^{BB} - \tilde{\pi}_\beta^{DBB}$ and $\tilde{s}_\beta^{DBB} = h^2\tilde{g}^{DBB} - \tilde{\pi}_\beta^{DBB}$.

One firm lobbying both regions and one lobbying the home region only ($BI$)

Suppose that firm $\alpha$ chooses $B$ and firm $\beta$ chooses $I$. Policy makers maximize:

$$V_a^{DBI} = \mu (W_a - S_{ab}) + (1 - \mu)S_{aa},$$
$$V_b^{DBI} = \mu (W_b + S_{ab}) + (1 - \mu)(S_{ab} + S_{\beta b}),$$

from which:

$$\tilde{g}_a^{DBI} = \hat{g}^D + 2mh^2,$$
$$\tilde{g}_b^{DBI} = \hat{g}^D + 2f^2 + 2m(h^2 + f^2).$$

Assume now that firm $\alpha$ does not lobby. Policy makers maximize:

$$V_a^{DNI} = \mu W_a,$$
$$V_b^{DNI} = \mu W_b + (1 - \mu)S_{\beta b},$$

and optimal public goods supplies are:

$$\tilde{g}_a^{DNI} = \hat{g}^D,$$
$$\tilde{g}_b^{DNI} = \hat{g}^D + 2mh^2.$$

Solving

$$V_a^{DBI}(\tilde{g}_a^{DBI}, \tilde{g}_b^{DBI}, \pi_{\alpha a}, \pi_{\alpha b}) = V_a^{DNI}(\tilde{g}_a^{DNI}, \tilde{g}_b^{DBI}),$$
$$V_b^{DBI}(\tilde{g}_a^{DBI}, \tilde{g}_b^{DBI}, \pi_{\beta b}, \pi_{\alpha b}) = V_b^{DNI}(\tilde{g}_a^{DBI}, \tilde{s}_\beta^{DNI}, \pi_{\beta b}).$$
for $\pi_{\alpha a}$ and $\pi_{\alpha b}$ we get the equilibrium profits that a firm lobbying both regions makes at home and abroad when the other firm is lobbying only at home (see Table 3). To compute the equilibrium profits of firm $\beta$, assume now that $\alpha$ chooses $B$ while $\beta$ chooses $N$. Policy makers maximize (48) and (49) and the solutions are (50) and (51). Solving the equation

$$V^DBI_b (\tilde{g}^DBI_a, \tilde{g}^DBI_b, \pi_{\beta b}, \pi_{\alpha b}) = V^DBN_b (\tilde{g}^DBN_a, \tilde{g}^DBN_b, \pi_{\alpha b}),$$

for $\pi_{\beta b}$ we get the equilibrium profits that a firm makes at home when lobbying only at home while the other firm is lobbying both regions (see Table 3). Finally, equilibrium contributions for the game $BI$ (see Table 4) are computed by substitutions of net profits and public good supplies into the compensating contribution schedules.

**One firm lobbying both regions and the other lobbying abroad (BO)**

Suppose that firm $\alpha$ chooses $B$ and firm $\beta$ chooses $O$. Policy makers maximize:

$$V^DBO_a = \mu(W_a - S_{\beta a} + S_{\beta a}) + (1 - \mu)(S_{\alpha a} + S_{\beta a}),$$

$$V^DBO_b = \mu(W_b - S_{\beta a} + S_{\beta a}) + (1 - \mu)S_{\beta a},$$

from which:

$$\tilde{g}^DBO_a = \hat{g}^D + 2f^2 + 2m(h^2 + f^2),$$

$$\tilde{g}^DBO_b = \hat{g}^D + 2(1 + m)f^2.$$  

Assume now that firm $\beta$ does not lobby. Hence the game is $BN$, policy makers maximize (48) and (49), and the solutions are (50) and (51). Solving the equation

$$V^DBO_a (\tilde{g}^DBO_a, \tilde{g}^DBO_b, \pi_{\alpha a}, \pi_{\beta a}, \pi_{\beta a}) = V^DBN_a (\tilde{g}^DBN_a, \tilde{g}^DBN_b, \pi_{\alpha a}, \pi_{\beta a}),$$

for $\pi_{\beta a}$ we get the equilibrium profits that a firm makes abroad when lobbying only abroad while the other firm is lobbying both regions (see Table 3). Assume now that firm $\alpha$ does not lobby. Policy makers maximize:

$$V^DNO_a = \mu(W_a + S_{\beta a}) + (1 - \mu)S_{\beta a},$$

$$V^DNO_b = \mu(W_b - S_{\beta a}),$$

from which:

$$\tilde{g}^DNO_a = \hat{g}^D + 2(1 + m)f^2,$$

$$\tilde{g}^DNO_b = \hat{g}^D.$$
Solving
\[ V_a^{DBO} (\hat{g}_a^{DBO}, \hat{g}_b^{DBO}, \pi_{\alpha a}, \pi_{\beta a}, \pi_{ab}) = V_a^{DNO} (\hat{g}_a^{DNO}, \hat{g}_b^{DBO}, \pi_{\beta a}), \]
\[ V_b^{DBO} (\hat{g}_a^{DBO}, \hat{g}_b^{DBO}, \pi_{\beta a}, \pi_{ab}) = V_b^{DNO} (\hat{g}_a^{DNO}, \hat{g}_b^{DNO}, \pi_{\beta a}), \]
for \( \pi_{\alpha a} \) and \( \pi_{ab} \) we get the equilibrium profits that a firm lobbying both regions makes at home and abroad when the other firm is lobbying only abroad (see Table 3). Finally, equilibrium contributions for the game \( BO \) (see Table 4) are computed by substitutions of net profits and public good supplies into the compensating contribution schedules.

One firm lobbying both regions and the other no lobbying (\( BN \))

Suppose that firm \( \alpha \) chooses \( B \) and \( \beta \) chooses \( N \). Policy makers maximize (48) and (49) and the solutions are (50) and (51). Assume now that firm \( \alpha \) is not lobbying. Policy makers maximize \( V_a^{DNN} = \mu W_a \) and \( V_b^{DNN} = \mu W_b \). The solution is the no-lobbying optimal public good supply \( \hat{g}^D \) for both \( g_a \) and \( g_b \). Solving
\[ V_a^{DBN} (\hat{g}_a^{DBN}, \hat{g}_b^{DBN}, \pi_{\alpha a}, \pi_{ab}) = V_a^{DNN} (\hat{g}_a^{DNN}, \hat{g}_b^{DBN}), \]
\[ V_b^{DBN} (\hat{g}_a^{DBN}, \hat{g}_b^{DBN}, \pi_{\beta a}, \pi_{ab}) = V_b^{DNN} (\hat{g}_a^{DNN}, \hat{g}_b^{DBN}), \]
for \( \pi_{\alpha a} \) and \( \pi_{ab} \) we get the equilibrium profits that a firm lobbying both regions makes at home and abroad when the other firm is not lobbying (see Table 3). Profits at home and abroad of the no-lobbying firm \( \beta \) are computed by substituting the optimal public good supplies into the corresponding profit functions. Finally, equilibrium contributions for the game \( BN \) (see Table 4) are computed by simple substitutions of net profits and public good supplies into the compensating contribution schedules of firm \( \alpha \).

Both firms lobbying only the home region (\( II \))

When both firms lobby only the home region, the policy makers’ objective functions are (26) and (27) in the text. By maximizing (26) with respect to \( g_a \) and (27) with respect to \( g_b \), we get the optimal public good supplies \( \hat{g}^{DII} \) in (28). To compute the equilibrium profits, assume that \( \beta \) lobbies at home (\( I \)), while \( \alpha \) does not lobby (\( N \)). Policy makers maximize (58) and (59), and the solutions are (60) and (61). Solving the equation
\[ V_a^{DII} (\hat{g}_a^{DII}, \hat{g}_b^{DII}, \pi_{aa}) = V_a^{DNI} (\hat{g}_a^{DNI}, \hat{g}_b^{DNI}), \]
for the home profits $\pi_{a\alpha}$, and then adding the “abroad” profits, $f^2\tilde{g}^{DII}$, we get total profits $\tilde{\pi}^{DII}$ in (29). Equilibrium contributions, shown in Table 4, are obtained by substituting (home) net profits and public good supply into firm $\alpha$’s compensating contribution schedule to region $a$.

One firm lobbying the home region and the other lobbying abroad ($IO$)

Suppose that firm $\alpha$ chooses $I$ and firm $\beta$ chooses $O$. Policy makers maximize:

$$V_a^{DIO} = \mu(W_a + S_{\beta a}) + (1 - \mu)(S_{\alpha a} + S_{\beta a}),$$

$$V_b^{DIO} = \mu(W_b - S_{\beta a}),$$

and the solutions are:

$$\tilde{g}_a^{DIO} = \hat{g}^D + 2f^2 + 2m(h^2 + f^2),$$

$$\tilde{g}_b^{DIO} = \hat{g}^D.$$  

Assume now that firm $\alpha$ does not lobby while firm $\beta$ lobbies at home, so that the game is $NO$. Policy makers maximize (66) and (67), and the solutions are (68) and (69). Solving the equation

$$V_a^{DIO} (\tilde{g}_a^{DIO}, \tilde{g}_b^{DIO}, \pi_{a\alpha}, \pi_{\beta a}) = V_a^{DNO} (\tilde{g}_a^{DNO}, \tilde{g}_b^{DNO}, \pi_{\beta a})$$

for $\pi_{a\alpha}$ we obtain the equilibrium profits that a firm makes at home when lobbying at home only while the other firm is lobbying away only. Next we assume that firm $\alpha$ lobbies at home while firm $\beta$ does not lobby. Policy makers maximize:

$$V_a^{DIN} = \mu W_a + (1 - \mu)S_{\alpha a},$$

$$V_b^{DIN} = \mu W_b,$$

and optimal public goods supplies are:

$$\tilde{g}_a^{DIN} = \hat{g}^D + 2mh^2,$$

$$\tilde{g}_b^{DIN} = \hat{g}^D.$$  

Solving the equation

$$V_b^{DIO} (\tilde{g}_a^{DIO}, \tilde{g}_b^{DIO}, \pi_{\beta a}) = V_b^{DIN} (\tilde{g}_a^{DIN}, \tilde{g}_b^{DIN})$$

for $\pi_{\beta a}$ we obtain the profits that a firm make abroad when lobbying only abroad whereas the other firm is lobbying only at home. Equilibrium contributions for the game $IO$ (see Table 4) are computed by substituting net profits and public good supplies into the corresponding contribution schedules.
One firm lobbying the home region and the other no lobbying (IN)

Suppose that firm $\beta$ chooses $I$ and firm $\alpha$ chooses $N$. Policy makers maximize (58) and (59) and the solutions are (60) and (61). Assuming that firm $\beta$ is not lobbying, policy makers maximize $V_a^{DNN} = \mu W_a$ and $V_b^{DNN} = \mu W_b$. The solution is the no-lobbying optimal public good supply $\hat{g}^D$ for both $g_a$ and $g_b$. Solving

$$V_b^{DIN}(\hat{g}^{DNI}_a, \hat{g}^{DNI}_b, \pi_{\beta b}) = V_b^{DNN}(\hat{g}^D_a, \hat{g}^D_b),$$

for $\pi_{\beta b}$ we get the equilibrium profits that a firm makes at home when lobbying only at home while the other firm is not lobbying (see Table 3). Firm $\beta$’s profits abroad, and profits at home and abroad of the no-lobbying firm $\alpha$ are computed by substituting the optimal public good supplies into the corresponding profit functions. Equilibrium contributions for the game $IN$ (see Table 4) are computed by substituting net profits and public good supplies into firm $\beta$’s compensating contribution schedule.

Both firms lobbying only abroad (OO)

When both firms lobby only abroad, the policy makers’ objective functions are (30) and (31) in the text. By maximizing (30) with respect to $g_a$ and (31) with respect to $g_b$, we get the optimal public good supplies $\hat{g}^{DOO}$ in (32). Assume now that firm $\alpha$ does not lobby while firm $\beta$ lobbies abroad. The game is $NO$, policy makers maximize (66) and (67), and the solutions are (68) and (69). Solving the equation

$$V_b^{DOO}(\hat{g}^{DOO}_a, \hat{g}^{DOO}_b, \pi_{ab}, \pi_{\beta a}) = V_b^{DNO}(\hat{g}^{DNO}_a, \hat{g}^{DNO}_b, \pi_{\beta a}),$$

for $\pi_{ab}$ we obtain the equilibrium profits that a firm makes abroad when both firms are lobbying abroad only (see Table 3). Adding the home profits, we get total profits $\bar{\pi}^{DOO}$ shown in (33). Equilibrium contributions, shown in Table 4, are computed by substituting (abroad) net profits and public good supply into firm $\alpha$’s compensating contribution schedule to region $b$.

One firm lobbying abroad and the other no lobbying (ON)

Suppose that firm $\beta$ chooses $O$ and firm $\alpha$ chooses $N$. Policy makers maximize (66) and (67) and the solutions are (68) and (69). Assuming that firm $\beta$ is not lobbying, policy makers maximize $V_a^{DNN} = \mu W_a$ and $V_b^{DNN} = \mu W_b$. The solution is the no-lobbying
optimal public good supply $\hat{g}^D$ for both $g_a$ and $g_b$. Solving

$$V_a^{DNO} (\hat{g}_a^{DNO}, \hat{g}_b^{DNO}, \pi_{\beta a}) = V_a^{DNN} (\hat{g}_a^D, \hat{g}_b^D)$$

dom for $\pi_{\beta a}$ we obtain the equilibrium profits that a firm earns abroad when lobbying only abroad while the other firm is no lobbying (see Table 3). Firm $\beta$'s profits at home, and profits at home and abroad of the no-lobbying firm $\alpha$ are computed by substituting the optimal public good supplies into the corresponding profit functions. Equilibrium contributions for the game $ON$ (see Table 4) are computed by substituting net profits and public good supply into firm $\beta$'s contribution schedule.

### A.3 Proof of Proposition 2

From the where-to-lobby game in Table 1, one can see that for $\mu \neq 1$ strategy $N$ is strictly dominated by strategy $I$, since $\Delta \pi^{Ij} - \Delta \pi^{Nj} = mh^4 > 0$ for all $j \in \{B, I, O, N\}$. Hence both firms never play strategy $N$. Since $\Delta \pi^{Bi} - \Delta \pi^{Ii} = (m - m^{-1})f^4$, $\Delta \pi^{Bi} - \Delta \pi^{Oi} = mh^4 - (1 + m^{-1})f^4$ and $\Delta \pi^{Ii} - \Delta \pi^{Oi} = mh^4 - (1 + m)f^4$ for all $i \in \{B, I, O\}$, all the Nash equilibria of the game are in dominant strategies. Next, it is $\Delta \pi^{Bi} = \Delta \pi^{Ii}$ iff $\mu = \frac{1}{2}$, $\Delta \pi^{Bi} = \Delta \pi^{Oi}$ iff $\mu = \mu^{BO}(\delta; c)$ defined in (34), and $\Delta \pi^{Ii} = \Delta \pi^{Oi}$ iff $\mu = \mu^{IO}(\delta; c)$ defined in (35). Plain algebra shows that both $\mu^{BO}(\delta; c)$ and $\mu^{IO}(\delta; c)$ are monotonically increasing in $\delta$, that $\mu^{BO}(1; c) = (3 - \sqrt{2})/2 \approx \frac{3}{4}$, $\mu^{BO}(\delta_{\text{max}}; c) = 1$, $\mu^{IO}(1; c) = 0$, $\mu^{IO}(\delta_{\text{max}}; c) = 1$, and that the three curves $\mu = \frac{1}{2}$, $\mu = \mu^{BO}(\delta; c)$ and $\mu = \mu^{IO}(\delta; c)$ have a unique intersection at $\delta = \tilde{\delta}(c)$ defined in (36). These properties imply that if $\Delta \pi^{Bi} - \Delta \pi^{Ii} \geq 0$ (i.e. $\mu \leq \frac{1}{2}$) and $\Delta \pi^{Bi} - \Delta \pi^{Oi} \geq 0$ (i.e. $\mu \leq \mu^{BO}$), then $B$ is the dominant strategy for each player. Thus $BB$ is the unique Nash equilibrium of the “where-to-lobby” game. If $\Delta \pi^{Bi} - \Delta \pi^{Ii} < 0$ (i.e. $\mu > \frac{1}{2}$) and $\Delta \pi^{Bi} - \Delta \pi^{Oi} \geq 0$ (i.e. $\mu \leq \mu^{IO}$) the unique Nash equilibrium in dominant strategies is $II$. Finally, if $\Delta \pi^{Bi} - \Delta \pi^{Oi} < 0$ (i.e. $\mu > \mu^{BO}$) and $\Delta \pi^{Ii} - \Delta \pi^{Oi} < 0$ (i.e. $\mu > \mu^{IO}$) the Nash equilibrium is $OO$. ■

### A.4 Proof of Corollary 1

From Table 1 it is $\Delta \pi^{BB} - \Delta \pi^{II} = 2(1 + m)h^2 f^2 + (m - m^{-1})f^4$ and $\Delta \pi^{BB} - \Delta \pi^{OO} = mh^4 + 2mh^2 f^2 - (1 + m^{-1})f^4$. For $\mu < \frac{1}{2}$, $\Delta \pi^{BB} > \Delta \pi^{II}$, since $m > m^{-1}$, and $\Delta \pi^{BB} > \Delta \pi^{OO}$, since $2m > 1 + m^{-1}$ and $h \geq f$, implying that when the strategy pair $BB$ is a Nash equilibrium it is also Pareto efficient. For $\mu \geq \frac{1}{2}$, we need to compare the equilibria $II$ and $OO$. From $\Delta \pi^{II} - \Delta \pi^{OO} \geq 0$ it is $\mu \leq \mu^E(\delta; c) \equiv \frac{h^4 - f^4}{h^4 + 2mf^4}$, where $\mu^E(\delta; c)$ is
monotonically increasing in $\delta$, with $\mu^E(1; c) = 0$, $\mu^E(\delta_{\text{max}}; c) = 1$, and such that, see eq. (35), $\mu^E(\delta; c) < \mu^{IO}(\delta; c)$ for all $\delta \in [1, \delta_{\text{max}}]$. Hence, since $\mu^{IO}(\delta; c)$ defines the boundary between the Nash equilibria $II$ and $OO$, the strategy pair $II$ is Pareto efficient if and only if $\mu \leq \mu^E(\delta; c)$, provided that $\mu^E \geq \frac{1}{2}$.}

**A.5 Proof of Proposition 3**

As for the comparison of net profits, using (20), (25), (29) and (33) it is $\tilde{\pi}^C - \tilde{\pi}^{DBB} = (2 + m^{-1})f^4 \geq 0$, $\tilde{\pi}^C - \tilde{\pi}^{DII} = 2(1 + m)h^2f^2 + (2 + m)f^4 \geq 0$ and $\tilde{\pi}^C - \tilde{\pi}^{DOO} = mh^4 + 2mh^2f^2 + f^4 > 0$, which shows that profits are higher under centralization. As for contributions, from (21) and Table 4, it is $\tilde{s}_{aa}^{DBB} + \tilde{s}_{ab}^{DBB} - \tilde{s}^C = (2 + m^{-1})f^4 \geq 0$, $\tilde{s}^C - \tilde{s}_{aa}^{DII} = mf^4 \geq 0$ and $\tilde{s}^C - \tilde{s}_{ab}^{DOO} = mh^4 - f^4$. From the latter one obtains that $\tilde{s}^C \geq \tilde{s}_{ab}^{DOO}$ iff $\mu \leq \mu^S(\delta; c) \equiv \frac{h^4}{h^4 + f^4}$. $\mu^S(\delta; c)$ is monotonically increasing in $\delta$, with $\mu^S(1; c) = \frac{1}{2}$, $\mu^S(\delta_{\text{max}}; c) = 1$, and that $\mu^S(\delta; c) > \mu^{IO}(\delta; c)$, meaning that the region in which $OO$ is a Nash equilibrium is divided into two areas: $\tilde{s}^C < \tilde{s}_{ab}^{DOO}$, for $\mu > \mu^S$; $\tilde{s}^C \geq \tilde{s}_{ab}^{DOO}$, otherwise.

As for public good provision and social welfare, from (18) and (24) it is $\tilde{g}^C = \tilde{g}^{DBB}$, which implies that aggregate social welfare is the same under centralization and under the equilibria $BB$. By the comparison of (18) and (28) it follows that $\tilde{g}^{DII} \leq \tilde{g}^C$; aggregate social welfare is larger in the decentralized equilibrium $II$ since $|\tilde{g}^{DII} - \tilde{g}^C| \leq |\tilde{g}^C - \tilde{g}^C|$, given that $\tilde{g}^C$ maximizes social welfare, which is quadratic in public goods supply. Finally, using (18) and (32) one can see that $\tilde{g}^{DOO} < \tilde{g}^C$; aggregate social welfare is larger in the decentralized equilibrium $OO$ since $0 \leq \tilde{g}^{DOO} - \tilde{g}^C \leq \tilde{g}^C - \tilde{g}^C$. ■

**B Appendix: Lobbying for the market**

**B.1 Proof of Lemma 1**

We first derive the optimal public goods levels by maximizing $W^{J_aK_b} = W^{J_aK_b} + W^{J_bK_b}$, $J, K = \{H, F\}$, as defined in (38)–(41), with respect to $g_a$ and $g_b$. This gives

$$
\begin{align*}
\hat{g}_a^{H_aH_b} &= \hat{g}_b^{H_aH_b} = \hat{g}_a^{H_aF_b} = \hat{g}_b^{H_aF_b} = 3H^2, \quad \hat{g}_a^{F_aF_b} = \hat{g}_b^{F_aF_b} = \hat{g}_a^{H_aF_b} = \hat{g}_b^{H_aF_b} = 3F^2.
\end{align*}
$$

Monopoly profits when supplying the home and the foreign region are $3H^4$ and $3F^4$, respectively. Thus, given $S^H$ and $S^F$, with $0 \leq S^H \leq 3H^4$ and $0 \leq S^F \leq 3F^4$, the
politician’s value functions in the four possible cases are
\[
V^{H_aH_b} = \mu \frac{9H^4}{2} + (1 - \mu)(S^H_\alpha + S^H_\beta) + 2\mu\bar{y},
\]
\[
V^{H_aF_b} = \mu \frac{9(H^4 + F^4)}{4} + (1 - \mu)(S^H_\alpha + S^F_\alpha) + 2\mu\bar{y},
\]
\[
V^{F_aH_b} = \mu \frac{9(H^4 + F^4)}{4} + (1 - \mu)(S^H_\beta + S^F_\beta) + 2\mu\bar{y},
\]
\[
V^{F_aF_b} = \mu \frac{9F^4}{2} + (1 - \mu)(S^F_\alpha + S^F_\beta) + 2\mu\bar{y}.
\]
Consider firm \(\alpha\) (the same argument holds true for firm \(\beta\)). Given \(S^H_\alpha\) and \(S^F_\beta\) the government chooses \(H_aH_b\) iff \(V^{H_aH_b} \geq V^{H_aF_b}, V^{H_aH_b} \geq V^{F_aH_b}, V^{H_aH_b} \geq V^{F_aF_b}\); after some algebra these inequalities reduce to \(S^H_\alpha \geq T^H(S^F_\beta)\) and \(S^F_\alpha \leq T^F(S^H_\beta)\), where
\[
T^H(S^F_\beta) = \max \left\{ -\frac{9\mu(H^4 - F^4)}{4(1 - \mu)} + S^F_\beta, 0 \right\},
\]
\[
T^F(S^H_\beta) = \min \left\{ \frac{9\mu H^4 - F^4}{4(1 - \mu)} + S^H_\beta, 3F^4 \right\}.
\]
Analogously one gets that the government chooses \(F_aF_b\) iff \(S^H_\alpha \leq T^H(S^F_\beta)\) and \(S^F_\alpha \geq T^F(S^H_\beta)\), \(H_aF_b\) iff \(S^H_\alpha > T^H(S^F_\beta)\) and \(S^F_\alpha > T^F(S^H_\beta)\), and \(F_aH_b\) iff \(S^H_\alpha < T^H(S^F_\beta)\) and \(S^F_\alpha < T^F(S^H_\beta)\). The profit function of the firm is then defined as
\[
\Pi(\alpha; S^H_\alpha, S^F_\alpha, S^H_\beta, S^F_\beta) = \begin{cases} 
3H^4 - S^H_\alpha & \text{if } S^H_\alpha \geq T^H(S^F_\beta) \text{ and } S^F_\alpha \leq T^F(S^H_\beta), \\
3F^4 - S^F_\alpha & \text{if } S^H_\alpha \leq T^H(S^F_\beta) \text{ and } S^F_\alpha \geq T^F(S^H_\beta), \\
3(H^4 + F^4) - S^H_\alpha - S^F_\alpha & \text{if } S^H_\alpha > T^H(S^F_\beta) \text{ and } S^F_\alpha > T^F(S^H_\beta), \\
0 & \text{if } S^H_\alpha < T^H(S^F_\beta) \text{ and } S^F_\alpha < T^F(S^H_\beta).
\end{cases}
\]
Profit maximization requires the firm to set \(S^H_\alpha = T^H(S^F_\beta) + \varepsilon\) and \(S^F_\alpha = T^F(S^H_\beta) + \varepsilon\), with \(\varepsilon > 0\) as close as possible to zero. Since the same profit maximizing behavior holds true for firm \(\beta\), the two firms will engage in a Bertrand-type competition in contributions, leading to the unique Nash equilibrium (pure) strategy profile: \(S^F_\rho = 3F^4\) and \(S^H_\rho = \max \{T^H, 0\}\), with \(\hat{T}^H = T^H(S^F_\rho)\) as defined in (43).

\[\text{B.2 Proof of Proposition 4}\]

From \(\hat{T}^H = 0\), with \(\hat{T}^H\) defined in (43), we get
\[
\mu_2(\delta; c) = \frac{4F^4}{F^4 + 3H^4}.
\]
Figure 3: An illustration of Proposition 4

where $H$ and $F$ are defined in (37). Eq. (78) divides the closed set $S = (\mu, \delta) \in [0, 1] \times [1, \delta_{\text{max}}]$ in two regions (see Figure 3): $\hat{S}H > 0$ for $\mu < \mu_2$, and $\hat{S}H = 0$ otherwise. $\mu_2(\delta; c) \in C^2$ is monotonically decreasing in $\delta$, with $\mu_2(1; c) = 1$ and $\mu_2(\delta_{\text{max}}; c) = \frac{4}{29} = .082$.

From
$$\hat{V}h^f - \mu \frac{9H^4}{2} - 2(1 - \mu)\hat{T}^H - 2\mu\bar{y} = 0$$
we derive
$$\mu_1(\delta; c) = \frac{12F^4}{3F^4 + [(h + f)^2 + 2(h^2 + f^2)]^2},$$
(79)
where $h$ and $f$ are defined in (5). One can see that $\mu_1(\delta; c) \in C^2$, $\mu_1(1; c) = \frac{372}{1207} \approx .767$, $\mu_1(\delta_{\text{max}}; c) = \frac{4}{29}$, and that $\mu_1(\delta; c)$ and $\mu_2(\delta; c)$ have a unique intersection at $\delta = \delta_1(c) \equiv \frac{5 + 17c}{22c}$ for $\delta \in [1, \delta_{\text{max}})$, for which $\mu = \frac{34084}{860299} \approx .425$. Thus, for $\delta \in [1, \delta_1]$, the locus defined by eq. (79) separates the subset of $S$ in which $\hat{S}H > 0$ into two subsets such that: $\hat{V}h^f > \hat{V}H$ for $\mu > \mu_1$ and $\hat{V}h^f \leq \hat{V}H$ otherwise, proving the first part of the proposition.

For $\delta \in (\delta_1, \delta_{\text{max}})$, if $\mu \leq \mu_1$ then $\hat{V}h^f < \hat{V}H$ since $\mu_1 > \mu_2$. If $\mu > \mu_2$ then $\hat{S}H = 0$. Define $\Psi(\mu, \delta; c) = \hat{V}h^f - \mu \frac{9H^4}{4}$, $\Psi \in C^2$. Since $\Psi(\mu, 1; c) > 0$, $\Psi(\mu, \delta_{\text{max}}; c) = 0$ and there is a unique root at $\delta = \delta_1$ for $\delta \in [1, \delta_{\text{max}})$, then $\Psi < 0$ for all $\delta \in (\delta_1, \delta_{\text{max}})$, proving that only the home firm enters the market without paying any contribution.

B.3 Proof of Lemma 2

The proof is conducted in three steps. The first step proves the first part of the lemma and derives the politicians’ value functions when one firm only is allowed into each regional
market. The second and third steps derive the politicians’ value functions in the remaining cases.

Step 1. Both regional governments admit one firm only. By deriving the optimal public goods levels through the maximization in \(g_a\) and \(g_b\), respectively, of \(W^{J_aK_b}_a\) and \(W^{J_bK_b}_b\), \(J, K = \{H, F\}\), as defined in (38)–(41), and given \(S^H\) and \(S^F\), \(\rho = \{\alpha, \beta\}\), with 
\[0 \leq S^H \leq 3H^4 \text{ and } 0 \leq S^F \leq 3F^4,\]
we obtain region \(a\) politician’s value functions in the four possible cases

\[
\begin{align*}
V_{aH_aH_b} &= \mu \frac{9H^4}{4} + (1 - \mu)S^H_a + \mu \bar{y}, \\
V_{aH_aF_b} &= \mu \left( \frac{9H^4}{4} + F^4 \right) - \mu S^F_a + (1 - \mu)S^H_a + \mu \bar{y}, \\
V_{aF_aH_b} &= \mu \frac{F^4}{4} + S^F_a + \mu \bar{y}, \\
V_{aF_aF_b} &= \mu \frac{5F^4}{4} - \mu S^F_a + S^F_a + \mu \bar{y}.
\end{align*}
\]

Given \(S^F\), it is a (weakly) dominant strategy for region \(a\) (and symmetrically the same holds true for region \(b\)) to choose the home firm iff \(V_{aH_aH_b} \geq V_{aF_aH_b}\) and \(V_{aH_aF_b} \geq V_{aF_aF_b}\).

These two inequalities are satisfied for the same condition, i.e.

\[
S^H \geq \max \left\{ -\frac{\mu(9H^4 - F^4)}{4(1 - \mu)} + \frac{S^F}{1 - \mu}, 0 \right\}.
\]

Bertrand competition in contributions implies that \(\hat{S}^F = F^4\) and thus it is

\[
\hat{S}^H = \max \left\{ -\frac{\mu(9H^4 - F^4)}{4(1 - \mu)} + \frac{F^4}{1 - \mu}, 0 \right\},
\]

proving equation (45) in the lemma. One needs to check that \(\hat{S}^H \leq 3H^4\). For \(\mu \neq 1\), this requires \(\mu (\delta) \leq \mu^T (\delta) \equiv 4\frac{3H^4 - F^4}{3H^4 + F^4}\). By recalling (37), it is immediate to show that it is \(\mu^T (1) = 2\) and \(\frac{\partial \mu^T (\delta)}{\partial \delta} > 0\). Hence \(\hat{S}^H\) is always smaller than the profits realized in the home region.

Thus, when only one firm is allowed to enter a regional market, the home firm wins the contest for the market and the politician’s value function (in each region) is \(V_{aH_aH_b}\) in Table 2.

Step 2. Both regional governments allow both firms in their market. This case has been examined in section 3, where policy without lobbying has been described. Using the optimal public good provision given in (13) and substituting it into (10), region \(a\) politician’s value function when both firms are allowed to enter their market is \(\hat{V}_{ah_{aH_aH_b}},\)
shown in Table 2.

Step 3. One regional government admits one firm only and the other one admits both.

Suppose, without loss of generality, that region $a$ lets both firms in, while region $b$ allows only one of them to enter its regional market. In the case in which firm $\beta$ gets region $b$’s market, social welfare becomes

$$W_{aHb} = \mu \frac{(h + f)^2 g_a + 2h^2 g_a - g_a^2}{2} - \frac{g_a^2}{4},$$

$$W_{bHb} = \mu \frac{3H^2 g_b + 2f^2 g_a - g_b^2}{2} - \frac{g_b^2}{4} + (1 - \mu) S_H^\beta.$$

On the other hand, in the case in which firm $\alpha$ gets region $b$’s market, the corresponding social welfare functions are

$$W_{aFb} = \mu \frac{(h + f)^2 g_a + 2h^2 g_a + 2F^2 g_b - 2S_F^\alpha}{2} - \frac{g_a^2}{4},$$

$$W_{bFb} = \mu \frac{F^2 g_b + 2f^2 g_a + 2S_F^\alpha}{2} - \frac{g_b^2}{4} + (1 - \mu) S_F^\alpha.$$

By maximizing each regional social welfare function in the local public good supply, one obtains the corresponding politicians’ value functions

$$\hat{V}_{aHb} = \mu \frac{[h + f]^2 + 2h^2]^2}{4},$$

$$\hat{V}_{bHb} = \mu \frac{9H^4 + 4f^2[(h + f)^2 + 2h^2]}{4} + (1 - \mu) S_H^\beta,$$

$$\hat{V}_{aFb} = \mu \frac{[h + f]^2 + 2h^2]^2 + 4F^4}{4} - \mu S_F^\alpha,$$

$$\hat{V}_{bFb} = \mu \frac{F^4 + 4f^2[(h + f)^2 + 2h^2]}{4} + S_F^\alpha.$$

Region $b$ allows firm $\beta$ in iff $V_{bHb} \geq V_{bFb}$ that requires

$$S_H^\beta (S_F^\alpha) \geq \max \left\{ -\frac{\mu(9H^4 - F^4)}{4(1 - \mu)} + \frac{S_F^\alpha}{1 - \mu}, 0 \right\}.$$

By Bertrand competition, $\hat{S}_F^\alpha = F^4$ and

$$\hat{S}_H^\beta = \max \left\{ -\frac{\mu(9H^4 - F^4)}{4(1 - \mu)} + \frac{F^4}{1 - \mu}, 0 \right\},$$

where $\hat{S}_H^\beta > 0$ for $\mu < \frac{4F^4}{9H^4 - F^4}$. Moreover, by the same argument in Step 2, $\hat{S}_H^\beta \leq 3H^4$.

Thus, substituting $\hat{S}_H^\beta$ into $V_{bHb}$ the region $b$ politician’s value function is $\hat{V}_{bHb}$ in Table 2. The same applies symmetrically when region $b$ let both firms in, while region $a$ allows only one of them to enter its regional market. 

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B.4 Proof of Proposition 6

Considering the game in Table 2, it is a (weakly) dominant strategy for both regions to admit one firm only iff \( \hat{V}_a^{hf_aH_b} \geq \hat{V}_a^{hf} \) and \( \hat{V}_a^{H_aH_b} \geq \hat{V}_a^{hf_aH_b} \). These inequalities imply (i) \( \mu \leq \mu_6(\delta; c) \equiv \frac{4F^4}{(h+f)^2+2h^2} \) for \( \mu < \mu_5(\delta; c) \), where \( \mu_5(\delta; c) \) is defined in (46), and (ii) \( \mu \{9H^4 - [(h+f)^2 + 2h^2]^2 \} \geq 0 \) for \( \mu \geq \mu_5(\delta; c) \). Condition (ii) is always satisfied for all \( \delta \in [1, \delta_{\text{max}}] \) and \( c \in (0, 1) \), since \( \mu_6(\delta; c) \geq \mu_5(\delta; c) \). The latter inequality follows by a continuity argument from \( \mu_6(1; c) = \frac{36}{55} > \mu_5(1; c) = \frac{1}{2} \), \( \mu_6(\delta_{\text{max}}; c) = \mu_5(\delta_{\text{max}}; c) = \frac{4}{143} \), and \( \mu_6(\delta; c) \neq \mu_5(\delta; c) \) for all \( \delta \in [1, \delta_{\text{max}}] \). Hence one-firm in each region is the unique Nash equilibrium also for \( \mu < \mu_5(\delta; c) \). In both cases, by Lemma 2, it is the home firm to gain access to the market. \( \Box \)
References


