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Dipartimenti e Istituti di Scienze Economiche

Pricing Policy and Partial Collusion

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IEF0090 - October - 2009

UNIVERSITA' CATTOLICA DEL SACRO CUORE
- Milano -

**QUADERNI DELL'ISTITUTO DI
ECONOMIA E FINANZA**

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n. 90 - ottobre 2009

Quaderni dell’Istituto di Economia e Finanza
numero 90 ottobre 2009

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* Esemplare fuori commercio per il deposito legale agli effetti della Legge n. 106 del 15 aprile 2004.

* La Redazione ottempera agli obblighi previsti dalla Legge n. 106 del 15.04.2006, Decreto del Presidente della Repubblica del 03.05.2006 n. 252 pubblicato nella G.U. del 18.08.2006 n. 191.

* I quaderni sono disponibili on-line all'indirizzo dell'Istituto http://www.unicatt.it/istituti/EconomiaFinanza

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Pricing Policy and Partial Collusion

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Abstract

We study the pricing policy equilibria emerging in a partial collusion duopolistic framework where firms in the first stage of the game choose non-cooperatively whether to price discriminate or not, and from the second stage onward collude on prices. When the discount factor is particularly high or particularly low both firms price discriminate in equilibrium. For intermediate discount factors and high firms' asymmetry, the unique equilibrium is characterized by only the smaller firm choosing price discrimination. In the case of intermediate discount factors and low firms' asymmetry, there are two possible equilibria: both firms price discriminate or no firm price discriminates.

JEL codes: D43; L13; L40
Keywords: Partial collusion; Pricing policy; Price discrimination.

[♦] I am indebted to Michele Grillo for useful suggestions. All remaining errors are my own.

1. Introduction

Price discrimination is a business practice which has received a lot of attention by economists (see for example the recent surveys by Stole, 2007, and Armstrong, 2006 and 2008). A consistent body of literature considers the implications of price discrimination in oligopolies with horizontal product differentiation (Thisse and Vives, 1988; Corts, 1998; Ulph and Vulkan, 2000; Armstrong and Vickers, 2001; Liu and Serfes, 2004). This literature emphasizes that under quite general conditions price discrimination may decrease equilibrium profits with respect to the uniform price regime. This occurs because "*competitive price discrimination may intensify competition by giving firms more weapons with which to wage their war*" (Corts, 1998, p.321). A different result emerges if firms are able to collude on discriminatory prices. In this case, firms maintain all the advantages of price discrimination (better targeting of prices on consumers' willingness to pay) without suffering the costs of increasing competition (Gupta and Venkatu 2002; Liu and Serfes, 2007; Miklòs-Thal, 2008).

Our work is aimed to extend the understanding of the implications of price discrimination on tacit collusion within the spatial competition framework. In particular, we move from a collusive set-up to a partial collusion context. Partial collusion (or semi-collusion) is defined as a situation in which long-run decisions are competitively made but with the understanding that collusion will follow in subsequent stages of the game (Grillo, 1999). Partial collusion models are particularly relevant for inference on tacit collusion. In fact, given that the equilibrium decisions on the long-run variables differ depending on whether collusion on short-run variables is sustainable or not, competition policy may look at the long-run decisions "*because of their informative value in telling whether firms expect that a collusive equilibrium behaviour will or will not follow in the market*" (Grillo, 1999, p.38). Partial collusion has received extensive attention by economists, which focused on different long-run variables. For example, Friedman and Thisse (1993) and Posada and Straume (2004) assume that firms choose non-collusively the locations in the product space, and then collude on prices; in Osborne and Pitchik (1987) and Davidson and Deneckere (1990) the strategic variable which is chosen non-cooperatively by firms before collusion is capacity; Fershtman and Gandal (1994) and Brod and Shivakumar (1999) consider the R&D expenditures as the long-run non-collusive variable; Deltas and Serfes (2002) assume that firms choose non-cooperatively the quality of the product they offer, while they collude on price. Our main departure from the partial collusion literature consists in the long-run variable we focus on. In this paper we follow Thisse and Vives (1988), which in a purely competitive model assume that firms first choose whether to commit not to price discriminate or not (they refer to this choice as the "*pricing policy*" choice) and then set prices. That is, pricing policy is the long-run variable, while price is the short-run variable. The partial collusion version of the Thisse and Vives (1988) approach is the following: firms choose non-cooperatively the pricing policy at the beginning of the game, but try to enforce a collusive agreement on prices from the second period of the game onward conditionally on the pricing policies chosen in the first stage of the game.

There are many ways in which a firm may commit to uniform pricing. First, one may imagine an explicit contract between the firm and the consumers. An example of such contract is the most-favoured-nation clause, which engages a firm to offer a consumer the same price as its other consumers: if the clause is not respected, the firm must pay back the consumer the difference between the price he effectively paid and the lowest price fixed by the firm. Other forms of commitment to uniform pricing are no-haggle policies, in which the firm promises the customers that the posted sticker price is the final price (Corts, 1998). Moreover, one may consider a more subtle type of commitment to uniform pricing. Since a firm can price discriminate only if it is able to identify consumers (or groups of them), when a firm has not such specific consumers' information, it is prevented from price discrimination. Therefore, when a firm does not acquire consumers' information, it commits to uniform pricing (Liu and Serfes, 2004). Finally, as Encaoua and Hollander (2008) argue, commitment "*may derive from sunk investments in a distribution channel that puts intermediaries between manufacturers and consumers and does not allow the former to ascertain individual consumer preferences*" (Encaoua and Hollander, 2007, p.6). In our paper, we assume that commitment is irreversible: once one firm has committed to uniform pricing, it will set a unique price for all consumers. If a firm has not committed, it maintains full flexibility in setting prices.

The pricing policy choice in oligopolistic setups has received some attention by economists. Thisse and Vives (1988) show that in a fully non-cooperative horizontal differentiation model with maximally differentiated firms, the unique equilibrium emerging in a two-stage game in which firms first choose the pricing policy and then set the prices is characterized by no commitment. Eber (1998) extends the Thisse and Vives (1988) framework to allow for endogenous product differentiation. In a three stage model where firms first simultaneously choose where to locate, then choose whether to price discriminate or not, and then set the price schedules, the unique equilibrium is characterized by both firms price discriminating. Instead, when the first two stages of the game are reversed, the unique equilibrium is characterized by both firms committing not to discriminate. Encaoua and Hollandar (2008) consider a two-stage game *a la* Thisse and Vives (1988) in a vertical differentiation model, and show that price discrimination emerges as the unique pricing policy equilibrium. The pricing policy choice has been studied also in a third-degree price discrimination model by Liu and Serfes (2004). The authors show that when the information quality about consumers' preferences is low, unilateral commitments not to price discriminate arise in equilibrium, while when the information quality is high enough price discrimination emerges as the unique equilibrium. In a second-degree price discrimination context, Corts (1998) derives conditions under which a unilateral commitment not to discriminate by one firm may suffice to implement the uniform pricing equilibrium. Cooper (1986) instead analyses a dynamic model of intertemporal price discrimination, and shows that a no-discriminatory pricing policy may be a device to sustain collusion.

In studying partial collusion we adopt a spatial competition framework *a la* Hotelling (1929). However, we introduce a relevant generalization with respect to other works employing the Hotelling model for studying collusion sustainability (see for instance Chang, 1991 and 1992, Hackner, 1994). In fact, using the framework developed in Colombo (2009), firms are not constrained to be symmetric: any degree of symmetry/asymmetry between firms is allowed. We obtain the following results. When the market discount factor is particularly high or particularly low the unique equilibrium is characterized by both firms price discriminating. Instead, in the case of intermediate values of the market discount factor and a low degree of symmetry, the unique equilibrium is characterized by the larger firm renouncing to price discriminate, while the smaller firm does not commit. Finally, in the case of intermediate market discount factors but high degree of symmetry, two equilibria arise: either both firms price discriminate or no firm price discriminates.

The paper proceeds as follows. In section 2 we describe the model. In section 3 we calculate the critical discount factor when both firms have committed not to price discriminate. In section 4 we calculate the critical discount factor when no firm has committed not to price discriminate. In section 5 the critical discount factor in the case of asymmetric pricing policies is studied. In section 6 the pricing policy equilibrium is derived. Section 7 concludes.

2. The model

Suppose that there are two firms, firm *A* and firm *B*. Suppose also that there are two pricing policy options available for firms: committing not to price discriminate (U) and not committing (D). At time 1 firms simultaneously select the pricing policy. Once chosen the pricing policy is fixed forever. Conditionally on the pricing policies chosen by firms, four situations are possible: UU (both firms commit not to price discriminate), DD (both firms do not commit), UD (firm *A* commits while firm *B* does not commit) and DU (firm *A* does not commit while firm *B* commits). From the second period onward and forever firms try to enforce a collusive agreement on prices. Since firms are not constrained to be symmetric, a question arises about how to define the collusive agreement in such asymmetric context[1]. Here we assume that in the collusive agreement firms share the market following the market sharing arising during the punishment phase. This collusive agreement is in line with Friedman and Thisse (1993) collusive profit-sharing rule. They assume that collusive profits are strictly proportional to the one-shot punishment profits. We simplify further by assuming that the collusive agreement replicates the market sharing during the punishment stage[2].
In supporting collusion, we assume that firms use the *grim trigger* strategy of Friedman (1971)[3]. Denote by $\Pi_i^{J,c}$, $\Pi_i^{J,d}$ and $\Pi_i^{J,n}$, with $J = A,B$ and $i = UU, DD, UD, DU$, respectively as the one-shot collusive, deviation and punishment (or Nash) profits of each firm in any possible situation. The market discount factor, δ, is exogenous and common for each firm. It is well known that collusion is sustainable as a sub-game perfect equilibrium in situation i if and only if: $\delta \geq \delta_i^* \equiv \max[\delta_i^A, \delta_i^B]$, where $\delta_i^J \equiv (\Pi_i^{J,d} - \Pi_i^{J,c}) / (\Pi_i^{J,d} - \Pi_i^{J,n})$, with $J = A,B$ and $i = UU, DD, UD, DU$. Therefore, δ_i^* measures the cartel sustainability: the higher is δ_i^* the smaller is the set of market discount factors supporting collusion (i.e. collusion is *less* sustainable) in situation *i*.
Firms sell a differentiated good. Following Hotelling (1929), the differentiated good is represented in the unit interval $[0,1]$. Consumers are uniformly distributed over the interval. Denote by $x \in [0,1]$ the location of each consumer. For a consumer positioned at a given point, the most preferred variety is represented by the point in which he is located. Each consumer consumes no more than 1 unit of the good. Define with *v* the maximum price that a consumer is willing to pay for buying his preferred variety. Fixed and marginal costs of firms are set equal to zero. Firm *A* is located at *a*, while firm *B* is located at *b*. Without loss of generality, we assume $0 \leq a < b \leq 1$. Denote by $k \equiv b - a$, with $k \in (0,b]$, the differentiation degree between the two firms: the higher is *k* (i.e. the more the firms are distant on the segment), the more the firms are differentiated[4]. Denote by p_i^J the

[1] As Hackner argues "*it is not obvious how to define collusive pricing in an asymmetric game*" (Hackner, 1994, p.161).
[2] This, of course, is not the only possible collusive agreement. The main attractiveness of the collusive agreement we propose consists in its simplicity: if one accepts the idea that colluding firms cannot enforce very complex collusive agreement, the collusive contract we assume is rational. For other possible "ad hoc" collusive contracts between asymmetric firms see for instance Hackner (1994).
[3] The *grim trigger* strategy is not optimal (Abreu, 1986). However, "*this is one of very realistic punishment strategies because of its simplicity*", as argued by Matsumura and Matsushima (2005, p.263). The most part of the papers which study collusion sustainability within the Hotelling framework adopt the *grim trigger* strategy. See for example, Chang (1991, 1992), Friedman and Thisse (1993), Hackner (1994, 1995), Lambertini *et al.* (1998), Matsumura and Matsushima (2005) and Liu and Serfes (2007). Exceptions are Hackner (1996) and Miklòs-Thal (2008), where optimal punishments are assumed.
[4] An alternative interpretation of the differentiation degree between the firms within the Hotelling model considers the transportation cost parameter (see for example Polo, 1991, and Schultz, 2005). Here we follow the interpretation adopted, among the others, by Chang (1991, 1992), Friedman and Thisse (1993) and Hackner (1995).

price charged by firm $J = A, B$, in situation $i = UU, DD, UD, DU$. The utility of a consumer depends on *v*, on the price set by the firm from which he buys, and on the distance between his most preferred variety and the variety produced by the firm. Following D'Aspremont *et al*. (1979), we assume quadratic disutility costs. The utility of a consumer located at *x* when he buys from firm *A* is given by: $u_x^A = v - p_i^A - t(x-b+k)^2$, while his utility when he buys from firm *B* is given by: $u_x^B = v - p_i^B - t(x-b)^2$.

As in Shaffer and Zhang (2002) and in Colombo (2009) we define firms' symmetry as a situation in which, all else equal, the share of the consumers that prefer firm *A* to firm *B* is equal to the share of the consumers that prefer firm *B* to firm *A*. This occurs when $a+b=1$ (Picture 1.A), which implies $b=(k+1)/2$. When firms are asymmetric, there are two possibilities. One possibility is that, all else equal, more consumers prefer firm *A* to firm *B*. This occurs when $a+b>1$ (Picture 1.B), which implies: $b>(k+1)/2$. The other possibility is that, all else equal, more consumers prefer firm *B* to firm *A*. This occurs when $a+b<1$ (Picture 1.C), which implies: $b<(k+1)/2$. In the rest of the article we consider only the case in which firm *B* is "larger" than firm *A*. Therefore, $b \in [k,(k+1)/2)$[5]. Parameter *b* measures the firms' symmetry. The lower is *b*, the more are the consumers that, all else equal, prefer firm *B* to firm *A*. When $b \to 0$, all consumers, all else equal, prefer firm *B* to firm *A*. Instead, when $b \to (k+1)/2$, the share of the consumers preferring firm *A* to firm *B* is equal to the share of the consumers preferring firm *B* to firm *A*. Therefore, the higher is *b* the higher is symmetry[6].

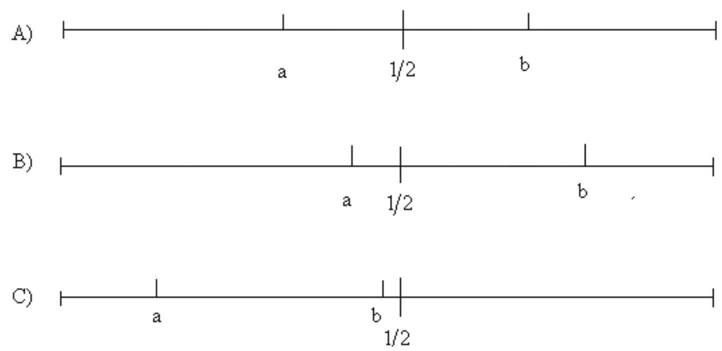
Finally, we assume that the market is sufficiently large. In particular, the following assumption on the parameters of the model is introduced:

<u>Assumption 1</u>: $v \geq 5t$.

Assumption 1 has some important implications for the rest of the analysis. Namely[7]:

Implication 1: there is full market coverage in any situation;
Implication 2: the deviating firm serves the whole market in any situation;
Implication 3: $\Pi_i^{J,c} > \Pi_{-i}^{J,n}$ with $J = A, B$ and $i = UU, DD, UD, DU$.

Picture 1



[5] Note that the assumptions on *b* and *k* can be written even in a compact way: $0 < k \leq b < (k+1)/2 \leq 1$.

[6] Clearly, when firm *A* is the larger firm (that is, when $0 < (k+1)/2 < b \leq 1$ holds) the higher is *b* the lower is symmetry.

[7] The derivation of implications 1,2 and 3 from Assumption 1 is trivial, but quite long. Therefore, it has been omitted. However, the Mathematica file with all calculations is available from the author upon request.

3. Collusion with symmetric pricing policies: case UU

Suppose that at time 1 both firms have decided to commit to set a uniform price by choosing U.

The punishment stage. The punishment price is the Nash-equilibrium price of the one-shot stage game. Denote by $p_{UU}^{A,n}$ and $p_{UU}^{B,n}$ the Nash price set by firm *A* and by firm *B* respectively. Denote by $\bar{x}$ the indifferent consumer (i.e. the consumer that receives the same utility buying from firm *A* or from firm *B*). Equating u_x^A and u_x^B and solving for *x*, it follows: $\bar{x} = (p_{UU}^{B,n} - p_{UU}^{A,n})/2tk + (2b-k)/2$. Since consumers are uniformly distributed, the demand of firm *A* is $D_{UU}^A = \bar{x}$, while the demand of firm *B* is $D_{UU}^B = 1-\bar{x}$. The profit function of firm *A* is therefore $p_{UU}^{A,n}\bar{x}$, while the profit function of firm *B* is $p_{UU}^{B,n}(1-\bar{x})$. Each firm maximizes its profits taking the rival's price as given. Straightforward calculations yield the following Nash equilibrium prices: $p_{UU}^{A,n} = tk(2+2b-k)/3$ and $p_{UU}^{B,n} = tk(4-2b+k)/3$. By substituting the equilibrium prices into the profit functions, we obtain the punishment profits:

$$\Pi_{UU}^{A,n} = tk(2+2b-k)^2/18 \tag{1}$$

$$\Pi_{UU}^{B,n} = tk(4-2b+k)^2/18 \tag{2}$$

Moreover, the equilibrium demand of firm *A* is $D_{UU}^A = (2+2b-k)/6$, while the equilibrium demand of firm *B* is: $D_{UU}^B = (4-2b+k)/6$.

The collusive stage. Firms share the market replicating the competitive market shares, and each firm serves its part of the market. Denote by $\hat{x}_{UU}^J$ the consumer paying the highest transportation costs for firm $J = A, B$. It turns out that:

<u>Lemma 1:</u>

$$\hat{x}_{UU}^A = \begin{cases} 0 & if \quad b \geq (2+11k)/10 \\ (2+2b-k)/6 & if \quad b \leq (2+11k)/10 \end{cases} \tag{3}$$

$$\hat{x}_{UU}^B = \begin{cases} (2+2b-k)/6 & if \quad b \geq (8-k)/10 \\ 1 & if \quad b \leq (8-k)/10 \end{cases} \tag{4}$$

Proof: Consider firm *A*. The middle point of its market is: $\tilde{x}_{UU}^A = (2+2b-k)/12$. If firm *A* is located at the right (left) of $\tilde{x}_{UU}^A$, the most distant consumer is located at the left (right) endpoint of firm *A* market. By solving $b-k \geq (\leq)\tilde{x}_{UU}^A$ the first part of Lemma 1 follows. Consider firm *B*. The middle point of its market is: $\tilde{x}_{UU}^B = (8+2b-k)/12$. If firm *B* is located at the right (left) of $\tilde{x}_{UU}^B$, the most distant consumer is located at the left (right) endpoint of firm *B*' market. By solving $b \geq (\leq)\tilde{x}_{UU}^B$ the second part of Lemma 1 follows. ■

Given Implication 1, firms set the highest price which allows to serve the whole market. The collusive prices therefore leave the most distant consumers without surplus. It follows that the collusive price set by firm A and firm B respectively results from the indifference conditions: $v - p_{UU}^{A,c} - t(\hat{x}_{UU}^{A} - b + k)^2 = 0$ and $v - p_{UU}^{B,c} - t(\hat{x}_{UU}^{B} - b)^2 = 0$. Solving with respect to the price, we get:

$$p_{UU}^{A,c} = \begin{cases} v - t(b-k)^2 & if \quad b \geq (2+11k)/10 \\ v - [(2-4b+5k)/6]^2 & if \quad b \leq (2+11k)/10 \end{cases} \tag{5}$$

$$p_{UU}^{B,c} = \begin{cases} v - t[(2-4b-k)/6]^2 & if \quad b \geq (8-k)/10 \\ v - t(1-b)^2 & if \quad b \leq (8-k)/10 \end{cases} \tag{6}$$

Therefore, the collusive profits of each firm are:

$$\Pi_{UU}^{A,c} = \begin{cases} [v - t(b-k)^2](2+2b-k)/6 & if \quad b \geq (2+11k)/10 \\ [v - [(2-4b+5k)/6]^2](2+2b-k)/6 & if \quad b \leq (2+11k)/10 \end{cases} \tag{7}$$

$$\Pi_{UU}^{B,c} = \begin{cases} [v - t[(2-4b-k)/6]^2](4-2b+k)/6 & if \quad b \geq (8-k)/10 \\ [v - t(1-b)^2](4-2b+k)/6 & if \quad b \leq (8-k)/10 \end{cases} \tag{8}$$

Clearly, the smaller firm (firm A) obtains lower collusive profits than the larger firm (firm B).

The deviation stage. The deviating firm lowers its price in order to steal the rival's consumers. Denote by $p_{UU}^{J,d}$ the deviation price of firm $J = A, B$. Given Implication 2, the deviating firm lowers its price until the consumer disliking it the most is indifferent between the firms. If the cheating firm is A, it sets the highest price which makes the consumer located at 1 indifferent between buying from it or from firm B. The indifference condition is therefore:

$$v - p_{UU}^{A,d} - t(1-b+k)^2 = v - p_{UU}^{B,c} - t(1-b)^2 \tag{9}$$

Substituting equation (6) into equation (9) and solving for $p_{UU}^{A,d}$ we get:

$$p_{UU}^{A,d} = \Pi_{UU}^{A,d} = \begin{cases} v - t(1-b+k)^2 + t(1-b)^2 - t[(2-4b-k)/6]^2 & if \quad b \geq (8-k)/10 \\ v - t(1-b+k)^2 & if \quad b \leq (8-k)/10 \end{cases} \tag{10}$$

If the cheating firm is B, it sets the highest price which makes the consumer located at 0 indifferent between buying from it or from firm A. The indifference condition is:

$$v - p_{UU}^{A,c} - t(b-k)^2 = v - p_{UU}^{B,d} - tb^2 \tag{11}$$

Substituting equation (5) into equation (11) and solving for $p_{UU}^{B,d}$ we get:

$$p_{UU}^{B,d} = \Pi_{UU}^{B,d} = \begin{cases} v - tb^2 & if \quad b \geq (2+11k)/10 \\ v + t(b-k)^2 - t[(2-4b+5k)/6]^2 - tb^2 & if \quad b \leq (2+11k)/10 \end{cases} \tag{12}$$

Note that firm *A* has lower deviation profits than firm *B*.

The critical discount factor. Inserting equations (1), (7) and (10) into δ_{UU}^{A} and equations (2), (8) and (12) into δ_{UU}^{B}, after some algebra we get:

$$\delta_{UU}^{A} = \begin{cases} \dfrac{[4-12b^3+68k+25k^2+6k^3+b^2(2+30k)-8b(2+5k+3k^2)]+6v(2b-4-k)}{[4+76k+29k^2+2k^3+8b^2(2+k)-8b(2+6k+k^2)]t-36v} & if \quad (b,k)\in S_1 \\ \dfrac{v+(2+2b-k)(tb^2-v)/6-t(1-b+k)^2}{v-tk(2+2b-k)/18-t(1-b+k)^2} & if \quad (b,k)\in S_2 \\ -\dfrac{(2b-k-4)[(4+16b^2+92k+25k^2-8b(2+5k))t-36v]}{6[(4+76k+29k^2+2k^3+8b^2(2+k)-8b(2+6k+k^2))t-36v]} & if \quad (b,k)\in S_3 \\ \dfrac{v+t(1-b+k)^2-(2+2b-k)(v-t(2-4b+5k)^2/36)/6}{v-tk(2+2b-k)^2/18-t(1-b+k)^2} & if \quad (b,k)\in S_4 \end{cases}$$

$$\delta_{UU}^{B} = \begin{cases} \dfrac{v+tb^2-(4-2b+k)(v-t(4b-2+k)^2/36)/6}{v-tb^2-tk(4b-2+k)^2/18} & if \quad (b,k)\in S_1 \\ \dfrac{v-tb^2-(4-2b+k)[v-t(1-b)^2]/6}{v-tb^2-tk(4b-2+k)^2/18} & if \quad (b,k)\in S_2 \\ \dfrac{(2+2b-k)[(4+16b^2+8b(k-2)+68k+k^2)t-36v]}{6[(4+52k+5k^2+2k^3+8b^2(2+k)-8b(2+k^2))t-36v]} & if \quad (b,k)\in S_3 \\ \dfrac{[-20+12b^3+14k-11k^2+44b(1+k)-2b^2(16+3k)]t+6v(k-2-2b)}{t[4+52k+5k^2+2k^3+8b^2(2+k)-8b(2+k^2)]t-36v} & if \quad (b,k)\in S_4 \end{cases}$$

where:

$$S_1 \equiv \{(b,k): b \geq \max[(2+11k)/10;(8-k)/10\}$$
$$S_2 \equiv \{(b,k): (8-k)/10 \geq b \geq (2+11k)/10\}$$
$$S_3 \equiv \{(b,k): (2+11k)/10 \geq b \geq (8-k)/10\}$$
$$S_4 \equiv \{(b,k): b \leq \min[(2+11k)/10;(8-k)/10\}$$

It can be shown that the discount factor of firm *A* is always higher than the discount factor of firm *B*. It follows that the critical discount factor is δ_{UU}^{A} for any *S*.

4. Collusion with symmetric pricing policies: case DD

Suppose that at stage 1 no firm has decided to commit to set a uniform price. That is, in the first stage of the game both firms choose *D*. We look for the conditions of collusion sustainability in this case.

The punishment stage. We assume that if the utility of the consumer is the same when he buys from firm A and when he buys from firm B, he buys from the nearer firm[8]. Denote by $p_{DD}^{A,n}$ and $p_{DD}^{B,n}$ the punishment price schedules in situation DD for firm A and firm B respectively. Suppose that consumer x is nearer to firm A than to firm B. For a given couple of firms' locations and for a given price set by firm B, the best thing firm A can do is setting a price that gives the consumer the same utility he receives from firm B: this is the highest possible price that guarantees that consumer x buys from A. Suppose instead that the consumer x is nearer to firm B. For a given couple of firms' locations and for a given price set by firm B, in order to serve consumer x the best thing firm A can do is giving him a slightly higher utility than the utility provided to him by firm B. Of course, an analogous reasoning holds for firm B.
The following proposition defines the Nash equilibrium price schedules for any couple of locations.

Proposition 1: *if both firms price discriminate, the Nash equilibrium prices are the following*:

$$p_{DD}^{A,n} = \begin{cases} t(x-b)^2 - t(x-b+k)^2 & \text{if} \quad x \le b-k/2 \\ 0 & \text{if} \quad x \ge b-k/2 \end{cases}$$
$$p_{DD}^{B,n} = \begin{cases} 0 & \text{if} \quad x \le b-k/2 \\ t(x-b+k)^2 - t(x-b)^2 & \text{if} \quad x \ge b-k/2 \end{cases}$$

Proof. Denote by $p_{DD,x}^{J,n}$ the Nash price set by firm $J = A, B$ on consumer x in situation DD. Suppose that x is near to firm A, that is, $x < b-k/2$. Consider firm B. First, we show that $p_{DD,x}^{B,n} > 0$ cannot be an equilibrium. When $p_{DD,x}^{B,n} > 0$, the best-reply of firm A consists in setting: $p_{DD,x}^{A,n} = p_{DD,x}^{B,n} + t(x-b)^2 - t(x-b+k)^2$: the consumer x obtains the same utility and buys from firm A. But now firm B has the incentive to undercut firm A by setting a price equal to: $p_{DD,x}^{B,n}{}' = p_{DD,x}^{B,n} - \varepsilon$, where ε is a positive and infinitely small number. Since $p_{DD,x}^{B,n}$ is higher than 0 by hypothesis and ε is a positive and infinitely small number by definition, $p_{DD,x}^{B,n}{}'$ is higher than 0. Therefore, $p_{DD,x}^{B,n} > 0$ cannot be an equilibrium, because firm B would obtain higher profits by setting $p_{DD,x}^{B,n}{}'$. We show instead that $p_{DD,x}^{B,n} = 0$ is an equilibrium. The best-reply of firm A is: $p_{DD,x}^{A,n} = t(x-b)^2 - t(x-b+k)^2$. With such a price firm B obtains zero profits from consumer x, which buys from firm A, but it has no incentive to change the price, because increasing the price it would continue to obtain zero profits, and setting a price lower than zero would entail a loss. It follows that $p_{DD,x}^{A,n} = t(x-b)^2 - t(x-b+k)^2$ and $p_{DD,x}^{B,n} = 0$ represents the (unique) price equilibrium. The proof for $x > b-k/2$ is symmetric to the proof for $x < b-k/2$. Finally, when the consumer is at the same distance from the two firms, that is $x = b-k/2$, the standard Bertrand's result holds: the unique price equilibrium when two undifferentiated firms compete on price is represented by both firms setting a price equal to the marginal costs (which are zero in this case). ■

[8] This assumption is common in spatial models, and it is necessary to avoid the technicality of ε-equilibrium concept when both firms price discriminate. For more details about this assumption, see among the others Hurter and Lederer (1985), Lederer and Hurter (1986), Thisse and Vives (1988), Hamilton *et al.* (1989), Hamilton and Thisse (1992).

Therefore, firm *A*'s demand is $D_{DD}^{A} = b - k/2$, while firm *B*'s demand is $D_{DD}^{B} = 1 - b + k/2$. The punishment profits are:

$$\Pi_{DD}^{A,n} = \int_{0}^{b-k/2} p_{DD}^{A,n} dx = tk(2b-k)^2/4 \tag{13}$$

$$\Pi_{DD}^{B,n} = \int_{b-k/2}^{1} p_{DD}^{B,n} dx = tk(2-2b+k)^2/4 \tag{14}$$

It can be easily noted that firm *A* (*B*) has lower (higher) punishment profits.

The collusive stage. Firm *A* serves all consumers from 0 to $x = b - k/2$, while firm *B* serves all consumers from $x = b - k/2$ to 1. The collusive price schedules are set in such a way to extract the whole consumer surplus. Therefore, firm *A* serves the consumers located at $x \le b - k/2$ using the following price schedule:

$$p_{DD}^{A,c} = v - t(b-k-x)^2 \tag{15}$$

Firm *B* serves the consumers located at $x \ge b - k/2$ using the following price schedule:

$$p_{DD}^{B,c} = v - t(b-x)^2 \tag{16}$$

Therefore, the collusive profits of each firm are:

$$\Pi_{DD}^{A,c} = \int_{0}^{b-k/2} p_{DD}^{A,c} dx = [12v(2b-k) - t(2b-k)(4b^2 - 10bk + 7k^2)]/24 \tag{17}$$

$$\Pi_{DD}^{B,c} = \int_{b-k/2}^{1} p_{DD}^{B,c} dx = [12v(2-2b+k) - t(8 - 24b + 24b^2 - 8b^3 + k^3)]/24 \tag{18}$$

Obviously, firm *A* obtains lower collusive profits than firm *B*.

The deviation stage. Denote by $p_{DD}^{J,d}$ the deviation price of firm $J = A, B$. We assume that when a consumer is indifferent between the deviating firm and the colluding firm, he buys from the deviating firm[9]. Given Implication 2, the deviating firm serves the whole market. If the cheating firm is *A*, the deviation price schedule is obtained solving the following indifference condition with respect to $p_{DD}^{A,d}$:

$$v - p_{DD}^{A,d} - t(x-b+k)^2 = v - p_{DD}^{B,c} - t(x-b)^2 \tag{19}$$

Inserting equation (14) into equation (19) and solving, we get:

[9] This assumption can be rationalized noting that the deviating firm can always offer to the consumer a utility which is strictly larger than the utility he receives from the colluding firm by setting a price equal to $p_{DD}^{J,d} - \varepsilon$, where ε is a positive small number and $J = A, B$.

$$p_{DD}^{A,d} = v - t(x - b + k)^2 \tag{20}$$

If the cheating firm is *B*, the deviation price schedule is obtained solving with respect to $p_{DD}^{B,d}$ the following indifference condition:

$$v - p_{DD}^{A,c} - t(x - b + k)^2 = v - p_{DD}^{B,d} - t(x - b)^2 \tag{21}$$

Inserting equation (15) into equation (21) and solving, we get:

$$p_{DD}^{B,d} = v - t(x - b)^2 \tag{22}$$

The deviation profits are therefore:

$$\Pi_{DD}^{A,d} = \int_0^1 [v - t(x - b + k)^2]dx = v - t(\frac{1}{3} - b + b^2 + k - 2bk + k^2) \tag{23}$$

$$\Pi_{DD}^{B,d} = \int_0^1 [v - t(x - b)^2]dx = v - t(\frac{1}{3} - b + b^2) \tag{24}$$

Note that firm *A* has lower deviation profits than firm *B*.

The critical discount factor. Inserting equations (13), (17) and (23) into δ_{DD}^{A} and equations (14), (18) and (24) into δ_{DD}^{B} we get:

$$\delta_{DD}^{A} = \frac{t(-2 + 2b - k)[(4 + 4b^2 + 10k + 7k^2 - 2b(4 + 5k)] - 12v(-2 + 2b - k)}{2t[4 + 12k + 12k^2 + 3k^3 + 12b^2(1 + k) - 12(1 + k)^2] - 24v}$$

$$\delta_{DD}^{B} = \frac{8tb^3 - tk^3 - 24bv + 12kv}{2t[4 + 12k + 12k^2 + 3k^3 + 12b^2(1 + k) - 12(1 + k)^2] - 24v}$$

A comparison between the discount factors shows that δ_{DD}^{A} is always higher than δ_{DD}^{B}. Therefore, the critical discount factor in situation DD is δ_{DD}^{A}.

5. Collusion with asymmetric pricing policies

We consider now the case of collusion when firms have adopted different pricing policies at the first stage of the game. First, we provide a unique proposition for the punishment prices in a situation of asymmetric pricing policies competition. Then, we consider the collusive profits and the deviation profits for situation UD. Finally, we turn to the collusive profits and the deviation profits for situation DU.

The punishment stage. In case of asymmetric pricing policies competition, we assume that if the utility of the consumer is the same when he buys from the discriminating firm and when he buys from the non discriminating firm, he buys from the discriminating firm[10]. We state the following proposition:

Proposition 2: *if firm A sets a uniform price while firm B price discriminates, the Nash equilibrium prices are the following:*

$$p_{UD}^{A,n} = tk(2b-k)/2$$

$$p_{UD}^{B,n} = \begin{cases} -tk(2b-k)/2 + 2tkx & \text{if} \quad x \geq (2b-k)/4 \\ 0 & \text{if} \quad x \leq (2b-k)/4 \end{cases}$$

If firm A price discriminates while firm B sets a uniform price, the Nash equilibrium prices are the following:

$$p_{DU}^{A,n} = \begin{cases} tk(2+2b-k)/2 - 2tkx & \text{if} \quad x \leq (2+2b-k)/4 \\ 0 & \text{if} \quad x \geq (2+2b-k)/4 \end{cases}$$

$$p_{DU}^{B,n} = tk(2-2b+k)/2$$

Proof. Suppose that firm *A* has committed while firm *B* has not committed. Denote by $p_{UD,x}^{B,n}$ the consumer-specific Nash price set by firm *B* in situation UD, and with $p_{UD}^{A,n}$ the uniform Nash price set by firm *A* in situation UD. Consider a generic consumer *x*. The best-reply of firm *B* consists in setting: $p_{UD,x}^{B,n} = p_{UD}^{A,n} + t(x-b+k)^2 - t(x-b)^2$. If firm *A* sets $p_{UD}^{A,n} > t(x-b)^2 - t(x-b+k)^2$, firm *B* can always serve the consumer *x* by undercutting the uniform price set by firm *A* without pricing below zero: therefore consumer *x* will always buy from firm *B* and firm *A* will obtain zero profits. In order to have a positive demand, firm *A* must set a uniform price such that: $p_{UD}^{A,n} \leq t(x-b)^2 - t(x-b+k)^2$, which cannot be undercut by firm *B*. Therefore, the highest uniform price that firm *B* cannot undercut is given by: $p_{UD}^{A,n} = t(x-b)^2 - t(x-b+k)^2$. Solving for *x*, we obtain the most at the right consumer served by firm *A*: $x^* = [t(-k^2+2bk) - p_{UD}^{A,n}]/2tk$. Given that consumers are uniformly distributed, the demand of firm *A* is $D_{UD}^{A} = x^*$, while the demand of firm *B* is $D_{UD}^{B} = 1 - x^*$. The profits of firm *A* are: $\Pi_{UD}^{A,n} = p_{UD}^{A,n} x^* = p_{UD}^{A,n}[2tbk - tk^2 - p_{UD}^{A,n}]/2tk$. Maximizing the profits with respect to $p_{UD}^{A,n}$, we get: $p_{UD}^{A,n} = t(2bk-k^2)/2$. Substituting $p_{UD}^{A,n}$ into the best-reply function of firm *B* we get the equilibrium price schedule of the discriminating firm. The proof for the case DU proceeds in the same way. ■

[10] This assumption is necessary to avoid ε-equilibria in the sub-games when only one firm price discriminates, and it can be easily rationalized noting that the discriminating firm can always offer to the consumer a utility which is strictly larger than the utility he receives from the non-discriminating firm simply by setting a price equal to $p_x - \varepsilon$, where p_x is the discriminatory price which makes the consumer *x* indifferent between the two firms and ε is a positive small number.

The equilibrium demands during the punishment stage are: $D_{UD}^{A} = (2b-k)/4$, $D_{UD}^{B} = (4-2b+k)/4$, $D_{DU}^{A} = (2+2b-k)/4$ and $D_{DU}^{B} = (2-2b+k)/4$. The equilibrium punishment profits are therefore:

$$\Pi_{UD}^{A,n} = [tk(2b-k)^2]/8 \tag{25}$$

$$\Pi_{UD}^{B,n} = [tk(4-2b+k)^2]/16 \tag{26}$$

$$\Pi_{DU}^{A,n} = [tk(2+2b-k)^2]/16 \tag{27}$$

$$\Pi_{DU}^{B,n} = [tk(2-2b+k)^2]/8 \tag{28}$$

Case UD. Now we focus on the case in which only the smaller firm (firm *A*) committed to set a uniform price at stage 1 of the game.

The collusive stage. Firm *A*'s market share goes from 0 to $x = (2b-k)/4$, while firm *B*'s market share goes from $x = (2b-k)/4$ to 1. Firm *B* sets the discriminatory pricing schedule in such a way to extract the whole consumer surplus from each consumer of its own market. Therefore:

$$p_{UD}^{B,c} = v - t(b-x)^2 \tag{29}$$

The collusive profits of firm *B* are therefore:

$$\Pi_{UD}^{B,c} = \int_{(2b-k)/4}^{1} p_{UD}^{B,c} dx = [(4-2b+k)[48v - t(16+28b^2+8b(k-5)-4k+k^2)]]/192 \tag{30}$$

Firm *A* sets the collusive uniform price in such a way to serve the most distant consumer. The most distant consumer from firm *A*, $\hat{x}_{UD}^{A}$, is defined in the following Lemma:

Lemma 2:

$$\hat{x}_{UD}^{A} = \begin{cases} 0 & if \quad b \geq 7k/6 \\ (2b-k)/4 & if \quad b \leq 7k/6 \end{cases}$$

Proof: The middle point of firm *A*' market is: $\tilde{x}_{UD}^{A} = (2b-k)/8$. If firm *A* is located at the right (left) of $\tilde{x}_{UD}^{A}$, the most distant consumer is located at the left (right) endpoint of firm *A* market. By solving $b-k \geq (\leq) \tilde{x}_{UD}^{A}$ Lemma 2 follows. ■

From Lemma 2, the collusive price set by firm *A* follows:

$$p_{UD}^{A,c} = \begin{cases} v - t(b-k)^2 & if \quad b \geq 7k/6 \\ v - t[(2b-3k)/4]^2 & if \quad b \leq 7k/6 \end{cases} \tag{31}$$

The collusive profits of firm *A* are:

$$\Pi_{UD}^{A,c} = \begin{cases} [v - t(b-k)^2](2b-k)/4 & if \quad b \geq 7k/6 \\ [v - t[(2b-3k)]^2](2b-k)/4 & if \quad b \leq 7k/6 \end{cases} \tag{32}$$

The deviation stage. If the cheating firm is *A*, the deviation uniform price is such that the most distant consumer is indifferent between buying from firm *A* and from firm *B*. The most distant consumer is located at 1. Since such consumer obtains no surplus when he buys from firm *B*, the optimal uniform deviation price and the deviation profits are:

$$p_{UD}^{A,d} = \Pi_{UD}^{A,d} = v - t(1-b+k)^2 \tag{33}$$

If the cheating firm is *B*, the deviation price schedule is obtained solving with respect to $p_{UD}^{B,d}$ the following indifference condition (for consumers located in firm *A*'s market):

$$v - p_{UD}^{A,c} - t(x-b+k)^2 = v - p_{UD}^{B,d} - t(x-b)^2 \tag{34}$$

Substituting equation (31) into equation (34) and solving, we get:

$$p_{UD}^{B,d} = \begin{cases} v - t(x-b)^2 - t(b-k)^2 + t(x-b+k)^2 & if \quad b \geq 7k/6 \\ v - t(x-b)^2 - t[(2b-3k)/4]^2 + t(x-b+k)^2 & if \quad b \leq 7k/6 \end{cases} \tag{35}$$

The deviation profits of firm *B* are therefore:

$$\Pi_{UD}^{B,d} = \Pi_{UD}^{B,c} + \int_0^{\frac{2b-k}{4}} p_{UD}^{B,d}\, dx$$

$$= \begin{cases} \dfrac{192v - t[64 + 40b^3 + b^2(192-84k) - 11k^3 + 6b(9k^2 - 32)]}{192} & if \quad b \geq \dfrac{7k}{6} \\ \dfrac{96v + t[-32 + 16b^3 - 5k^3 - 12b^2(8+4k) + 24b(4+3k^2)]}{96} & if \quad b \leq \dfrac{7k}{6} \end{cases} \tag{36}$$

The critical discount factor. Inserting equations (25), (32) and (33) into δ_{UD}^{A} and equations (26), (30) and (36) into δ_{UD}^{B}, we get:

$$\delta_{UD}^{A} = \begin{cases} \dfrac{v - t(1-b+k)^2 - (2b-k)[v - t(b-k)^2]/4}{v - t(1-b+k)^2 - tk(k-2b)^2]/8} & if \quad b \geq \dfrac{7k}{6} \\ -\dfrac{(2b-4-k)[t(16+4b^2+28k+9k^2-12b(2+k)) - 16v]}{8[t(2+k)(4+4b^2+6k+k^2-4b(2+k)) - 8v]} & if \quad b \leq \dfrac{7k}{6} \end{cases}$$

$$\delta_{UD}^{B}=\begin{cases}\dfrac{12(2b-k)(4tb^{2}-2tbk+tk^{2}-4v)}{(64+40b^{3}+b^{2}(192-36k)+192k+96k^{2}+k^{3}+6b(k^{2}-32k-32))t-192} & if \quad b\geq\dfrac{7k}{6}\\ \dfrac{3(2b-k)(4tb^{2}+12tbk-3tk^{2}-16v)}{2[(-32+16b^{3}-96k-48k^{2}-11k^{3}-12b^{2}(8+5k)+48b(2+2k+k^{2}))t+96v]} & if \quad b\leq\dfrac{7k}{6}\end{cases}$$

It can be shown that δ_{UD}^{A} is always larger than δ_{UD}^{B}. It follows that the critical discount factor in situation UD is δ_{UD}^{A}.

Case DU. Now we consider the case in which only the larger firm (firm *B*) has chosen to commit in the first stage of the game.

The collusive stage. The collusive agreement implies that firm *A* serves all the consumers from 0 to $x=(2+2b-k)/4$, while firm *B* serves the consumers from $x=(2+2b-k)/4$ to 1. Firm *A* sets the discriminatory pricing schedule in such a way to extract the whole consumer surplus from its own market share. Therefore:

$$p_{DU}^{A,c}=v-t(b-k-x)^{2} \tag{37}$$

Then, the collusive profits of firm *A* are:

$$\Pi_{DU}^{A,c}=\int_{0}^{(2+2b-k)/4}p_{DU}^{A,c}dx=\frac{(2+2b-k)[48v-t(4+28b^{2}-16b(4k+1)+20k+37k^{2})]}{192} \tag{38}$$

Firm *B* sets the collusive uniform price in such a way to serve the most distant consumer. Denote by $\hat{x}_{DU}^{B}$ the most distant consumer from firm *B*. The following lemma characterizes $\hat{x}_{DU}^{B}$:

Lemma 3:

$$\hat{x}_{DU}^{B}=\begin{cases}(2+2b-k)/4 & if \quad b\geq 1-k/6\\ 1 & if \quad b\leq 1-k/6\end{cases}$$

Proof: The middle point of firm *B*'s market is: $\tilde{x}_{DU}^{B}=(6+2b-k)/8$. If firm *B* is located at the right (left) of $\tilde{x}_{DU}^{B}$, the most distant consumer is located at the left (right) endpoint of firm *B*' market. By solving $b\geq(\leq)\tilde{x}_{DU}^{B}$ Lemma 3 follows. ■

The collusive price set by firm *B* follows directly from Lemma 3:

$$p_{DU}^{B,c}=\begin{cases}v-t[(2b-2+k)/4]^{2} & if \quad b\geq 1-k/6\\ v-t(1-b)^{2} & if \quad b\leq 1-k/6\end{cases} \tag{39}$$

The collusive profits of firm *B* are:

$$\Pi_{DU}^{B,c} = \begin{cases} [v - t[(2b-2+k)/4]^2][(2-2b+k)/4] & if \quad b \geq 1-k/6 \\ [v - t(1-b)^2][(2-2b+k)/4] & if \quad b \leq 1-k/6 \end{cases} \tag{40}$$

The deviation stage. If the cheating firm is *A*, the deviation price schedule is obtained solving with respect to $p_{DU}^{A,d}$ the following indifference condition (for consumers located in firm *B*'s market)

$$v - p_{DU}^{A,d} - t(x-b+k)^2 = v - p_{DU}^{B,c} - t(x-b)^2 \tag{41}$$

Substituting equation (39) into equation (41) and solving, we get:

$$p_{DU}^{A,d} = \begin{cases} v - t[(2b-2+k)/4]^2 + t(x-b)^2 - t(x-b+k)^2 & if \quad b \geq 1-k/6 \\ v - t(1-b)^2 + t(x-b)^2 - t(x-b+k)^2 & if \quad b \leq 1-k/6 \end{cases} \tag{42}$$

The deviation profits of firm *A* are therefore:

$$\Pi_{DU}^{A,d} = \Pi_{DU}^{A,c} + \int_{\frac{2+2b-k}{4}}^{1} p_{DU}^{A,d} dx =$$

$$= \begin{cases} \dfrac{96v - t[16+16b^3 - 12b^2(k-4) + 84k + 96k^2 + k^3 - 24b(2+7k)]}{96} & if \quad b \geq 1 - \dfrac{k}{6} \\ \dfrac{192v - t[104 - 40b^3 + 12b^2(3k+26) + 228k + 198k^2 - k^3 - 6b(52+76k+k^2)]}{192} & if \quad b \leq 1 - \dfrac{k}{6} \end{cases} \tag{43}$$

If the cheating firm is *B*, the deviation uniform price is such that the most distant consumer is indifferent between buying from firm *A* and from firm *B*. The most distant consumer is located at 0. Since it obtains no surplus when it buys from firm *A*, the optimal uniform deviation price and the deviation profits of firm *B* are:

$$p_{DU}^{B,d} = \Pi_{DU}^{B,d} = v - tb^2 \tag{44}$$

The critical discount factor. Inserting equations (27), (38) and (43) into δ_{DU}^{A} and equations (28), (40) and (44) into δ_{DU}^{B} we get:

$$\delta_{DU}^{A} = \begin{cases} -\dfrac{3(2b-2-k)[t(4+4b^2+20k+13k^2-4b(2+5k))-16v]}{2[(16+16b^3+108k+72k^2+7k^3+12b^2(4+k)-24b(2+5k+k^2))t-96v]} & if \quad b \geq 1-\dfrac{k}{6} \\ \dfrac{12(2b-k-2)[t(4+4b^2+6k+3k^2-2b(4+3k))-4v]}{(-104+40b^3-276k-150k^2-11k^3-12b^2(26+7k)+6b(52+60k+9k^2))t+192} & if \quad b \leq 1-\dfrac{k}{6} \end{cases}$$

$$\delta_{DU}^{B} = \begin{cases} \dfrac{v - tb^2 - (2-2b+k)[v - t(2b+k-2)^2/16]/4}{v - tb^2 - tk(2b+k-2)^2/8} & if \quad b \geq 1-\dfrac{k}{6} \\ \dfrac{v - tb^2 - (2-2b+k)[v - t(1-b)^2]/4}{v - tb^2 - tk(2b+k-2)^2/8} & if \quad b \leq 1-\dfrac{k}{6} \end{cases}$$

It can be shown that the larger discount factor is always δ^B_{DU}. Therefore, the critical discount factor in situation DU is δ^B_{DU}.

Before proceeding, note that the collusive and the punishment profits satisfy the following relationships: $\Pi^{A,c}_{DU} > \Pi^{A,c}_{DD} > \Pi^{A,c}_{UU} > \Pi^{A,c}_{UD}$, $\Pi^{B,c}_{UD} > \Pi^{B,c}_{DD} > \Pi^{B,c}_{UU} > \Pi^{B,c}_{DU}$, $\Pi^{A,n}_{DU} > \Pi^{A,n}_{UU} > \Pi^{A,n}_{DD} > \Pi^{A,n}_{UD}$ and $\Pi^{B,n}_{UD} > \Pi^{B,n}_{UU} > \Pi^{B,n}_{DD} > \Pi^{B,n}_{DU}$.

6. The pricing policy equilibrium

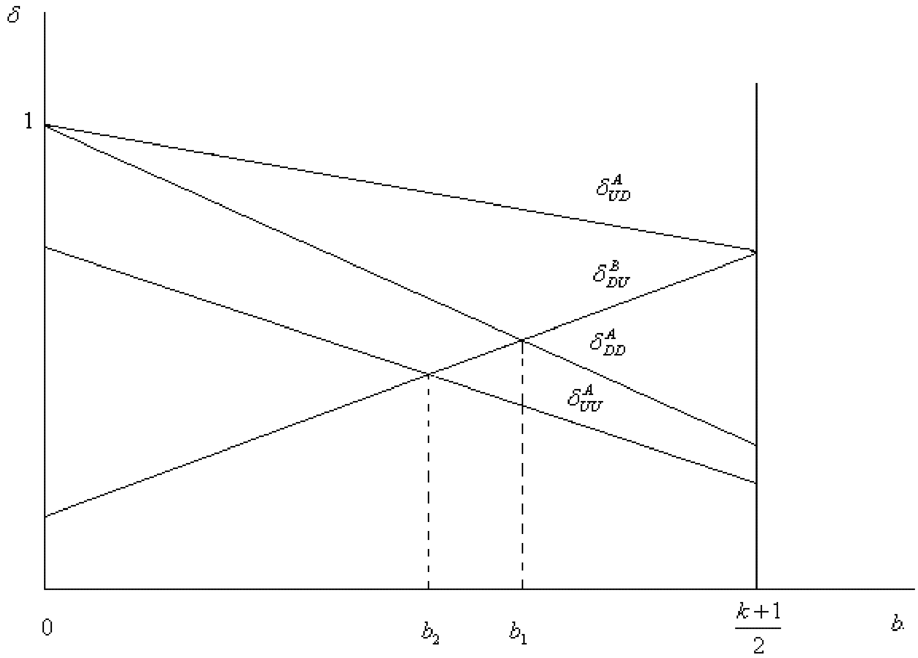
In order to describe the pricing policy equilibrium emerging in the semi-collusive game, first we characterize the shape of the critical discount factors as a function of *b* in the next proposition:

Proposition 3: The critical discount factor in situations UU, DD and UD is decreasing in *b*, while the critical discount factor in situation DU is increasing in *b*.

Proof. The proof consists in verifying the following inequalities: $\partial\delta^A_{UU}/\partial b < 0$, $\partial\delta^A_{DD}/\partial b < 0$, $\partial\delta^A_{UD}/\partial b < 0$ and $\partial\delta^B_{DU}/\partial b > 0$, which always hold. ■

Moreover, it can be easily shown by simple substitutions that under perfect symmetry (i.e. when $b = (k+1)/2$) the following relationships hold: $\delta^A_{UD} = \delta^B_{DU} > \delta^A_{DD} > \delta^A_{UU}$. Instead, under maximal asymmetry (i.e. when $b = 0$), the following relationships hold: $1 = \delta^A_{UD} = \delta^A_{DD} > \delta^A_{UU} > \delta^B_{DU}$. These relationships, together with Proposition 3, allow us to depict the critical discount factors as in Picture 2:

Picture 2



Time 1

Consider now the first stage of the game. Each firm has to decide whether to choose uniform pricing (U) or price discrimination (D). Looking at picture 2, three possible situations can be identified.

Case 1) when $b < b_2$, the following relationships hold: $\delta^A_{UD} > \delta^A_{DD} > \delta^A_{UU} > \delta^B_{DU}$
Case 2) when $b_2 < b < b_1$, the following relationships hold: $\delta^A_{UD} > \delta^A_{DD} > \delta^B_{DU} > \delta^A_{UU}$
Case 3) when $b > b_1$, the following relationships hold: $\delta^A_{UD} > \delta^B_{DU} > \delta^A_{DD} > \delta^A_{UU}$.

In what follows we study the first stage sub-game equilibrium. We start from case 1).

- If $\delta > \delta^A_{UD}$, all collusive schemes are sustainable. The payoff table at stage 1 of the game is therefore:

Table 1

Π^B / Π^A	U	D
U	$\Pi^{A,c}_{UU} ; \Pi^{B,c}_{UU}$	$\Pi^{A,c}_{UD} ; \Pi^{B,c}_{UD}$
D	$\Pi^{A,c}_{DU} ; \Pi^{B,c}_{DU}$	$\Pi^{A,c}_{DD} ; \Pi^{B,c}_{DD}$

It turns out that the unique equilibrium in the first stage of the game is DD.

- If $\delta^A_{UD} > \delta > \delta^A_{DD}$, all collusive schemes apart from the collusive scheme UD are sustainable. The payoff table in the first stage of the game is therefore:

Table 2

Π^B / Π^A	U	D
U	$\Pi^{A,c}_{UU} ; \Pi^{B,c}_{UU}$	$\Pi^{A,n}_{UD} ; \Pi^{B,n}_{UD}$
D	$\Pi^{A,c}_{DU} ; \Pi^{B,c}_{DU}$	$\Pi^{A,c}_{DD} ; \Pi^{B,c}_{DD}$

Again, the unique equilibrium in the first stage is DD.

- If $\delta^A_{DD} > \delta > \delta^A_{UU}$, the collusive schemes DD and UD are not sustainable. The payoff table at time 1 of the game is therefore:

Table 3

Π^B / Π^A	U	D
U	$\Pi^{A,c}_{UU} ; \Pi^{B,c}_{UU}$	$\Pi^{A,n}_{UD} ; \Pi^{B,n}_{UD}$
D	$\Pi^{A,c}_{DU} ; \Pi^{B,c}_{DU}$	$\Pi^{A,n}_{DD} ; \Pi^{B,n}_{DD}$

The unique equilibrium in the first stage of the game is DU[11].

- If $\delta_{UU}^{A} > \delta > \delta_{DU}^{B}$, the only sustainable collusive scheme is DU. The payoff table in the first stage of the game is therefore:

Table 4

Π^B \ Π^A	U	D
U	$\Pi_{UU}^{A,n};\Pi_{UU}^{B,n}$	$\Pi_{UD}^{A,n};\Pi_{UD}^{B,n}$
D	$\Pi_{DU}^{A,c};\Pi_{DU}^{B,c}$	$\Pi_{DD}^{A,n};\Pi_{DD}^{B,n}$

Again, the unique equilibrium in the first stage is DU.

- If $\delta < \delta_{DU}^{B}$, no collusive scheme is sustainable. The payoff table in the first stage of the game is therefore:

Table 5

Π^B \ Π^A	U	D
U	$\Pi_{UU}^{A,n};\Pi_{UU}^{B,n}$	$\Pi_{UD}^{A,n};\Pi_{UD}^{B,n}$
D	$\Pi_{DU}^{A,n};\Pi_{DU}^{B,n}$	$\Pi_{DD}^{A,n};\Pi_{DD}^{B,n}$

The unique equilibrium is DD[12].

Case 2). The only difference with respect to case 1) is that the order between δ_{UU}^{A} and δ_{DU}^{B} is reverted. This implies that Table 3 is now defined for values of the market discount factor such that $\delta_{DD}^{A} > \delta > \delta_{DU}^{B}$, while Table 5 is defined for values of the market discount factor such that $\delta < \delta_{UU}^{A}$ More importantly, when the critical discount factor is such that $\delta_{DU}^{B} > \delta > \delta_{UU}^{A}$ collusion is sustainable only in situation UU. Therefore, Table 4 is substituted by Table 6:

Table 6

Π^B \ Π^A	U	D
U	$\Pi_{UU}^{A,c};\Pi_{UU}^{B,c}$	$\Pi_{UD}^{A,n};\Pi_{UD}^{B,n}$
D	$\Pi_{DU}^{A,n};\Pi_{DU}^{B,n}$	$\Pi_{DD}^{A,n};\Pi_{DD}^{B,n}$

Therefore, when $\delta_{DU}^{B} > \delta > \delta_{UU}^{A}$ there are two equilibria at stage 1: UU and DD.

Case 3) The only difference with respect to case 2) is that the order between δ_{DD}^{A} and δ_{DU}^{B} is reverted. As a consequence, Table 2 is now defined for values of the market discount factor such

[11] In fact, given Implication 3, *all* collusive profits are strictly larger than *all* Nash profits.
[12] See also Thisse and Vives (1988, p.131), where firms behave in a non-cooperative way.

that $\delta^A_{UD} > \delta > \delta^B_{DU}$ and Table 6 is now defined for values of the market discount factor such that $\delta^A_{DD} > \delta > \delta^A_{UU}$. More importantly, when the critical discount factor is such that $\delta^B_{DU} > \delta > \delta^A_{DD}$, collusion is sustainable in situation UU and in situation DD. It follows that Table 3 is now substituted by Table 7:

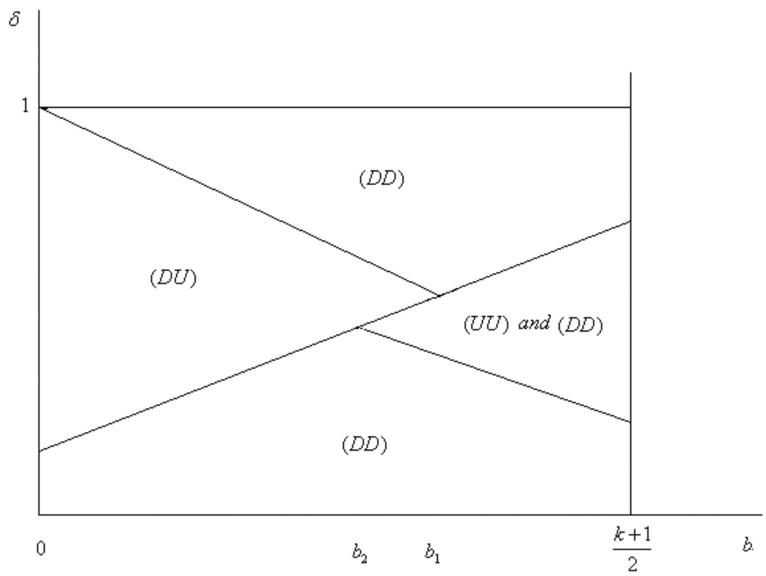
Table 7

Π^A \ Π^B	U	D
U	$\Pi^{A,c}_{UU}\,;\Pi^{B,c}_{UU}$	$\Pi^{A,n}_{UD}\,;\Pi^{B,n}_{UD}$
D	$\Pi^{A,n}_{DU}\,;\Pi^{B,n}_{DU}$	$\Pi^{A,c}_{DD}\,;\Pi^{B,c}_{DD}$

Hence, when $\delta^B_{DU} > \delta > \delta^A_{DD}$, two equilibria emerge in the first stage of the game: UU and DD.

Picture 3 summarizes the equilibria emerging in the different cases:

Picture 3



When the market discount factor is particularly high or particularly low the unique equilibrium is characterized by both firms price discriminating. This occurs because when collusion is always sustainable (the market discount factor is particularly high), price discrimination allows the firms to perfectly target the price on consumers' willingness to pay. Each firm has the possibility to extract the whole consumer surplus from its own market without the threat of competition: it follows that each firm chooses to price discriminate. A different mechanism works when collusion is never sustainable (the market discount factor is particularly low). In this case each firm chooses not to commit because being unconstrained in setting prices guarantees more flexibility in responding to rival's action. Since collusion will never arise, each firm wants to preserve all "weapons" to defend itself from competition of the rival. Note that the equilibrium which emerges in this case is "bad" for firms. Both firms would be better off by adopting a commitment strategy in the first stage of the game. The individual incentives lead both firms to choose not to commit in the first stage, and this situation yields to a typical Prisoner Dilemma problem. Consider instead the case of intermediate values of the market discount factor and *low* symmetry degrees. In this case the unique equilibrium is characterized by firm B renouncing to price discriminate, while firm A does not commit. That is, DU emerges as the unique equilibrium. This occurs because, for low symmetry degrees, firm A (the smaller firm) has a great incentive to deviate. Intuitively, this occurs because the lower is firm A with respect to firm B the higher is the additional market share it can capture by cheating (Motta, 2004). If both firms price discriminate, the critical discount factor turns out to be high and collusion is difficult to sustain. Instead, if the larger firm does not discriminate, the critical discount factor decreases, because the collusive profits of the smaller firm are higher (compare equation (17) with equation (38)). Firm B anticipates this fact, and it prefers to renounce to price discriminate in order to maintain collusion sustainability. Instead, firm A has no incentive to commit. In fact, choosing a commitment strategy firm A would make collusion *less* sustainable and less profitable. Finally consider the case of intermediate market discount factors but *high* degree of symmetry. Two equilibria arise: UU and DD. This occurs because when firms are similar in terms of market shares, the critical discount factors in the symmetric pricing policies collusive schemes are lower, and eventually they fall below the DU collusive scheme critical factor. Therefore, for intermediate values of the market discount factor, asymmetric pricing policies collusive agreements are not more sustainable than symmetric pricing policies collusive agreements. In this case, it is never optimal for a firm to choose a pricing policy different from the pricing policy chosen by the rival, because this would imply no collusion sustainability. In other words, each firm always prefers to match the rival's pricing policy in order to guarantee collusion sustainability. This conduces to the symmetric pricing policies equilibria.

7. Conclusions

We study the pricing policy equilibria emerging in a partial collusion framework where firms in the first stage of the game choose non-cooperatively whether to price discriminate or not, and collude on prices from the second stage of the game onward. By adopting a spatial competition setup *a la* Hotelling (1929) with no symmetry restriction (Colombo, 2009), we obtain the following results. When the market discount factor is particularly high or particularly low the unique equilibrium is characterized by both firms price discriminating. Instead, in the case of intermediate values of the market discount factor and low symmetry degree, the unique equilibrium is characterized by the larger firm renouncing to price discriminate in order to keep collusion sustainability. On the contrary, the smaller firm does not commit. Finally, in the case of intermediate market discount factors but high degree of symmetry, there are two possible equilibria: both firms price discriminate, or no firm price discriminates. These results provide new rationale for commitment not to price discriminate strategies. Renouncing to price discrimination by a large firm can be seen as a rational strategy as long as it provides better conditions for collusion sustainability by reducing the incentive of a smaller firm to deviate from the collusive agreement.

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