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Is the Friedman Rule Stabilizing? Some Unpleasant Results in a Heterogeneous Expectations Framework*

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Abstract

The recent economic crisis gave proof of the fact that the Taylor rule is no more that good instrument as it was thought to be just ten years ago; this might be due to the fact that agents acting in the economy hold Heterogeneous Expectations (HE).

In a recent paper Anufriev et al. (2013) suggest that a way to force stability on the economic system is to adopt a more aggressive Taylor rule.

In the present paper a standard NK-DSGE is considered in order to investigate whether a Friedman k-percent monetary policy rule may be a valid instrument to counteract the

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instability created by the presence of HE in a framework à la Brock and Hommes (1997). The model here presented suggests that when such a money supply rule is adopted by the Central Bank, stability strongly depends on the intensity of choice, which represents the ability of the agents to switch toward the best available predictor.

JEL Classification: E37, E52, E58.

Keywords: Heterogeneous Expectations, Friedman Monetary Policy Rule, Macroeconomic Stability.

Contents

1 Introduction

The rational, representative agent approach is still the most widely used in all contemporary macroeconomic literature. The possibility of admitting and considering the existence of heterogeneity in expectations have been less investigated,even if there are some hot spots[1] where the production of works that take into account the presence of heterogeneous agents has been abundant.

The aim of this paper is to contribute on this relatively new theory of expectations by investigating whether the introduction of heterogeneous agents can cause deviations from the long run inflation and output gap steady states. In order to accomplish my objectives, I use as starting points of my research the works by Evans and Honkapohja (2003) and Anufriev et al. (2013).

During the years following the works by Taylor (1993) and Clarida et al. (1999) the Friedman monetary policy rule has been put aside in favour of the more appealing Taylor rule that is based on interest rate setting. This approach seemed to provide a good way to stabilize macroeconomic variables to their steady state values or long run equilibria.

Nevertheless the recent financial crisis gave proof of the fact that the interest rate setting is no more that good instrument as it was thought to be just ten years ago; this pushed a number of economists toward the attempt of understanding how structural disequilibrium, such as the one we are experimenting

[1] Just to mention two of them: the CeNDEF (Center for Nonlinear Dynamics in Economics and Finance) at the University van Amsterdam with its major exponent Cars Hommes and the Stanford University with the group of economists following Mordecai Kurz's guidelines.

these days all over the world, could have emerged; the presence of heterogeneous agents, is one possible answer. Analysing the work by Anufriev et al. (2013) we can clearly see that HE are the perfect habitat for instability to develop and emerge, if their existence is not taken into account by the central bankers while deciding on the monetary policy to be adopted.

For that reason my work tries to investigate if a different monetary policy - such as the *Friedman k-percent rule* - may be a valid instrument to contrast the instability due to heterogeneity in expectations.

Thus I here present, by means of dynamic numerical simulations, what are the effects on inflation and output gap if the adopted policy rule is a Friedman k-percent money supply rule of the form (in logs):

$$M_t = M + kt + \omega_t,$$

where M_t is money supply, M is a constant, t is simply the time period and k is the percentage increase (decrease) of money; ω_t is a random, exogenous noise term which will be assumed to be a white noise for simplicity.

As already proved by Evans and Honkapohja (2003)[2], *"the Friedman k-percent rule leads to determinacy of equilibria"* and *"the Rational Expectation Equilibrium (REE) is stable under learning"*. In the following I will proceed verifying if the validity of these statements still hold in an heterogeneous expectation framework.

The paper is organized as it follows. Section 2 describes how the Friedman k-percent money supply rule is included in the New-Keynesian framework;

[2]They designed a model in which although learning was possible, it was still based on a representative rational agent.

section 3 introduces heterogeneity of agents by means of the Adaptive Belief System à la Brock and Hommes (1997) which was used also in Anufriev et al. (2013); section 4 contains the investigation by means of numerical simulations[3] on the stability of the macroeconomic variables and on their dynamics; finally, section 5 concludes.

2 The Friedman Rule in the New-Keynesian Framework

The structural form of the workhorse NK-DSGE framework consists of two behavioural (dynamic) equations for the private sector: namely the IS, derived from the Euler equation for consumer optimization,

$$x_t = E_t x_{t+1} - \frac{1}{\sigma}\left[i_t - E_t \pi_{t+1}\right] + g_t \qquad IS \tag{1}$$

and the New-Keynesian Phillips Curve (NKPC), which emerges from the price setting of monopolistic competitive firms

$$\pi_t = \beta E_t \pi_{t+1} + \lambda x_t + u_t \qquad NKPC \tag{2}$$

where, x_t represents output gap, i_t is the nominal interest rate, π_t stands for inflation and E_t is the expected value operator; moreover g_t and u_t are assumed to be observable AR(1) random shocks of the form

[3]MATLAB and E&F Chaos are the softwares used for this purpose and that allowed me to deal with the nonlinear dynamics emerged when adding heterogeneous agents.

$$\begin{pmatrix} g_t \\ u_t \end{pmatrix} = V \begin{pmatrix} g_{t-1} \\ u_{t-1} \end{pmatrix} + \begin{pmatrix} \tilde{g}_t \\ \tilde{u}_t \end{pmatrix} \qquad \text{with} \qquad V = \begin{pmatrix} \mu & 0 \\ 0 & \rho \end{pmatrix},$$

where μ and ρ are inside the unit circle and $\tilde{g}_t \sim iid(0, \sigma_g)$, $\tilde{u}_t \sim iid(0, \sigma_u)$ are independent white noises.

To close the model, it is customary to introduce the Central Bank, whose role has been largely discussed by generations of macro and monetary economists. During the last decade two competing views have provided different answers to the vexata questio: *"What the Central Bank should do?"* The American view is that the Federal Reserve (FED) should pursue two objectives, namely low inflation and low output gap[4] while in Europe it is thought that the European Central Bank (ECB) should react only to inflation, keeping it under (but close to) a reasonably low level which has been set at 2%.

Analytically, if we look at the Taylor rule[5]

$$i_t = r_t^* + \phi_\pi(\pi_t - \pi^*) + \phi_y(y_t - y_t^*),$$

we can specify the different prescriptions looking at the parameter ϕ_y; $\phi_y \neq 0$ in the US while in Europe the ECB implicitely sets $\phi_y = 0$. Technically this means that the FED reacts to variations in inflation and output by means of interest rate adjustments; on the other hand in Europe the interest rate will not be adjusted as long as inflation is under the 2% threshold, even if output is far from its target level y_t^*.

The instrument of the central banks is the interest rate i_t (both for the FED and for the ECB): however the interest rate is a sort of an indirect instrument for regulating the money supply and thereafter inflation; henceforth it can be

[4]This second objective is a sort of "road to low unemployment".

[5]Starred letters represent target levels of the corresponding variable.

noticed that the ECB objective of low inflation can theoretically be reached also by a policy that directly affects money supply[6]. This money supply rule in logs is

$$M_t^S = M + kt + \omega_t, \tag{3}$$

meaning that money supply is increased in each period[7] by a $k\%$ rate, and that can deviate also due to a white noise shock ω_t.

Now take the money supply rule in Equation 3 and money demand[8] $M_t^D = \theta x_t - \frac{1}{\eta} i_t + \varepsilon_t$, which is a function increasing in output gap and decreasing in the interest rate; it is then possible to write the equation of the LM curve - *the money market equilibrium* - $\frac{M_t^S}{P_t} = M_t^D$ as

$$i_t = \eta \left[\theta x_t + p_t - kt - M + (\varepsilon_t - \omega_t)\right],$$

which can be plugged into the IS curve (1) to derive what I will call the "*Friedman IS*"[9]

$$x_t = E_t x_{t+1} - \frac{1}{\sigma} \left\{\eta \left[\theta x_t + p_t - kt - M + (\varepsilon_t - \omega_t)\right] - E_t \pi_{t+1}\right\} + g_t \tag{4}$$

[6]The Friedman's k-percent rule is a policy of that type: it strikes money supply without any intermediate instrument and hence, according to Friedman statement *"Inflation is always and everywhere a monetary phenomenon in the sense that it is and can be produced only by a more rapid increase in the quantity of money than in output..."* this kind of policy may (theoretically) be a good method to counteract inflation.

[7]As an example, year or a quarter can be taken as time intervals.

[8]This money demand form may be justified at the micro-level by means of a (MIU) model where M enters the utility function capturing the liquidity services provided by money; otherwise it can be derived by means of a cash-in-advance model, in which agents are restricted to carrying out a volume of transactions equal to or less than their money holdings. See Walsh (2010) chapters 2 and 8.

[9]This "*Friedman IS*" basically is an expectations augmented AD curve.

The model to be analysed is then closed by means of the definition of inflation

$$p_t = \pi_t + p_{t-1}. \tag{5}$$

Therefore the model can be summarized as a system of three equations:

i) Friedman IS (4)

ii) New-Keynesian Phillips-Curve (2)

iii) Definition of inflation (5)

3 Heterogeneous Beliefs

Before analysing the model numerically, we have to specify the expectation formation mechanism. In this section I will present the Adaptive Belief System by Brock and Hommes (1997) that allows to model heterogeneous expectations across agents.

Assume there are H forecasting strategies, to predict both output gap and inflation, denoted by the subscript $h = 1, 2, ..., H$ so that $\hat{E}_{h,t}x_{t+1}$ and $\hat{E}_{h,t}\pi_{t+1}$ are today's predictions of future output and future inflation for an agent who has chosen the forecasting strategy h. I will denote with $n^x_{h,t}$ ($n^\pi_{h,t}$) the fraction of agents using output gap (inflation) prediction rule h. Assuming that individual expectations can be linearly aggregated, as generally done in the literature[10], the *Friedman IS* and the NKPC can now be written as:

$$x_t = \sum_{h=1}^{H} n^x_{h,t}\hat{E}_{h,t}x_{t+1} - \frac{1}{\sigma}\{\eta\left[\theta x_t + p_t - kt - M + (\varepsilon_t - \omega_t)\right] - \sum_{h,t}^{H} n^\pi_{h,t}\hat{E}_{h,t}\pi_{t+1}\} + g_t \tag{6}$$

[10]This assumption can be considered a first-order approximation of the true non-linear system.

$$\pi_t = \beta \sum_{h=1}^{H} n^{\pi}_{h,t} \hat{E}_{h,t} \pi_{t+1} + \lambda x_t + u_t. \qquad (7)$$

The beliefs on output gap and inflation are updated over time and so the fractions of agents using a particular rule h are updated too, according to an evolutionary fitness measure that in every period evaluates the past performance of the rule. The *fitness measures*, for output and for inflation respectively, are publicly available but subject to noises and are expressed in utility terms as

$$\tilde{U}_{h,t} = U_{h,t} + \xi_{h,i,t} \qquad \text{for output gap}$$

and

$$\tilde{V}_{h,t} = V_{h,t} + \xi_{h,i,t} \qquad \text{for inflation}$$

where $U_{h,t}$ and $V_{h,t}$ are the deterministic parts of the fitness measures (we assume they can be found in a freely available newspaper) and $\xi_{h,i,t}$ is the stochastic part of the fitness measures, more precisely it collects independent and identically distributed noises, idiosyncratic across each time t, each strategy and each agent i.

A theorem on discrete choice by Mansky and McFadden (1981) states that, assuming that the noises $\xi_{h,i,t}$ are drawn from a double exponential distribution, as the number of agents i goes to infinity the probability that an agent picks the strategy h is given by the discrete choice fractions

$$n^{x}_{h,t} = \frac{e^{\psi U_{h,t-1}}}{\sum_{h=1}^{H} e^{\psi U_{h,t-1}}}, \qquad \text{for output gap rules} \qquad (8)$$

and

$$n^{\pi}_{h,t} = \frac{e^{\psi V_{h,t-1}}}{\sum_{h=1}^{H} e^{\psi V_{h,t-1}}}, \qquad \text{for inflation rules.} \qquad (9)$$

The parameter ψ is related to the stochastic part of the fitness measure and has its support in $[0, +\infty)$. This parameter in the literature is called intensity of choice and is inversely related to the standard deviation of ξ. The case $\sigma_\xi = \infty$ implies $\psi = 0$ meaning that differences across the h fitness measures $\tilde{U}_h$ and $\tilde{V}_h$ can not be observed, agents do not switch between stategies and the fractions are constant and equal to $1/H$. The opposite case $\sigma_\xi = 0$ is the neoclassical case and implies $\psi = \infty$; there is no noise, the fitness measure for each strategy is perfectly observable and in each period all agents pick the forecasting rule with the highest performance in the previous period. The deterministic part of the fitness measure enters directly in the equations for the determination of the two fractions and affects the fraction size in a positive way, meaning that the higher the previous period utilities[11] $U_{h,t-1}$ and $V_{h,t-1}$, the greater is the fraction of agents that use rule h in period t.
As in Anufriev et al. (2013) a natural and easily tractable performance measure is past squared forecast error, but in the present framework it is important to remember that these should be calculated both for output gap and for inflation predictions. Therefore

$$U_{h,t-1} = -(x_{t-1} - \hat{E}_{t-2}x_{t-1})^2 - C_h \qquad \text{for output gap} \tag{10}$$

and

$$V_{h,t-1} = -(\pi_{t-1} - \hat{E}_{t-2}\pi_{t-1})^2 - C_h \qquad \text{for inflation} \tag{11}$$

where in both cases C_h is the information gathering cost of strategy h. Agents have the possibility to choose among different predictors both for the output

[11] In the literature the past fitness measure is used by assumption, because today's outcome are not yet realized when agents take their decisions.

gap and for the inflation. They broadly know the fundamental steady state but their rationality is bounded; hence, forecasting the fundamental steady states of output gap and inflation requires some efforts or some information gathering costs $C_h \geq 0$[12].

In all the simulations that follow I will focus on the simplest case where only H=3 constant prediction rules are available to the agents both for inflation and output gap forecasts, namely fundamental, optimistic and pessimistic predictors:

$$\hat{E}_{1,t}x_{t+1} = 0, \qquad \hat{E}_{2,t}x_{t+1} = d, \qquad \hat{E}_{3,t}x_{t+1} = -d$$

$$\hat{E}_{1,t}\pi_{t+1} = 0, \qquad \hat{E}_{2,t}\pi_{t+1} = b, \qquad \hat{E}_{3,t}\pi_{t+1} = -b.$$

4 Evolutionary Dynamics with Few Constant Beliefs

It is useful to find the normal form of the 3D system and write the model in deviations from the fundamental steady state, as shown in Appendix.

Then in its final form, the model to be analysed is

[12]It is here necessary to state clearly that a fundamentalist agent is different from the rational one (except in one particular case): as a matter of fact the former does not know that in the economy some other non rational, heterogeneous agents are present and so, he predicts the equilibrium thinking that all the agents are as he is; differently the *"true"* rational agents, must be aware of the presence of all the other non rational agents and also of their biased predictions, taking them into account while predicting the equilibrium.

$$\begin{pmatrix} x_t \\ \pi_t \\ p_t \end{pmatrix} = r \begin{pmatrix} 1 & \frac{(1-\beta\eta)}{\sigma} & 0 \\ \lambda & \frac{\lambda}{\sigma} + \beta(1+\frac{\eta\theta}{\sigma}) & 0 \\ \lambda & \frac{\lambda}{\sigma} + \beta(1+\frac{\eta\theta}{\sigma}) & 0 \end{pmatrix} \begin{pmatrix} \hat{E}_t x_{t+1} \\ \hat{E}_t \pi_{t+1} \\ \hat{E}_t p_{t+1} \end{pmatrix} + \\ + r \begin{pmatrix} 0 & 0 & -\frac{\eta}{\sigma} \\ 0 & 0 & -\frac{\eta\lambda}{\sigma} \\ 0 & 0 & (1+\frac{\eta\theta}{\sigma}) \end{pmatrix} \begin{pmatrix} x_{t-1} \\ \pi_{t-1} \\ p_{t-1} \end{pmatrix} + r \begin{pmatrix} 1 & -\frac{\eta}{\sigma} \\ \lambda & 1+\frac{\eta\theta}{\sigma} \\ \lambda & 1+\frac{\eta\theta}{\sigma} \end{pmatrix} \begin{pmatrix} \tilde{g}_t \\ u_t \end{pmatrix} \tag{12}$$

with $r = \dfrac{1}{1+\frac{\eta}{\sigma}(\theta+\lambda)}$ and where the expected value operator with an hat (i.e. $\hat{E}_t$) denotes a convex combination of heterogeneous expectations, as defined in the previous section. There is a hidden non-linearity in the system, due to the fact that the weights of the convex combination of heuristics that yield the average expectation; i.e. the fractions of agents that choose the different heuristics, are itself non-linear functions of the lagged, current and expected inflation and output gap.

Looking at system (12) it is evident that it would be difficult to obtain general theoretical results on stability by analytical means. Therefore I examine the dynamics numerically. In the numerical analysis I use two different sets of calibrated parameter values, respectively suggested by Woodford (1996) and by McCallum and Nelson (1996). The two parameter configurations are:

- Woodford: $\sigma = 1, \eta = 0.053, \theta = 1, \lambda = 0.3, \beta = 0.95$ and hence $r = 0.93554$;
- McCallum and Nelson: $\sigma = 6.0976, \eta = 0.09, \theta = 0.93, \lambda = 0.3$ and finally $\beta = 0.99$ implying that $r = 0.98217$.

Since both configurations lead to very similar conclusions, for the sake of brevity, in what follows I will present only the results obtained using Woodford's calibrated parameters.

4.1 Stability Analysis

In this section I analyse how the dynamical behaviour of the system changes when a parameter is changed; in particular I will investigate how the system changes as ψ varies since the intensity of choice parameter determines the non linearity of the system. In order to investigate the relation between the value of a parameter and the stability of a system I present the bifurcation dyagram for π_t when the parameter that varies is the intensity of choice ψ. Another useful tool, strictly related to the bifurcation diagram, that allows to measure the average exponential rate of divergence of nearby initial states of a dynamical system is the (largest) *Lyapunov exponent*: when the Lyapunov exponent is positive we have sensitive dependence on initial condition and hereby a chaotic dynamics while, when the Lyapunov exponent is negative we have stable dynamics.

Figures 1 and 2 show that the dynamics may be particularly complicated for some values of the intensity of choice. The first figure suggests the presence of two different behaviours: for $\psi \in [0, 0.585)$ (i.e. when the intensity of choice is relatively low) we have a single line, meaning that the system is stable and convergence toward the fundamental steady state occurs while for $\psi \in (0.585, +\infty)$ more complicated dynamics emerge. The Lyapunov exponent plot helps to understand the properties of this complicated dynamics; indeed if we look at Figure 2 we can classify three (rather than two) different

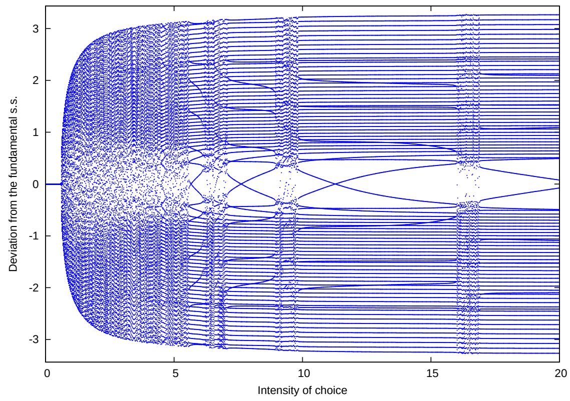

Figure 1: Bifurcation Diagram for inflation. For relatively low values of the intensity of choice the system converges to the steady state while for higher values the dynamics become much more complex.

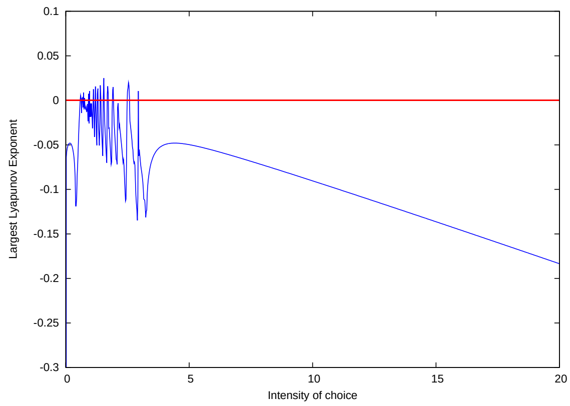

Figure 2: Largest Lyapunov Exponent plot. As long as the parameter ψ is relatively low or relatively high, we have negative Lyapunov exponent and a stable map; at $\psi = 0.585$ an Hopf bifurcation occurs and for $\psi \in (0.585, 2.979)$ there is a quasi-periodic dynamics.

areas: two of them suggest that stability is reached (the areas where the Lyapunov exponent is significantly below the zero level) while one of them (the area in which the Lyapunov exponent fluctuates around the zero level) suggests the emergence of a quasi-periodic (or quasi-chaotic) dynamics, being halfway between stability and chaos.

Two straightforward questions arise:

i) *why the dynamics has this sudden and relevant behavioural change for* $\psi = 0.585$*?*;

ii) *why the bifurcation diagram suggests an unstable behaviour for relatively high values of* ψ *while the Lyapunov exponent suggests that the system is stable in the same region?*

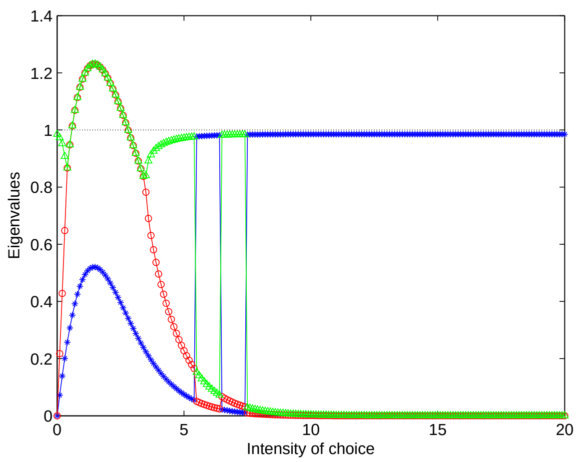

Figure 3: A representation of the three eigenvalues (λ_1 in blue, λ_2 in red and λ_3 in green) as a function of the intensity of choice parameter.

To answer the first question it is useful to evaluate and plot (Figure 3) the eigenvalues $\lambda_1, \lambda_2, \lambda_3$ of the (3X3) Jacobian matrix evaluated at the steady state. What can be seen is that for $\psi < 0.585$, λ_1, λ_2 and λ_3 are in absolute value lower than 1 meaning that the system is stable; when $\psi \approx 0.585$, $|\lambda_1| < 1$ but λ_2 and λ_3 become a pair of coniugate complex eigenvalues which cross the unit circle, causing the system to be unstable. Nonlinear dynamics theory suggests that this behaviour is typical when a Hopf[13] bifurcation is going to emerge. We can conclude that for $\psi \approx 0.585$ a Hopf bifurcation occurs, changing subtantially the dynamics of the system. Differently, for $\psi \approx 2.979$ the two complex eigenvalues get back to levels which are inside the unit circle and therefore the system becomes stable again, as can be seen in Figure 3.

As to the second question, it has to be said that a simplistic interpretation of the bifurcation diagram may lead to wrong conclusions; as a matter of fact, in the area right to $\psi = 2.979$ there is stability, as confirmed by the eigenvalues and by the Lyapunov exponent plots, but it is local, meaning that for some sets of initial conditions or even if a small shock is present, the system may enter a quasi-periodic dynamics never converging to the steady state (as shown in the bifurcation diagram in Figure 1).

These results are qualitatively robust for different sets of initial conditions and therefore we can conclude that for relatively low values of the intensity of choice ψ the system is stable and a Friedman k-percent money supply rule leads to stability; otherwise, if agents in the economy have a relatively intermediate or high switching behaviour[14], the Friedman k-percent money supply rule may not be a good instrument for counteracting inflation, and

[13]More precisely a Neimar-Sacker bifurcation in this discrete timing case.

[14]It is exactly this switching behaviour the endogenous force that causes the instability, but more about that can be found in the following section.

might never reach the objective of keeping it under a reasonable low level.

The model under analysis is a three dimensional system, therefore in order to better understand the complete dynamics it is useful to produce a *phase plot*.

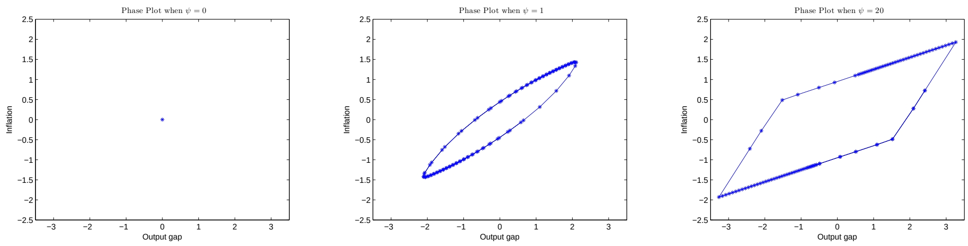

Figure 4: Phase plots for $\psi = 0$, $\psi = 1$ and $\psi = 20$ respectively. Output gap is plotted along the abscissa and inflation along the ordinate. It has to be noted that as the intensity of choice increases, the attractor changes, converging (possibly) to a limit cycle.

Figure 4, which consists of three different panels, each representing the phase plot of inflation and output gap for three different values of the intensity of choice, confirms all the results already stated above. As a matter of fact, when the intensity of choice is low ($\psi = 0$), both output and inflation converge to their steady states, the attractor is a single point in $(0, 0)$ (as shown in the top-left panel) meaning that there is a zero deviation from the two fundamental equilibrium (x^*, π^*). Anyway, when the intensity of choice is increased to an intermediate value and overcomes the 0.585 critical point, the dynamics becomes quasi-chaotic and the attractor consists of an invariant circle, as shown in the top-right panel. Increasing ψ even further i.e. allowing agents to be even more reactive in their switching behaviour toward the most performing predictor, we see that the results do not change qualitatively and the attractor is again an invariant circle; the unique differences are that now the attractor is more sharp and that both the variables are more volatile, but the result is that the long run dynamics are still quasi-periodic.

In order to have more complete results I have also investigated what happens to the dynamics of the system when the agents are assumed to be endowed with an infinite intensity of choice, meaning that we get back to the the neoclassical case in which the agents are always perfectly aware about which one is the predictor that outperforms the other two. In such a situation, in every period, all the agents immediately switch toward the predictor that in the previous period had shown the lowest forecast error.

The results, are depicted in Figure 5 and Figure 6 and show that the sys-

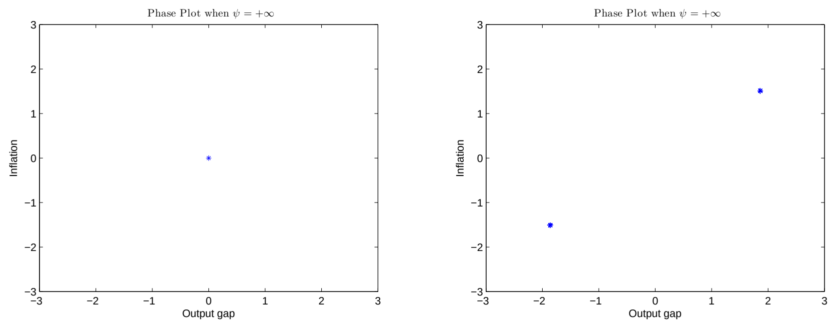

Figure 5: Phase plots for the case $\psi = \infty$ in which it is shown convergence to the fundamental steady state (left panel) against convergence to a 2-cycle (right panel) depending on the set of initial condition.

tem, depending on whether the initial conditions (x_0, π_0) are nearby or far from the fundamental steady state, may have two different dynamics. The

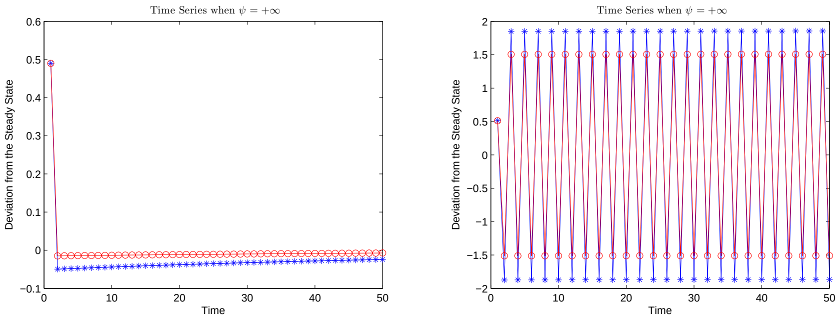

Figure 6: Inflation (red) and output (blue) time series for the case $\psi = \infty$ confirming the results presented in the two above phase plots.

first one is represented in the left panels and show that the system converges to the fundamental steady state (x^*, π^*) for a set of initial conditions $(x_0, \pi_0) \in [-0.5, 0.5]$; the second case is represented in the right panel, it shows a convergence to a 2-cycle $\{(x_{high}, \pi_{high}), (x_{low}, \pi_{low})\}$ where periods of high inflation and overproduction alternate with periods of low inflation and underproduction.

4.2 Inflation and Fractions Dynamics

It is interesting to better understand which are the economic motives of the dynamic behaviours presented above; this could be investigated by looking at the time series of inflation π and of the fraction of agents using the fundamental, the optimistic and the pessimistic rules $n_1^\pi, n_2^\pi, n_3^\pi$ respectively. In order to understand what is the economic interpretation of such behaviours it is useful to recall the three equations of the model and then to plot time series for the previously analysed values of ψ.

The deterministic skeleton of the model is:

$$x_t = r\hat{E}_t x_{t+1} + r\frac{(1-\beta\eta)}{\sigma}\hat{E}_t\pi_{t+1} - r\frac{\eta}{\sigma}p_{t-1}; \tag{13}$$

$$\pi_t = r\lambda\hat{E}_t x_{t+1} + r\left[\frac{\lambda}{\sigma} + \beta(1+\frac{\eta\theta}{\sigma})\right]\hat{E}_t\pi_{t+1} - r\frac{\eta\theta}{\sigma}p_{t-1}; \tag{14}$$

$$p_t = r\lambda\hat{E}_t x_{t+1} + r\left[\frac{\lambda}{\sigma} + \beta(1+\frac{\eta\theta}{\sigma})\right]\hat{E}_t\pi_{t+1} + r(1+\frac{\eta\theta}{\sigma})p_{t-1}. \tag{15}$$

Assuming an infinite number of agents, according to the Law of Large Numbers, we can say that each rule $h = \{1,2,3\}$ (both for inflation and output gap in the initial state) is chosen with probability 1/3. This will be the initial situation in all the studied cases.

When the intensity of choice is null ($\psi = 0$) agents do not perceive any performance difference across the three predictors thus they never change their initially chosen predictor; the fractions are therefore always constant, symmetric and equal to $0.\bar{3}$, implying that $\hat{E}_t\pi_{t+1} = 0$ and also $\hat{E}_t x_{t+1} = 0$, for all the t periods.

Therefore, when looking at the set of Equations 13 - 15 it is easy to see that the first two terms on the RHS always disappear and the equations reduce only to the third term, meaning that output gap, inflation, and price dynamics are governed by past period price only.

In all the three equations, the parametric term multiplying p_{t-1} is in absolute value lower than the unity, leading the system to slowly converge toward its steady state. In Figure 7 it is possible to observe this convergence from below for inflation[15].

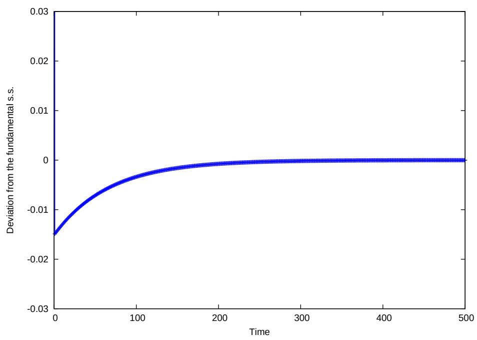

Figure 7: Simulated inflation time series when the intensity of choice is null. When $\psi = 0$ inflation slowly converges to the steady state level.

Let's consider the case in which the intensity of choice is set to an intermediate value (i.e. $\psi = 1$), in which case agents are somehow able to understand which forecasting strategy performs better and try (when they are able) to switch toward the best performing predictor.

After some transient periods where the dynamics path depends on the definition of the initial conditions, for any initial vector $[x_0, \pi_0, p_0]$ the dynamics follows the same path which can be defined as slow up and down oscillating process, without never reaching the steady state. In certain moments the expectations are reinforced while in others they are not; this can be seen by looking at the dynamics reported in Figure 8. For example, as soon as realized inflation cross the zero level from above (period 121), the most performing forecasting strategy is the fundamental one and a number of agents abandon the positive bias predictor: such a behaviour pushes infla-

[15]Convergence occurs from below because in $t = 0$, price deviation is assumed to be positive. Giving as initial condition $p_0 < 0$ would cause a convergence from above.

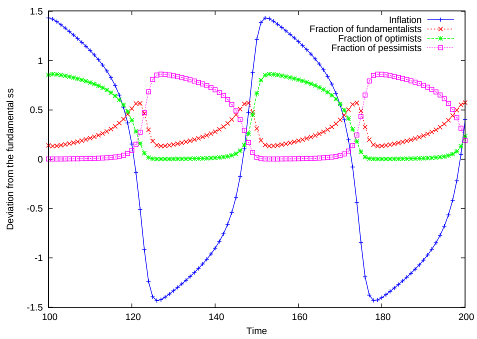

Figure 8: Simulated inflation (blue), fraction of fundamentalists (red), optimists (green) and pessimists (pink) when $\psi = 1$. Here inflation slowly oscillates and never converges to its steady state value.

tion to even more negative values and this in turn causes the negative bias predictor being the best. Therefore in periods 122 to 126 individual expectations are reinforced by actual realizations of the endogenous variables.

However this process affects also price level (Equation 15) and as time goes on the effect (in absolute value) on p_t is larger than the one on π_t; at a certain point therefore price deviation from its steady state value is so highly negative that its (negative) effect on actual inflation (the third term in Equation 14) becomes positive and so relatively high to overcome the negative expectations effects (the first two terms in Equation 14), causing inflation to change its trend in period 127 and starting to push it upward. Therefore in periods 127 the reinforcing expectations process stops and until period 146 the reverse occur and inflation increases.

As soon as agents understand this change in trend, they start abandoning the pessimistic predictor in favour of the fundamental rule and later, when inflation crosses the zero level from below (period 147) and continues increasing, in favour of the optimistic predictor accelerating the opposite process of increased inflation. The same process is then repeated infinitely many times causing inflation to slowly oscillate up and down as it is shown in Figure 8.

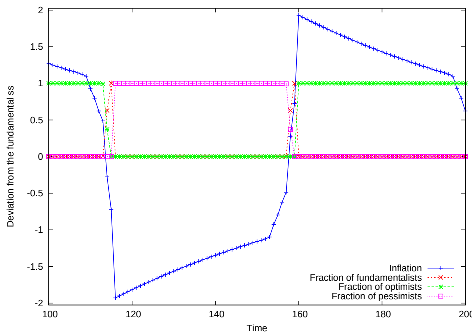

Figure 9: Simulated inflation (blue), fraction of fundamentalists (red), optimists (green) and pessimists (pink) when $\psi = 20$. Here inflation slowly oscillates and never converges to its steady state value while the fractions are updated only during the critical periods when inflation crosses the zero deviation level.

The third case to be analysed, is the one in which the intensity of choice is high (i.e. $\psi = 20$); in such a case agents are highly reactive in understanding which predicting rule is the best one and they actually switch to it in the very next period. The dynamics in this case are qualitatively similar to the case of an intermediate value of intensity of choice. There is however, an important difference to be noticed when looking at Figure 9: the fastness of the switching behaviour of all the agents causes inflation to be more volatile with respect to the previous case and indeed oscillations around the steady state values are more pronounced.

Hence the three presented graphs containing inflation time series for different values of the intensity of choice, confirm the previous results and allow an economic explanation for the emergence of more complicated dynamics when the intensity of choice overcomes the bifurcation value.

5 Conclusions

The paper presented a New-Keynesian model in which expectations are heterogeneous, in line with the seminal paper by Brock and Hommes (1997) and in sharp contrast with the mainstream economic literature which is still anchored with the idea of a rational representative agent. This chapter adds to the literature on monetary policy by investigating whether a Friedman k-percent money supply rule, as an alternative to the standard Taylor rule, may be helpful to stabilize the economy in presence of heterogeneous expectations. The Friedman rule may in fact be seen as an alternative for the Taylor rule which rather than adjusting the interest rate, it copes direclty with money supply.

Simulations with calibrated parameters suggest that this monetary policy rule may be a good instrument to counteract deviations from the steady state level of inflation, as long as agents do not have a pronounced switching behaviour (i.e. when the intensity of choice is null or very low); this situation may be relevant when each individual has a preference for one particular predictor and actually never abondons it, even if the related performance is worse than those of the others forecasting strategies[16].

Differently, when allowing for an higher intensity of choice and a greater reactiveness to differences in performances (i.e. when the intensity of choice is intermediate or high), the agents may switch toward what they perceive to be the best performing predictor; in such a case, agents' expectations strongly affect future realizations: periods of reinforcing phropecies are therefore followed by periods of trend reversals and the dynamics of the whole economic system consists of slow quasi-chaotic oscillations that never converge to the steady state level.

A particular case in which a very high intensity of choice leads to stability is the neoclassical limit case, in which subjects in the economy have perfect knowledge about predictors performances' ($\psi = \infty$) and immediately switch toward the predictor with the lowest forecasting error in the last period. In such a situation indeed convergence may occur, but only if the system starts from a condition close enough to the fundamental steady state; otherwise when the system begins "not so close" to the steady state, the attractor of the system consists of a stable 2-cycle and the emerging dynamics is explained by an oscillating behaviour in which periods of high inflation and overproduction alternate periods of low inflation and underproduction.

Recent studies such those of Branch (2004), Cornea et al. (2013) and Assenza et al. (2011) provide empirical and laboratory evidence for this switching behaviour, meaning that in a real world it is very likely that a Friedman k-percent rule does not lead to stability.

The results obtained in the present paper are therefore in contrast with Evans

[16] See Branch (2004) for a detailed discussion of this "predisposition effect".

and Honkapohja (2003) who stated that *"the Friedman k-percent rule leads to determinacy of equilibria"*; as just explained, in the presence of heterogeneous expectations, the Friedman k-percent rule does not always lead to determinacy; in fact when the intensity of choice is relatively high, it may lead to quasi-chaotic dynamics where neither inflation nor output-gap settle to their steady state values.

6 Appendix

Appendix A - Derivation of the normal form of the system

To compute simulations with MATLAB and E&F Chaos it is necessary to rewrite the system in the normal form, meaning that today's variables have to be functions of past values and expectations (which are conditional on the information set).

To begin with, I find the fundamental steady state of the 3-D map

$$\begin{cases} x_t = \hat{E}_t x_{t+1} - \dfrac{1}{\sigma}\left\{\eta\left[\theta x_t + p_t - kt - M + (\varepsilon_t - \omega_t)\right] - \hat{E}_t \pi_{t+1}\right\} + g_t \\ \pi_t = \beta E_t \pi_{t+1} + \lambda x_t + u_t \\ p_t = \pi_t + p_{t-1}. \end{cases}$$

Knowing that the steady state for inflation is $\pi^* = k$[17], the steady states for x and p are easily computed from the first two equations of the system and they are respectively $x^* = \lambda^{-1}(1-\beta)k$ and $p^* = a + kt$ [18].

Then using a change of notation, defining x_t, π_t, p_t as deviations from the fundamental steady state, the system becomes

$$\begin{cases} x_t = -\dfrac{1}{\sigma}\eta\theta x_t - \dfrac{1}{\sigma}\eta p_t + \dfrac{1}{\sigma}\hat{E}_t \pi_{t+1} + \hat{E}_t x_{t+1} + \tilde{g}_t \\ \pi_t = \lambda x_t + \beta \hat{E}_t \pi_{t+1} + u_t \\ p_t = \pi_t + p_{t-1} \end{cases}$$

[17] Because it is equal to the percentage increase in money supply.

[18] With $a = \dfrac{k}{\eta} + M - \theta\lambda^{-1}(1-\beta)k$.

where $\tilde{g}_t = g_t - \frac{1}{\sigma}(\varepsilon_t - \omega_t)$.

Adjusting terms and playing with algebra it is possible to obtain

$$A \begin{pmatrix} x_t \\ \pi_t \\ p_t \end{pmatrix} = B \begin{pmatrix} \hat{E}_t x_{t+1} \\ \hat{E}_t \pi_{t+1} \\ \hat{E}_t p_{t+1} \end{pmatrix} + C \begin{pmatrix} x_{t-1} \\ \pi_{t-1} \\ p_{t-1} \end{pmatrix} + I \begin{pmatrix} \tilde{g}_t \\ u_t \\ 0 \end{pmatrix}$$

where

$$A = \begin{pmatrix} 1 + \frac{1}{\sigma}\eta\theta & 0 & \frac{1}{\sigma}\eta \\ -\lambda & 1 & 0 \\ 0 & -1 & 1 \end{pmatrix}, \qquad B = \begin{pmatrix} 1 & \frac{1}{\sigma} & 0 \\ 0 & \beta & 0 \\ 0 & 0 & 0 \end{pmatrix}, \qquad C = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

and I is the identity matrix.

Then calculating A^{-1} and multiplying it on the RHS, the system to be analysed finally becomes

$$\begin{pmatrix} x_t \\ \pi_t \\ p_t \end{pmatrix} = r \begin{pmatrix} 1 & \frac{(1-\beta\eta)}{\sigma} & 0 \\ \lambda & \frac{\lambda}{\sigma} + \beta(1 + \frac{\eta\theta}{\sigma}) & 0 \\ \lambda & \frac{\lambda}{\sigma} + \beta(1 + \frac{\eta\theta}{\sigma}) & 0 \end{pmatrix} \begin{pmatrix} \hat{E}_t x_{t+1} \\ \hat{E}_t \pi_{t+1} \\ \hat{E}_t p_{t+1} \end{pmatrix} + \\ + r \begin{pmatrix} 0 & 0 & -\frac{\eta}{\sigma} \\ 0 & 0 & -\frac{\eta\lambda}{\sigma} \\ 0 & 0 & (1 + \frac{\eta\theta}{\sigma}) \end{pmatrix} \begin{pmatrix} x_{t-1} \\ \pi_{t-1} \\ p_{t-1} \end{pmatrix} + r \begin{pmatrix} 1 & -\frac{\eta}{\sigma} \\ \lambda & 1 + \frac{\eta\theta}{\sigma} \\ \lambda & 1 + \frac{\eta\theta}{\sigma} \end{pmatrix} \begin{pmatrix} \tilde{g}_t \\ u_t \end{pmatrix} \tag{16}$$

where $r = \dfrac{1}{1 + \frac{\eta}{\sigma}(\theta + \lambda)}$.

References

Mikhail Anufriev, Tiziana Assenza, Cars H. Hommes, and Domenico Massaro. Interest Rate Rules and Macroeconomic Stability under Heterogeneous Expectations. *forthcoming in Macroeconomic Dynamics*, 2013.

Tiziana Assenza, Peter Heemeijeer, Cars H. Hommes, and Domenico Massaro. Individual Expectations and Aggregate Macro Behavior. *CeNDEF Working Paper*, 2011.

William A. Branch. The Theory of Rationally Heterogeneous Expectations: Evidence from Survey Data on Inflation Expectations. *The Economic Journal*, 2004.

William A. Brock and Cars H. Hommes. A Rational Route to Randomness. *Econometrica*, 1997.

Richard Clarida, Jordi Gali, and Mark Gertler. The Science of Monetary Policy: A New Keynesian Perspective. *Journal of Economic Literature*, 1999.

Adriana Cornea, Cars H. Hommes, and Domenico Massaro. Behavioural Heterogenity in U.S. Inflation Dynamics. *Tinbergen Institute Discussion Paper*, 2013.

George W. Evans and Seppo Honkapohja. Freidman's Money Supply Rule vs. Optimal Interest Rate Policy. *Scottish Journal of Political Economy*, 2003.

Charles F. Mansky and Daniel L. McFadden. Alternative Estimators and Sample Designs for Discrete Choice Analysis. *Structural Analysis of Discrete Data*, 1981.

Bennet T. McCallum and Edward Nelson. Performance of Operational Policy Rules in an Estimated Semi-Classical Model. *In Taylor (1999) "Monetary Policy Rules", University of Chicago Press*, 1996.

John Taylor. Discretion versus Policy Rules in Practice. *Carnegie-Rochester Conference Series on Public Policy*, 1993.

Carl E. Walsh. *Monetary Theory and Policy.* MIT Press, 2010.

Michael Woodford. Control of the Public Debt: A Requirement for Price Stability? *NBER Working Paper*, 1996.

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