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Fighting Mobile Crime*

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Abstract

We develop a model in which two countries choose their enforcement levels non-cooperatively, in order to deter native and foreign individuals from committing crime in their territory. We assume that crime is mobile, both ex ante (migration) and ex post (fleeing), and that criminals who hide abroad after having committed a crime in a country must be extradited back. We show that, when extradition is not too costly, countries overinvest in enforcement compared to the cooperative outcome: insourcing foreign criminals is more costly than paying the extradition cost. By contrast, when extradition is sufficiently costly, a large enforcement may induce criminals to flee the country in which they have perpetrated a crime. Surprisingly, the fear of extraditing criminals enables countries to coordinate on the efficient (cooperative) outcome.

JEL Classification: K14, K42

Keywords: Crime, Enforcement, Extradition, Fleeing, Migration.

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1. Introduction

Globalization has substantially contributed to improving living standards over the last decades. At the same time, however, it has also influenced the way in which criminals behave, by giving them the possibility to move some of their illegal interests in foreign countries. For example, many criminal organizations, including Mafia, have progressively expanded their sphere of influence and relocated some activities abroad (Varese, 2006, 2011). Sociologists have long debated about crime mobility, and recognized that it is a salient aspect to be taken into account when designing policies aimed at deterring crime (see, e.g., Bernasco, 2014, and Morselli and Royer, 2008, among others). Governments have also realized the threat from mobile crime and decided to react accordingly, by signing international conventions to coordinate collective responses to such a common threat.¹

Surprisingly, despite the existence of an established literature on the economics of crime, no formal economic model exists that studies the effects of crime mobility on the optimal enforcement of criminal law. Although many papers have investigated the decision by criminals about whether to commit a crime (extensive margin) and about the amount of crime to commit (intensive margin), little is known about the effects of crime mobility in settings in which countries design their enforcement systems in a non-cooperative way. How should enforcement policies be designed when crime is mobile? What type of mobility matters? Do countries make inefficient choices when they behave non-cooperatively? If so, why? Answering these questions is of paramount importance to better understand the bright and the dark side of enforcement policies, and to interpret the existing patterns of crime migration. Moreover, understanding the logic behind countries’ enforcement decisions may help governments better coordinate their fight against crime.

The mobility margins studied in this paper differ from the usual extensive margin, in that they are intrinsically associated with the idea of competition (see, e.g., Lehman et al., 2014). Indeed, crime mobility is a special phenomenon, which could involve not only ex-ante mobility — i.e., felons moving across borders to perpetrate crime abroad (migration) — but also ex-post mobility — i.e., felons escaping from the country where they have perpetrated crime, in order to shield themselves against the risk of apprehension (fleeing). Extraditing back these criminals often involves cumbersome bureaucratic procedures, which make extradition very costly for the demanding country.² These costs

¹For example, the United Nations Convention against Transnational Organized Crime entered into force in 2003 with this objective.
²See, e.g., People ex rel. Westbrook v. O’Neill, 378 Ill. 324 (Ill. 1941). Extradition is the act by which one nation delivers up an individual, accused or convicted of an offense outside its own territory.
could influence Governments’ strategic behavior in the process of setting up their enforcement systems, over and above the standard effects associated with ex-ante migration. Why should civilized nations pose similar obstacles to the implementation of extradition?

To study these issues, we set up a model in which two countries (Governments) choose their enforcement levels non-cooperatively, in order to deter native and foreign individuals from committing crime in their territory. Criminals are heterogeneous along two migration-related dimensions: a migration cost, which is borne when individuals decide to commit the crime abroad (ex-ante mobility); and a fleeing cost, which is borne when individuals hide abroad after having committed a crime in a country (ex-post mobility). Upon observing the countries’ enforcement decisions, individuals choose whether to commit the crime, where to operate, and whether to flee the country whose law they have infringed. The analysis builds on the following trade-off. On the one hand, if a country sets a higher enforcement level than the other country, in equilibrium it will outsource crime (i.e., some natives will decide to perpetrate crime abroad). However, the individuals who stay in the country and break the law, will subsequently try to flee and hide abroad. As a consequence, the country will have to pay the cost of extraditing back these criminals. On the other hand, if a country sets a lower enforcement level than the other country, in equilibrium it will be targeted by foreign criminals (i.e., it will insource crime). However, the country will save on extradition costs.

We show that the extradition cost plays a key role in the analysis. Specifically, as long as extradition is costly, the game features a continuum of symmetric equilibria, in which both countries choose the same enforcement level. In these equilibria, there is neither migration nor fleeing. Each equilibrium must be robust to two types of deviations. First, no country must have an incentive to set enforcement above the equilibrium level with the aim of outsourcing criminals (upward deviation). Second, no country must have an incentive to set enforcement below the equilibrium level with the aim of saving on extradition costs (downward deviation). A symmetric equilibrium featuring too large an enforcement level is likely to be undercut, since a downward deviation would lead the country to save on enforcement costs. On the other hand, a symmetric equilibrium featuring too low an enforcement level is not robust to upward deviations, which allow the country to save on extradition costs. The tension between these two opposing forces shapes the equilibrium set. Interestingly, when extradition is costless, only upward deviations matter, since the migration effect is not balanced by the presence of a fleeing concern. Hence, a race to the top takes place: there is a unique symmetric equilibrium in which to another nation or state, which is competent to try and punish the criminal and demands him.
both countries set the highest enforcement level in the set mentioned before.

Next, we study the efficiency properties of these equilibria. We first characterize the cooperative solution, which minimizes the sum of the two countries’ loss function. We show that this solution is symmetric (and, as such, it features neither migration nor fleeing) and corresponds to the autarkic benchmark. Moreover, we show that the cooperative solution cannot be decentralized when the extradition cost is too small. Indeed, in this region of parameters, countries tend to overinvest in enforcement when playing non-cooperatively, since insourcing foreign criminals is more expensive than paying the extradition cost. By contrast, when the extradition cost is sufficiently large, setting a high enforcement level may induce fleeing, which requires countries to pay for the extradition procedures. The fear of incurring the extradition cost enables countries to coordinate on equilibria featuring an enforcement level even lower than the cooperative (efficient) solution. As a result, in this region of parameters, the cooperative solution can be decentralized as an equilibrium of the non-cooperative game. Interestingly, this result implies that, when extradition is relatively cheap, international agreements that set a common enforcement standard are required to achieve a cooperative solution. With a sufficiently high extradition cost, instead, these agreements may not be necessary. In other words, countries that are under the threat of mobile crime may wish to commit to costly and long extradition procedures, in order to achieve efficiency without the need of setting up an explicit enforcement treaty. In this sense, our model offers a novel (economic) rationale for the controversially costly and cumbersome extradition procedures observed around the world (see, e.g., Bassiouni, 2014, Margolies, 2011, and Moore, 1911, among others).

Our analysis is mainly related to the literature studying the relation between expected penalties on an illegal activity and the harm that it inflicts to society (see, e.g., Becker, 1968; Landes and Posner, 1975; Polinsky and Shavell, 1984; Friedman, 1981; Stigler, 1970; Friedman and Sjostrom, 1991; Mookherjee and Png, 1992, 1994; Polinsky and Shavell, 1992; Shavell, 1991, 1992, and Wilde, 1992). All these models have overlooked the role of crime outsourcing and criminal fleeing. As a consequence, they are silent on how potential mobility by criminals may affect the design of optimal enforcement policies by competing governments. This is the starting point of our analysis.

The paper is also connected to, and motivated by, the empirical literature on migration and crime. Buonanno and Pazzona (2014) and Scognamiglio (2018) look at the effect of the geographical relocation of Mafia members on crime. These studies find evidence that the geographical mobility of Mafia members has contributed to the diffusion of organized crime. Further, these studies also show that the geographical mobility of Mafia members has contributed to the diffusion of organized crime.

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\(^3\)See also Crinò, Immordino and Piccolo (2017) for empirical evidence on this relationship.
crime in Italy. There is also a growing body of evidence on the relationship between foreign immigration and crime. Borjas, Grogger and Hanson (2010) and Alonso-Borrego, Garoupa and Vázquez (2012) find that immigration increased crime in the US and Spain, respectively. Bell, Fasani and Machin (2013) find that the wave of asylum seekers in the UK caused a significant increase in crime, whereas the post-2004 inflow of people from the EU accession countries did not.\footnote{Other less closely related papers have tested the effect of a change in the legal status of immigrants on crime (see Baker, 2015; Mastrobuoni and Pinotti, 2015; Pinotti, 2017).} The link between migration and crime documented by this empirical literature represents the starting point of our analysis.

The rest of the paper is organized as follows. Section 2 sets up the baseline model. Section 3 characterizes the optimal policy both for symmetric and asymmetric equilibria. Section 4 presents some extensions. Section 5 concludes. All proofs are in the Appendix.

2. The model

Players. Consider two countries — or two states within the same federal country — denoted by $i \in \{A, B\}$. In each country there is a continuum of potential criminals, which can move across the border to carry out an illegal activity (crime). The crime imposes an harm $h > 0$ to the country where it is perpetrated. Conditional on the target (home or foreign) country, agents decide whether to commit the crime. If they do so, they obtain a random (monetary) benefit $\pi \in [0, 1]$. This can be interpreted as the result either of ability or of some contingencies (unknown to Governments when enforcement is set) that can make a crime relatively more or less profitable.\footnote{For simplicity, we ignore the possibility for agents to also choose among different types of crime. See, e.g., Mookerjee and Png (1994) for a model (without crime mobility) in which this possibility is taken into account.} Agents are heterogeneous along two migration-related dimensions: (i) a random migration cost $m \in [0, M]$, which they bear when deciding to commit the crime abroad; and (ii) a random fleeing cost $l \in [0, L]$, which they bear if they decide to flee the country after having committed the crime.

Mobility costs and crime profitability. For simplicity, we assume that the three (random) characteristics described above are identically and independently distributed across individuals and countries. The ex ante mobility cost corresponds to a loss in utility due, for instance, to the need of setting up a new illegal network, transporting people and weapons, learning a different language, and adapting to another culture. Denote by $G(m)$ the cumulative distribution function of the migration cost — i.e., the mass of criminals with migration costs below $m$ — whose density is $g(m)$. We also consider the
possibility that an individual who commits a crime in country $A$ might choose to flee that country and hide abroad (in country $B$) to avoid the sanction imposed by country $A$. When this happens, country $B$ has to help catching the criminal and extradite him back to country $A$. In this case, country $A$ incurs the extradition cost $x > 0$. We denote by $Z(l)$ the cumulative distribution function of the fleeing cost — i.e., the mass of criminals with fleeing costs below $l$ — whose density is $z(l)$. Finally, the cumulative distribution function of returns from crime is $F(\pi)$, with density $f(\pi)$.

**Sanctions and enforcement.** In keeping with the ‘territorial principle’in criminal law (see, e.g., Perkins, 1971), we assume that the country where the crime is committed has jurisdiction on the offence. We assume that each Government always sanctions the offence with the highest possible penalty. This is without loss of generality in our model, since each individual chooses whether to commit a single harmful act (see, e.g., Becker, 1968; Landes and Posner, 1975; Polinsky and Shavell, 1984; Friedman, 1981) and Governments always set the sanction at the maximum possible level. For simplicity, and to save on notation, we also normalize the maximum possible penalty in each country to 1. The (endogenous) probability of apprehension in country $i$ is denoted by $p_i \in [0, 1]$. The cost of enforcement is linear and given by $cp_i$ for every country (see, e.g., Mookherjee and Png, 1994). All players are risk neutral. Following the literature, all sanctions will be interpreted as the monetary equivalent of the imprisonment terms, fines, damages, and so forth, to which criminals expose themselves. We assume that governments are unable or unwilling to base sanctions on migration cost, fleeing status, benefit from crime or native country.

**Timing.** The timing of the game is as follows:

$t = 0$ Governments simultaneously commit to an enforcement level $p_i$.

$t = 1$ Knowing each country’s enforcement level, the crime profitability and the migration cost (but being uncertain about the fleeing cost) agents decide whether to commit the crime and in which country.

$t = 2$ Criminals learn their fleeing cost and decide whether to flee the country.

$t = 3$ Sanctions are imposed to criminals who get caught. Extradition costs (if any) are paid.

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6For instance, this the case in the US, where the extradition cost is borne by the demanding State.
The idea that criminals learn their fleeing costs after committing the crime seems natural. Indeed, various contingencies, unexpected at the time a criminal decides to break the law, can influence these costs (e.g., the ability of the police officers in charge of the case, the possibility of getting injured during the crime, an unexpected reaction by the victims, and the presence of a witness on the crime scene).

In line with an intuitive reputation argument, we assume that countries extradite back criminals who have infringed their laws.

**Equilibrium.** Each Government chooses the enforcement level that minimizes a loss function determined by the sum of: (i) the expected harm from domestic crime (which can be caused both by residents and immigrants); (ii) the cost of extraditing back criminals who have fled to the other country after having committed the crime; and (iii) the enforcement costs, taking as given the enforcement level chosen by the other government. Criminals make decisions along three margins: whether to commit the crime, where to operate, and whether to flee the country after having committed the crime. The solution concept is Subgame Perfect Nash Equilibrium.

**Assumptions.** We impose the following assumptions:

**A1** \( f' (\cdot) < 0 \) and \( f (1) < \frac{c}{h} < f (0) \).

This assumption ensures that the cooperative benchmark — i.e., the enforcement level that minimizes the joint loss function of the two countries — has a unique internal solution.

**A2** The inverse hazard rate \( \frac{1-Z(l)}{z(l)} \) is decreasing.

In the Appendix, we impose additional technical requirements by studying the (sufficient) conditions under which the countries’ objective functions are well-behaved (i.e., they are strictly convex).

3. **Preliminaries**

Before characterizing the equilibrium of the game, it is useful to determine criminals’ behavior for given enforcement policies. The game is solved by backward induction. Hence, we begin with the analysis of the fleeing decisions occurring in stage \( t = 2 \).
Lemma 1. An agent who has committed a crime in country $i$ flees that country and hides in country $j$ if and only if

$$p_i \geq p_j + l \iff l \leq l_i = p_i - p_j.$$  \hfill (3.1)

As intuition suggests, criminals flee a country if the cost of doing so is sufficiently small and only if the destination country sets a lower enforcement level.

Moving backward, we can now determine criminals’ expected utilities and optimal decisions in stage $t = 1$. Suppose (without loss of generality) that $p_i \geq p_j$. Then, the expected utility of a criminal who is resident in $i$ and decides to commit the crime in his home country is

$$u_{ii} (p_i, p_j) \triangleq \pi - p_i (1 - Z (l_i)) - p_j Z (l_i).$$  \hfill (3.2)

This expression takes into account the expected cost of fleeing. Notice that, even if a criminal does not migrate, the enforcement of the foreign country affects his utility through the ex post fleeing decision. Hence, fleeing criminals also are responsive to the enforcement policy set abroad.

By contrast, the expected utility of a criminal who is resident in country $i$ and migrates to country $j$ is

$$u_{ij} (p_i, p_j) \triangleq \pi - p_j - m.$$  \hfill (3.3)

Moreover, as implied by Lemma 1, such a criminal will not return back to country $i$ since $p_i \geq p_j$.

Comparing (3.2) with (3.3), we can show the following result.

Lemma 2. A criminal who is resident in country $i$ immigrates to country $j$ if and only if

$$u_{ii} (p_i, p_j) \leq u_{ij} (p_i, p_j) \iff m \leq m_i \triangleq (1 - Z (l_i)) l_i.$$  \hfill (3.4)

The threshold $m_i$ identifies the marginal migrant, i.e., the criminal who is indifferent between migrating and committing the crime in his home country. Similarly to the fleeing decision (condition 3.1), the decision to migrate also depends on the difference between the two enforcement levels. Specifically, when $p_i = p_j$, there is no migration (and, of course, no fleeing), because expected sanctions are the same in the two countries. Notice that, the effect of $p_i$ on the marginal migrant is ambiguous, as stated in the following lemma.

Lemma 3. There exists a threshold $l_i^* \in (0, L)$ such that $\frac{\partial m_i}{\partial p_i} \geq 0$ if and only if $l_i \leq l_i^*$,
with $l_i^*$ being the unique solution of

$$\frac{1 - Z(l_i)}{z(l_i)} = l_i. \quad (3.5)$$

By contrast, $\frac{\partial m_i}{\partial p_j} \geq 0$ if and only if $l_i \geq l_i^*$.

The effect of a change in $p_i$ on the marginal migrant $m_i$ is determined by two contrasting forces. On the one hand, as $p_i$ increases, the expected utility obtained by a criminal who commits the crime in his home country $(u_{ii} (\cdot))$ drops, because the expected sanction in that country is higher: other things being equal, more natives migrate to country $j$. On the other hand, the incentive to flee country $i$ also magnifies, since $l_i$ is increasing in $p_i$. Other things being equal, the strategy of committing the crime at home and then fleeing becomes relatively more attractive, making ex ante migration less appealing. The first effect dominates if $l_i$ is not too large, that is, when $p_i$ is close to $p_j$. The second effect, by contrast, dominates when $l_i$ is large enough, that is, when $p_i$ is sufficiently higher than $p_j$. Of course, the same ambiguity (with an opposite sign) holds when considering the effect of changing $p_j$ on $m_i$.

Let $\pi_i$ be the level of $\pi$ above which criminals who do not migrate (but flee with some probability) commit the crime in country $i$,

$$u_{ii} (\cdot) \geq 0 \iff \pi \geq \pi_i \triangleq p_i (1 - Z(l_i)) + p_j Z(l_i) = p_j + m_i. \quad (3.6)$$

Similarly, since we assumed $p_i \geq p_j$ (offenders who commit the crime in country $j$ will never flee that country), let $\pi_j$ be the level of $\pi$ above which natives of country $j$ commit the crime at home,

$$u_{jj} (\cdot) \geq 0 \iff \pi \geq p_j. \quad (3.7)$$

Finally let $\pi_j (m)$ be the level of $\pi$ above which an offender who is resident in country $i$ and has migration cost $m \leq m_i$ commits the crime in country $j$,

$$u_{ij} (\cdot) \geq 0 \iff \pi \geq \pi_j (m) \triangleq m + p_j. \quad (3.8)$$

Clearly, the higher is the migration cost, the higher the benefit $\pi$ has to be for the crime to be profitable for a migrant. Moreover, the higher is the enforcement level $p_j$ implemented by country $j$, the less profitable is migration to that country.

Summing up, under the hypothesis that $p_i \geq p_j$, conditions (3.1), (3.4), (3.6), (3.7) and (3.8) describe criminals’ optimal response to the enforcement policies implemented
by the two countries (see Figure 3.1).

4. Basic insights: small economy

In order to better understand the forces driving a country’s choice of enforcement, it is useful to start with the analysis of a small economy, which takes as given the enforcement level in the rest of the world. Building on the insights offered by this analysis, we will then extend the logic to the case of strategic interaction between the two countries. Hence, without loss of generality, in the rest of the section we focus on the decision making problem solved by country $A$ and take as given $p_B$, which can be interpreted as the average enforcement level taken worldwide.

As explained before, the difference $p_A - p_B$ determines the flows of criminals that migrate ex ante and flee ex post. Hence, country $A$’s loss function is

$$L^A(p_A, p_B) \triangleq c p_A + \begin{cases} \left(1 - G(m_A)\right) \left(1 - F(\pi_A)\right) \times \frac{h + xZ(I_A)}{h + extradition\ cost} & \text{if } p_A \geq p_B \\ h \int_{p_A}^1 dF(\pi) + h \int_0^{m_B} \left(1 - F(\pi_A(m))\right) dG(m) & \text{if } p_A < p_B \end{cases}$$
This loss function reflects how criminals move across the borders as a best response to A’s policy. When \( p_A \geq p_B \), country A sets a tougher policy than the rest of the world (country B). Hence, A is the outsourcing country, while the rest of the world is insourcing country: some criminals resident in A migrate to B \((m < m_A)\), while others commit the crime at home and flee afterwards \((m \geq m_A \text{ and } l \leq l_A)\). This implies that A will have to bring the latter criminals back, which costs \( xZ (l_A) \). By contrast, when \( p_A < p_B \), country A sets a more lenient policy than the rest of the world, so it saves on the fleeing cost but bears the additional harm produced by foreign criminals. Notice that this function is continuous and piecewise differentiable, with a kink at \( p_A = p_B \) (see the Appendix).

Hence, in order to characterize A’s optimal policy we have to consider each case in turn. From now on, we posit that \( \mathcal{L}^A (\cdot) \) is (strictly) convex in either case (we will derive in the Appendix sufficient conditions under which this conjecture holds).

Suppose first that \( p_A \geq p_B \). In this case, country A solves the following minimization problem:

\[
\min_{p_A \geq p_B} \{(h + xZ (l_A)) (1 - G (m_A)) (1 - F (\pi_A)) + cp_A \}.
\]

(4.1)

Differentiating with respect to \( p_A \) we have

\[
\frac{\partial \mathcal{L}^A (p_A, p_B)}{\partial p_A} \bigg|_{p_A \geq p_B} = c + x (1 - G (m_A)) (1 - F (\pi_A)) z (l_A) \left( \frac{\partial l_A}{\partial p_A} + \text{Fleeing effect (+)} \right) - (h + xZ (l_A)) f (\pi_A) \left( \frac{\partial \pi_A}{\partial p_A} \right) - (h + xZ (l_A)) g (m_A) \left( \frac{\partial m_A}{\partial p_A} (1 - F (\pi_A)) \right) \left( \text{Deterrence on natives (-)} \right) + (h + xZ (l_A)) j (m_A) \left( \frac{\partial j_A}{\partial p_A} (1 - F (\pi_A)) \right) \left( \text{Migration effect (?)} \right)
\]

When A is the outsourcing country, increasing \( p_A \) has the following effects over and above the obvious direct cost of enforcement: first, a higher enforcement induces more criminals to flee ex post, which is detrimental to country A because extradition is costly; second, a higher enforcement reduces the amount of crime by deterring natives to commit the crime at home; third, there is an ambiguous effect on migration, because a higher level of enforcement \( p_A \) has an ambiguous effect on the marginal migrant \( m_A \) (see equation 3.5 and its interpretation). In particular, it may well be the case that a higher enforcement lowers so much the fleeing cost to make ex ante migration a worst option than committing the crime at home and fleeing ex post.

Therefore, A’s problem features an interior solution \( p_A > p_B \) if and only if the deriva-
tive of the (convex) loss function is negative at $p_A \to p_B^+$,

$$\lim_{p_A \to p_B^+} \frac{\partial L^A(\cdot)}{\partial p_A} < 0 \iff \frac{c + x (1 - F(p_B)) z(0)}{\text{Marginal cost}} < \frac{h [f(p_B) + (1 - F(p_B)) g(0)]}{\text{Marginal benefit}}.$$  \hfill (4.2)

Otherwise, the function is minimized for some $p_A \leq p_B$. In brief, country $A$ has an incentive to set an enforcement level tougher than the rest of the world if the sum of the enforcement and fleeing (marginal) costs is small compared to the (marginal) benefit in terms of deterrence that such a policy would generate for $p_A$ sufficiently close to $p_B$.

Next, suppose that $p_A \leq p_B$. In this case, country $A$’s minimization problem is

$$\min_{p_A \leq p_B} \left\{ h (1 - F(p_A)) + h \int_0^{m_B} (1 - F(\pi_A(m))) dG(m) + cp_A \right\}. \quad (4.3)$$

Differentiating with respect to $p_A$ we have

$$\frac{\partial L^A(p_A,p_B)}{\partial p_A} \bigg|_{p_A \leq p_B} = c - \left[ \frac{h f(p_A)}{\text{Deterrence on natives (–)}} + \frac{h \int_0^{m_B} f(\pi_A(m)) dG(m) + h (1 - F(\pi_A(m_B))) g(m_B) \frac{\partial m_B}{\partial p_A}}{\text{Deterrence on immigrants (–)}} + \frac{\partial m_B}{\partial p_A}}{\text{Migration effect (?)}} \right].$$

When $A$ is the insourcing country, increasing $p_A$ has the following effects over and above the obvious direct cost of enforcement: first, it clearly deters both native and immigrants from committing crime, since a higher $p_A$ reduces the profitability of perpetrating crime in country $A$; second, since a higher $p_A$ increases the cost of fleeing to foreign criminals (see again equation 3.5), the effect on the marginal migrant is ambiguous. In other words, on the extensive margin, a higher $p_A$ may induce more criminals to migrate from abroad to $A$.

$A$’s problem features an interior solution $p_A < p_B$ if and only if the derivative of the (convex) loss function is positive at $p_A \to p_B^-$,

$$\lim_{p_A \to p_B^-} \frac{\partial L^A(\cdot)}{\partial p_A} > 0 \iff \frac{c > h [f(p_B) + (1 - F(p_B)) g(0)]}{\text{Marginal benefit}}.$$  \hfill (4.4)

In brief, $A$ has an incentive to set an enforcement level more lenient than the rest of the world if the marginal cost of enforcement ($c$) is larger than the marginal benefit in terms of deterrence that such a policy would generate when $p_A$ is sufficiently close to $p_B$. 

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Gathering (4.2) and (4.4), we can state the following result:

**Proposition 1.** There exist two thresholds \( p_L^* \in (0, 1) \) and \( p_H^* \in (0, 1) \), with \( p_L^* < p_H^* \), such that country A’s optimal enforcement level, say \( p_A^* \), has the following features:

- \( p_A^* > p_B \) if and only if \( p_B < p_L^* \). The threshold \( p_L^* \) is the unique solution of
  \[
  c = h \left[ (1 - F(p)) g(0) + f(p) \right] - x (1 - F(p)) z(0).
  \]

- \( p_A^* < p_B \) if and only if \( p_B > p_H^* \). The threshold \( p_H^* \) is the unique solution of
  \[
  c = h \left[ f(p) + (1 - F(p)) g(0) \right].
  \]

- \( p_A^* = p_B \) for every \( p_B \in P = [p_L^*, p_H^*] \subseteq [0, 1] \).

This result illustrates how country A sets its enforcement level when it takes as given the enforcement in the rest of the world. There are two main forces that shape this choice. On the one hand, country A would like to shield itself against migration of foreign criminals, which requires a relatively high enforcement level (high \( p_A \)). On the other hand, such a strong enforcement may induce natives to flee ex post, which would raise extradition costs. This novel trade-off determines the optimal enforcement level set by A and the extent to which a small country is tougher or more lenient with criminals compared to the rest of the world.

In order to better understand the logic behind the result, consider first the case where \( p_B \) is sufficiently low: in this case, A has an incentive to raise its enforcement level above \( p_B \). The reason is that saving on the extradition cost would require \( p_A \) smaller than \( p_B \). This would both attract foreign criminals and (since \( p_B \) is already small) sensibly weaken the deterrence on natives. Hence, it is relatively too costly for A to avoid paying the extradition cost, and it is optimal to strengthen deterrence above \( p_B \) in order to induce as many natives as possible to perpetrate their crimes abroad, while also lowering natives’ incentive to commit crimes.

By contrast, when \( p_B \) is sufficiently large, A has an incentive to lower its enforcement level below \( p_B \). In this region of parameters, it is relatively too costly for A to avoid migration (say by setting \( p_A \) above \( p_B \)). Hence, A is mainly concerned with discouraging native criminals from fleeing the home country, thus avoiding paying the extradition cost.

Finally, when \( p_B \) takes intermediate values, the two forces described above offset each other, so that it is optimal for a small country to keep up with the international standard, i.e., it is optimal for A to set \( p_A^* = p_B \).
The following comparative statics offers some interesting implications of the model.

**Proposition 2.** The optimal enforcement chosen by Country A is such that:

- the region of parameters in which \( p_A^* < p_B \) expands as \( c \) grows large and shrinks as \( h \) grows large.

- the region of parameters in which \( p_A^* = p_B \) expands as \( x \) grows large; the effect of \( h \) is ambiguous.

- the region of parameters in which \( p_A^* > p_B \) shrinks as \( c \) and \( x \) grow large and expands as \( h \) grows large.

The intuition behind this comparative statics is straightforward. When the cost of enforcement increases (higher \( c \)), country A is less willing to invest public funds into enforcement activities; hence, the region of parameters in which \( p_A^* \) falls short of \( p_B \) expands, while the region of parameters in which \( p_A^* \) exceeds \( p_B \) shrinks. The comparative statics on \( h \) is also rather intuitive. As the harm produced by the crime becomes more serious (higher \( h \)), country A is ceteris paribus more willing to deter both native and foreign individuals from breaking the law; hence, the region of parameters in which \( p_A^* \) falls short of \( p_B \) shrinks, while the region of parameters in which \( p_A^* \) exceeds \( p_B \) expands.

The comparative statics on \( x \) is the most interesting. When extraditing criminals becomes more costly (e.g., because of long bureaucratic procedures) country A has a lower incentive to choose a policy more lenient than the rest of the world. Indeed, if it does so, criminals committing the crime in A will be more likely to flee the country, which is costly because they will need to be extradited back.

**5. Strategic Interaction**

We now turn to study the strategic interaction between countries, i.e., the case in which \( p_A \) and \( p_B \) are both endogenous and determined simultaneously in equilibrium. We first characterize the cooperative solution in which the two enforcement levels maximize the countries’ joint welfare (i.e., minimize their joint loss function) and then turn to the non-cooperative solution. The objective of the analysis is to study the efficiency properties of the equilibria, the role played by the model’s underlying parameters (e.g., fleeing costs) and the scope (if any) for international cooperation between countries.
5.1. Cooperative benchmark

Suppose that $p_A$ and $p_B$ are chosen cooperatively, i.e., as a solution of the following problem

$$\min_{(p_A, p_B) \in [0,1]^2} \sum_{i=A,B} L^i(p_i, p_{-i}).$$

We can show the following preliminary result.

**Lemma 4.** *The cooperative solution never features asymmetric enforcement levels, i.e., the optimal policy is such that $p^c_i = p^c$ for every $i \in \{A, B\}.*

Intuitively, when the countries choose cooperatively, it is never optimal to set two different enforcement levels, because an asymmetric solution would generate fleeing and thus extradition costs, which are a pure waste from a joint welfare point of view. By contrast, the enforcement of a symmetric outcome rules out both fleeing and ex ante migration. Therefore, the enforcement level (say $p^c$) that maximizes the countries’ joint welfare solves

$$\min_{p \in [0,1]} 2 [h(1 - F(p)) + cp].$$

In words, in a symmetric solution, the joint loss induced by crime is equal to twice the sum of the cost of enforcement and the harm caused by criminals who decide to break the law, i.e., those for whom $\pi \geq p$. We can thus show the following intuitive result.

**Proposition 3.** *When countries play cooperatively, they choose a symmetric enforcement level $p^c \in (0,1)$ that solves the following first-order condition*

$$hf(p) = c.$$

Intuitively, the cooperative solution — like the well-known autarkic solution — must balance the marginal cost of enforcement with the marginal benefit that the reduction in crime driven by the higher enforcement level produces.

5.2. Non-cooperative outcome

We now turn to the analysis of the non-cooperative game. Since countries are identical we consider symmetric equilibria, i.e., $p_A = p_B = p^*$. In order for $p^*$ to be a symmetric equilibrium, it must be immune to upward and downward deviations. Hence, each country must have no incentive either to undercut $p^*$ or to choose an enforcement level above $p^*$.

Consider, without loss of generality, a deviation by country $A$, and assume first that $p_A > p^*$, so that criminals flee and migrate from $A$ to $B$. The best possible deviation is
the solution of the minimization problem (4.1) with $p_B = p^*$. Evaluating the first-order condition at $p_A = p_B = p^*$, an upward deviation is never profitable if and only if

$$\lim_{p_A \to p^+} \frac{\partial \mathcal{L}_A (\cdot)}{\partial p_A} < 0 \iff c + x (1 - F(p^*)) z(0) > h [(1 - F(p^*)) g(0) + f(p^*)]. \quad (5.1)$$

This condition reflects the trade-off discussed in the case of a small economy for $p_A > p_B$. In words, the reduction of crime induced by a marginal increase in the enforcement level (above $p^*$) must not be worth the cost of strengthening enforcement and the waste of public resources needed to extradite back criminals who manage to flee the country.

By the same token, $p^*$ is an equilibrium if it is immune to downward deviations, i.e., such that $p_A < p^*$. The most profitable of such deviations is the solution to the minimization problem (4.3) with $p_B = p^*$. Evaluating the first-order condition at $p_A = p_B = p^*$, deviating downward is never profitable if and only if

$$\lim_{p_A \to p^-} \frac{\partial \mathcal{L}_A (\cdot)}{\partial p_A} > 0 \iff c < h [(1 - F(p^*)) g(0) + f(p^*)]. \quad (5.2)$$

Once again, this condition reflects the trade-off discussed in the case of a small economy for $p_A < p_B$. Intuitively, $p^*$ is an equilibrium if the enforcement costs is small compared to the benefit in terms of deterrence that such a deviation would generate.

Summing up, an equilibrium candidate in which both countries choose the same level of enforcement must satisfy simultaneously (5.1) and (5.2), which yields exactly the set $P$ characterized before. Hence:

**Proposition 4.** The game features a continuum of symmetric equilibria, i.e., any enforcement $p^* \in P$. The equilibrium is unique when extradition is costless, i.e., if $x = 0$. In this limiting case, $p^* = p^*_H$.

The set of symmetric equilibria is bounded from below and from above (see Figure 5.1). A symmetric equilibrium featuring too large an enforcement level is likely to be undercut, since such a deviation would lead the deviating country to save on enforcement costs. On the other hand, a symmetric equilibrium featuring too low an enforcement level is not robust to upward deviations, which allow the deviating country to save on the extradition cost.

Notice that as the extradition cost $x$ grows large, the equilibrium set widens (since $p^*_L$ is decreasing in $x$, as discussed before). Interestingly, when extradition is costless, there is a unique symmetric equilibrium in which both countries set the enforcement level to the highest level within the set $P$: a race to the top. The reason is that only upward
deviations matter when extradition is costless, since the migration effect is not balanced by the presence of a fleeing concern.

5.3. Selection and efficiency

In Proposition 4, we have shown that the game may feature a continuum of symmetric equilibria. One may wonder which equilibrium will be selected. To address this issue, we use a selection criterion based on Pareto dominance. In particular, we assume that countries select the equilibrium that maximizes joint welfare, i.e., the \( p^* \in P \) that minimizes the sum of their expected losses. This equilibrium need not be the cooperative outcome \( p^c \), since it is not clear a priori whether this solution lies within the equilibrium set \( P \).

In the next proposition, we show that the cooperative outcome can be decentralized as an equilibrium of the game if and only if the extradition cost is sufficiently large compared to the harm,

**Proposition 5.** \( p^c \in P \) if and only if

\[
x > x^2 \triangleq h \frac{g(0)}{z(0)}.
\]

Otherwise, \( p^c < p^*_L \) and the countries coordinate on \( p^*_L \).

Hence, the cooperative solution cannot be decentralized when the extradition cost is relatively small. Indeed, in this region of parameters, countries tend to overinvest in enforcement when playing non-cooperatively, since insourcing foreign criminals is relatively more costly than paying the extradition cost. By contrast, when the extradition cost is sufficiently large, setting a high enforcement level may induce fleeing, which requires countries to pay for the extradition procedures. The fear of incurring this cost may then lead countries to coordinate on equilibria featuring an enforcement level below the cooperative solution, which does not take extradition costs into account. As a result, in this
region of parameters, the cooperative solution can be decentralized as an equilibrium of the non-cooperative game. Interestingly, this result implies that international agreements that set a common enforcement standard are required to achieve a cooperative solution when extradition is relatively cheap, while these agreements may not be necessary with a sufficiently high extradition cost.

6. Conclusion

We have presented the first formal economic model studying the effects of crime mobility both ex ante (migration) and ex post (fleeing) on the optimal enforcement of criminal law. We have shown that, when extradition is not too costly, countries overinvest in enforcement compared to the cooperative outcome: insourcing foreign criminals is more costly than paying the extradition cost. By contrast, when extradition is sufficiently costly, a large enforcement may induce criminals to flee the country in which they have perpetrated a crime. Then, the fear of extraditing these criminals back enables countries to coordinate on the cooperative (efficient) outcome. These results contribute to better understand how enforcement systems should be designed when crime is mobile. In particular, the model offers an explanation for the complexity of many international treaties.

We have assumed that the two countries are identical, so that in equilibrium there is neither migration nor fleeing. However, in reality, we do observe both phenomena. This might be due either to cross-country asymmetries, which we did not model, or to a failure of coordination, as our model implies. Introducing sources of heterogeneity between countries in our model would complicate the analysis without altering its qualitative results. For instance, when countries differ in terms of the enforcement and the extradition costs, our comparative statics already suggests the effect of these potential asymmetries on the equilibrium of the game. Indeed, we guess that a country that is little efficient in enforcing the law (i.e., a country that has a high enforcement cost) is likely to set a low enforcement level in equilibrium, thereby attracting foreign criminals. Similarly, countries with high extradition costs would be more likely to set a relatively low enforcement level in equilibrium, in order to shield themselves against fleeing. Although these asymmetries are certainly relevant in real life, the implications of our analysis are rather general. In this sense, our analysis should be seen as a normative benchmark to evaluate the effects of crime mobility in the design of optimal enforcement policies.
Appendix

**Proof of Lemma 1.** The proof of this result follows immediately from the comparison between the utility that an individual who has committed a crime in country $i$ obtains when he does not leave that country — i.e., $p_i$ — and the utility that he obtains when he flees the country — i.e., $l + p_j$. ■

**Proof of Lemma 2.** The proof of this result follows immediately from the comparison between $u_{ii}(p_i, p_j)$ and $u_{ij}(p_i, p_j)$. ■

**Proof of Lemma 3.** The proof of this result is simple. Recall that

$$\frac{\partial m_i}{\partial p_i} = 1 - Z(l_i) - z(l_i) l_i.$$  

This expression is positive if and only if

$$\frac{1 - Z(l_i)}{z(l_i)} \geq l_i,$$

which always holds for $l_i$ sufficiently small — i.e., $\frac{\partial m_i}{\partial p_i} > 0$ for $l_i = 0$. Next, it is also immediate to verify that $\frac{\partial m_i}{\partial p_i} < 0$ for $l_i = L$. Hence, since $\frac{1 - Z(l_i)}{z(l_i)}$ is decreasing by assumption A2, it follows that the solution of

$$\frac{1 - Z(l_i)}{z(l_i)} = l_i,$$

is unique.

The same argument can be used to sign $\frac{\partial m_j}{\partial p_j}$. ■

**Proof of Proposition 1.** In order to study the behavior of the optimal enforcement chosen by country $A$, it is useful to study the sign of the following derivatives

$$\lim_{p_A \to p_B^+} \frac{\partial \mathcal{L}^A(\cdot)}{\partial p_A} = c - h [f(p_B) + (1 - F(p_B)) g(0)] + x (1 - F(p_B)) z(0),$$

and

$$\lim_{p_A \to p_B^-} \frac{\partial \mathcal{L}^A(\cdot)}{\partial p_A} = c - [f(p_B) + (1 - F(p_B)) g(0)].$$

Where clearly, for given $p_B$, one has

$$\lim_{p_A \to p_B^+} \frac{\partial \mathcal{L}^A(\cdot)}{\partial p_A} > \lim_{p_A \to p_B^-} \frac{\partial \mathcal{L}^A(\cdot)}{\partial p_A}. 19$$
Hence, 
\[
\lim_{p_A \to p_B^+} \frac{\partial \mathcal{L}^A (\cdot)}{\partial p_A} < 0 \Rightarrow \lim_{p_A \to p_B^+} \frac{\partial \mathcal{L}^A (\cdot)}{\partial p_A} < 0,
\]
and 
\[
\lim_{p_A \to p_B^+} \frac{\partial \mathcal{L}^A (\cdot)}{\partial p_A} > 0 \Rightarrow \lim_{p_A \to p_B^+} \frac{\partial \mathcal{L}^A (\cdot)}{\partial p_A} > 0.
\]

Therefore, since we assumed that the loss function $\mathcal{L}^A (\cdot)$ is strictly convex, it is optimal for country $A$ to set $p_A^* > p_B$ if and only if 
\[
c < \Phi (p_B) \triangleq h \left[ f (p_B) + (1 - F (p_B)) g (0) \right] - x (1 - F (p_B)) z (0). \tag{1}
\]

Notice that, $\Phi' (p_B) < 0$ by assumption $A1$. Moreover, by $A1$ it must also be $\Phi (1) < c < \Phi (0)$. Hence, there exists a unique value $p_L^* \in (0, 1)$, which solves $c = \Phi (p_B)$, such that $(1)$ holds for every $p_B < p_L^*$. As a result, $p_A^* > p_B$ for every $p_B < p_L^*$. 

By the same token, it is optimal for country $A$ to set $p_A^* < p_B$ if and only if 
\[
c > \overline{\Phi} (p_B) \triangleq h \left[ f (p_B) + (1 - F (p_B)) g (0) \right]. \tag{2}
\]

Notice that $\overline{\Phi}' (p_B) < 0$ by assumption $A1$. Moreover, by $A1$ it must also be $\overline{\Phi} (1) < c < \Phi (0)$, there exists a unique value $p_L^* \in (0, 1)$, which solves $c = \Phi (p_B)$, such that $(2)$ holds for every $p_B > p_L^*$. Hence, $p_A^* < p_B$ for every $p_B > p_L^*$. 

Finally, it is easy to verify that $\overline{\Phi} (p_B) > \overline{\Phi} (p_B)$ so that $p_A^* > p_L^*$. Hence, $p_A^* = p_B$ for every $p_B \in [p_L^*, p_H^*]$. 

**Proof of Proposition 2.** The proof of this result follows immediately from the fact that the functions $\Phi (p_B)$ and $\Phi (p_B)$ are decreasing in $p_B$, increasing in $h$ and non-increasing in $x$. 

**Proof of Lemma 4.** Consider a point $(p_A, p_B) \in [0, 1]^2$, with $p_A \geq p_B$ without loss of generality. Let $\hat{p} = \frac{1}{2} p_A + \frac{1}{2} p_B$, we want to show that 
\[
\mathcal{L}^A (p_A, p_B) + \mathcal{L}^B (p_B, p_A) \geq \mathcal{L}^A (\hat{p}, \hat{p}) + \mathcal{L}^B (\hat{p}, \hat{p}) = 2 \left[ h (1 - F (\hat{p})) + c \hat{p} \right].
\]

To begin with notice that $\mathcal{L}^A (p_A, p_B) + \mathcal{L}^B (p_B, p_A) \geq 
\[
[h (1 - F (p_A)) + c p_A] + [h (1 - F (p_B)) + c p_B] - h G (m_A) [F (\pi_B (m_A)) - F (\pi_A)].
\]

Next, recall that $\pi_B (m_A) \triangleq m_A + p_B > m_A$ and that $\pi_A \triangleq m_A + p_B$, so that $\pi_B (m_A) \triangleq \pi_A$. Hence,
\[
[h (1 - F (\pi_A)) + c p_A] + [h (1 - F (p_B)) + c p_B] - h G (m_A) [F (\pi_B (m_A)) - F (\pi_A)] = [h (1 - F (\pi_A)) + c p_A] + [h (1 - F (p_B)) + c p_B].
\]
Moreover, since \( \pi_A \Delta p_A (1 - Z(l_A)) + p_B Z(l_A) = p_A - l_A Z(l_A) < p_A \), it follows that
\[
[h (1 - F (\pi_A)) + cp_A] + [h (1 - F (p_B)) + cp_B] > [h (1 - F (p_A)) + cp_A] + [h (1 - F (p_B)) + cp_B].
\]
Finally, since \( f'(\cdot) < 0 \) by A1 it follows that
\[
\frac{1}{2} [h (1 - F (p_A)) + cp_A] + [h (1 - F (p_B)) + cp_B] > [h (1 - F (\hat{p})) + c\hat{p}],
\]
which proves the result.

**Proof of Proposition 3.** Differentiating \( h (1 - F (p)) + cp \) with respect to \( p \) yields immediately the first-order condition \( hf'(p) = c \). By assumption A1 the objective function is strictly convex. Moreover, A1 also implies that the solution is interior since \( hf'(0) > c > hf'(1) \).

**Proof of Proposition 4.** The proof of this result follows immediately from the proof of Proposition 1. Any \( p^* < p_A^* \) cannot be a symmetric equilibrium of the game because it is always profitable for a country to deviate by choosing an enforcement level strictly larger than \( p_A^* \) since \( \frac{\partial^2 \pi_A(p,p)}{\partial p^2} < 0 \). Similarly, any \( p^* > p_B^* \) cannot be an equilibrium because it is always profitable for a country to deviate by choosing an enforcement level strictly lower than \( \bar{p} \) since \( \frac{\partial^2 \pi_B(p,p)}{\partial p_A^2} > 0 \).

**Proof of Proposition 5.** Let \( \mathcal{L}(p) \triangleq h (1 - F (p)) + cp \). Then, using the definition of \( p_A^* \), notice that
\[
\frac{\partial \mathcal{L}(p)}{\partial p} = -hf(p_A^*) + c = h (1 - F (p_B)) (hg(0) - xz(0)),
\]
which by the convexity of \( \mathcal{L}(p) \) directly implies the result — i.e., \( p^c > p_A^* \) if and only if \( hg(0) < xz(0) \). Using the definition of \( p_H^* \), notice also that,
\[
\frac{\partial \mathcal{L}(\bar{p})}{\partial p} = -hf(p_H^*) + c = h (1 - F (p_B)) hg(0) > 0,
\]
implying, again by the convexity of \( \mathcal{L}(p) \), that \( p^c < p_H^* \).

**Convexity of the loss functions.** We now characterize sufficient conditions under which the loss function \( \mathcal{L}_A (\cdot) \) is strictly convex in \( p_A \).

Consider first the case \( p_A \geq p_B \). Recall that \( l_A \triangleq p_A - p_B \), \( m_A \triangleq l_A (1 - Z(l_A)) \) and
\[
\pi_A \triangleq p_A (1 - Z(l_A)) + p_B Z(l_A) = p_B + m_A,
\]
so that
\[
\frac{\partial^2 \pi_A}{\partial p_A^2} = \frac{\partial^2 m_A}{\partial p_A^2} = -2z(l_A) - z'(l_A) l_A.
\]
Denote now $\beta (p_A) \triangleq (1 - G (m_A)) (1 - F (\pi_A)),$
$$\phi (p_A) \triangleq g (m_A) (1 - F (\pi_A)) + f (\pi_A) (1 - G (m_A)),$$ and
$$\varepsilon (p_A) \triangleq [g' (m_A) (1 - F (\pi_A)) - 2f (\pi_A) g (m_A) + f' (\pi_A) (1 - G (m_A))].$$
Then, $\beta' (p_A) = -\frac{\partial m_A}{\partial p_A} \phi (p_A)$ and
$$\beta'' (p_A) = -\frac{\partial^2 m_A}{\partial p_A^2} \phi (p_A) - \left(\frac{\partial m_A}{\partial p_A}\right)^2 \varepsilon (p_A).$$
Similarly, let $\alpha (p_A) \triangleq h + xZ (l_A),$ with $\alpha' (p_A) = xz (l_A) > 0$ and $\alpha'' (p_A) = xZ' (l_A).$ Hence,
$$\frac{\partial^2 \mathcal{L}^A (\cdot)}{\partial p_A^2} > 0 \iff \beta'' (\cdot) \alpha (\cdot) + 2\alpha' (\cdot) \beta' (\cdot) + \beta (\cdot) \alpha'' (\cdot) > 0,$$
Rearranging terms we have
$$-\alpha (\cdot) \varepsilon (\cdot) \left(\frac{\partial m_A}{\partial p_A}\right)^2 - \phi (\cdot) \left[\frac{\partial^2 m_A}{\partial p_A^2} \alpha (\cdot) + 2xz (\cdot) \frac{\partial m_A}{\partial p_A}\right] + \beta (\cdot) xZ' (\cdot) > 0,$$
Assume $z' (\cdot) \geq 0 \geq g' (\cdot).$ Then, $\varepsilon (\cdot) < 0$ and $\mathcal{L}^A (\cdot)$ is convex if
$$\frac{\partial^2 m_A}{\partial p_A^2} \alpha (\cdot) + 2xz (\cdot) \frac{\partial m_A}{\partial p_A} \leq 0,$$
substituting terms we have
$$2xz (l_A) (1 - Z (l_A) - z (l_A) l_A) \leq (h + xZ (l_A)) (2z (l_A) + z' (l_A)).$$ (5)
Notice that
$$(h + xZ (l_A)) (2z (l_A) + z' (l_A)) > 2hz (l_A).$$
It then follows that a sufficient condition for (5) to hold is
$$2hz (l_A) \geq 2xz (l_A) (1 - Z (l_A) - z (l_A) l_A),$$
which is implied by $h \geq x$ since $z' (\cdot) > 0.$ Summing up, $\mathcal{L}^A (\cdot)$ is convex if $z' (\cdot) \geq 0 \geq g' (\cdot)$ and $h \geq x.$ Notice that $h \geq x$ is not in contradiction with Proposition 5 as long as $g(0) z(0) < 1.$

Next, consider the case $p_A \leq p_B.$ Recall that $l_B \triangleq p_B - p_A,$ $m_B = (1 - Z (l_B)) l_B$ and
\[ \pi_A(m) \triangleq p_A + m \]

\[ \pi_A(m) \triangleq m + p_A, \]

so that

\[ \frac{\partial m_B}{\partial p_A} = -(1 - Z(l_B)) + z(l_B)l_B, \]

and

\[ \frac{\partial^2 m_B}{\partial p_A^2} = -2z(l_B) - z'(l_B)l_B, \]

which is strictly negative if \( z'(\cdot) \geq 0 \). Recall that \( \pi_A(m_B) \triangleq m_B + p_A \), hence

\[ \frac{\partial \pi_A(m_B)}{\partial p_A} = Z(l_B) + z(l_B)l_B, \]

and

\[ \frac{\partial^2 \pi_A(m_B)}{\partial p_A^2} = \frac{\partial^2 m_B}{\partial p_A^2}. \]

Differentiating \( L^A(\cdot) \) with respect to \( p_A \)

\[
\frac{\partial^2 L^A(\cdot)}{\partial p_A^2} = -hf'(p_A) - h \int_0^{m_B} f'(\pi_A(m)) dG(m) + \\
- h f(\pi_A(m_B)) g(m_B) \frac{\partial m_B}{\partial p_A} (1 + Z(l_B) + z(l_B)l_B) + \\
+ h (1 - F(\pi_A(m_B))) \left[ g'(m_B) \left( \frac{\partial m_B}{\partial p_A} \right)^2 + g(m_B) \frac{\partial^2 m_B}{\partial p_A^2} \right].
\]

Assume as before \( z'(\cdot) \geq 0 \geq g'(\cdot) \). Moreover, suppose that \( L \geq 1 \) and \( \frac{1 - Z(1)}{z(1)} > 1 \), so that \( \frac{\partial m_B}{\partial p_A} < 0 \). Then \( L^A(\cdot) \) is convex if

\[ |f'(p_A)| > -\sup \left[ g'(m_B) \left( \frac{\partial m_B}{\partial p_A} \right)^2 + g(m_B) \frac{\partial^2 m_B}{\partial p_A^2} \right]. \]

In order to show that this inequality does not define an empty set, suppose for example that \( G(\cdot) \) and \( Z(\cdot) \) are uniform and that \( F(\pi) = \pi^{\lambda} \). The above condition rewrites as

\[ p_A^{\lambda - 2} > \frac{2\lambda^2}{ML(\lambda - 1)}. \]
Hence, since $p_A^{\frac{1}{\lambda^2}}$ is decreasing in $p_A$, it is enough to impose

$$1 > \frac{2\lambda^2}{ML(\lambda - 1)}.$$ 

Summing up, we have shown that sufficient conditions under which the countries’ loss function is strictly convex can be found. ■
References


60. E. Brenna, *Healthcare tax credits: financial help to taxpayers or support to higher income and better educated patients? Evidence from Italy*, giugno 2017.