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Dipartimenti e Istituti di Scienze Economiche

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IEF0095 – May - 2010



UNIVERSITA' CATTOLICA DEL SACRO CUORE
- Milano -

**QUADERNI DELL'ISTITUTO DI
ECONOMIA E FINANZA**

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n. 95 - maggio 2010



Quaderni dell'Istituto di Economia e Finanza
numero 95 maggio 2010

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* Esemplare fuori commercio per il deposito legale agli effetti della Legge n. 106 del 15 aprile 2004.

* La Redazione ottempera agli obblighi previsti dalla Legge n. 106 del 15.04.2006, Decreto del Presidente della Repubblica del 03.05.2006 n. 252 pubblicato nella G.U. del 18.08.2006 n. 191.

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Bertrand and Cournot in the unidirectional Hotelling model

Stefano Colombo[♦]

Abstract

The unidirectional Hotelling model where consumers can buy only from firms located on their right (left) is extended to allow for elastic demand functions. A Bertrand-type model and a Cournot-type model are considered. If firms choose location and then set prices, agglomeration never arises; instead, if firms choose location and then set quantities, agglomeration arises at one endpoint of the segment when transportation costs are low enough. Equilibrium distance between firms is lower in Cournot than Bertrand under the whole parameters' set. We also study the impact of firms' location on perfect collusion sustainability. We show that when consumers can buy only from firms located on their right (left), the incentive to deviate of each firm decreases the more the firm is located to the right (left) and the more the rival is located to the left (right).

JEL codes: D43; L11; L41

Keywords: Unidirectional Hotelling model; Location equilibrium; Collusion; Bertrand; Cournot

1. Introduction

Spatial models have received consistent attention by economists in the last decades. The most famous ones are probably the linear-city model (Hotelling, 1929) and the circular-city model (Vickrey, 1964, Salop, 1979). However, other spatial representations have been introduced recently to analyze economic phenomena for which the spatial dimension plays a relevant role. For example, Hwang and Mai (1990) and Gross and Holahan (2001) considers a barbell model where there are two cities connected by a highway; Takahashi and De Palma (1993) and Ebina et al. (2009) develop a quasi-linear city model where consumers incur a costs when pass through a certain point (which may represent a mountain, a river, a congested bridge) in the space; Huang (2009) introduces a two-lines Hotelling model, where firms are located in one line, while consumers are located in the other line.

In this paper, we build on a model which has been introduced recently by Kharbach (2009). He develops a unidirectional Hotelling model (UHM henceforth), which differentiates from the standard bidirectional Hotelling model (BHM henceforth) for this reason: while in the BHM consumers have a bidirectional purchasing ability, in the UHM a consumer can buy only from firms located at his right or only from firms located at his left. The UHM can be used to describe spatial situations like highways or one way roads, or non-revertible flows in gas and oil pipelines (Kharbach, 2009). In a location-price game with uniform pricing and quadratic transportation costs, Kharbach (2009) shows that when consumers can buy only from firms located on their right (left), one firm locates in position $3/5$ from the left (right) endpoint of the linear market, while the other firm locates at the right (left) endpoint. Colombo (2009a) extends the UHM to allow for spatial price discrimination and a general class of transportation costs. Firms

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are able to set different prices for consumers located at different locations in the space. In a location-price game, Colombo (2009a) obtains that one firm always locates at one extremity of the market, while the other locates in the middle of the segment. Also, Colombo (2009a) considers collusion between firms, and obtains that when consumers can buy only from firms located on their right (left), the maximum collusive profits sustainable in equilibrium monotonically increase (decrease) with the location of the firm located at the right (left), while initially increase and then decrease with the location of the firm located at the left (right).¹

Both Kharbach (2009) and Colombo (2009a) assume that consumers have inelastic demand functions. To aim of this paper is to extend the analysis of the UHM to the case of elastic demand functions and spatial discrimination. Two different two-stage games are supposed. In one game (Bertrand), firms simultaneously choose location, and then set the price schedule, where prices may be different across locations; in the other game (Cournot), firms simultaneously choose location, and then set the quantity schedule, where quantities may be different across locations. We show that when consumers can buy only from firms located on their right, in the location-price game one firm always locates at one endpoint of the market, while the other locates in the middle of the market when the transportation costs approximate to zero: when the transportation costs increase, the equilibrium distance between the two firms decreases, but it is never maximal. Instead, in the location-quantity game, one firm still localizes at one endpoint of the segment, but the rival locates in the same endpoint when transportation costs are sufficiently low: when transportation costs increase, the equilibrium distance between the two firms increases. These results are substantially different from the location equilibria emerging within the BHM. In the case of Bertrand competition, firms tend to maximally differentiate in the BHM if demand functions are inelastic and there is no price discrimination (D'Aspremont et al., 1979), while they localize at the first and the third quartile in case of perfect price discrimination (Lederer and Hurter, 1986). Colombo (2009b) shows that the equilibrium distance between firms discontinuously decreases with the degree of imperfectness of price discrimination. Finally, Hamilton et al. (1989) find that, with elastic demand function and perfect price discrimination, firms locate between the first and the third quartile. In the case of Cournot competition within the BHM, Hamilton et al. (1989), Anderson and Neven (1991) and Shimizu (2002) obtain that the equilibrium location of firms is characterized by agglomeration in the middle of the segment. Moreover, we compare welfare in the Bertrand equilibrium with welfare in the Cournot equilibrium. We obtain that, unless the transportation costs are very low, Bertrand equilibrium is characterized by higher welfare than Cournot equilibrium.

In the second part of the article, we consider the impact of firms' location on the sustainability of profit-maximizing collusion. This issue has received considerable attention within the BHM. For example, Chang (1991), Chang (1992), Ross (1992) and Hackner (1995), in a uniform price model, find that the more the firms are located near in the space, the more collusion is difficult to sustain in equilibrium. Gupta and Venkatu (2002) and Colombo (2009c), in a spatial discrimination model, show that the relationship between firms distance and collusion sustainability may be negative when

¹ The unidirectional Hotelling model has been considered also by Cancian et al. (1995), Nilssen (1997) and Lai (2001). However, in these papers, firms are assumed to maximize the size of their own market (i.e. there is no competition on price or quantity). This substantially differentiates these models from ours, where firms set the price or the quantity.

firms use discriminatory prices instead of uniform prices. When studying collusion, we leave apart the two-stage game and we introduce an infinitely repeated game. In the case of price-setting firms, we obtain that when consumers can buy only from firms located on their right (left), the incentive to deviate of each firm decreases the more the firm is located to the right (left) and the more the rival is located to the left (right). In the case of quantity-setting firms, we need to adopt numerical computations, which however confirm the results we obtained within the Bertrand framework.

This paper is structured as follows. In Section 2 the UHM is introduced. In Section 3 we analyse the location-price equilibrium and the location-quantity equilibrium, and we compare equilibrium welfare in the two-cases. In Section 4 we introduce the infinitely repeated game and we analyse the impact of firms' location on the sustainability of perfect collusion as a sub-game perfect equilibrium. Section 5 summarizes.

2. The model

Assume a linear market of length 1. Consumers are uniformly distributed along the market. Denote by $x \in [0, 1]$ the location of each consumer. We depart from the traditional bidirectional Hotelling model (BHM) by assuming that a consumer can buy only from a firm located on his right-hand-side (Kharbach, 2009, and Colombo, 2009a).² There are two firms, firm A and firm B , whose location is identified respectively by a and b . Let us denote by A the firm which in equilibrium locates at the left, and with B the firm which locates at the right. Each firm produces at constant marginal costs, which are normalized to zero. Fixed costs are nil, but the firms pay the transportation costs to ship the good from the plant to consumers' location. We assume linear transportation costs as in Hamilton et al. (1989) and others. That is, to ship one unit of the product from its plant a (resp. b) to a consumer located at x , firm A (resp. B) pays a transport cost equal to: $t|a-x|$ (resp. $t|b-x|$), where t is the (strictly positive) unit transport cost. Firms set location-specific prices in the Bertrand game, while they set location-specific quantities in the Cournot game. Arbitrage between consumers is excluded. Denote by $p^A(x)$ and $p^B(x)$ the price schedule set by firm A and firm B respectively, and by $q^A(x)$ and $q^B(x)$ the quantity schedule set by firm A and firm B respectively. The term "price schedule" has the same meaning as in Encaoua and Hollander (2007): it refers to a positive valued function $p^J(\cdot)$ defined on $[0, 1]$ that specifies the price $p^J(x)$ set by firm $J = A, B$ to consumer x . Similarly, the term "quantity schedule" refers to a positive valued function $q^J(\cdot)$ defined on $[0, 1]$ that specifies the quantity $q^J(x)$ sold by firm $J = A, B$ to consumer x . In order to save notation, in the rest of the article the argument x in the price schedule and in the quantity schedule shall be omitted. At each location x , the demand function is assumed to be linear, and it is given by: $Q_x = 1 - p_x$, where p_x is the lower delivered price offered to consumers at x (that is, $p_x \equiv \min[p^A(\cdot), p^B(\cdot)]$). For the Cournot game, we use the inverse demand function, and the price at each location shall be determined by the

² The case where a consumer can buy only from a firm located on his left-hand-side is symmetric: therefore, we will only report the relevant results.

market-clearing condition. Therefore, the inverse demand function is $p_x = 1 - Q_x$, where Q_x is the total amount of quantity offered by firms at location x (that is, $Q_x \equiv q^A(\cdot) + q^B(\cdot)$). We assume that $t \leq 1/2$: this condition guarantees that there are no local monopolies and that no location is left without a positive quantity in equilibrium. This assumption is standard in spatial price discrimination literature (see, among the others, Hamilton et al., 1989, and Anderson and Neven, 1991).³

Finally, in order to distinguish the analysis within the Bertrand framework from the analysis within the Cournot framework, we shall identify with an upper bar the variables when the analysis is performed within the Cournot framework.

3. Location equilibrium

3.1. Bertrand

In this sub-section we study the location equilibrium emerging in a two-stage game where in the first stage the firms choose simultaneously where to locate and in the second stage choose simultaneously the price schedule.⁴ In this subsection and in the following, the sub-game Nash equilibrium concept is used in solving the game.

In the second stage of the game firms choose the price schedules given the locations. Consider the consumers located at $x \in [0, a]$. The consumers located at $x \in [0, a]$ can buy from both firms. In order to avoid ε -equilibria, we assume that if the two firms set the same price at location $x \in [0, a]$, the consumer buys from the nearer firm; if the two firms are located in the same point (so that the two firms are both “the nearer firm” for any given consumer), the market is shared evenly.⁵ Assume for the moment that $a \neq b$. The equilibrium prices on a consumer located at $x \in [0, a]$ have the following characteristics: both firms set the same price, which corresponds to the higher transportation costs to location x .⁶ Since when $x \in [0, a]$ firm A is nearer than firm B , the equilibrium price schedule is:

$$p^A * (a, b) = p_{x \in [0, a]}^B * (a, b) = t(b - x) \quad (1)$$

Both firms set the same price, and all consumers located at $x \in [0, a]$, in equilibrium, buy from firm A . Consider now consumers $x \in (a, b]$. The consumers located between the two firms can buy only from firm B . Therefore, firm B sets the monopolistic price at each location x . The equilibrium price schedule set by firm B on consumers $x \in (a, b]$ is therefore:

³ The only exceptions we are aware of are Chamorro-Rivas (2000) and Benassi et al. (2007).

⁴ As it will clear later, the results we obtain here hold also for a sequential-location game, where one firm enters the market first and locates, the other firm enters later and locates, and finally the firms compete simultaneously on price.

⁵ This assumption is standard in spatial models. For more details about this assumption, see among others Hurter and Lederer (1985), Lederer and Hurter (1986), Thisse and Vives (1988), Hamilton *et al.* (1989), Hamilton and Thisse (1992).

⁶ For a formal and general proof, see Lederer and Hurter (1986).

$$p_{x \in (a,b]}^B * (a,b) = [1 + t(b-x)]/2 \quad (2)$$

The consumers located at $x \in (b, 1]$ cannot buy any product. Therefore, the profits functions of the two firms are:⁷

$$\Pi_N^A(a,b) = \int_0^a [p^A * (a,b) - t(a-x)] Q_x(p^A * (a,b)) dx = [a(b-a)(2+at-bt)]/2 \quad (3)$$

$$\begin{aligned} \Pi_N^B(a,b) &= \int_a^b [p_{x \in (a,b]}^B * (a,b) - t(b-x)] Q_x(p_{x \in (a,b]}^B * (a,b)) dx = \\ &= (b-a)[3-t(b-a)(3+at-bt)]/12 \end{aligned} \quad (4)$$

Note that the profits functions are concave in the locations for the relevant range of a , b and t . First, observe that:

$$\frac{\partial \Pi_N^B(a,b)}{\partial b} = \frac{(1+ta-tb)^2}{4} > 0$$

It follows that the firm B locates at the right endpoint of the linear market.⁸ Therefore,

$$b^* = 1$$

Substituting $b^* = 1$ into $\partial \Pi_N^A(a,b)/\partial a = 0$ and solving with respect to a , we get the equilibrium location of firm A . That is:

$$a^* = 1 - \frac{2 - \sqrt{4 - 6t + 3t^2}}{3t}$$

It remains to verify that choosing the same location of the rival is never the optimal strategy for any firm. But this is immediate, as when the firms are located at the same point Bertrand competition leads prices to zero, which yield zero equilibrium profits. Therefore, the two firms will differ in equilibrium ($a \neq b$).

We sum up in the following proposition:

Proposition 1. In equilibrium, firm A locates at $a^* = 1 - \frac{2 - \sqrt{4 - 6t + 3t^2}}{3t}$ and firm B locates at $b^* = 1$.⁹

⁷ Let us use the subscript N to indicate the equilibrium profits. This will become useful in Section 4 when collusion will be introduced.

⁸ The fact that firm the derivative of firm B 's profits increases with b for any value of a excludes any incentive for firm B to "leapfrog" firm A by positioning at its left. For the same reason, firm A has no the possibility to "leapfrog" firm B by positioning at its right.

⁹ Note that, given the asymmetry of the model, there are two pure Nash equilibria. One is indicated in Proposition 1, and the other is simply obtained by reversing the firms' indices. In this sense, the

Note that the equilibrium location of firm A depends on the transportation costs. In fact:

$$\frac{\partial a^*}{\partial t} = \frac{3t - 4 + 2\sqrt{4 - 6t + 3t^2}}{3t^2\sqrt{4 - 6t + 3t^2}} > 0$$

Since the equilibrium location of firm A increases with t , the lower bound of the equilibrium location of firm A is $\lim_{t \rightarrow 0} a^* = 0.5$, while the upper bound is $a^*(t = 1/2) = (\sqrt{7} - 1)/3 \approx 0.55$. Moreover, Proposition 1 is valid also in a sequential-location game. Suppose firm A enters the market first, while firm B enters second. Solving by backward induction, it is immediate to observe that firm B chooses to locate at the right endpoint of the segment (recall that $\partial \Pi_N^B(a, b)/\partial b$ is strictly positive for any a). It follows that firm A chooses to locate at a^* .¹⁰

Therefore, when the consumers can buy only from firms located on the right-hand-side, one firm locates at the right endpoint of the market, while the other locates in the proximity of the middle of the market (by symmetry, when the consumers can buy only from firms located on the left-hand-side, one firm locates at the left endpoint of the market, while the other locates near the middle of the market – namely, between 0.45 and 0.5). The intuition is the following. Consider firm B . When both firms set location-specific prices, firm B monopolistically serves the consumers located between the two firms, but, in equilibrium, it does not serve the consumers located at the left of the rival. Therefore, firm B has the incentive to maximize the number of consumers that patronize it. It follows that firm B locates as far as possible from firm A in order to maximize its own market. Consider now firm A . Firm A serves only consumers located on its left. Therefore, the higher is a , the higher is the number of consumers which patronize firm A . Let us call this effect as the *demand effect*. At the same time, firm A 's profits depend also on the distance between the two firms. In fact, at each location, the mark-up of firm A is: $\mu^A \equiv p^A(a, b) - t(a - x) = t(b - a)$. This pushes firm A far from firm B in order to increase the mark-up. Let us call this effect as the *strategic effect*. The equilibrium between the *demand effect* and the *strategic effect* occurs at a^* . Moreover, a^* increases with t . In fact, when t increases, two opposite effects are at work. On one hand, the intensity of the *strategic effect* increases (in fact, $\partial^2 \mu^A / \partial a \partial t = -1 < 0$, which implies that the losses caused by a movement to the right of firm A go up); on the other hand, the intensity of the *demand effect* increases as well (in fact, the higher is t , the higher is the equilibrium price paid by those consumers which start to buy from firm A after a movement of firm A to the right). This second effect dominates, and therefore the equilibrium distance between the two firms is lower when t is high.

unidirectional Hotelling model shares the same “coordination problem” of the vertical differentiation model *a la* Hotelling (see Tirole, 1988, p.297, for a discussion about this issue).

¹⁰ Clearly, in a sequential-location game, there is *one* pure Nash equilibrium, where the second entrant locates at the endpoint. Therefore, the coordination problem arising in the case of simultaneous moves (“which firm of the two firms locates at the endpoint?”, see footnote 9) disappears in the sequential-location game.

3.2. Cournot

In this sub-section we study the location equilibrium emerging in a two-stage game where in the first stage the firms choose simultaneously where to locate and in the second stage choose simultaneously the quantity schedule.

In the second stage of the game each firm chooses the quantity schedule given the locations. Consider the consumers located at $x \in [0, a]$, which can buy from both firms. Given that firms spatially discriminate, each location can be treated as an independent market. At location x , firm A 's profits are: $\bar{\pi}_x^A = [1 - \bar{q}^A - \bar{q}^B - t(a-x)]\bar{q}^A$, while firm B 's profits are: $\bar{\pi}_x^B = [1 - \bar{q}^A - \bar{q}^B - t(b-x)]\bar{q}^B$. Straightforward calculations show that the equilibrium quantity schedules are:

$$\bar{q}^A * (a, b) = [1 - 2t(a-x) + t(b-x)]/3 \quad (5)$$

$$\bar{q}_{x \in [0, a]}^B * (a, b) = [1 - 2t(b-x) + t(a-x)]/3 \quad (6)$$

Note that, differently from the Bertrand framework, both firms sell positive quantities to consumers located at $x \in [0, a]$. Consider now consumers located at $x \in (a, b]$, which can buy from firm B only. Since firm B is a monopolist, it maximizes $\bar{\pi}^B = [1 - \bar{q}^B - t(b-x)]\bar{q}^B$, which yields:

$$\bar{q}_{x \in (a, b]}^B * (a, b) = [1 - t(b-x)]/2 \quad (7)$$

Using (5), (6) and (7), we can write each firm's profits as a function of locations. That is:

$$\begin{aligned} \bar{\Pi}_N^A(a, b) &= \int_0^a \bar{\pi}_x^A(\bar{q}^A * (a, b), \bar{q}_{x \in [0, a]}^B * (a, b)) dx = \\ &= \int_0^a [1 - \bar{q}^A * (a, b) - \bar{q}_{x \in [0, a]}^B * (a, b) - t(a-x)] \bar{q}^A * (a, b) dx = \\ &= \frac{a[7t^2a^2 - 9ta(1+tb) + 3(1+tb)^2]}{27} \end{aligned} \quad (8)$$

$$\begin{aligned} \bar{\Pi}_N^B(a, b) &= \int_0^a \bar{\pi}_x^B(\bar{q}^A * (a, b), \bar{q}_{x \in [0, a]}^B * (a, b)) dx + \int_a^b \bar{\pi}_x^B(\bar{q}_{x \in (a, b]}^B * (a, b)) dx = \\ &= \int_0^a [1 - \bar{q}^A * (a, b) - \bar{q}_{x \in [0, a]}^B * (a, b) - t(b-x)] \bar{q}_{x \in [0, a]}^B * (a, b) dx + \\ &+ \int_a^b [1 - \bar{q}_{x \in (a, b]}^B * (a, b) - t(b-x)] \bar{q}_{x \in (a, b]}^B * (a, b) dx = \\ &= \frac{19t^2a^3 + 9ta^2(1-5tb) + 9b(3-3tb+3t^2b^2) - 3a(5-2tb-7t^2b^2)}{108} \end{aligned} \quad (9)$$

Both functions are concave in the locations for the relevant range of a , b and t . Note that:

$$\frac{\partial \bar{\Pi}_N^B(a, b)}{\partial b} = \frac{9(1-tb)^2 - 15t^2a^2 + 2ta(1+7tb)}{36} > 0$$

It follows that, as in the Bertrand game, firm B locates at the right endpoint of the segment.¹¹ Therefore:

$$\bar{b}^* = 1$$

Substituting $\bar{b}^* = 1$ into $\partial \bar{\Pi}_N^A(a, b)/\partial a$, we get:

$$\frac{\partial \bar{\Pi}_N^A(a, b=1)}{\partial a} = \frac{1+t(2-6a)+t^2(1-6a+7a^2)}{9}$$

Note that when $t \leq 1 - \sqrt{2}/2$, we obtain: $\partial \bar{\Pi}_N^A(a, b=1)/\partial a > 0$, while when $t \geq 1 - \sqrt{2}/2$ an interior solution of $\partial \bar{\Pi}_N^A(a, b=1)/\partial a = 0$ exists. Therefore, the equilibrium location of firm A is given by:

$$\bar{a}^* = \begin{cases} 1 & \text{if } t \leq 1 - \sqrt{2}/2 \\ [3(1+t) - \sqrt{2(1+2t+t^2)}]/7t & \text{if } t \geq 1 - \sqrt{2}/2 \end{cases}$$

We sum up the equilibrium location in the following proposition:

Proposition 2. In equilibrium, firm A locates at $\bar{a}^* = \begin{cases} 1 \\ [3(1+t) - \sqrt{2(1+2t+t^2)}]/7t \end{cases}$

if $t \leq 1 - \sqrt{2}/2$ and firm B locates at $\bar{b}^* = 1$.¹²
if $t \geq 1 - \sqrt{2}/2$

Therefore, when the transportation costs are sufficiently low, firm A locates at the right endpoint of the segment, as firm B . Hence, agglomeration arises. Instead, when transportation costs are sufficiently high, firm A locates in a different point with respect to firm B . Moreover, when $t \geq 1 - \sqrt{2}/2$, we get: $\partial \bar{a}^*/\partial t = -(3 - \sqrt{2})/7t^2 < 0$. Therefore, the lowest equilibrium location for firm A occurs when $t = 1/2$. That is: $\bar{a}^*(t = 1/2) = 3(3 - \sqrt{2})/7 \approx 0.67$. It follows that the equilibrium location of firm A decreases with the transportation costs parameter and it is comprised between 0.67 and 1, while firm B always locates at 1. As for the Bertrand model, it is immediate to verify that the equilibrium locations in Proposition 2 arise also in a sequential-location game. In this case, the second entrant always locates at the right endpoint of the segment.

Summing up, when the consumers can buy only from firms located on the right-hand-side, one firm locates at the right endpoint of the market, while the other locates in the

¹¹ As for the Bertrand model, this eliminates the possibility of “leapfrogging” by firms (see footnote 8).

¹² As noticed in footnote 9, another equilibrium, where the indices of the two firms are reverted, also exists.

proximity of the right endpoint of the market (by symmetry, when the consumers can buy only from firms located on the left-hand-side, one firm locates at the left endpoint of the market, while the other locates in the proximity of the left endpoint of the market, between 0 and 0.33).

Comparing the equilibrium configuration under the Cournot framework with the equilibrium configuration under the Bertrand framework, we observe that under quantity competition equilibrium distance between firms is lower than under price competition. Moreover, for low levels of the transportation costs agglomeration arises at the most eastern point of the segment within the Cournot framework, while agglomeration never arises within the Bertrand framework. The reason of the difference between the Bertrand and the Cournot game is the following. While in the Bertrand game only firm A serves the consumers localized at the left, in the Cournot game both firms serve consumers localized at the left of firm A . As a consequence, the *strategic effect* (see sub-section 3.1) is less strong, since a movement to the right by firm A does not fully reflect in a decrease of the mark-up.¹³ It follows that firm A has a greater incentive to locate to the right. Moreover, \bar{a}^* decreases with t . The intuition is the following. On one hand, the intensity of the *strategic effect* increases;¹⁴ on the other hand, the intensity of the *demand effect* increases too, because the higher is t , the higher is the equilibrium price paid by the consumers which start to buy from firm A after that firm A moves to the right. This second effect is less strong in Cournot than in Bertrand (any location served by firm A is also served by firm B): in contrast with the Bertrand framework, in the Cournot framework the former effect dominates, and therefore the equilibrium distance between the two firms is lower when t is low: when transportation costs are particularly low, agglomeration of firms occurs in equilibrium.

3.3. Welfare

In this sub-section we compare welfare in the Bertrand equilibrium with the welfare in the Cournot equilibrium. Let w_x and \bar{w}_x denote the welfare (consumer surplus plus the profits of the two firms) at location x in the Bertrand equilibrium and in the Cournot equilibrium respectively. Given the linearity of demand, welfare at x in Bertrand equilibrium is:

$$w_x = \frac{(1 + p^{B*})Q_x(p^{B*})}{2} - t(a^* - x)q^A(p^{B*}) - t(b^* - x)q^B(p^{B*})$$

where:

¹³ In fact, at each location, the mark-up of firm A is: $\bar{\mu}^A \equiv p_x(\bar{q}^A(a, b), \bar{q}_{x \in [0, a]}^B(a, b)) - t(a - x) = (1 + tb - 2ta + tx)/3$. Comparing the impact of a movement to the right by firm A in the Cournot framework with the analogous in the Bertrand framework, we observe that: $|\partial \bar{\mu}^A / \partial a| = 2t/3 < t = |\partial \mu^A / \partial a|$, that is, the *strategic effect* is less strong under Cournot than under Bertrand.

¹⁴ In fact, the *strategic effect* is: $|\partial \bar{\mu}^A / \partial a| = 2t/3$, which increases with t .

- $p^{B*} = \begin{cases} p_{x \in [0, a]}^B & \text{if } x \in [0, a^*] \\ p_{x \in (a, b]}^B & \text{if } x \in (a^*, b^*] \end{cases}$
- $q^A(p^{B*}) = \begin{cases} Q_x(p^{B*}) & \text{if } x \in [0, a^*] \\ 0 & \text{if } x \in (a^*, b^*] \end{cases}$
- $q^B(p^{B*}) = \begin{cases} 0 & \text{if } x \in [0, a^*] \\ Q_x(p^{B*}) & \text{if } x \in (a^*, b^*] \end{cases}$

In Cournot equilibrium, welfare at x is:

$$\bar{w}_x = \frac{(1 + p_x(\bar{Q}_x^*))\bar{Q}_x^*}{2} - t(\bar{a}^* - x)\bar{q}^A - t(\bar{b}^* - x)\bar{q}^B$$

where:

- $\bar{q}^{B*} = \begin{cases} \bar{q}_{x \in [0, a]}^B & \text{if } x \in [0, a^*] \\ \bar{q}_{x \in (a, b]}^B & \text{if } x \in (a^*, b^*] \end{cases}$

Therefore, total welfare in the two models is given by: $W = \int_0^{b^*} w_x dx$ and $\bar{W} = \int_0^{\bar{b}^*} \bar{w}_x dx$. Substituting the equilibrium values, we obtain the following equations:

$$W = \frac{-206 + 738t - 531t^2 + 108t^3 + (103 - 150t + 75t^2)\sqrt{4 - 3t(2 - t)}}{648t}$$

$$\bar{W} = \begin{cases} \frac{4(3 - 3t + t^2)}{27} & \text{if } t \leq 1 - \frac{\sqrt{2}}{2} \\ \frac{86\sqrt{2} - 27 + t(28836 + 762\sqrt{2}) - t^2(25596 - 1266\sqrt{2}) + t^3(10368 + 590\sqrt{2})}{74088t} & \text{if } t \geq 1 - \frac{\sqrt{2}}{2} \end{cases}$$

Let us define: $\Lambda \equiv W - \bar{W}$. We can observe that Λ is always positive unless $t \leq 0.03$. Therefore, welfare is always higher in Bertrand equilibrium than in Cournot equilibrium, unless the transportation costs parameter is very low. The following figures illustrate the relationship between total welfare in the Bertrand equilibrium and in the Cournot equilibrium (Λ). In Figure 1 we consider the case where $t \leq 1 - \sqrt{2}/2$, while in Figure 2 we consider the case where $t \geq 1 - \sqrt{2}/2$.

Figure 1

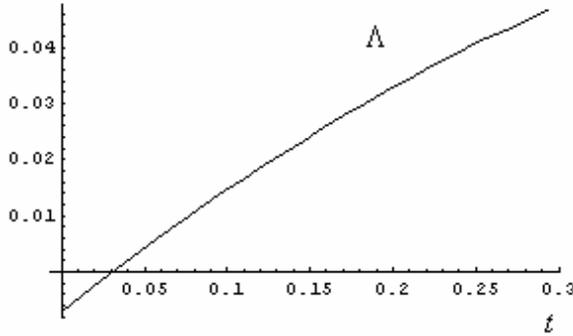
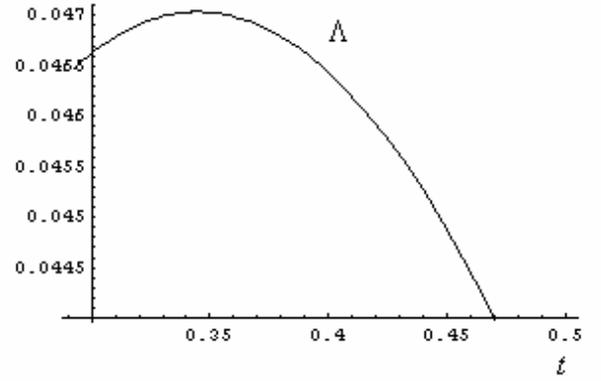


Figure 2



Therefore, welfare tends to be higher when firms compete with prices with respect to the case where firms compete with quantities. The reason is the following. Bertrand competition is fiercer than Cournot competition. As a consequence, *for equal locations*, equilibrium prices are lower under Bertrand than under Cournot. In our model, equilibrium locations in Bertrand are different from equilibrium locations in Cournot. However, it can be easily verified that prices continue to be lower in Bertrand than in Cournot at locations $x \in [0, a^*]$, that is, in those locations where both firms are active in both models.¹⁵ *Ceteris paribus*, this tends to make welfare higher in Bertrand than in Cournot. Consider now equilibrium locations. In Bertrand, firms are more distant in equilibrium than in Cournot. This has two opposite effects on welfare. On one hand, higher distance between firms decreases total transportation costs, and this increases welfare. On the other hand, higher distance between firms increases the monopoly area of firm B . More locations are served in monopoly, and this tends to reduce welfare. For a wide range of parameters of the model, this last effect is outweighed by the other effects, and welfare is higher in the Bertrand equilibrium than in the Cournot equilibrium. However, when the transportation costs are very low ($t \leq 0.03$), firms are maximally distant in Bertrand equilibrium, while they agglomerate in Cournot equilibrium (recall that a^* increases with t , while \bar{a}^* decreases with t). Therefore, the monopolist area served by firm B is maximal in Bertrand, while is nil in Cournot. In this case, the detrimental effect in terms of welfare due to the monopolistic area outweighs the fact that transportation costs and prices in the competitive area are lower in Bertrand than in Cournot: as a consequence, welfare is lower in Bertrand than in Cournot.

4. Collusion

In this section, we consider collusion within the UHM, both under the assumption of price-setting firms (Bertrand) and under the assumption of quantity-setting firms (Cournot). The two-stage game used in Section 3 is substituted by an infinitely repeated game, which is needed in order to assess the conditions for collusion as a sub-game perfect equilibrium. We focus in particular on the role of firms' locations (which,

¹⁵ One has to check that $p_{x \in [0, a^*]}^B \leq p_x(\bar{Q}_x^*)$, $\forall x \in [0, a^*]$. Details of the calculations are available from the author upon request.

therefore, are kept exogenous in this section). Moreover, in order to maintain tractability, we follow Gupta and Venkatu (2002) and Matsumura and Matsushima (2005) and we consider only perfect collusion. A grim strategy is assumed (Friedman, 1971)¹⁶ and there is perfect monitoring. Denote by Π_C^i , Π_D^i and Π_N^i respectively the one-shot collusive profits, the one-shot deviation profits and the one-shot punishment (or Nash) profits for firm $i = A, B$: obviously, it must be: $\Pi_D^i > \Pi_C^i > \Pi_N^i$. Denote by δ the market discount factor, which is assumed to be exogenous and common for each firm. It is well known that collusion is sustainable as a sub-game perfect Nash equilibrium if and only if the discounted value of the profits that each firm obtains under collusion exceeds the discounted value of the profits that each firm obtains deviating from the agreement. Formally, the following incentive-compatibility constraint must be satisfied: $\sum_{t=0}^{\infty} \delta^t \Pi_C^i \geq \Pi_D^i + \sum_{t=1}^{\infty} \delta^t \Pi_N^i$, for $i = A, B$. After rearranging, the condition for collusion as sub-game perfect equilibrium of the super-game is:

$$\delta \geq \delta^* \equiv \max[\delta^{A*}, \delta^{B*}]$$

where $\delta^{A*} = (\Pi_D^A - \Pi_C^A) / (\Pi_D^A - \Pi_N^A)$ and $\delta^{B*} = (\Pi_D^B - \Pi_C^B) / (\Pi_D^B - \Pi_N^B)$. Define δ^* as the critical discount factor. If the market discount factor is greater than the critical discount factor collusion is sustainable in equilibrium, otherwise it is not sustainable. Then, the critical discount factor measures the sustainability of the collusive agreement: the greater is δ^* the smaller is the set of market discount factors which support collusion.

4.1. Bertrand

In this sub-section we consider price-setting firms. The consumers located between a and b can only buy from firm B , which sets monopolistic prices. Recall that under competition (see sub-section 3.1), firm B sets monopolistic prices on consumers located at the right of firm A . Therefore, the collusive profits must coincide with Nash profits at locations $x \in (a, b]$. In other words, a collusive agreement cannot generate higher profits than competitive profits over consumers located at the right of firm A . Consider now consumers located at $x \in [0, a]$. Here, a collusive agreement may be profitable for both firms. Suppose a perfect collusive agreement of this type. In change for renouncing to compete with firm A on consumers located at $x \in [0, a]$, firm B receives a fraction $\theta \in (0, 1)$ of the collusive profits obtained by firm A .¹⁷ The collusive (monopolistic)

¹⁶ The grim trigger strategy is not optimal (Abreu, 1986). However, “*this is one of very realistic punishment strategies because of its simplicity*”, as argued by Matsumura and Matsushima (2005, p.263). The most part of the papers which study collusion sustainability in spatial models adopt the grim trigger strategy. See for example, Chang (1991), Chang (1992), Friedman and Thisse (1993), Hackner (1994, 1995), Matsumura and Matsushima (2005).

¹⁷ The cases $\theta = 0$ and $\theta = 1$ must be excluded, since they imply $\Pi_C^B = \Pi_N^B$ and $\Pi_C^A < \Pi_N^A$ respectively.

price schedule set by firm A on consumers located at $x \in [0, a]$ is $p_C^A = [1 + t(a - x)]/2$. This yields the following collusive profits for firm A and firm B respectively:

$$\Pi_C^A = (1 - \theta)G \quad (10)$$

$$\Pi_C^B = \theta G + \Pi_N^B \quad (11)$$

where

$$G = \int_0^a [p_C^A - t(a - x)]Q_x(p_C^A)dx = \frac{a(3 - 3ta + t^2a^2)}{12} \quad (12)$$

G can be interpreted as the collusive profits that firm A would obtain if $\theta \rightarrow 0$, i.e. if firm A would keep for itself all the profits it obtains when firm B does not compete over consumers located at $x \in [0, a]$. Note that firm A participates to the collusive agreement only if $\Pi_C^A > \Pi_N^A$, which implies: $\theta < \hat{\theta} = 1 - \Pi_N^A/G < 1$.

Suppose now that firm B deviates from the collusive agreement. The optimal strategy for firm B consists in undercutting firm A at each location $x \in [0, a]$. Instead, no deviation is profitable at locations $x \in (a, b]$, and here deviation profits coincide with punishment profits. Therefore, the overall firm B 's deviation profits can be written as:¹⁸

$$\begin{aligned} \Pi_D^B &= \int_0^a [p_C^A - t(b - x)]Q_x(p_C^A)dx + \Pi_N^B = \\ &= \int_0^a [p_C^A - t(a - x) + t(a - x) - t(b - x)]Q_x(p_C^A)dx + \Pi_N^B = \\ &= \int_0^a [p_C^A - t(a - x)]Q_x(p_C^A)dx - \int_0^a [t(b - x) - t(a - x)]Q_x(p_C^A)dx + \Pi_N^B = \\ &= G - \Pi_N^A + \Pi_N^B \end{aligned} \quad (13)$$

Substituting (11) and (13) into δ^{B*} we get:¹⁹

$$\delta^{B*} = 1 - \frac{\theta G}{G - \Pi_N^A} \quad (14)$$

Suppose now that firm A deviates from the collusive agreement. Clearly, firm A has never the incentive to deviate from the collusive agreement *through prices*: there are monopolistic prices on consumers located at $x \in [0, a]$ and therefore no deviation is profitable. Moreover, consumers located at $x \in (a, b]$ cannot be served by firm A whatever the price it sets. However, firm A can deviate from the collusive agreement by

¹⁸ Note that $\theta < \hat{\theta}$ also guarantees that deviation is *a priori* profitable for firm B . In fact, solving $\Pi_D^B > \Pi_C^B$ yields $\Pi_C^A = (1 - \theta)G > \Pi_N^A$, which is satisfied when $\theta < \hat{\theta}$.

¹⁹ Note that Π_N^B simplifies and therefore plays no role on the sustainability of the collusive agreement.

refusing to transfer a fraction of the collusive profits to firm B . Since *any* deviation induces the same punishment, firm A chooses the best deviation. Therefore, if deviates, firm A refuses to transfer the whole fraction θ of the collusive profits over consumers located at $x \in [0, a]$ to firm B . In other words, the deviation profits of firm A coincides with G , that is: $\Pi_D^A = G$. Plugging deviation profits, collusive profits and punishment profits into δ^{A*} , we get:

$$\delta^{A*} = \frac{\theta G}{G - \Pi_N^A} \quad (15)$$

Let us define $\Gamma \equiv G/(G - \Pi_N^A)$ and $\theta^* = 1/2\Gamma$. Note also that $\theta^* = \hat{\theta}/2$. We can state the following proposition:

Proposition 3. The critical discount factor is the following: $\delta^* = \begin{cases} \delta^B * & \text{if } \theta \in (0, \theta^*] \\ \delta^A * & \text{if } \theta \in [\theta^*, \hat{\theta}] \end{cases}$

Moreover, it must be: $\theta^* \in (0, 1/2]$, with $\partial\theta^*/\partial a \geq 0$ and $\partial\theta^*/\partial b \leq 0$.

Proof. Recall that the critical discount factor coincides with the highest value between $\delta^A *$ and $\delta^B *$. The first part of the proposition comes from the comparison between (14) and (15). In order to prove the second part of the proposition, we substitute (3) and (12) into Γ , and then we substitute again into θ^* . After simplifications, we get:

$$\theta^*(a, b, t) = \frac{7t^2 a^2 + ta(1 - 2tb) + 3(1 - 2tb)^2}{2(3 - 3ta + t^2 a^2)}$$

Then, taking the derivative of θ^* with respect to a and b respectively, we observe:

$$\frac{\partial\theta^*(a, b, t)}{\partial a} = \frac{3t[6 - 15tb + 6t^2 b^2 - 5t^2 a^2(5 - 3tb) + 2ta(3 + 2tb - 2t^2 b^2)]}{(3 - 3ta + t^2 a^2)^2} \geq 0$$

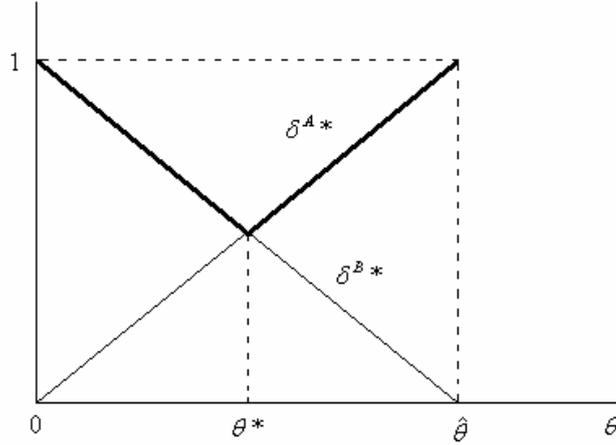
$$\frac{\partial\theta^*(a, b, t)}{\partial b} = -\frac{3t(2 + 3ta - 4tb)}{3 - 3ta + t^2 a^2} \leq 0$$

In order to find the range of the admissible values of θ^* , note that as a consequence of $\partial\theta^*/\partial a \geq 0$, the maximum of θ^* occurs at $a = b$. Substituting, we get: $\theta^*(a = b) = 1/2$. The minimum of θ^* occurs when $a \rightarrow 0$ (note that $a = 0$ has to be excluded, since it implies that collusive profits of firm A are zero as the punishment profits, thus making collusion meaningless). Substituting, we get: $\theta^* = (1 - 2tb)^2/2$.

Since θ^* decreases with b and with t , we substitute $b = 1$ and $t = 1/2$ into θ^* , and we obtain $\theta^*(b = 1, t = 1/2) = 0$. ■

Figure 3 illustrates the first part of Proposition 3. The bold line indicates the shape of the critical discount factor δ^* , which coincides with δ^{B*} when $\theta \leq \theta^*$ and with δ^{A*} when $\theta \geq \theta^*$. Proposition 1 has a straightforward intuition. When the fraction of the collusive profits on the consumers located at $x \in [0, a]$ that goes to firm B is high, firm A has more incentive to deviate than firm B (thus, $\delta^* = \delta^{A*}$), while the reverse occurs when the fraction of the collusive profits on consumers located at $x \in [0, a]$ that goes to firm B is low (thus, $\delta^* = \delta^{B*}$).

Figure 3



Now, we turn to the effect of firms' locations on collusion sustainability. We can state the following proposition:

Proposition 4. The impact of a on the critical discount factor is the following:

- If $\theta \in [\theta^*, \hat{\theta})$, $\forall a$, then $\partial \delta^* / \partial a \leq 0$.
- If $\theta \in (0, \theta^*]$, $\forall a$, then $\partial \delta^* / \partial a \geq 0$.
- If $\tilde{a} \in (0, b]$ exists such that $\theta = \theta^*(\tilde{a})$, then $\partial \delta^* / \partial a \leq 0$, $\forall a \in (0, \tilde{a}]$ and $\partial \delta^* / \partial a \geq 0$, $\forall a \in [\tilde{a}, b]$.

The impact of b on the critical discount factor is the following:

- If $\theta \in [\theta^*, \hat{\theta})$, $\forall b$, then $\partial \delta^* / \partial b \geq 0$.
- If $\theta \in (0, \theta^*]$, $\forall b$, then $\partial \delta^* / \partial b \leq 0$.
- If $\tilde{b} \in [a, 1)$ exists such that $\theta = \theta^*(\tilde{b})$, then $\partial \delta^* / \partial b \leq 0$, $\forall b \in [a, \tilde{b}]$ and $\partial \delta^* / \partial b \geq 0$, $\forall b \in [\tilde{b}, 1)$.

Proof. Consider the first part of Proposition 4. Suppose that $\theta \in [\theta^*, \hat{\theta})$, $\forall a$. From Proposition 1 (first part), it follows that $\delta^* = \delta^{A*} = \theta \Gamma$. Since $\theta^* = 1/2\Gamma$, Proposition 3 (second part) implies $\partial \Gamma / \partial a \leq 0$, which in turn implies $\partial \delta^* / \partial a \leq 0$. Suppose $\theta \in (0, \theta^*]$, $\forall a$. From Proposition 3 (first part), it follows that $\delta^* = \delta^{B*} = 1 - \theta \Gamma$. Since

$\theta^* = 1/2\Gamma$, Proposition 3 (second part) implies $\partial\Gamma/\partial a \leq 0$, which in turn implies $\partial\delta^*/\partial a \geq 0$. Consider the case where $\tilde{a} \in (0, b]$ exists such that $\theta = \theta^*(\tilde{a})$. Since $\partial\theta^*/\partial a \geq 0$ (Proposition 3, second part), it must be $\theta \in [\theta^*, \hat{\theta})$ if $a \leq \tilde{a}$ and $\theta \in (0, \theta^*]$ if $a \geq \tilde{a}$. Consider the second part of Proposition 4. Suppose that $\theta \in [\theta^*, \hat{\theta}), \forall b$. From Proposition 3 (first part), it follows that $\delta^* = \delta^{A*} = \theta\Gamma$. Since $\theta^* = 1/2\Gamma$, Proposition 3 (second part) implies $\partial\Gamma/\partial b \geq 0$, which in turn implies $\partial\delta^*/\partial b \geq 0$. Suppose that $\theta \in (0, \theta^*], \forall b$. From Proposition 3 (first part), it follows that $\delta^* = \delta^{B*} = 1 - \theta\Gamma$. Since $\theta^* = 1/2\Gamma$, Proposition 3 (second part) implies $\partial\Gamma/\partial b \geq 0$, which in turn implies $\partial\delta^*/\partial b \leq 0$. Consider the case where $\tilde{b} \in [a, 1)$ exists such that $\theta = \theta^*(\tilde{b})$. Since $\partial\theta^*/\partial b \leq 0$ (Proposition 3, second part), it must be $\theta \in (0, \theta^*]$ if $b \leq \tilde{b}$ and $\theta \in [\theta^*, \hat{\theta})$ if $b \geq \tilde{b}$. ■

Proposition 4 can be summarized as follows: when consumers can buy only from firms located on their right, the incentive to deviate of each firm decreases the more the firm is located to the right and the more the rival is located to the left. By symmetry, when consumers can buy only from firms located on their left, the incentive to deviate of each firm decreases the more the firm is located to the left and the more the rival is located to the right.

The intuition is the following. Consider the impact of a higher a on δ^{B*} . When a marginally increases, collusive profits of firm B increase by $\theta \partial G/\partial a$. Instead, if firm B deviates, it undercuts firm A : therefore, when a marginally increases, deviation profits of firm B increase by $\partial G/\partial a$. It follows that when firm A moves to the right, deviation profits of firm B increase more than collusive profits (*undercutting effect*), thus making collusion less sustainable. Consider now the case of higher b . When b increases, firm B has to pay higher transportation costs when it deviates, and this makes deviation less profitable for firm B and collusion more sustainable. Consider now the impact of higher a on δ^{A*} . Differently from firm B , punishment profits of firm A are affected by the locations of firms. In particular, when firms are nearer (a is higher) competition is fiercer and punishment profits of firm A are lower. Therefore, this effect (let us call it *punishment effect*) increases collusion sustainability. Even if the *undercutting effect* is at work also for firm A (when it moves to the right and deviates, firm A appropriates all the collusive profits by refusing to transfer the quota pertaining to firm B), the *punishment effect* outweighs the *undercutting effect*: then, the temptation to deviate of firm A decreases when a increases. Finally, consider the impact of an higher b on δ^{A*} . Clearly, neither collusive nor deviation profits of firm A are affected. Instead, higher b means that competition between the two firms is less fierce, i.e. punishment profits of firm A are higher. This increases the temptation to deviate of firm A . Of course, as we have shown in Proposition 1, whether the critical discount factor coincides with δ^{A*} or δ^{B*} depends on the sharing rule adopted by the colluding firms.

4.2. Cournot

In this section, we analyse collusion sustainability when firms are assumed to set quantities. Unfortunately, no easy relationship between the incentive to deviate of firm

A and the incentive to deviate of firm B can be derived. However, we can proceed as follows. Consider collusive quantities. The joint-profits maximizing quantity schedule consists in a series of monopolistic quantities, one at each location. Moreover, consumers located at $x \in [0, a]$ are served by firm A only, since this minimizes transportation costs, while consumers located at $x \in (a, b]$ are served by firm B only, since this is the only feasible solution in the UHM. The monopolistic quantity sold by firm A on consumers located at $x \in [0, a]$ is $\bar{q}_C^A = [1 - t(b - x)]/2$. As for Bertrand, in change for renouncing to compete with firm A on consumers located at $x \in [0, a]$, firm B receives a fraction $\bar{\theta} \in (0, 1)$ of the collusive profits of firm A . Since under collusion, both in Bertrand and Cournot, all points in the space are monopolized by one firm, the collusive profits must be the same in the two models (for a monopolist there is no difference between choosing a quantity schedule or a price schedule). In other words, $\bar{\Pi}_C^B = \Pi_C^B$ and $\bar{\Pi}_C^A = \Pi_C^A$. Consider a deviation by firm B .²⁰ When firm A sells \bar{q}_C^A and firm B deviates from the collusive agreement, firm B maximizes $\bar{\Pi}^B(\bar{q}_C^A, \bar{q}^B)$ with respect to \bar{q}^B , which yields the following deviation quantity schedule:

$$\bar{q}_D^B = \frac{1 - t(b - a - x)}{4}$$

In order to obtain a positive mark-up when it serves a consumer located at $x \in [0, a]$, the following condition must hold: $1 - \bar{q}_C^A - \bar{q}_D^B - t(b - x) \geq 0, \forall x$, or $x \geq x^* = 2b - a - 1/t, \forall x$. Since $t \leq 1/2$ by hypothesis and the maximum distance between firms is one, it follows that $x^* \leq 0$, which implies that the mark-up of firm B is always positive. Therefore, the deviating firm sells a positive amount everywhere. The deviation profits of firm B are:

$$\begin{aligned} \bar{\Pi}_D^B &= \int_0^a [p_x(\bar{q}_C^A, \bar{q}_D^B) - t(b - x)]\bar{q}_D^B dx + \int_a^b [p_x(0, \bar{q}_C^B) - t(b - x)]\bar{q}_C^B dx = \\ &= \frac{3t^2 a^2 - 3ta^2(1 + 2tb) - 3a(3 - 4tb) + 4b(3 - 3tb + 3t^2 b^2)}{48} \end{aligned} \quad (16)$$

Let us consider the discount factor $\bar{\delta}^{B*} = (\bar{\Pi}_D^B - \bar{\Pi}_C^B) / (\bar{\Pi}_D^B - \bar{\Pi}_N^B)$. The sign of the derivative of $\bar{\delta}^{B*}$ with respect to a and b is difficult to be derived analytically. However, we undertake a series of computations for different values of $t \in (0, 1/2]$ and they all show that $\partial \bar{\delta}^{B*} / \partial a \geq 0$ and $\partial \bar{\delta}^{B*} / \partial b \leq 0$.²¹ This confirms that the results derived for the Bertrand game hold also with quantity-setting firms.

Consider now firm A . Firm A cannot profitably deviate through quantities, because it is already setting monopolistic quantities, but it can deviate refusing to give the fraction $\bar{\theta}$ of its collusive profits to firm B . In this case, it obtains $\bar{\Pi}_D^A = G$ (note that the

²⁰ We are implicitly assuming that deviation is not *a priori* unprofitable for firm B , that is, $\bar{\theta}$ is assumed to be sufficiently low.

²¹ The complete expressions of the derivatives are relegated in the appendix.

deviation profits of firm A in Cournot coincide with the deviation profits in Bertrand, because firm A sets monopolistic quantities; this was not true for firm B). Therefore, the discount factor of firm A is:

$$\bar{\delta}^{A*} = \frac{\bar{\Pi}_D^A - \bar{\Pi}_C^A}{\bar{\Pi}_D^A - \bar{\Pi}_N^A} = \frac{\bar{\theta}G}{G - \bar{\Pi}_N^A} \quad (17)$$

Note that the only difference with respect to the discount factor of firm A in the Bertrand model, δ^{A*} , regards the punishment profits. We can state the following proposition.

Proposition 5. The following inequalities hold: $\partial \bar{\delta}^{A*} / \partial a \leq 0$ and $\partial \bar{\delta}^{A*} / \partial b \geq 0$.

Proof. Plugging (8) and (12) into (17) and then taking the derivative of $\bar{\delta}^{A*}$ with respect to a , we get:

$$\frac{\partial \bar{\delta}^{A*}}{\partial a} = - \frac{108\bar{\theta}t[6 + 3tb - 6t^2b^2 + t^2a^2(4 - 3tb) - 2ta(6 - 2tb - t^2b^2)]}{[19t^2a^2 - 9ta(1 + 4tb) - 3(5 - 8tb - 4t^2b^2)]^2} \leq 0$$

Similarly, taking the derivative of $\bar{\delta}^{A*}$ with respect to b , we get:

$$\frac{\partial \bar{\delta}^{A*}}{\partial b} = \frac{108\bar{\theta}t(2 - 3ta + 2tb)(3 - 3ta + t^2a^2)}{[19t^2a^2 - 9ta(1 + 4tb) - 3(5 - 8tb - 4t^2b^2)]^2} \geq 0 \quad \blacksquare$$

Proposition 5 shows that the incentive to deviate of firm A in the Cournot model depends on the location parameters in the same way as in the Bertrand model: higher a decreases the incentive to deviate, while higher b increases the incentive to deviate. In fact, when a increases, the competition between firms over the consumers located at $x \in [0, a]$ during the punishment stage is fiercer, and the punishment profits of firm A are lower (*punishment effect*, see Section 4.1), and therefore the temptation to deviate of firm A decreases. At the opposite, when b increases, the punishment profits of firm A are higher, thus increasing the temptation to deviate of firm A .

Also, note that $\bar{\delta}^{A*}$ increases with $\bar{\theta}$, while $\bar{\delta}^{B*}$ decreases with $\bar{\theta}$. Therefore, as in the Bertrand framework, when the fraction of the collusive profits that goes to firm B is high, firm A has a greater incentive to deviate from the agreement than firm B , while the opposite holds when the fraction of the collusive profits that goes to firm B is low (see Proposition 3).

5. Conclusion

This paper considers a unidirectional Hotelling model, which differentiates from the standard Hotelling model because consumers are assumed to have a unidirectional purchasing ability, i.e. they can buy only from firms located at their right or only from firms located at their left. In the first part of the article we analyse the equilibrium which

emerges in two different two-stage games: in one game (Bertrand), we suppose that firms first choose location, and then set price schedules, where prices may be different across locations; in the other game (Cournot), we suppose that firms first choose location, and then set quantity schedules, where quantities may be different across locations. We show that in both games, one firm locates at one endpoint of the market in order to maximize the number of consumers which are served monopolistically. However, the equilibrium distance between the firms is quite different between the two models. In Bertrand, agglomeration never arises and the firm which is not located at the endpoint locates near to the middle of the segment. Instead, in Cournot, when transportation costs are sufficiently low, agglomeration arises: when there is no agglomeration, equilibrium distance between the firms is lower than in Bertrand.

In the second part of the article, we study the impact of firms' location on the ability of firms to preserve a joint-profits maximizing collusive agreement (on prices or on quantities) from defection of one of the members of the cartel. In Bertrand, we obtain that when consumers can buy only from firms located on their right (left), the incentive to deviate of each firm decreases the more the firm is located to the right (left) and the more the rival is located to the left (right). In Cournot, numerical computations show that the results obtained in the Bertrand model can be extended to the case of quantity-setting firms.

Appendix

In this appendix, we report the complete expression of $\bar{\delta}^{B*}$ and the derivate of $\bar{\delta}^{B*}$ with respect to a and with respect to b . Plugging (9), (11) and (16) into $\bar{\delta}^{B*}$, after simplifications, we get:

$$\bar{\delta}^{B*} = \frac{9[t^2a^3(1+4\bar{\theta}) + 3ta^2(5-4\bar{\theta}-2tb) - 8b(3-3tb+t^2b^2) + 3a(7+4\bar{\theta}-12tb+t^2b^2)]}{7a[7t^2a^2 + 9ta(1-2tb) + 3(1-2tb)^2]}$$

$$\frac{\partial \bar{\delta}^{B*}}{\partial a} = \frac{\left[\begin{aligned} &-216[-t^4b^5 + t^3b^4(4+3ta) - t^2b^3(25+42ta+13t^2a^2) + \\ &+ tb^2(15+54ta+6t^2a^2(7+\bar{\theta}) + 2t^3a^3(3-2\bar{\theta})) + \\ &+ b(-3-18ta-3t^2a^2(14+5\bar{\theta}) - 4t^3a^3(5-\bar{\theta}) - 4t^4a^4(1-3\bar{\theta})) + \\ &+ ta^2(6+12ta+4t^2a^2 + \bar{\theta}(6+ta-5t^2a^2))] \end{aligned} \right]}{7a[7t^2a^2 + 9ta(1-2tb) + 3(1-2tb)^2]^2}$$

$$\frac{\partial \bar{\delta}^{B*}}{\partial b} = \frac{\left[\begin{aligned} &216[-3+6tb-3t^2b^2 + 8t^3b^3 - 4t^4b^4 - t^4a^4(1-3\bar{\theta}) + \\ &-3ta(1+5t^2b^2 - 4t^3b^3 - 2\bar{\theta}(1-2tb)) + t^2a^2(2+8tb + \\ &-13t^2b^2 + 3\bar{\theta}(1+atb)) - t^3a^3(1-6tb + \bar{\theta}(7+4tb))] \end{aligned} \right]}{7a[7t^2a^2 + 9ta(1-2tb) + 3(1-2tb)^2]^2}$$

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