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The role of productivity and financial fragility**

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# Heterogeneous Firms and International Trade: The role of productivity and financial fragility\*

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## Abstract

In his seminal paper, starting from the premise that productivity is heterogeneous across firms, Melitz (2003) nicely accounts for the stylized fact that the level of individual productivity is key in determining the capability of a firm to export. In this paper we build a model along Melitz's lines to show that also financial capacity, captured by the level of individual net worth, affects the behaviour of firms on international markets. In our framework, in fact, the decision to export depends on both productivity and net worth, and both are heterogeneous across firms. We show that firms with low productivity may still be able to penetrate foreign markets provided they have enough net worth to incur the cost of exporting. However, even a really high net worth may not guarantee the presence in both domestic and foreign markets if the firm does not have a *minimum* level of productivity. Finally, we explore the effects of changes in transport costs, fixed costs for exporters and the financial constraints.

**Key words:** Productivity; Net Worth; International trade; Heterogeneous firms.

**JEL codes:** E44; F12; F14; F21.

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# 1 Introduction

Since the '90s, increasing access to firm-level data has allowed to explore the role that the heterogeneity of firms' characteristics (e.g. productivity, size, cost of inputs etc.) plays in driving behaviour on domestic and international markets. Only a fraction of firms is capable of exporting and this capability is somehow related to the intrinsic features of the exporting firms, e.g. their productivity. These crucial microeconomic stylized facts were not explained by international trade models available at the time.

To account for these stylized facts, in his seminal paper, Melitz (2003) assumes that productivity is heterogeneous across firms and the distribution of firms' productivity is right skewed. Starting from this premise and adding some new features to Krugman's model – an entry/exit decision mechanism and fixed/sunk costs of exporting – Melitz nicely accounts for some of the above mentioned stylized facts. In his model, in fact, only the most productive firms are able to incur the cost of exporting and therefore to access international markets. Less productive firms are confined to domestic markets. Hence the level of individual productivity is key in determining the capability of a firm to export. Moreover, he delivers interesting predictions on the role of the extensive margin in determining aggregate trade flows.

In the last decade the literature on the role of firms' heterogeneity in affecting both the origin and the patterns of international trade has grown rapidly (see Melitz and Redding 2014 for an extensive survey). However, there is still ample room for improvement. As Redding (2010) pointed out, *'one area for further research [consists in] gaining a deeper understanding of the origins of firms' heterogeneity and the role of internal firm organization'*.

In this paper we claim that there are other sources of heterogeneity which affect the behaviour of firms in domestic and international markets. In particular, we single out the heterogeneity of financial conditions, captured by the firm's net worth. In our framework, in fact the decision to export depends on both productivity and net worth, and both are heterogeneous across firms.

The role of financial factors – broadly speaking – in international trade is the focus of a relatively small but expanding literature. For instance Chaney (2005) analyzes a model in which firms' decision to export is affected by a liquidity constraint. He shows that only the most productive firms which generate sufficient internal resources are capable of relaxing the liquidity constraint and penetrating foreign markets. On the other hand, exporting firms, facing the higher entry costs of foreign markets, may be hindered by a random liquidity shock. Differently from his paper, we explicitly model the financial system. In our streamlined framework, financial intermediaries charge heterogeneous interest rates to borrowing firms depending on the firms' net worth. Financially fragile firms are charged higher interest rates.

Along similar lines, Caggese and Cuñat (2012) develop a dynamic model in which

firms may accumulate wealth as a form of precautionary saving, given the constraints on the credit market. Their model departs from Melitz assuming that fixed cost are heterogeneous and paid in advance and that firms engaging exports have a higher volatility of profits, that is they are more prone to bankruptcy.

A number of scholars have analysed the relation between financial constraints and exports empirically. Manova (2013) analyzes, mostly empirically, the relation between a country's financial system and international trade, exploring how the constraints and weaknesses of the former may affect the latter. To organize the empirical analysis, she introduces heterogeneity across countries in financial systems in a Melitz framework. Assuming that firms need external financial support to cover the fixed costs to export, she shows that more developed financial systems allow firms to sell a higher number of products on an higher number of markets, increasing the total country exports. Along the same line of research, Crinò and Ogliari (2014) develop a trade model with heterogeneity in both financial constraints and financial vulnerability. They highlight that financial development has an effect on both trade flow and export structure through changes in quality.

We develop a static model à la Melitz in which heterogeneity is not confined only to productivity but regards also the firm's net worth. In section 2 we present the model; section 3 is devoted to the analysis of the output of simulations; in section 4 we present and discuss some empirical evidence. Section 5 concludes.

## 2 The model

We build a static model à la Melitz with heterogeneous firms that face an export decision. Differently from Melitz, where firms show heterogeneous productivity only, we introduce a further degree of heterogeneity. In fact, firms in our model are characterized also by heterogeneous financial robustness. For the sake of clarity, we consider two different scenarios: i) a closed economy (section 2.1) and then we extend the model to the case of ii) an open economy (section 2.2).

### 2.1 Closed economy

In this section, we consider a closed economy populated by  $H$  households and  $N$  firms. Households are homogeneous. Firms are heterogeneous (more on this later) and produce differentiated goods (varieties) in a Dixit-Stiglitz monopolistic competition setting. There is one firm per variety. Hence the set of varieties has the same cardinality as the set of firms.

The representative household's preferences over varieties are represented by the fol-

lowing utility function:

$$U_h = C_h = \left[ N^{\frac{1}{1-\sigma}} \sum_1^N (q_{ih})^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \quad (1)$$

The utility function coincides with a CES aggregator of the quantities consumed (i.e. with aggregate consumption in real terms)  $C_h$ ,  $q_{ih}$  is the quantity consumed by household  $h$  of variety  $i$  where  $i = 1, 2, \dots, N$ ,  $\sigma > 1$  is the constant elasticity of substitution among varieties.<sup>1</sup>

The budget constraint of the representative household is:

$$\sum_1^H p_i q_{ih} \leq R_h = wl_h + \Pi_h \quad (2)$$

where  $p_i$  is the price of variety  $i$  and  $R_h$  denotes “resources” available to the household. Resources consist of the wage bill  $wl_h$  ( $w$  is the nominal wage,  $l_h$  is the household’s labour effort) and of dividends accruing to the household  $\Pi_h$ .<sup>2</sup>

Moreover, by definition:

$$\sum_1^H p_i q_{ih} = PC_h \quad (3)$$

where  $P$  is the aggregate price level (to be defined below).

The household maximizes (1) by choosing the optimal quantity of each variety under the constraints (2) and (3). Solving the household’s maximization problem we get the demand for variety  $i$ :

$$q_{ih} = \left( \frac{p_i}{P} \right)^{-\sigma} \frac{Q_h}{N} \quad (4)$$

where  $Q_h := R_h/P$  are total resources available in the households sector in real terms, such that the term  $\frac{Q_h}{N}$  in equation 4 represents resources available to each (homogeneous) households in real terms. Summing over the total population of households we determine the market demand for the  $i$  –  $th$  variety (coming from all the households in the economy):

$$q_i = \left( \frac{p_i}{P} \right)^{-\sigma} \frac{Q}{N} \quad (5)$$

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<sup>1</sup>Leisure does not show up in the utility function, hence we are implicitly assuming that the household supplies inelastically its endowment of labour.

<sup>2</sup>As usual, we assume that households own firms and all the profits earned by firms are distributed to households as dividends. With  $\Pi_h = \sum_1^N \Pi_{ih}$  where  $\Pi_{ih}$  denotes dividends paid out by firm  $i$  to household  $h$ .

where  $q_i := \sum_1^H q_{ih}$  and  $Q$  is real GDP.<sup>3</sup>

Following Melitz, we assume that *firms are heterogeneous in terms of productivity*. In particular, the  $i$  – *th* firms has a linear technology, which uses only labor as an input of production:

$$q_i = \varphi_i l_i \quad (6)$$

where  $\varphi_i$  is the productivity of labour and  $l_i$  is the amount of labor employed by firm  $i$ . The nominal wage is given and normalized to unity:  $w = 1$ . Moreover, we assume that the firm must incur a fixed production cost  $F$  (homogeneous for all firms). Total *Production Costs* ( $PC_i$ ) therefore are:

$$PC_i = \frac{q_i}{\varphi_i} + F \quad (7)$$

We assume that the firm must advance funds to cover production cost before carrying out production. In a setting of incomplete and imperfect financial markets, a financing hierarchy emerges. The firm will finance production using first “internal finance” i.e., a (given) endowment of financial resources  $a_i > 0$ .<sup>4</sup> *Firms are heterogeneous in terms of internal finance*. The endowment  $a_i$  can be interpreted as the firm’s *equity base or net worth*. In other words  $a_i$  is an idiosyncratic indicator of firms’ financial robustness, the higher  $a_i$  the more financially robust the firm is and vice-versa.<sup>5</sup>

If internal finance is insufficient to cover total production costs – i.e. if a “financing gap” must be filled, equal to the difference between total production costs and internal finance – the firm will raise external funds which will be provided by the financial system in the form of bank loans. In the following, for simplicity, we will refer to the financial system as the bank.<sup>6</sup>

We assume that internal finance is available to the firm at zero costs while the bank charges differentiated interest rates to each firm depending on its financial robustness

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<sup>3</sup>By definition  $Q = \frac{R}{P}$  where  $R = \sum_1^H R_h$ . In words:  $Q$  denotes total resources in real terms available to households, i.e. total nominal resources  $R$  deflated by the price index  $P$ . Total nominal resources are the sum of the economy wide wage bill and profits, hence they coincide with nominal GDP. In fact by denoting total employment with  $l = \sum_1^H l_h$  so that  $wl$  is the economy wide wage bill and total profits by  $\Pi = \sum_1^H \Pi_h$ , summing resources over households we get  $R = wl + \Pi$ .

<sup>4</sup>Since we have assumed that internal finance is always positive, we are ruling out actual bankruptcy which occurs when net worth becomes negative. We leave this interesting scenario for further research.

<sup>5</sup>This is a simplifying shortcut. A more satisfactory measure of the firm’s financial robustness could be a financial ratio such as the equity ratio, i.e. the reciprocal of leverage:  $\alpha_i := \frac{a_i}{f_i}$ , where  $f_i = \frac{q_i}{\varphi_i} + F - a_i$  is the financing gap. This assumption, however, would make calculations much more complicated.

<sup>6</sup>The amount of loans extended by the bank to the firm is equal to the financing gap,  $f_i$ . For simplicity, we also assume that the financing gap will always be filled by the bank, i.e. there is no credit rationing.



indicator, i.e., the interest rate charged to firm  $i$  is a function of the net worth  $a_i$ :

$$r_i = \phi(a_i), \quad (8)$$

where  $\phi(a_i)$  is a strictly positive and continuous function, with  $d\phi/da_i < 0$ . The absolute value of  $d\phi/da_i$  determines the reactivity of the bank to a change in the financial robustness of the firm. The more financially robust the firm, the lower the interest rate charged by the bank on the extended loan and vice-versa.<sup>7</sup>

In the case in which production is totally self financed by the  $i$  –  $th$  firm, total costs coincide with total production costs as defined in equation (7). If, on the contrary, production is not entirely covered by net worth total costs must include financial costs, i.e. debt commitments.

The main novelty of our paper w.r.t. the models described in Melitz (2003) and by Caggese and Cuñat (2013) is that firms in our setting must face a financial cost to produce (and export). Therefore, in the following we will assume that it is always true that  $a_i < PC_i \forall i$ , i.e., we will take into account only the case in which all firms do not have enough internal finance to carry out production, hence they need external funds. Since we had assumed that the bank charges idiosyncratic interest rates to each firm depending on the firm's financial robustness (see equation (8)), the financial cost, faced by each firm, is heterogeneous and depends on the specific features of firms' financial robustness and of the financial system. Hence, total costs become:

$$TC_i = PC_i(1 + r_i) - a_i r_i. \quad (9)$$

Plugging (7) into (9) and rearranging we get :

$$TC_i = \left( \frac{q_i}{\varphi_i} + F \right) [1 + \phi(a_i)] - a_i \phi(a_i) \quad (10)$$

Total costs are decreasing in net worth  $a_i$ , the more financially robust the firm, the lower the interest rate charged by the bank and therefore the lower the financial costs, given total production costs.<sup>8</sup> The marginal costs are:

$$c_i = \frac{1}{\varphi_i} [1 + \phi(a_i)] \quad (11)$$

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<sup>7</sup>An alternative, richer but more complex definition of the interest rate would be  $r_i = \phi(\alpha_i)$ . In this case, OTBE, the interest rate would depend not only on net worth as in (8) but also on productivity.

<sup>8</sup>It is straightforward to show that  $\frac{dTC_i}{da_i} < 0$ . The condition holds for  $r_i > \frac{d\phi}{da_i} f_i$ , that being  $r_i > 0$ ,  $\frac{d\phi}{da_i} < 0$  and  $f_i > 0$  is always verified.

Marginal costs are decreasing in the firm's net worth and in the productivity (respectively  $a_i$  and  $\varphi_i$ ). Given the CES utility function and the demand function as defined in equation (5), the optimal price charged by the  $i$ -th firm is:

$$p_i = \frac{\sigma}{\sigma - 1} c_i \quad (12)$$

Substituting eq.(11) into eq.(12) the individual price  $p_i$  can be written as a (decreasing) function of individual productivity ( $\varphi_i$ ) and individual equity ( $a_i$ ):

$$p(\varphi_i, a_i) = \frac{\sigma}{\sigma - 1} \frac{1}{\varphi_i} [1 + \phi(a_i)]. \quad (13)$$

Profits ( $\pi_i$ ) of the  $i$ -th firm are equal to total revenues ( $p(\varphi_i, a_i)q_i$ ) net of total costs ( $TC_i$ ), i.e.:

$$\pi_i(\varphi_i, a_i) = p(\varphi_i, a_i)q_i - TC_i \quad (14)$$

substituting equations (12) and (10) and rearranging we obtain:

$$\pi_i(\varphi_i, a_i) = \frac{1}{\sigma - 1} \frac{1}{\varphi_i} [1 + \phi(a_i)]q_i - F[1 + \phi(a_i)] + a_i\phi(a_i). \quad (15)$$

Individual profits are a function not only of individual productivity and equity, but also of the productivity and equity of all other firms in the economic system through the demand function and, specifically, the price index (that enters the definition of  $q_i$  as shown in equation 5).

### 2.1.1 Participation Constraint

Firms will enter the market and start production if the participation constraint is satisfied, i.e.  $\pi_i \geq 0$ . Profits defined in equation (15) can be rewritten substituting the demand function (equation (5)), after some tedious algebra and rearranging we get:

$$\pi_i = \left[ \frac{(\sigma - 1)\varphi_i}{1 + \phi(a_i)} \right]^{\sigma-1} \frac{P^{\sigma-1} R}{\sigma^\sigma N} - F[1 + \phi(a_i)] + a_i\phi(a_i). \quad (16)$$

Assuming that the  $i$ -th firm takes as given the price index  $P$ , we can determine the threshold for  $\varphi_i$ , denoted with  $\bar{\varphi}_i$ , such that given  $a_i$  and  $P$ , the participation constraint is satisfied. The  $i$ -th firm is willing to enter the market and produce if:

$$\varphi_i \geq \bar{\varphi}_i(P, a_i) := \frac{1}{P} \Omega \left\{ F[1 + \phi(a_i)]^\sigma - a_i\phi(a_i)[1 + \phi(a_i)]^{\sigma-1} \right\}^{\frac{1}{\sigma-1}}, \quad (17)$$

with  $\Omega = (\sigma - 1)^{-1} \sigma^{\frac{\sigma}{\sigma-1}} \left( \frac{N}{R} \right)^{\frac{1}{\sigma-1}}$ .

It is important to stress that the threshold value for individual productivity,  $\bar{\varphi}_i$ , depends on the idiosyncratic interest rate on loans charged by the bank,  $r_i = \phi(a_i)$ , that in turn is a decreasing function of the firm's net worth and therefore of its financial robustness. Moreover the threshold value for individual productivity,  $\bar{\varphi}_i$ , is a decreasing function of the aggregate price index ( $P$ ) that in turn is affected by firm's individual decision to participate in the market. Furthermore the aggregate price index influences the decision of the other firms to participate in the market. In other words, there is a two way feedback from the aggregate price index to the firm's individual decision to enter the market and vice-versa. The intuition can be explained with a simple example. Let's assume that firms are homogeneous in terms of financial robustness, i.e. they have the same net worth. If a firm characterized by low productivity (and therefore high individual price  $p_i$ , as shown in equation (13)), decides not to participate in the market then the aggregate price index will decrease. From eq.(17) it is clear that the productivity threshold in the participation constraint for all the other firms will increase because of the decrease of the aggregate price index <sup>9</sup> hence the productivity level needed to be willing to produce ( $\bar{\varphi}_i$ ) becomes higher.

It is important to note that in equilibrium the aggregate price index  $P$  depends on the decisions of each firm, and vice-versa, the decision of each firm depends on the aggregate price. The following two equations must be satisfied simultaneously:

$$P = \left[ \frac{1}{N} \sum_{i=1}^N I_{\pi_i \geq 0} (p_i)^{1-\sigma} \right]^{\frac{1}{1-\sigma}} \quad (18)$$

$$I_{\pi_i \geq 0} = \begin{cases} 1, & \text{if } \varphi_i \geq \bar{\varphi}_i(P, a_i) \\ 0, & \text{if } \varphi_i < \bar{\varphi}_i(P, a_i) \end{cases} \quad (19)$$

where  $I_{\pi_i \geq 0}$  is an indicator function taking the value 1 if firm  $i$  participates to the market and zero otherwise. Due to non-linearity in the system of equations (18)-(19), it is difficult to find an analytical solution, but it is relatively easy to solve the system, finding the equilibrium decision for each firm, numerically.

## 2.2 Open economy

In the following we will consider an economy populated by two symmetric countries, therefore we extend the model by assuming that domestic firms can decide to export to a symmetric country, and similarly, that foreign firms from the symmetric country can

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<sup>9</sup>From equation (17) smaller aggregate price index, implies, *ceteris paribus*, higher  $\bar{\varphi}_i$ .

decide to export on the domestic market. This implies a change on the domestic market, since domestic firms must compete also with the exporters from the foreign country. We want to analyze the macroeconomic effect of trade, and the influence of the financial system on trade decisions.

We consider an open economy populated by  $H$  households and  $\tilde{N}$  firms <sup>10</sup>, being  $\tilde{N}$  the number of firms operating within the country (domestic and foreign). Proceeding as in section 2.1, the market demand for the  $i$  –  $th$  variety is:

$$\tilde{q}_i = \left( \frac{\tilde{p}_i}{\tilde{P}} \right)^{-\sigma} \frac{Q}{\tilde{N}}. \quad (20)$$

The production function has the same functional form as in equation (6), therefore it is  $\tilde{q}_i = \varphi_i l_i$ . In an open economy we need to distinguish between domestic and exporter firms in each country. We model domestic firms as in section 2.1 taking into account the presence of foreign firms operating in the domestic economy. In the following we will describe the exporters' decision procedure. Let 'us assume that firms who want to export has to face a fixed production cost  $F^o > F$ , where  $F^o$  already incorporates costs related to export activity. As it is quite common in this literature, each exporter has to incur in an iceberg cost,  $\tau > 1$ . Therefore total *Production Costs* for exporters are:

$$\tilde{P}C_i^o = \frac{\tilde{q}_i}{\varphi_i} \tau + F^o. \quad (21)$$

An exporter firm may finance its production activity both through internal resources and, if they are not sufficient to cover production costs, she may ask for a loan to a bank located in her country. We model the interest rate decision of the bank as in section 2.1. Defining  $a_i^o$  as the internal liquidity available to produce on the foreign market, total costs (including financial costs) for an exporter are:

$$\tilde{T}C_i^o = \tilde{P}C_i^o(1 + r_i) - a_i^o r_i. \quad (22)$$

The marginal cost is:

$$\tilde{c}_i^o = \frac{\tau}{\varphi_i} (1 + r_i), \quad (23)$$

from which we obtain the following individual price for an exporter:

$$\tilde{p}_i^o(\varphi_i, a_i) = \frac{\sigma}{\sigma - 1} \frac{\tau}{\varphi_i} (1 + r_i) \quad (24)$$

Profits equal total revenues net of total costs, i.e.:

$$\tilde{\pi}_i^o(\varphi_i, a_i) = \tilde{p}_i^o(\varphi_i, a_i) \tilde{q}_i - \tilde{T}C_i^o \quad (25)$$

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<sup>10</sup>Hereafter variables with a tilde refer to the open economy set up.

Given that we are assuming symmetric countries and that  $F^o > F$  it follows that firms who decide to export will also produce domestically. Moreover, for the sake of simplicity, we assume that firms that decide to export use all their internal resources in the domestic production, i.e.  $a_i^o = 0$ . Making use of equations (8), (20) and (24) after substitutions profits for an exporter become:

$$\tilde{\pi}_i^o(\varphi_i, a_i) = \left\{ \frac{(\sigma - 1)\varphi_i}{[1 + \phi(a_i)]\tau} \right\}^{\sigma-1} \frac{\tilde{P}^{\sigma-1} R}{\sigma^\sigma \tilde{N}} - F^o[1 + \phi(a_i)] \quad (26)$$

A firm will enter the domestic market if  $\tilde{\pi} \geq 0$ , and will enter also the foreign market if  $\tilde{\pi}_i^o \geq 0$ .

The participation constraints for a domestic and an exporter firm respectively are:

$$\varphi_i \geq \tilde{\varphi}_i(\tilde{P}, a_i) := \frac{1}{\tilde{P}} \tilde{\Omega} \left\{ F[1 + \phi(a_i)]^\sigma - a_i \phi(a_i) [1 + \phi(a_i)]^{\sigma-1} \right\}^{\frac{1}{\sigma-1}} \quad (27)$$

$$\varphi_i \geq \tilde{\varphi}_i^o(\tilde{P}, a_i) := \frac{\tau}{\tilde{P}} \tilde{\Omega} \{ F^o [1 + \phi(a_i)]^\sigma \}^{\frac{1}{\sigma-1}}, \quad (28)$$

$$\text{with } \tilde{\Omega} = (\sigma - 1)^{-1} \sigma^{\frac{\sigma}{\sigma-1}} \left( \frac{\tilde{N}}{R} \right)^{\frac{1}{\sigma-1}}.$$

Note that given the assumptions that  $\tau > 1$  and  $F^o > F$ , the participation constraint in equation (28) lies always above the participation constraint in equation (27), meaning that a firm that decides to export, will also decide to produce domestically. It is worth noting that the aggregate price index ( $\tilde{P}$ ) in an open economy includes also the prices of the foreign firms exporting to the domestic market, therefore it is defined as it follows:

$$\tilde{P} = \left\{ \frac{1}{\tilde{N}} \left[ \sum_{i=1}^{\tilde{N}} I_{\tilde{\pi}_i \geq 0} \tilde{p}_i^{1-\sigma} + \sum_{i=1}^{\tilde{N}} I_{\tilde{\pi}_i^o \geq 0} (\tilde{p}_i^o)^{1-\sigma} \right] \right\}^{\frac{1}{1-\sigma}} \quad (29)$$

with

$$I_{\tilde{\pi}_i \geq 0} = \begin{cases} 1 & \text{if } \varphi_i \geq \tilde{\varphi}_i(\tilde{P}, a_i) \\ 0 & \text{if } \varphi_i < \tilde{\varphi}_i(\tilde{P}, a_i) \end{cases} \quad (30)$$

$$I_{\tilde{\pi}_i^o \geq 0} = \begin{cases} 1 & \text{if } \varphi_i \geq \tilde{\varphi}_i^o(\tilde{P}, a_i) \\ 0 & \text{if } \varphi_i < \tilde{\varphi}_i^o(\tilde{P}, a_i) \end{cases} \quad (31)$$

where  $I_{\tilde{\pi}_i \geq 0}$  and  $I_{\tilde{\pi}_i^o \geq 0}$  are indicator functions taking the value 1 if firm  $i$  participates respectively to the domestic and the foreign market and zero otherwise. The solution

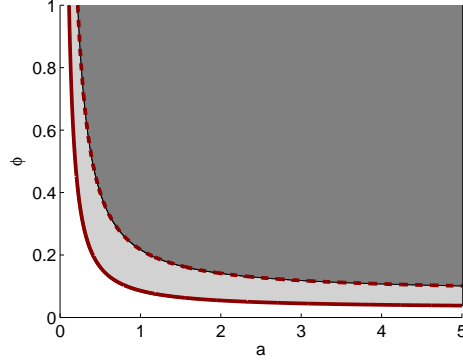


Figure 1: Participation decision space. The white area, are combinations of  $a_i$  and  $\varphi_i$  implying non participation. The light gray area shows the set of firms participating only to the domestic market, the dark grey represents firms participating both to the domestic and the foreign market.

of the system of equations (29)-(31) identifies both domestic and exporting firms at the equilibrium.<sup>11</sup>

In the next section we will show the above results through numerical simulations.

### 3 Getting the equilibrium through simulations

As we have seen above, the solution of the model is implicit. Therefore, in the following section we will present the main results of the numerical simulations we run to find the solution of the model and describe some comparative statics of both the case with symmetric and asymmetric countries.

First of all we need to specify an explicit functional form for the interest rate on loans charged by the bank (see equation (8)). In particular, we assume that the bank determines the interest rate according to the following simple rule:

$$r_i = \frac{b}{a_i^c} \quad (32)$$

where  $b$  and  $c$  are parameters that describe the characteristics of the financial system. The model shows 7 free parameters that we assume to take the values in Table 1. The number of firms in each country is set to 40000.

<sup>11</sup>Note that the closed economy scenario is a special case of the open economy one when  $\tau \rightarrow \infty$  and/or  $(F^o - F) \rightarrow \infty$ . Indeed, assuming a high enough  $\tau$  such that firms are prevented to export,  $\tilde{P} \rightarrow P$  implying that  $\tilde{\varphi}_i \rightarrow \bar{\varphi}_i$  and  $\tilde{\varphi}_i^o \rightarrow \infty$ .

Table 1: Parameters' value in numerical simulations

$F$	$F^o$	$\tau$	$b$	$c$	$a_i$	$\varphi_i$
1	4	1.5	1	1	$\in (0, 1]$	$\in (0, 1]$

### 3.1 Comparative statics with symmetric countries

In the present subsection we will describe the main findings of numerical simulations for the case of an open economy with symmetric countries. In fig. 1, we plot the participation decision space for domestic firms, the two countries are symmetric, therefore the graph represents the foreign market too. The figure shows combinations of  $a_i$  and  $\varphi_i$  implying non participation (white area), participation only in the domestic market (light gray area) and participation in both the domestic and foreign market (dark gray area). Hence, the continuous curve describes the zero profit condition that firms have to satisfy to produce domestically (see equation (27)). While firms that lie on and above the dashed curve produce also for the foreign market (see equation (28)). As Melitz (2003) and Chaney (2005) pointed out we find that firms with high productivity (having a lower individual price) may export. However, differently from their models, in our model we explicitly take into account the role of firms financial robustness and how credit market conditions may affect firms' production capability. Therefore, in our set up exporting firms may be also those showing a low productivity but high net worth. Indeed, as the firm's net worth increases the interest rate on loans applied by the bank decreases and, in turn, marginal costs and individual price decrease as well. Moreover, it is worth noting that even if a firm is characterized by high productivity (net worth), it needs to have a minimum threshold net worth (productivity) to enter both domestic and foreign markets.

In what follows, we study both the effects of changes in the interest rate ( $r_i$ ) and trade barriers ( $\tau$ ) on mean productivity and net worth for domestic firms, number of firms and exporters, and real GDP.

In Fig. 2 we show the effects of an increase of the parameter  $b$  on mean productivity and net worth for domestic firms, number of firms and exporters, and real GDP. It is worth noting that an higher  $b$  induces an increase of the interest rate on loans charged by the bank for all firms (see equation (3)), implying an upward shift of the interest rate function, reflecting a higher cost of lending. Indeed, with a higher  $b$ , only the most productive and least fragile firms are able to produce and export and therefore both mean productivity and net worth rises as  $b$  increases (see upper left panel in Fig. 2). At the same time, as  $b$  rises both marginal costs and prices increase and we observe a reduction of real GDP (lower panel in Fig. 2). Intuitively, as shown in the upper right panel of Fig. 2, as  $b$  (and therefore the interest rate on loans) increases both the number of firms and exporters decrease.

In Fig. 3 we show the effects of an increase of iceberg cost  $\tau$  on mean productivity and net worth for domestic firms, number of firms and exporters, and real GDP. Notice

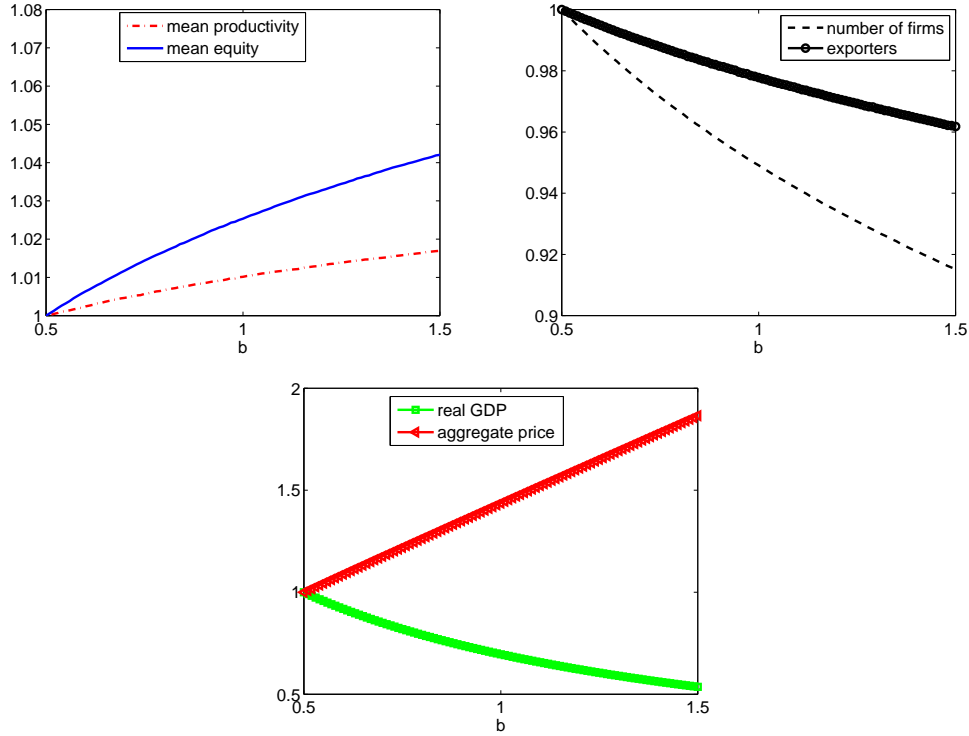


Figure 2: Effects on average productivity, average equity, number of firms and exporters, real GDP and aggregate price of a change in  $b$

that an higher  $\tau$  translates into an increase of the trade barriers. As shown in Fig. 3, as  $\tau$  increases (and trade barriers are less stringent) mean productivity (as in Melitz, 2003), mean net worth (upper left panel), real GDP (lower panel) and the number of exporters (upper right panel) decrease. The total number of firms on the domestic market decreases, but at a lower rate with respect to the decrease of exporters. This means that the domestic firms struggle with the foreign firms entering in the market, and the number of domestic firms producing only for the domestic market decreases.

Finally, in Fig. 4, we report the effects of changes of  $F^o$  on mean productivity and net worth for domestic firms, number of firms and exporters, and real GDP. It is worth noting that an increase of  $F^o$  shows very small effects on mean productivity and net worth (upper left panel) as it changes the zero profit condition only indirectly through its (small) effect on the aggregate price index. In fact, as shown in the lower panel of Fig. 4, as  $F^o$  increases the aggregate price rises slightly. Moreover, as  $F^o$  increases, the number of firms on the domestic market and the number of exporters decrease (upper right panel) inducing a reduction of real GDP as well (lower panel).



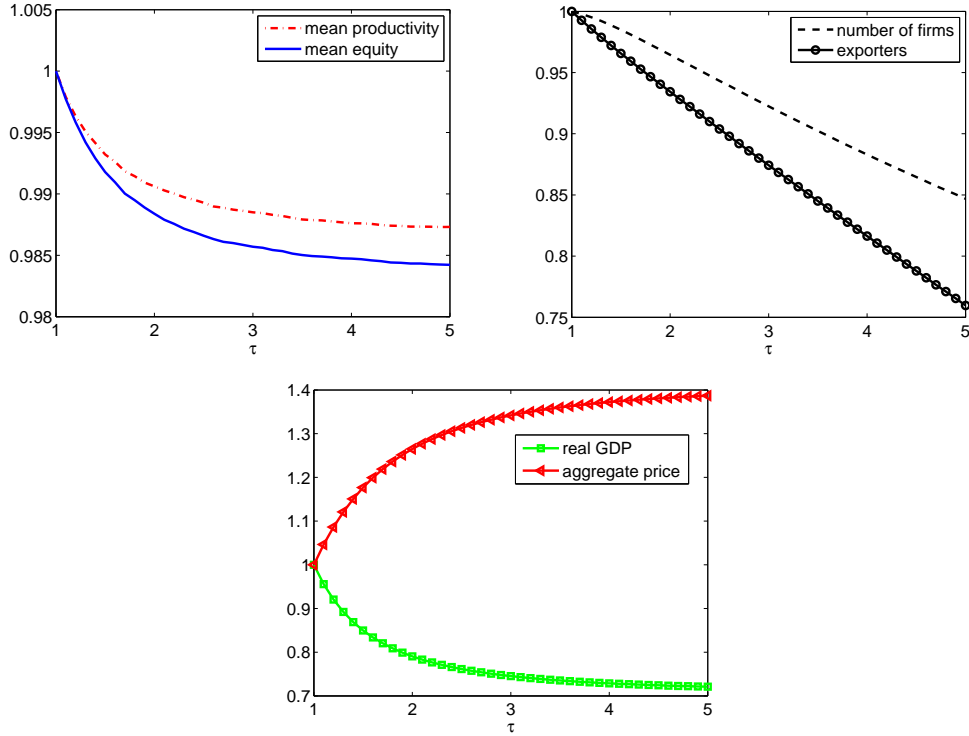


Figure 3: Effects on average productivity, average equity, number of firms and exporters, real GDP and aggregate price of a change in  $\tau$

### 3.2 Comparative statics with asymmetric countries

Following the existing literature (see Manova, 2013 as an example) we can extend our model to study the effects of heterogeneous financial systems performing an analogous set of simulations as in the previous subsection. Figure 5 plots the relationship between private credit (as a proxy of the health of the financial system, as in Crinò (2014)) and the share of export over GDP. It shows how countries with healthy financial system, i.e. where private credit is cheaper, are characterized by a higher percentage of exporters.

In the following, we present results of numerical simulations in an open economy characterized by asymmetric countries in terms of health of the financial system. In particular, in order to evaluate the role of the whole financial system, we introduce heterogeneity in the interest rate function (see equation (32)) assuming that parameters  $b$  and  $c$  differ in the two countries and, specifically, we will consider two countries “d” and “f” such that  $b_d > b_f$  and  $c_d > c_f$ .

Let us start considering heterogeneous parameters  $b$ . Notice that a higher  $b$  translates

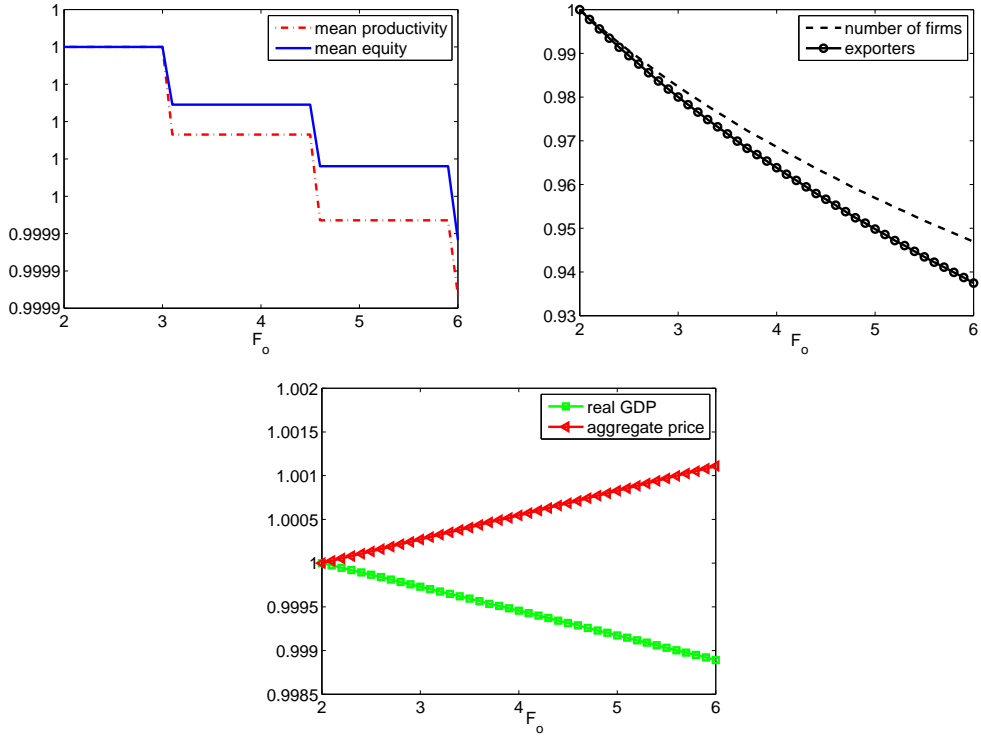


Figure 4: Effects on average productivity, average equity, number of firms and exporters, real GDP and aggregate price of a change in  $F_o$ .

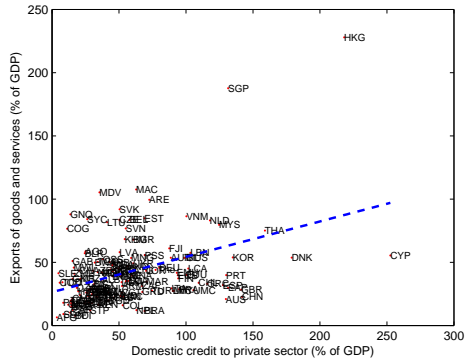


Figure 5: Private credit and export

into an upward shift of equation 32: domestic firms borrow at higher costs. In Fig. 6, we report the effects of an increasing value of  $b_d$ , while  $b_f$  is fixed and equal to one, on the key variables of the domestic country.

An higher cost of borrowing leads to a fall of domestic GDP, as well as in the number of both domestic and exporting firms. However, the reduction of total number of firms is small revealing that foreign firms substitute domestic ones. Finally, there is a self-selection of the most productive and most financially robust domestic firms, in fact, they are able to *self-finance* their production and compete on the market.

Let us now consider the effects of heterogeneous parameter  $c$ . It is worth noting that parameter  $c$  represents the constant interest rate elasticity of  $a_i$ . Hence, while the change in  $b$ , shifting upward the interest rate function, has the same effect on all firms, the effect of asymmetric parameter  $c$  depends critically on the support of the net worth  $a_i$ . In particular, assuming  $b_d = b_f = 1$ , for those firms with  $a_i \in (0, 1)$  (as assumed above) an increase in  $c$  implies higher costs of borrowing; on the other hand, with  $a_i \in (1, \infty)$  all firms face a lower interest rate.

In fig. 7, 9 and 8 we report the effects of heterogeneous  $c$  (keeping  $c_f = 1$ ) on the main domestic variables and for different supports of  $a_i$ . We find that when  $a_i \in (0, 1)$ , the effects of assuming heterogeneous parameter  $c$  on the main domestic variables are similar to those induced by an increase of the parameter  $b_d$  (see Fig. 7). On the contrary, an increase in the elasticity, once we change the support of the net worth from  $a_i \in (0, 1)$  to  $a_i \in [1, 2]$ , shows opposite effects. In general, we can infer that the effects depend strictly on the quota of domestic firms for which an increase in  $c$  translates into a lower cost of borrowing.

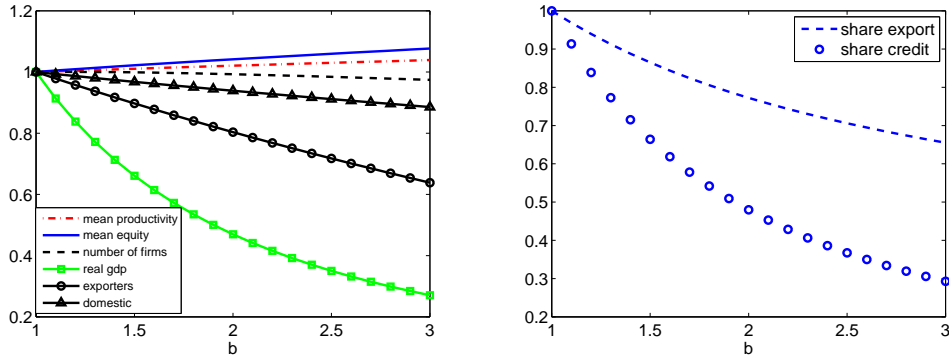


Figure 6: Main results with asymmetric values of  $b$

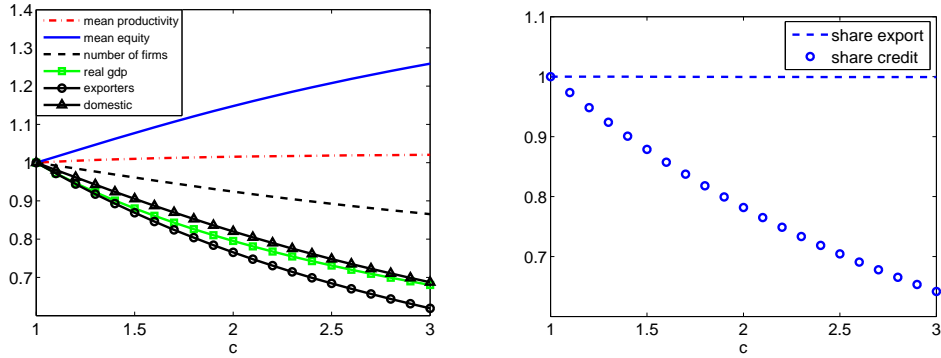


Figure 7: Main results with asymmetric values of  $c$  and  $a_i \in (0, 1)$

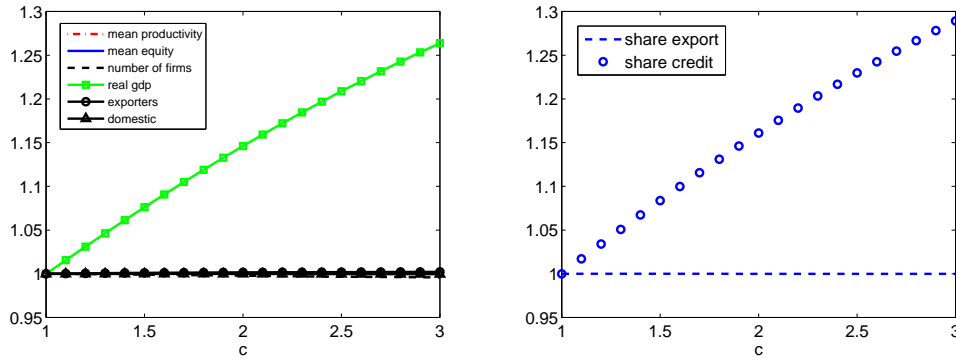


Figure 8: Main results with asymmetric values of  $c$  and  $a_i \in [1, 2]$ .

## 4 Empirical evidence

In this section, we analyze the relationship between the decision to export and both productivity level and financial constraints. To conduct the analysis we employ the Mediocredito-Capitalia Survey which collected data on Italian firms from 1995 to 2003. Capitalia's Observatory on Small and Medium Size Firms is a survey on a representative sample of 3564 Italian firms, providing information on many different aspects, such as R&D, innovation and destination markets for exports. The sample includes all firms with more than 500 employees and, among firms with less than 500 employees, a representative sample selected using a stratified design on location, industrial activity and size.

Our empirical analysis is explorative and we neither pretend to calibrate the model discussed above nor to state causal effects among the variables.

We use the 2000 survey, specifically the question on whether the firm has exported in

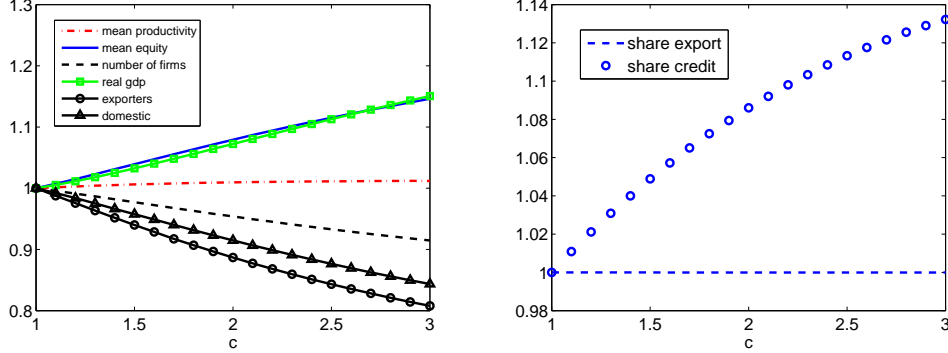


Figure 9: Main results with asymmetric values of  $c$  and  $a_i \in [0.01, 2]$ .

the previous year. More than two thirds of firms (68%) within the sample declare to be an exporter in year 2000. Unfortunately this question is available only for the last year of the survey. However, for each firm we have balance sheet data from 1991 to 2000. We firstly calculate the productivity, using all sample period, and then we run a simple probit model on the relation between being an exporter and both productivity and financial robustness, controlling for the technological level of the sector.

Firms' productivity is usually calculated as the well-known total factor productivity (TFP). Let's start from a Cobb-Douglas production function for the firm:

$$Y_{it} = A_{it} L_{it}^{\beta_l} K_{it}^{\beta_k} \quad \beta_l, \beta_k > 0, \quad (33)$$

where  $i$  and  $t$  are firms and year subscripts respectively;  $Y$  is output (value added);  $L$  is labor;  $K$  is capital and  $A$  is a Hicksian neutral technology multiplier (unobservable).

We assume that firms share the same technology, except than for the neutral parameter  $A$ , that is  $\beta_l$  and  $\beta_k$  are the same for all firms. Transforming equation 33 in logarithm, we get:

$$y_{it} = \beta_0 + \beta_l l_{it} + \beta_k k_{it} + \varepsilon_{it} \quad (34)$$

where the sum of the constant and the error term gives the Hicksian technology:  $a_{it} = \beta_0 + \varepsilon_{it}$ . After the estimation, the value of the estimated TFP is as follows:

$$\hat{a}_{it} = y_{it} - \hat{\beta}_l l_{it} - \hat{\beta}_k k_{it} \quad (35)$$

We use the Levinsohn-Petrin (LP) procedure (2003), that employs intermediate inputs as an instrument for unobservable productivity shocks. LP procedure assumes that the firm demand for intermediate inputs depends on firms state variables, namely capital and the predictable component of the error term,  $m_{it} = m(k_{it}, v_{it})$ . Under the assumption of monotonicity, the latter function can be inverted and we can write  $v_{it} = v(k_{it}, m_{it})$ ,

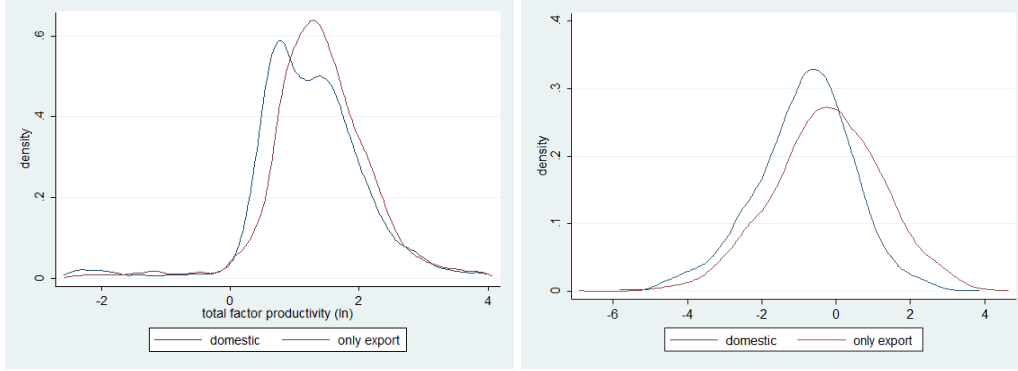


Figure 10: Left panel: TFP for exporters and non exporters. Right panel: net worth for exporters and non exporters

so that the unobservable productivity is a function of two observable variables. However, the functional form is unknown, and LP take a semi-parametric approach by approximating the function  $\varphi(k_{it}, m_{it}) = \beta_0 + \beta_k k_{it} + v(k_{it}, m_{it})$  with a third-order polynomial. Therefore the production function estimated can now be written as follows:

$$y_{it} = \beta_l l_{it} + \varphi(k_{it}, m_{it}) + u_{it} \quad (36)$$

LP is a two-steps procedure, the first stage involves the above equation 36 to achieve  $\hat{\beta}_l$ ; while, introducing additional assumptions on the  $v_{it}$  term,  $\hat{\beta}_k$  is obtained in the second stage. And then, we get the TFP for each firm. In Fig. 10 we report the kernel density of the TFP (left panel) and net worth (right panel) for both exporters and non-exporters. It is worth noting that, on average, exporters present higher TFP and net worth. However, the distributions reveal a great area of overlapping. We believe that this could indicate the coexistence of different forces, beyond TFP, that may explain the decision to export.

Given the high correlation of both TFP and Net Worth with their own lags and given that our model is static, we run the following specification:

$$Export_{i,t} = \alpha + \beta_0 TFP_{i,t} + \beta_1 NetWorth_{i,t} + \beta_3 PAVITT + e_{i,t} \quad (37)$$

where Pavitt is the well-known Pavitt's taxonomy, which distinguish between supplier-dominated, scale-intensive, specialized suppliers and science-based firms. As mentioned above, we do not intend to study any causal effect between the variables but only the existence of the correlations emerged from the theoretical framework.

In table 2 we report the results of our simple econometric model. As expected, both net worth and TFP have a positive effect on the probability to be an exporter, but it is worth noting the magnitude of the former is twice that of the latter. Hence, firms' financial

Table 2: Probit, baseline

Variable	Export
Net worth (log)	0.172*** (0.017)
TFP	0.934 *** (0.032)
Pavitt	0.126 *** (0.026)
Constant	0.191 *** (0.071)

*s.e.* in parentheses

\*  $p < 0.1$ , \*\*  $p < 0.05$ , \*\*\*  $p < 0.01$

robustness seems to play a key role in the decision to export. Finally, it is confirmed that firms in high-tech sectors have higher probability to be an exporter.

## 5 Conclusions

The recent burgeoning literature on globalization has brought about a change in perspective from the analysis of patterns of international trade at the industry level to firms' characteristics, stressing the role of firms' heterogeneity in the fragmentation of production. Melitz (2003) has pioneered the exploration of the role of heterogeneity, in terms of firms' productivity, in shaping the decision to export, showing that only the most productive firms export abroad.

However, empirical studies have shown that heterogeneity among firms regards also size, wages paid, the ratio between skilled and unskilled workers, the investments in R&D, the strategies adopted to conquer foreign markets and credit constraints. Therefore, there is still room for studying '*the origins of firms' heterogeneity and the role of internal firm organization*'.

In this paper, we develop a static model in which firms are heterogeneous in terms of both productivity and financial robustness. Indeed, firms may borrow funds from the banking system to cover total production costs and the banking system charges different interest rates to heterogeneous firms according to their own level of net worth.

Our framework is different from that put forward by Chaney (2005) because we explicitly model the financial system. We show that firms with low productivity may be exporters if they have enough net worth to incur the higher exporting costs. Moreover, even a really high net worth may not guarantee the presence in both domestic and foreign

markets if the firm does not have a *minimum* level of productivity. Finally, the effects of changes in transport costs, fixed costs for exporters and the financial constraints with both symmetric and asymmetric countries are analysed.

We believe that three directions should be followed to further improve the analysis. Firstly, an endogenous firms' default should be introduced. Secondly, a more complex financial sector may be modelled determining the equilibrium interest rate matching supply and demand of credit. Finally, both productivity and net worth should be endogenously determined. All these research goals can be achieved only moving towards a dynamic model.



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