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L. Colombo G. Femminis

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L. Colombo (*) G. Femminis (•)

(*)Istituto di Economia e Finanza, Università Cattolica del Sacro Cuore, Largo Gemelli 1 – 20123 Milano, e-mail: lucava.colombo@unicatt.it

(•)Istituto di Teoria Economica e Metodi Quantitativi, Università Cattolica del Sacro Cuore, Largo Gemelli 1 – 20123 Milano, e-mail: gianluca.femminis@unicatt.it

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Istituto di Economia e Finanza
Università Cattolica del S. Cuore
Largo Gemelli 1
20123 Milano
tel.: 0039.02.7234.2976
fax: 0039.02.7234.2781
e-mail: ist.ef@unicatt.it

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# The Welfare Implications of Costly Information Provision*

Luca Colombo[a, †] Gianluca Femminis[a, ‡]

[a] Università Cattolica del Sacro Cuore, Largo Gemelli 1, I-20123 Milano, Italy

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**Abstract**

We study information acquisition in a framework characterized by strategic complementarity or substitutability. Agents' actions are based on costly public and private signals, the precisions of which are set by a policy maker and by private agents, respectively. The policy maker – acting as a von Stackelberg leader – takes into account that an increase in the precision of public information reduces the incentives for private information acquisition. The precisions of both the public and private information available to each agent are shown to depend crucially on the degree of strategic complementarity or substitutability. We explore the welfare and policy implications of our results in economies with beauty contests, price setting complementarities, and negative externalities entailing strategic substitutability.

**Keywords:** Incomplete information, strategic complementarity, strategic substitutability, welfare
**JEL classification:** C72, D62, D83, E50

## 1 Introduction

Coordination issues play a key role in many economic environments. Whenever strategic interactions matter, agents' actions depend not only on their own expectations about the fundamental state of the economy but also on their expectations about other agents' beliefs. These depend in turn on the information agents are endowed with. As available information typically differs across agents, different beliefs on other agents' actions emerge, which affect equilibrium outcomes and welfare. A common assumption is that information is freely available. In practice, however, the process of information

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†E-mail: lucava.colombo@unicatt.it. Tel.: +39.02.7234.2637; Fax.: +39.02.7234.2781.

‡E-mail: gianluca.femminis@unicatt.it. Tel.: +39.02.7234.3808; Fax.: +39.02.7234.2923.

acquisition/provision is often far from costless, which constrains the precision of the information acquired by private agents, or provided by a policy maker. Understanding how costly information influences agents' expectations and actions, and evaluating the welfare and policy implications of public information provision versus private information acquisition, are the main goals of this paper.

In a beauty contest framework, Morris and Shin (2002) have shown that an heightened precision of the information provided by a policy maker may have a detrimental effect on social welfare. Subsequent contributions have proved that Morris and Shin's results are not conclusive as to the social value of information. In particular, Angeletos and Pavan (2007) have shown that the effects of public information on welfare depend crucially on the degree of strategic complementarity or substitutability of agents' actions.

Following Angeletos and Pavan (2007), we allow for both strategic complementarity and substitutability in actions. However, our setup differs from their in two important respects. First, we model explicitly both the acquisition of private information and the provision of public information as being costly and endogenously determined. Second, we model decision making in a sequential way: private agents choose the precision of their private information only after the precision of the public signal has been set by a policy maker. The latter acts as a von Stackelberg leader, who optimally exploits the fact that an increase in the precision of the public signal reduces agents' incentives to acquire private information, thereby inducing socially valuable savings of private resources.[1]

Focusing on symmetric linear equilibria, we show that the endogenous level of precision of public information increases in the degree of strategic complementarity among agents' actions. This is due to the fact that the more strategic complementarity there is, the larger are the incentives for agents to align their actions, and thus the larger is the weight assigned to the precision of public information. Exactly the opposite occurs for the demand of private information precision, which is therefore decreasing in the degree of strategic complementarity.

In a welfare perspective, allowing for information precision to be costly leads to novel insights on the interplay between public and private signals. We show that, when the costs of information precision are linear, only one type of information of positive precision (either public or private) is provided to the market.[2] In particular, a welfarist policy maker chooses to provide public information (of optimal precision) if the cost ratio of public to private information precision is below a threshold. We find that this threshold depends crucially on the degree of coordination among agents' actions. Indeed, an increase in the equilibrium degree of agents' actions coordination has two countervailing effects on information precision. On the one hand, it reduces the value each individual attaches to the precision of her private signal. This, by

[1]The provision of public information is typically based on complex and standardized procedures of data collection and processing performed by a statistics authority. As private agents are not bound to follow the same procedures but can focus just on the piece of information they need, they can be much faster in acquiring additional information. Therefore, it seems natural to assume that private agents take as given the precision of the information provided by the statistics authority before deciding whether to collect more precise information.

[2]The assumption of linear costs is a useful benchmark. In Appendix C we extend our analysis to the case of convex cost functions, showing that our results are not specific to a linear cost setup.

augmenting the relative value of public information, increases the cost threshold below which only public information is provided. On the other hand, however, it may induce individuals to inefficiently overweight public information (as it is typically the case in a beauty contest), so that an increase in the equilibrium degree of coordination widens the distance between first best and equilibrium outcomes. Interestingly, we find that the first effect dominates the second one even when the cost of public information precision exceeds that of private information. This suggests that in the many instances in which strategic complementarities matter (so that agents have an incentive to coordinate their actions) a more precise public information is almost always welfare improving, unless of course the cost of public information precision is much larger than that of private information; a case in which the cost effect dominates the more interesting strategic effect. In the presence of strategic complementarities, the stronger is the coordination motive behind agents' actions, the more an increase in the precision of public information crowds-out private information.

These findings send a strong pro-transparency message to policy makers. The implications of strategic complementarities for public information transparency become evident when focusing on monopolistic competition monetary economies in which the substitution elasticity between goods induces a 'taste' for coordination among agents. In economies of this type, the dispersion in prices induces scattering in the production levels of specific varieties; a phenomenon that reduces the marginal utility of aggregate consumption and increases the marginal disutility of labor supply. Accordingly, the optimal level of public information precision is typically positive and larger than that of private information precision, even when public signals are much costlier than private ones. This suggests that a policy maker – e.g., a Central Bank – should (almost) always provide precise statistics for the key macroeconomic variables. Furthermore, in the presence of strategic complementarity, the provision of a more precise public signal tends to crowd out the acquisition of private information, since agents obtain better information from a source they value more. Hence, if public signals are sufficiently precise (i.e., optimally provided from a welfare stand point), consumers/firms have no incentives to acquire private information. Whenever this is the case, the business cycle is unlikely to be amplified by the dispersion of private information. This leads to the implication that the effects of active monetary (and/or fiscal) policies are unlikely to be significantly affected by information dispersion as far as the sources of the cycle lie in aggregate shocks.

The strong pro-transparency message of the paper rests on the welfare implications of the link between information provision and strategic complementarities. Quite obviously, different conclusions are reached when agents' actions are strategic substitutes rather than complements, as it is typically the case when negative externalities (such as pollution or congestion) are at play. We show that in these circumstances agents fail to fully internalize the effects of the externalities, which induces them to choose an excessively high level of actions' alignment. Therefore, they tend to systematically overweight the importance of public information in taking their decisions. Accordingly, an increase in the precision of private information, by augmenting the dispersion in agents' actions, is welfare improving; an effect that becomes stronger the larger is the degree of strategic substitutability in equilibrium. Conversely, providing a more precise public information is detrimental for welfare since it leads agents to excessively coordinate their actions, worsening the misalignment of private and social costs.

*Related literature*

Our paper belongs to the literature investigating the welfare implications of information provision. In a highly debated article, Morris and Shin (2002) have shown that public information may have a detrimental effect on welfare in a beauty contest framework. In the presence of strategic complementarity in actions, agents exploit public information to coordinate, relying on it to estimate a fundamental. However, when actions complementarity is not warranted in a social welfare perspective, more precise public information can be detrimental, as it inefficiently reduces agents' reliance on idiosyncratic signals. We show that this result is theoretically unwarranted when information is costly. The low reliance on private signals typical of a beauty contest, together with the assumption that information acquisition is costly, imply in fact that agents have little incentives to acquire more precise private information. In this scenario, an increase in the precision of the public signal turns out to be always beneficial, but for the cases in which the cost of public information greatly exceeds that of private information.

Morris and Shin's (2002) seminal paper has stimulated a series of contributions on the channels through which information provision affects welfare.[3] Cornand and Heinemann (2008), again in a beauty contest setup, focus on the diffusion of public information, showing that social welfare rises whenever more precise public information reaches a fraction only of market participants, so to weaken the coordination role of public information. Morris and Shin (2007) consider an economy characterized by a 'semi-public' signal reaching a fraction of agents, which adds to the usual public information directed to all market participants. The authors find that, whenever agents have no taste for dispersion, the fragmentation of information leads necessarily to a welfare loss. The welfare implications of both papers rest essentially on the fact that some relevant information reaches a share only of market participants. In our setup, the welfare effects of information depend instead on the strategic interactions between private and public signals that are available to all market participants.

In Morris and Shin (2005), public information is the result of a data collection process that reflects private actions, depending in turn upon private information. A higher precision of public information reduces the reliance of agents' actions on private information, so that future public signals are obtained by extracting information from 'less informative' actions. Hence, an increase in the precision of public information decreases the current use of private information.[4] In our paper, we focus on an entirely different setup in which the policy maker directly observes the aggregate shocks hitting the economy and strategically exploits the implications on private information acquisition of the public information she provides.

[3]There has been a debate also on the empirical plausibility of Morris and Shin's (2002) result, which has been questioned by Svensson (2006) and reaffirmed by Morris, Shin and Tong (2006).

[4]The information externality considered by Morris and Shin (2005) has been previously studied by, e.g., Vives (1993), and Amato and Shin (2006). More recently, Amador and Weill (2009) have built on an analogous learning externality that induces households to put less weight on their private forecasts following a release of public information, which may reduce welfare by increasing agents' uncertainty about fundamentals.

Hellwig (2005) and Roca (2006), among others, investigate the welfare implications of information provision/acquisition in monetary economies with monopolistic competition *à la* Dixit and Stiglitz. The main claim of this literature is that disregarding some private information can be socially valuable as firms partly neglect their contribution to aggregate risk. In our setup, monopolistic competition – by implying strategic complementarity of agents' actions – reduces the reliance of price setters on their private signals, which in turn is shown to increase the incentives of the policy maker to provide more precise public information.

Lorenzoni (2010) focuses on a setup similar to that of Hellwig (2005) but for the fact that agents face idiosyncratic productivity shocks, and that monetary policy affects the weight agents assign to the (exogenously given) precision of public information in estimating productivity differentials. He shows that an increase in the precision of public information has two opposite effects on welfare. On the one hand, it reduces welfare by increasing aggregate volatility; on the other hand, it increases welfare since it helps producers to set relative prices reflecting more closely the underlying productivity differentials across sectors. While for a given monetary policy rule the possibility of welfare-decreasing public information depends on the balance between aggregate and cross-sectional effects, the latter always dominate the first if monetary policy is set optimally, so that more precise public information has unambiguously a positive effect on welfare. We obtain similar results focusing, however, on an entirely different channel. In our setup, the effects of public information on welfare hinge in fact upon the substitutability of costly public and private information, which increases in the degree of complementarity of agents' actions.

The key role of strategic complementarities and substitutabilities in actions is investigated by Angeletos and Pavan (2007) in a framework that allows a complete welfare analysis of the effects of information provision/acquisition for a rich class of economies with payoff externalities and dispersed information. Looking at the same class of economies, we allow for information precision to be costly and for a sequential timing in the information acquisition process. To the best of our knowledge, only a few papers have focused on costly processes of signals acquisition. Among them, Hellwig and Veldkamp (2009) investigate optimal individual information choices in a model where agents acquire information within a collection of signals of given precision. They show that strategic complementarities in actions induce coordination motives in private information acquisition (i.e., "agents who want to do what others do, want to know what others know", Hellwig and Veldkamp, 2009, p. 223), which may lead to multiple equilibria.[5] Similarly to us, Hellwig and Veldkamp explicitly consider the stage in which agents can improve the quality of their information at a cost. Differently from us, however, they do not focus on the choice of public information precision by the policy maker, which we show to have a significant impact on private information acquisition.

Myatt and Wallace (2010) consider endogenous information acquisition in a beauty contest framework, allowing players to access a variety of information sources.[6] The

[5]A similar point is made by Maćkowiak and Wiederholt (2009) who, in a framework with rational inattention, show that strategic complementarity in price setting leads to strategic complementarity in the price setters' allocation of attention.

[6]A similar problem is addressed by Myatt and Wallace (2008), who however do not focus explicitly on the issue of information acquisition by agents.

precision of each signal depends on how much attention an agent pays to the corresponding information source. The greater care she exerts, the higher the cost she carries and the larger the signal clarity she achieves. If agents pay careful attention to the same sources of information, the correlation of their signals endogenously increases, which in turn implies an increase in the degree of information publicity. Differently from Hellwig and Veldkamp (2009) who develop a framework in which agents choose whether to observe or not a signal and multiple equilibria may arise, Myatt and Wallace (2010) focus on a setup in which agents choose how carefully to consider different information sources, which turns out implying that the information equilibrium is unique. Differently from us, by moving away from the public-private signal distinction, they do not study the implications of information provision by a policy maker and the strategic interplay between private and public sources of information.

Wong (2008) focuses on a more specialized framework addressing the role of transparency of a monetary policy authority. The policy maker may costlessly provide a signal of given precision to agents, who can additionally purchase at a given cost a private signal of infinite precision.[7] The main difference between our setup and that of Wong in terms of information provision is that Wong does not study the strategic dimension of the choice of information precision by the policy maker, which rests at the core of our contribution. Finally, Demerzis and Hoeberichts (2007) adapt Morris and Shin (2002) framework to investigate costly information acquisition in a macroeconomic setup in which agents receive a public signal from a monetary authority and need to forecast inflation. Differently from us, in their model public information provision by the policy maker (i.e., a central bank) and private information acquisition by agents occur simultaneously, which again weakens the strategic implications of the problem faced by the policy maker.

The rest of the paper is organized as follows. We setup our model and discuss its timing in Section 2. In Section 3 we introduce the equilibrium concept and we investigate the use of information in equilibrium, while in Section 4 we study the individual problem of information acquisition. Section 5 presents our efficiency benchmark, and Section 6 deals with the provision of public information by the policy maker. Section 7 applies our framework to economies characterized by the presence of beauty contests, price setting complementarities, and negative production externalities. Section 8 concludes the paper. All proofs and technical details omitted in the main text are contained in Appendix A. Appendix B fully details the price-setting complementarities model presented in Section 7.2, and Appendix C extends the analysis of our applications to the case in which the costs of information precisions are convex.

[7]In Wong's model, agents play a beauty contest and are heterogeneous in their costs of information acquisition, so that only those who face costs that are sufficiently low invest in a signal about the fundamental. An increase in the precision of the public signal reduces the share of agents that purchase the information about the fundamental. This may lead to an increase of the dispersion in firms' actions, which could be detrimental for welfare.

## 2 The Setup

We study a two periods economy populated by a continuum of agents indexed by the unit interval $[0,1]$ and characterized by incomplete information. Each agent $i$ observes noisy private and public signals on an underlying fundamental $\theta$. In period $-1$, every agent knows the state of the economy $\theta_{-1}$, which represents the common *ex ante* expectation on the state variable $\theta$.[8] The fundamental evolves according to the stochastic process

$$\theta = \theta_{-1} + \varphi.$$

The shock $\varphi$, occurring at the beginning of period 0, is normally distributed with mean zero, variance $\sigma_\theta^2$, and precision $p_\theta \equiv \sigma_\theta^{-2}$. After the realization of the shock, every agent $i$ receives a public signal, $y$, and a private signal, $x_i$, such that

$$y = \theta + \varepsilon,$$

$$x_i = \theta + \xi_i,$$

where $\varepsilon$ is normally distributed, independent of $\theta$, with mean zero and precision $p_y$, and the noise terms $\xi_i$ are normally distributed, independent of $\theta$, $\varepsilon$, and $\xi_j$ $(j \neq i)$, with mean zero and precision $p_{x_i}$. While $y$ is common knowledge to all agents, $x_i$ is an idiosyncratic shock specific to agent $i$ and not observable by the other agents. The precision of the private signal may vary across agents.

The common posterior on $\theta$ given public information is normally distributed, with mean $E[\theta|\, y] = \frac{p_\theta \theta_{-1} + p_y y}{p_\theta + p_y}$ and precision $p[\theta|\, y] = p_\theta + p_y$. To ease notation, we define $z \equiv E[\theta|\, y]$, and $p_z \equiv p[\theta|\, y]$. Private posteriors are normally distributed, with mean $E[\theta|\, y, x_i] = \frac{p_z z + p_{x_i} x_i}{p_z + p_{x_i}}$ and precision $p[\theta|\, y, x_i] = p_z + p_{x_i}$. Letting $\delta_i$ be the Bayesian weight of the public signal $\left(\text{i.e., } \delta_i \equiv \frac{p_z}{p_z + p_{x_i}}\right)$, we write the private posterior on $\theta$ as $E[\theta|\, y, x_i] = \delta_i z + (1-\delta_i) x_i$.[9]

Every agent's preferences are described by the quadratic utility function

$$\begin{aligned} U\left(k_i, K, \sigma_k^2, \theta\right) &= U_0 + \begin{bmatrix} U_k & U_K & U_\theta \end{bmatrix} \begin{bmatrix} k_i \\ K \\ \theta \end{bmatrix} + \\ &+ \frac{1}{2} \begin{bmatrix} k_i & K & \theta \end{bmatrix} \begin{bmatrix} U_{kk} & U_{kK} & U_{k\theta} \\ U_{Kk} & U_{KK} & U_{K\theta} \\ U_{\theta k} & U_{\theta K} & U_{\theta\theta} \end{bmatrix} \begin{bmatrix} k_i \\ K \\ \theta \end{bmatrix} + \\ &+ \frac{1}{2} U_{\sigma\sigma} \sigma_k^2 - C\left(p_{x_i}\right) - C\left(p_z\right), \end{aligned} \tag{1}$$

where $k_i \in \mathbb{R}$ denotes the action of agent $i$, $K \equiv \int_i k_i di$ is the mean, and $\sigma_k^2 \equiv \int_i [k_i - K]^2 \, di$ is the dispersion of individual actions in the population. As it is standard, we let dispersion having only a second-order non strategic external effect by assuming

[8] This allows us to compute the same ex ante expected utilities for every agent, and hence to provide a common evaluation for welfare.

[9] Our definition of information precision resembles that of 'accuracy' of agents' forecasts introduced by Angeletos and Pavan (2007).

that $U_{k\sigma} = U_{K\sigma} = U_{\theta\sigma} = 0$ and that $U_{\sigma}(k, K, 0, \theta) = 0$, for all $(k, K, \theta)$. Furthermore, we impose symmetry on the second-order effects of the fundamental and of agents' actions on utility; i.e., $U_{k\theta} = U_{\theta k}$, $U_{K\theta} = U_{\theta K}$, $U_{kK} = U_{Kk}$.

Our setup follows closely that of Angeletos and Pavan (2007), but for the fact that we model information precision as being costly. More precisely, $C(p_{x_i})$ denotes the cost suffered by agent $i$ to improve the precision of her private signal, and $C(p_z)$ is the cost to be suffered for improving the precision of the public signal. Throughout the paper, we assume that the cost $C(p_z)$ is financed by means of lump sum taxes $T_i = T$ for all $i$.

The quadratic specification of the utility function allows us to ensure the linearity of agents' best responses, and of the structure of efficient allocations. The following assumptions on partial derivatives are needed to guarantee that the utility maximization problem is well defined and that the equilibrium is unique.

**Assumption 2.1** *(i)* $U_{kk} < 0$, *(ii)* $-1 < -U_{kK}/U_{kk} < 1$, *(iii)* $U_{kk} + 2U_{kK} + U_{KK} < 0$, *(iv)* $U_{kk} + U_{\sigma\sigma} < 0$ *and (v)* $U_{k\theta} \neq 0$.

**Assumption 2.2** $C'(p_{x_i}) > 0$ *and* $C''(p_{x_i}) \geq 0$; $C'(p_z) > 0$ *and* $C''(p_z) \geq 0$.

Condition (i) in Assumption 2.1 imposes concavity at the individual level, so that best responses are well defined, while Condition (ii) guarantees that the equilibrium is unique. Conditions (iii) and (iv) ensure concavity at the aggregate level, and that the first-best allocation is unique and bounded. Finally, Condition (v) guarantees that the fundamental $\theta$ affects equilibrium behavior. Assumption 2.2 states that the marginal costs of both private and public information precision are non-decreasing.

The timing of the model, illustrated in Figure 1, is as follows.

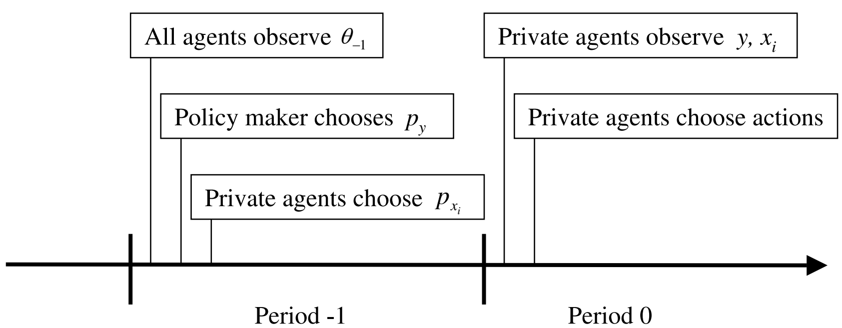


Figure 1: The timing of the model

In period $-1$, all agents observe the state of the economy $(\theta_{-1})$ and the benevolent (utilitarian) social planner chooses the precision of next period public information by maximizing welfare, defined as the sum of individual utilities. Subsequently, private agents decide how much to invest in the precision of their private signals. In period 0, each agent receives her signals (public and private), and chooses an action that affects both her utility and that of all other individuals.

We solve the model by backward induction: in period 0, given the precisions of information signals, private agents choose their actions. In period $-1$, private agents choose the precision of their information given that of the public signal, and the policy maker chooses the precision of public information by fully taking into account its effects on private agents' actions.

In the next two sections, we focus on the decisional problems of private agents, studying the choice of individual actions for given information precisions at time 0 (in Section 3), and that of private information precision (in Section 4) at time $-1$. In Sections 5 and 6 we turn to the definition of efficient allocations and to the related choice of the precision of the public signal at time $-1$, respectively.

# 3 The equilibrium use of information

In period 0, for signal precisions that are given, each individual chooses her action $k_i$ in order to maximize

$$E\left[U\left(k_i, K, \sigma_k^2, \theta\right) \mid y, x_i\right].$$

In the symmetric case in which all agents receive signals of the same precision (i.e., $p_{x_i} = p_x$, $i \in [0,1]$), we can apply the notion of linear equilibrium adopted by Morris and Shin (2002), and Angeletos and Pavan (2007).

**Definition 3.1** *A linear equilibrium is a strategy $k : \mathbb{R}^2 \to \mathbb{R}$, linear in $x$ and $y$, such that for all $(x, y)$,*

$$k(x,y) = \arg\max_k \; E\left[U\left(k, K(\theta,y), \sigma_k^2(\theta,y), \theta\right) \mid y, x\right], \tag{2}$$

*where $K(\theta,y) \equiv \int_i k_i di = \int_x k(x,y)\, dP(x \mid \theta, y)$, $P(x \mid \theta, y)$ denotes the cumulative distribution function of $x$ conditional on $\theta$ and $y$, and $\sigma_k(\theta,y) \equiv \left(\int_i [k_i - K]^2 di\right)^{1/2} = \left(\int_x [k(x,y) - K(\theta,y)]^2 dP(x \mid \theta,y)\right)^{1/2}$ for all $(\theta, y)$.*

Following the same logic of Proposition 1 in Angeletos and Pavan (2007), we can prove the following

**Proposition 3.1**

*Let $\alpha \equiv -\frac{U_{kK}}{U_{kk}}$, $\kappa_1 \equiv -\frac{U_{k\theta}}{U_{kk}+U_{kK}}$ and $\kappa_0 \equiv -\frac{U_k}{U_{kk}+U_{kK}}$.*

*(i) A strategy $k(x,y)$ is a linear equilibrium if and only if, for all $(x,y)$*

$$k(x,y) = E\left[(1-\alpha)\kappa_0 + (1-\alpha)\kappa_1\theta + \alpha K(\theta,y) \mid x, y\right]. \tag{3}$$

*(ii) There exists a unique linear equilibrium, which is given by*

$$k(x,y) = \kappa_0 + \kappa_1 (\gamma z + (1-\gamma) x),$$

*where*

$$\gamma = \frac{\delta}{1 - \alpha(1-\delta)}, \tag{4}$$

*and $\delta \equiv \frac{p_z}{p_z + p_x}$.*

Observe that $\alpha$ denotes the slope of every agent's best response to aggregate activity, which represents the equilibrium degree of coordination of agents' actions, with $\alpha \in (-1, 1)$ by Assumption 2.1. Note also that $\delta$ captures the relative precision of public information, and it is therefore the correlation across agents' forecasting errors on $\theta$, which is labeled by Angeletos and Pavan (2007) as the 'commonality' of information. It is evident from (4) that the sensitivity of the equilibrium to the two types of information (i.e., $\gamma$) depends both on the commonality of information and on the degree of coordination $\alpha$. As pointed out by Angeletos and Pavan (2007), it can easily be seen that when $\alpha \neq 0$ the equilibrium action is biased toward either private or public information depending on the degree of strategic substitutability or complementarity in agents' actions. More precisely, when agents' actions are strategic complements (i.e., $\alpha > 0$), the equilibrium is more sensitive to public information as $\gamma > \delta$; instead, when actions are strategic substitutes (i.e., $\alpha < 0$), the equilibrium strategy is tilted towards private information as $\gamma < \delta$.[10]

As we investigate the problem of agents' information acquisition, we need to determine the impact of individual information precision on agents' actions and thus on their expected utility. In order to do so, we assume that all agents other than $j$ – for given and identical information precisions – play according to their (linear) equilibrium strategy

$$k(x, y) = \kappa_0 + \kappa_1(\gamma z + (1 - \gamma) x), \tag{5}$$

where $\gamma$ is determined in the unique linear equilibrium of Proposition 3.1 by taking into account that the deviating agent $j$ has zero measure. The following proposition characterizes the best response of agent $j$ deviating from the equilibrium strategy played by all other agents $i \neq j$.

**Proposition 3.2** *A strategy $k_j(x_j, y)$ is the best response to the unique linear equilibrium strategy $k(x, y)$ played by all agents $i \neq j$ if and only if*

$$k_j(x_j, y) = \kappa_0 + \kappa_1(\gamma_j z + (1 - \gamma_j) x_j), \tag{6}$$

*where*

$$\gamma_j = \frac{(1 - \alpha)\delta_j + \alpha\delta}{1 - \alpha(1 - \delta)}, \tag{7}$$

*and* $\delta_j \equiv \frac{p_z}{p_z + p_{x_j}}$.

Note that $\gamma_j = \gamma$ if $\delta_j = \delta$ (or, equivalently, if $p_{x_j} = p_x$), and that $\gamma_j < \gamma$ if $\delta_j < \delta$. Therefore, an increase in the precision of agent $j$'s private signal (i.e., a reduction of $\delta_j$) implies a smaller weight on the public signal. Furthermore, an increase in the equilibrium degree of coordination $\alpha$ reduces the impact of $\delta_j$ on $\gamma_j$ since the larger is the degree of coordination between agents' actions the smaller is the marginal effect on actions of the precision of public information.

[10] It is also immediate to see that, when $\alpha = 0$, the weights on $x$ and $y$ are the Bayesian weights; i.e., $\gamma = \delta$.

# 4 The acquisition of private information

Given the equilibrium strategy (6), the ex-ante (period $-1$) expected utility of agent $j$ conditioned only on the past realization of the fundamental ($\theta_{-1}$) can be written as (see Appendix A)

$$E\left[U\left(k_j, K, \sigma_k^2, \theta \left|\theta_{-1}, p_{x_j}, p_x, p_z\right.\right)\right] = \tilde{\kappa} + \frac{U_{kk}}{2}\kappa_1^2\left(\frac{\gamma_j^2}{p_z} + \frac{\left(1-\gamma_j\right)^2}{p_{x_j}}\right) +$$
$$+U_{kK}\kappa_1^2\left(\gamma_j - 1\right)\frac{\gamma}{p_z} + \frac{U_{KK}}{2}\kappa_1^2\left(\gamma - 2\right)\frac{\gamma}{p_z} - U_{K\theta}\kappa_1\frac{\gamma}{p_z} + \frac{1}{2}U_{\sigma\sigma}\frac{\kappa_1^2\left(1-\gamma\right)^2}{p_x} - C\left(p_{x_j}\right) - T_j, \tag{8}$$

where $\tilde{\kappa}$ collects all the terms that are independent of information precisions $p_{x_j}$, $p_x$, $p_z$, and $T_j$ denotes the fraction of the cost of public information precision borne by agent $j$.

Notice that ex-ante utility is influenced by the precision of both public and private information. Agent $j$ chooses the precision of her private information by maximizing (8). In order to keep the problem analytically tractable, we assume that the cost of private information is linear.[11]

**Assumption 4.1** $C\left(p_{x_i}\right) = c_{pr} \cdot p_{x_i}$, *with* $c_{pr} > 0$, $\forall i$.

The effects of the precision of public and private information on agent $j$'s ex-ante utility are illustrated in the following proposition.

**Proposition 4.1**

*(i) A marginal increase in the weight $\gamma_j$ of the precision of public information has no effects on ex-ante individual expected utility.*

*(ii) The precision of private information that maximizes ex-ante individual expected utility is given by*

$$p_{x_j} = \max\left\{\sqrt{-\frac{U_{kk}\kappa_1^2}{2c_{pr}}\frac{\left(p_x + p_z\right)\left(1-\alpha\right)}{p_z + \left(1-\alpha\right)p_x}} - p_z, 0\right\}. \tag{9}$$

By differentiating (9) with respect to $p_x$, we obtain

$$\frac{\partial p_{x_j}}{\partial p_x} = \begin{cases} \sqrt{-\frac{U_{kk}\kappa_1^2}{2c_{pr}}\frac{\alpha(1-\alpha)p_z}{(p_z+(1-\alpha)p_x)^2}} & \text{if } p_{x_j} \geq 0 \\ 0 & \text{otherwise} \end{cases}.$$

[11]This is consistent with the view of, e.g., Myatt and Wallace (2010). Focusing as an illustrative example on a market research framework, they note that any survey by which a supplier aims at investigating demand conditions is subject to a sampling error. The precision of the information acquired through a survey is directly proportional to the sample size; i.e., to the number of individuals interviewed. Assuming that there is a fix price-per-interview, it is natural to assume that the cost of information acquisition is linear.

Therefore, when there is strategic complementarity among agents' actions (i.e., $\alpha \in (0,1)$), an increase in the precision of other agents' private information raises the precision of agent $j$'s information as well. The opposite occurs in the case of strategic substitutability (i.e., $\alpha \in (-1,0)$). These results are fully consistent with those of Hellwig and Veldkamp (2009) who note that, when actions are complements, information acquisition is complementary, while when actions are substitutes, agents want to know what others do not. To better understand the intuition of the findings above, observe that an increase in the precision of agents' private information implies that their signals become more concentrated. Hence, with strategic complementarity, agent $j$ has an incentive to increase the precision of her signal, so to better align it with the private information available to other agents. Conversely, in the case of strategic substitutability, agent $j$ aims at reducing the degree of alignment between her private signal and those of other agents. Thus, she has an incentive to reduce the precision of her information.

In a symmetric equilibrium it must be that $p_{x_j} = p_x$. Therefore, Equation (9) can be rewritten as

$$p_x = \max\left\{\sqrt{-\frac{U_{kk}\kappa_1^2}{2c_{pr}}} - \frac{p_z}{1-\alpha}, 0\right\}. \tag{10}$$

It is easy to see that $p_x$ is decreasing in its cost ($c_{pr}$) as well as in the precision of the public signal ($p_z$), and increasing in the effect of information precision on the individually perceived marginal utility of actions ($U_{kk}$) weighed by the agent's reaction to changes in the fundamental ($\kappa_1^2$). Perhaps less obviously, $p_x$ is decreasing in the degree of strategic complementarity of agents' actions ($\alpha$). This follows from the fact that a higher degree of strategic complementarity increases agents' incentives to align their actions, and hence it reduces the value they attach to the precision of their idiosyncratic information for any $c_{pr} > 0$. Analogous observations apply to the case of strategic substitutability.

Equation (10) highlights also the negative relationship between the precisions of private and public information, indicating the existence of substitutability between the two types of information. The extent of this substitutability depends crucially on the degree of strategic complementarity/substitutability among agents' actions. In the presence of strategic complementarity ($\alpha > 0$), an increase of $\alpha$ enhances the substitutability between public information and private information precisions. The higher is the equilibrium degree of coordination among agents' actions, the larger is the weight $\gamma$ assigned to public information in choosing actions (see Equation (4)) and the smaller is that assigned to private information precision.[12] Accordingly, public information carries more weight than private information, which increases its effectiveness in substituting for private signals. Conversely, in the presence of strategic substitutability that reduces agents' incentives to align actions, the negative relationship between the precision of public information and that of private information is weakened.

[12] Also Myatt and Wallace (2010) find that an increase in strategic complementarity reduces the precision of the private signals acquired by agents.

## 5 The efficiency benchmark

After having characterized the actions of private agents and their choices of private signals precisions, we turn to the characterization of the precision of the public signal. In order to understand the welfare implications of information volatility, and those of dispersion arising from strategic effects, we need first to establish an efficiency benchmark.

We define welfare in period 0 by means of the utilitarian aggregator

$$W\left(K,\sigma_k^2,\theta\right)=\int_i U\left(k_i,K,\sigma_k^2,\theta\right)di=\int_x U\left(k_i,K,\sigma_k^2,\theta\right)dP\left(x\mid\theta,y\right). \tag{11}$$

By using the definition of $U\left(k_i,K,\sigma_k^2,\theta\right)$ given in Equation (1), $W\left(K,\sigma_k^2,\theta\right)$ can be written as

$$\begin{aligned}W\left(K,\sigma_k^2,\theta\right) &= U_0+W_K K+U_\theta\theta+\frac{1}{2}W_{KK}K^2+\frac{1}{2}W_{\sigma\sigma}\sigma_k^2+ \\ &\quad +W_{K\theta}K\theta+U_{\theta\theta}\theta^2-\int_i\left[C\left(p_{x_i}\right)+T_i\right]di,\end{aligned} \tag{12}$$

where $W_K\equiv U_k+U_K$, $W_{KK}\equiv U_{kk}+2U_{kK}+U_{KK}$, $W_{K\theta}\equiv U_{k\theta}+U_{K\theta}$, and $W_{\sigma\sigma}\equiv U_{kk}+U_{\sigma\sigma}$. Note that (12) can also be interpreted as the complete information welfare, under the constraint that the dispersion of agents' actions is $\sigma_k^2$. Assumption 2.1, by requiring that $W_{KK}<0$ and $W_{\sigma\sigma}<0$, guarantees that welfare is finite.[13]

The following lemma characterizes the first best allocation for an economy in which the fundamental $\theta$ is common knowledge.

**Lemma 5.1** *The unique efficient allocation under complete information is given by*

$$\kappa^*\left(\theta\right)=\kappa_0^*+\kappa_1^*\theta, \tag{13}$$

*where* $\kappa_0^*\equiv-\frac{W_K}{W_{KK}}$, *and* $\kappa_1^*\equiv-\frac{W_{K\theta}}{W_{KK}}$.

It is straightforward to observe that $\kappa_1^*$ captures the impact of the fundamental on the first-best allocation.

We now need to determine the (second best) efficient allocations in a framework where the fundamental $\theta$ is not common knowledge (i.e., at the beginning of period 0). In order to do so, we define an efficient allocation as a strategy that maximizes ex ante utility under the constraint that information can not be transferred among agents. As noted by Angeletos and Pavan (2007), this amounts to identify "the best a society could do if its agents were to internalize their payoff interdependencies and appropriately adjust their use of available information without communicating with one another" (p. 1114). Formally:

[13]It is immediate to note that if Assumption 2.1 were not satisfied, infinite welfare could be achieved by reducing the precision of information at time −1. Such a decrease in precision would enlarge the dispersion of agents' actions in period 0 (see Equation 3) raising welfare.

**Definition 5.1** *An efficient allocation is a strategy* $k : \mathbb{R}^2 \to \mathbb{R}$ *that maximizes*

$$\int_\theta W\left(K, \sigma_k^2, \theta\right) dP\left(\theta \mid y\right)$$

*under the constraints that*

$$K\left(\theta, y\right) = \int_x k\left(x, y\right) dP\left(x \mid \theta, y\right)$$

*and*

$$\sigma_k^2\left(\theta, y\right) = \int_x \left[k\left(x, y\right) - K\left(x, y\right)\right]^2 dP\left(x \mid \theta, y\right).$$

The following proposition (equivalent to Proposition 2 in Angeletos and Pavan, 2007) characterizes the efficient allocation that solves the problem stated in Definition 5.1.

**Proposition 5.1**

*(i) An allocation* $k : \mathbb{R}^2 \to \mathbb{R}$ *is efficient under incomplete information if and only if, for almost all* $(x, y)$

$$k\left(x, y\right) = E\left[\left(1 - \alpha^*\right) \kappa^*\left(\theta\right) + \alpha^* K\left(\theta, y\right) | x, y\right], \tag{14}$$

*where*

$$\alpha^* \equiv 1 - \frac{W_{KK}}{W_{\sigma\sigma}}. \tag{15}$$

*(ii) There exists a unique efficient allocation for almost all* $(x, y)$*, which is given by*

$$k^*\left(x, y\right) = \kappa_0^* + \kappa_1^*\left(\left(1 - \gamma^*\right) x + \gamma^* z\right), \tag{16}$$

*where*

$$\gamma^* = \frac{\delta}{1 - \alpha^*\left(1 - \delta\right)}. \tag{17}$$

It is easy to see that Condition (14) is the analogue for efficiency of the best response function (3) in the characterization of the complete information equilibrium, once substituting $\kappa_0$, $\kappa_1$ and $\alpha$ with $\kappa_0^*$, $\kappa_1^*$ and $\alpha^*$, respectively. Therefore, $\alpha^*$ can be interpreted as the optimal degree of coordination, in that it is the level of complementarity ($\alpha^* > 0$) or substitutability ($\alpha^* < 0$) that a planner would like agents to perceive in order for the equilibrium of the economy to coincide with the efficient allocation. Equation (15) shows that $\alpha^*$ is decreasing with social aversion to volatility ($-W_{KK}$) and increasing with social aversion to dispersion ($-W_{\sigma\sigma}$). Moreover, since $W_{\sigma\sigma} \equiv U_{kk} + U_{\sigma\sigma}$, an increase of $U_{\sigma\sigma}$ reduces the social aversion to dispersion, implying a smaller $\alpha^*$. Observe finally that $\gamma^*$ indicates the relative sensitivity of the efficient allocation to public and private information, just as $\gamma$ does for the equilibrium allocation. By comparing $\gamma$ and $\gamma^*$, it is therefore possible to see that the sensitivity of the equilibrium allocation to public noise is inefficiently large whenever the equilibrium degree of coordination is higher than the optimal one; i.e., $\gamma \geq \gamma^* \iff \alpha \geq \alpha^*$.

## 6 The provision of public information

In period $-1$ the policy maker chooses optimally the precision of public information $p_y$ (and hence $p_z$) by taking into account agents' best responses to the choice of public information precision, both in terms of their signals precisions and of their actions (see Equations (10) and (5), respectively).

Consistently with the welfare criterion discussed in Section 5, we focus on a welfarist policy maker that chooses the precision of the public signal by maximizing the expected value of the integral sum over agents of individual utility functions, evaluated at the symmetric equilibrium in which $\gamma_i = \gamma$, for all $i$; i.e.,

$$W(p_x, p_z, \theta_{-1}) = E\left[W\left(K, \sigma_k^2, \theta\right)|\theta_{-1}\right] = \int_y \int_\theta \int_i U\left(k_i, K, \sigma_k^2, \theta\right) \ di \ dP(\theta, y|\theta_{-1}). \tag{18}$$

In Appendix A it is shown that the welfare criterion (18) can be restated as

$$\begin{aligned} W(p_x, p_z, \theta_{-1}) = \tilde{W} + W_{KK}\kappa_1^2\left(\frac{1}{2}\frac{\gamma^2}{p_z} + \frac{\kappa_1^* - \kappa_1}{\kappa_1}\frac{\gamma}{p_z}\right) + \frac{1}{2}W_{\sigma\sigma}\kappa_1^2\frac{(1-\gamma)^2}{p_x} \\ - \int_i C(p_{x_i})\,di - C(p_z), \end{aligned} \tag{19}$$

where $\tilde{W} \equiv \bar{W} + \frac{W_{KK}}{p_\theta}\kappa_1^2\left(\frac{1}{2} - \frac{\kappa_1^*}{\kappa_1}\right)$ collects all the terms that are independent of public and private signals precisions, and $C(p_z)$ denotes the cost of increasing the precision of public information, with $C(p_z) = \int_i T_i di$.[14] As for private information precision, we assume that the cost faced by the policy maker to increase the precision of public information is linear. More precisely:

**Assumption 6.1** $C(p_y) = c_{pb} \cdot p_y$, $c_{pb} > 0$.

Note that, since $p_z \equiv p_\theta + p_y$, and $p_\theta$ is a structural characteristic of the evolution of the fundamental, the cost of public information provision is completely defined by the cost of a more precise public signal $y$.

In the following, we also assume that

**Assumption 6.2** $W' \equiv W_{KK}\left(\frac{\kappa_1^*}{\kappa_1} - \frac{1}{2}\right) < 0$.

Given that $W_{KK} < 0$ by Assumption 2.1, Assumption 6.2 obviously requires that $\frac{\kappa_1^*}{\kappa_1} > \frac{1}{2}$. To better understand the implications of the assumption, focus on the special case in which $p_x = 0$ (and thus $\gamma = 1$), so that Equation (19) reduces to

$$W(0, p_\theta, p_y, \theta_{-1}) = \tilde{W} + \kappa_1^2 W_{KK}\left(\frac{\kappa_1^*}{\kappa_1} - \frac{1}{2}\right)\frac{1}{p_\theta + p_y} - c_{pb}p_y. \tag{20}$$

Equation (20) makes it evident that Assumption 6.2 provides a sufficient condition guaranteeing that the marginal utility of public information precision is positive at least when the precision of private information is zero.

[14]Recall that the cost of public information provision is financed by means of a lump sum tax $T_i$.

Focusing on Equation (19), we highlight several effects of the precision of public information on welfare. An increase of $p_z$ given $\gamma$ exerts a twofold direct effect on $W(p_x, p_z, \theta_{-1})$, which is captured by the term $W_{KK}\kappa_1^2\left(\frac{1}{2}\frac{\gamma^2}{p_z}+\frac{\kappa_1^*-\kappa_1}{\kappa_1}\frac{\gamma}{p_z}\right)$. First, note that when $p_z$ increases, each agent becomes better able to align her individual strategy $k(x, y)$ to the one that she would choose under perfect information (i.e., to $\kappa_0 + \kappa_1\theta$). This implies a reduction in the ex-ante variance of each individual's action, which improves welfare since the agent's objective function is concave. This first effect is captured by term $W_{KK}\kappa_1^2\frac{\gamma^2}{2p_z}$, which is increasing in $p_z$ since $W_{KK} < 0$ by Assumption 2.1. Second, recall that individual strategies are selected via a maximization procedure that disregards the contribution of each individual to the aggregate action $K$, so that each individual strategy (3) differs from the efficient allocation (16). Whenever $\kappa_1^* > \kappa_1$, it would be optimal for private agents to agree on a strategy profile involving a stronger reaction to the fundamental, and therefore to the public signal. Nonetheless, the fact that a weaker response to (the expected realization of) the fundamental is implemented has a positive second-order effect on welfare, as it induces a lower variance of each agent's action, which is captured by the welfare term $W_{KK}\kappa_1^2\frac{\kappa_1^*-\kappa_1}{\kappa_1}\frac{\gamma}{p_z}$.

An increase of $p_z$ determines also an increase in the weight of the public signal $\gamma$, which exerts three additional indirect effects on welfare. The first two are the exact counterparts of the effects of an increase in $p_z$ discussed above, and they are again captured by the second addendum on the right hand side of Equation (19). The third effect is instead captured by the third addendum on the right hand side of Equation (19), which summarizes the implications of the reduction in the weight $(1-\gamma)$ assigned by each agent to her private signal. Such a reduction has an overall positive effect on welfare since $W_{\sigma\sigma} = U_{kk} + U_{\sigma\sigma} < 0$. Note in fact that, although the effect of increasing $\gamma$ on the dispersion of agents' actions (captured by $U_{\sigma\sigma}$) is undetermined, its direct effect on welfare is certainly positive (since $U_{kk} < 0$) and sufficient to increase welfare overall.

In what follows, it is useful to state Equation (19) as

$$\begin{aligned} W(p_x, p_\theta, p_y, \theta_{-1}) &= \tilde{W} + \kappa_1^2 W_{KK}\left(\frac{\kappa_1^*}{\kappa_1} - \frac{1}{2}\right)\frac{1}{p_z + (1-\alpha)p_x} + \\ &\quad + \kappa_1^2\frac{((1-\alpha)W_{\sigma\sigma} - W_{KK})}{2}\frac{(1-\alpha)p_x}{(p_z+(1-\alpha)p_x)^2} + \\ &\quad - c_{pr}p_x - c_{pb}p_y, \end{aligned} \tag{21}$$

where (when analytically convenient) we use the fact that $p_z \equiv p_\theta + p_y$ and $\gamma = \frac{\delta}{1-\alpha(1-\delta)} = \frac{p_z}{p_x(1-\alpha)+p_z}$. By using the definition in Assumption 6.2 and by recalling that $\alpha^* \equiv 1 - \frac{W_{KK}}{W_{\sigma\sigma}}$ (see Equation (15)), Equation (21) can be rewritten in a more compact way as

$$\begin{aligned} W(p_x, p_\theta, p_y, \theta_{-1}) &= \tilde{W} + \kappa_1^2 W' \frac{1}{p_z + (1-\alpha)p_x} \\ &\quad + \frac{\kappa_1^2}{2}(\alpha^* - \alpha)W_{\sigma\sigma}\frac{(1-\alpha)p_x}{(p_z+(1-\alpha)p_x)^2} - c_{pr}p_x - c_{pb}p_y. \end{aligned} \tag{22}$$

Note that the term $(\alpha^* - \alpha)$ in the equation captures the relationship between the first best and the market equilibrium degrees of coordination between agents' actions.

When the equilibrium degree of coordination falls short of the optimal one, an increase in the precision of the public signal – by increasing the weight associated to public information – raises welfare (gross of the effect of the cost of information precision). This follows from the fact that a more precise public information favors the alignment of agents' actions.[15]

In the following, we assume that

**Assumption 6.3** $c_{pb} \leq \bar{c}_{pb} \equiv -\frac{\kappa_1^2 W'}{p_\theta^2}$; $c_{pr} \leq \bar{c}_{pr} \equiv -\frac{(1-\alpha)^2 U_{kk} \kappa_1^2}{2p_\theta^2}$.

Assumption 6.3 guarantees that there are incentives to provide public information of positive precision when no private information is acquired (see Equation 20), as well as to acquire private information of positive precision when no public information is provided (see Equation 10).[16] It is worth noting that both the upper-bound levels $\bar{c}_{pb}$ and $\bar{c}_{pr}$ are decreasing in $p_\theta$ since additional information becomes less valuable when the ex–ante information $\theta_{-1}$ already provides a good estimate of the fundamental. Furthermore, both $\bar{c}_{pb}$ and $\bar{c}_{pr}$ are increasing in $\kappa_1^2$ because actions respond more to both public and private signals when $\kappa_1^2$ is large, so that the precision of information becomes more valuable. Moreover, $\bar{c}_{pb}$ and $\bar{c}_{pr}$ are increasing in the absolute value of $W'$ and $U_{kk}$, respectively. This follows from the fact that the larger are $W'$ and $U_{kk}$ (in absolute value) the higher is the impact of the precision of information on utility. Finally, $\bar{c}_{pr}$ is decreasing in $\alpha$. An increase in the degree of coordination reduces the weight that agents assign to private information when selecting their equilibrium actions, which in turn reduces the maximum cost they are willing to pay in order to acquire additional information.

The following proposition characterizes the optimal choice of the precision of information by the policy maker, who (given the von Stackelberg structure of the information acquisition game) takes into account the optimal choices of private information precisions by agents for any given level of public information precision.

**Proposition 6.1**

*Suppose that Assumption 6.3 holds, and define*

$$c' \equiv \frac{2W'}{(1-\alpha)^2 U_{kk}} \text{ and } c'' \equiv \frac{1}{1-\alpha}\left(1 + \frac{\alpha^* - \alpha}{1-\alpha}\frac{W\sigma\sigma}{U_{kk}}\right).$$

[15] Differentiating

$$\frac{\kappa_1^2}{2}(\alpha^* - \alpha) W_{\sigma\sigma} \frac{(1-\alpha)p_x}{(p_z + (1-\alpha)p_x)^2}$$

with respect to $p_z$, it is immediate see that there is a positive contribution of the precision of public information to welfare whenever $\alpha^* > \alpha$.

[16] Assumption 6.3 allows us to disregard some uninteresting cases, simplifying the technicalities without loss of generality. It is in fact intuitive that when the cost of public information precision exceeds $\bar{c}_{pb}$, only private information is acquired. Similarly, when the cost of private information becomes larger than $\bar{c}_{pr}$, agents have no incentives to acquire private information.

*Case 1.* $c' > c''$.

*There exists a threshold* $\tilde{c}(c_{pb})$*, with* $c'' < \tilde{c}(c_{pb}) < c'$*, such that the optimal choices of the policy makers and of the private agents are:*

$$\begin{aligned} p_y^* &= \sqrt{-\frac{W'\kappa_1^2}{c_{pb}}} - p_\theta, \quad \textit{and} \quad p_x^* = 0 & \textit{for } \frac{c_{pb}}{c_{pr}} < \tilde{c}(c_{pb}); \\ p_y^* &= 0, \quad \textit{and} \quad p_x^* = \sqrt{\frac{-U_{kk}\kappa_1^2}{2c_{pr}}} - \frac{p_\theta}{1-\alpha} & \textit{for } \frac{c_{pb}}{c_{pr}} \geq \tilde{c}(c_{pb}). \end{aligned}$$

*Case 2.* $c' \leq c''$.

*The policy makers and the private agents optimal choices are:*

$$\begin{aligned} p_y^* &= \sqrt{-\frac{W'\kappa_1^2}{c_{pb}}} - p_\theta, & \textit{and} \quad p_x^* = 0 & \quad \textit{for } \frac{c_{pb}}{c_{pr}} < c' \leq c''; \\ p_y^* &= (1-\alpha)\sqrt{-\frac{U_{kk}\kappa_1^2}{2c_{pr}}} - p_\theta, & \textit{and} \quad p_x^* = 0 & \quad \textit{for } c' \leq \frac{c_{pb}}{c_{pr}} \leq c''; \\ p_y^* &= 0, & \textit{and} \quad p_x^* = \sqrt{\frac{-U_{kk}\kappa_1^2}{2c_{pr}}} - \frac{p_\theta}{1-\alpha} & \quad \textit{for } \frac{c_{pb}}{c_{pr}} \geq c''. \end{aligned}$$

To understand the intuition of the proposition, focus on the meaning and determinants of the relative cost thresholds $c''$ and $c'$. The threshold $c''$ determines whether it is optimal for the policy maker letting agents to acquire private information; i.e., $\{p_x^* > 0, p_y^* = 0\}$ if $c_{pb}/c_{pr} > c''$, and $\{p_x^* = 0, p_y^* > 0\}$ otherwise. It is easy to see that $c''$ is increasing in the spread between the socially optimal and the equilibrium degree of coordination (i.e., $\alpha^* - \alpha$). Note in fact that when $\alpha^* - \alpha > 0$, a greater degree of coordination is desirable. Therefore, as an increase in the precision of the public signal improves the alignment of agents' actions, the relative cost threshold above which no private information is provided increases. Additionally, the equilibrium degree of coordination $\alpha$ exerts two direct effects on the threshold $c''$. First, focus on the term $\frac{1}{1-\alpha}$ in the expression of $c''$. By reducing the weight of private information in the estimation of the fundamental due to the coordination motive in agents' actions, an increase in $\alpha$ diminishes more than proportionally agents' incentives to acquire private information (see Equations 4 and 10). This determines socially valuable savings of resources, inducing the policy maker to provide more precise public information for higher costs of public information precision, thus raising $c''$. Second, focus on the term $\frac{\alpha^*-\alpha}{(1-\alpha)^2}\frac{W\sigma\sigma}{U_{kk}}$ in the expression of $c''$. For a given $\alpha^* - \alpha > 0$, an increase in $\alpha$ induces a closer alignment of agents' actions, which raises the value of a more precise public signal – hence increasing the threshold $c''$.[17] Observe finally that, in the opposite case in which $\alpha^* - \alpha < 0$, it would be optimal to reduce the alignment of agents' actions, which creates a tension between the saving effect and that of actions alignment. The overall impact of $\alpha$ on $c''$ depends then on which of the two effects prevails.

As for $c'$, it defines the relative cost threshold below which the policy maker finds it optimal to supply a public signal having a precision larger than that offsetting the acquisition of private information (see the proof of Proposition 6.1 in Appendix A). This occurs when the relative cost of public to private information is lower than the marginal welfare effect of an increase in the precision of public information, for any given level of private information precision. It is easy to see from the definition of $c'$ in

[17] Recall that the improved alignment of agents' actions is beneficial since $\alpha^* - \alpha > 0$, and $\frac{W\sigma\sigma}{U_{kk}} > 0$.

Proposition 6.1 that $c'$ is increasing in the equilibrium degree of coordination of agents' actions. This follows again from the fact that a more precise public signal, by fostering actions alignment, raises welfare.

Having clarified the meaning of the thresholds $c''$ and $c'$, Case 2 in Proposition 6.1 is easily understood. As for Case 1, given that $c' > c''$, it is apparent from the discussion above that for $c'' < c_{pb}/c_{pr} < c'$ the policy maker can either provide all the needed information precision through a public signal, or allow private agents to acquire it by means of a private signal. Therefore, the decision problem of the policy maker becomes whether to provide a public signal entirely offsetting the acquisition of private information, or to let private agents acquiring information on their own. The threshold $\tilde{c}\left(c_{pb}\right)$ discriminates between these two cases.

# 7 Applications

In this section, we specialize the setup introduced in the previous sections to study the implications of strategic complementarities or substitutabilities for the provision/acquisition of information in general equilibrium frameworks that allow for meaningful welfare comparisons. In particular, we investigate the effects of information provision on welfare in beauty contests, the implications of price setting complementarities for transparency, and the impact of negative externalities entailing strategic substitutability on information acquisition.

Recall that in our setup, due to the assumption of linear costs of information precision, only one type of information of positive precision exists in equilibrium. In Appendix C, we extend our analysis to the case in which the costs of public and private information are convex so that public and private information of positive precision typically coexist.

## 7.1 'Beauty Contests'

Morris and Shin (2002) have shown that public information may have a detrimental effect on welfare in beauty contest frameworks. Their result builds on the twofold role of public information that, on the one hand, provides information on the fundamental and, on the other hand, serves as a focal point for agents' beliefs. As it is now well understood, the detrimental effect of public information arises from a coordination motive inducing agents to place a weight on the public signal that is larger than the socially optimal one.

Morris and Shin consider a game in which the payoff function of agent $i$ is

$$U\left(k_i, K, \sigma_k^2, \theta\right) = -\left(1-r\right)\left(k_i-\theta\right)^2 - r\left(L_i - \bar{L}\right), \tag{23}$$

where $k_i \in \mathbb{R}$ denotes agent $i$'s action, $\theta$ represents the state of the economy, and $r \in (0,1)$. Furthermore, $L_i = \int_0^1 \left[k_h - k_i\right]^2 dh = \left(k_i - K\right)^2 + \sigma_k^2$, where $K$ denotes the mean of agents' actions, and $\bar{L} = \int_0^1 L_i di = 2\sigma_k^2$. The first term in (23) captures the value for an agent of aligning her action to the fundamental $\theta$, $L_i$ is the 'beauty contest' term representing the private value of taking an action close to those of other agents, and $\bar{L}$ is an externality ensuring that there is no social value in reducing $L_i$.

Taking into account the costs of information provision, agent $i$'s utility can readily be rewritten as

$$U\left(k_i, K, \sigma_k^2, \theta\right) = -\left(1-r\right)\left(k_i-\theta\right)^2 - r\left(k_i-K\right)^2 + r\sigma_k^2 - c_{pr}p_x - c_{pb}p_y. \quad (24)$$

Morris and Shin's setup can easily be embedded into the framework of the present paper. In particular, it is immediate to note that $U_{kk} = -2$, $U_{kK} = 2r$, $U_{KK} = -2r$, $U_{k\theta} = 2\left(1-r\right)$, $U_{K\theta} = 0$, $U_{\sigma\sigma} = 2r$, and that $\alpha = r$, $\alpha^* = 0$, and $\kappa_1 = \kappa_1^* = 1$. Accordingly, in a social perspective, the private incentives to coordinate are excessively large since $\alpha > \alpha^*$. Note also that the term $\bar{L}$ introduces a taste for dispersion (i.e., $U_{\sigma\sigma} > 0$) into the model. Observe finally that $W_{KK} = W_{\sigma\sigma} = 2\left(r-1\right) < 0$, and $W' = W_{KK}/2 < 0$, so that Assumptions 2.1 and 6.2 hold.

The thresholds $c''$ and $c'$ of Proposition 6.1 become $c'' = 1$, and $c' = \frac{1}{1-r} > c''$. Therefore, when $c_{pb}/c_{pr} < \tilde{c}\left(c_{pb}\right)$ with $c'' < \tilde{c}\left(c_{pb}\right) < c'$, agents do not acquire private information and the best policy of the policy maker is to increase the precision of public information until the marginal utility of the latter equalizes its marginal cost.[18] Conversely, when $c_{pb}/c_{pr} \geq \tilde{c}\left(c_{pb}\right)$ and Assumption 6.3 holds, the cost of private information precision is so high that the only welfare maximizing policy is to provide the public signal alone.

Figure 2 highlights the shapes of the threshold $\tilde{c}\left(c_{pb}\right)$ for different values of $\alpha$.[19]

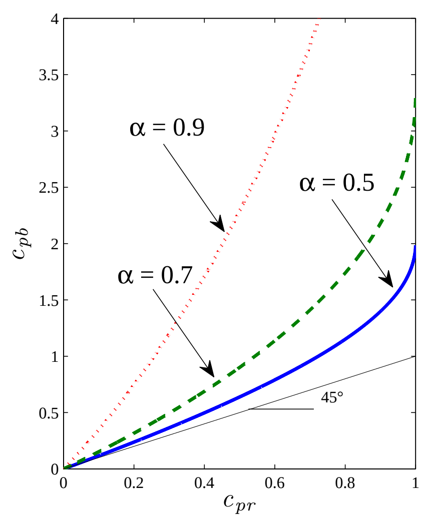


Figure 2: Behaviors of $\tilde{c}\left(c_{pb}\right)$ in Morris and Shin's beauty contest for different values of $\alpha$.

The continuous line in Figure 2 is drawn for $\alpha = 0.5$, which corresponds in Morris and Shin's (2002) framework to the smallest value of the degree of coordination allowing for a negative welfare effect of an increase in public information precision. It is

[18] Consistently with Colombo and Femminis (2008), whenever $c_{pb} = c_{pr}$, only public information of positive precision is provided.

[19] In this figure (as well as in Figures 3 and 4) the value of $c_{pr}$ is defined as a share of $\bar{c}_{pr}$, and the domain of $c_{pb}$ is determined consistently.

immediate to see from the figure that, even in this limiting case, only public information is provided whenever $c_{pb}/c_{pr} \leq \tilde{c}(c_{pb})$, despite the fact that the marginal cost of the public signal is larger than that of private information.

To better understand the intuition behind the interesting case in which $c_{pb}/c_{pr} \leq \tilde{c}(c_{pb})$, recall that the analysis we developed in the previous sections suggests that there are two countervailing forces affecting the provision of public and private information. First, the stronger is the coordination motive the less worthwhile is the acquisition of private information. In fact, the higher is $\alpha$, the lower is the weight assigned to the private signal in the assessment of the fundamental. Accordingly, an increase of $p_y$ determines a larger reduction in the demand of private information precision the higher is $\alpha$ (Equation 10), inducing a saving effect. However, there is a second effect going in the opposite direction, as the excessive incentive to coordinate that characterizes a beauty contest setup implies that the public signal is overweighted. Therefore, an increase in the precision of public information induces each agent to give more prominence to a signal the weight of which is already disproportionately large. This overweighting effect has a negative impact on welfare that contrasts with the positive effect of the savings in private information acquisition. Figure 2 shows that in Morris and Shin's (2002) beauty contest the saving effect is stronger than the overweighting effect (inducing agents not to acquire private information at all) also when the ratio of public to private information cost is larger than one. Interestingly, the comparison of the three lines in Figure 2 shows that an increase in $\alpha$ makes the cost saving effect stronger than the overweighting one.

Altogether these results strongly support the pro-transparency view that improving the accuracy of the signals provided by policy makers is beneficial, despite the negative effects of the beauty contest term and even when the cost of public information precision exceeds that of private information acquisition.

### 7.2 Price setting complementarity and transparency

A recent literature investigates the welfare implications of information provision in monetary economies (see, e.g., Hellwig, 2005; Roca, 2006; Adam, 2007; Lorenzoni, 2010). We address this issue in the framework of a simple consumer-producer economy, in which a continuum of agents $i \in [0,1]$ derives utility $J(C_i)$ from the consumption of a composite good $C_i$ – obtained via the usual Dixit-Stiglitz aggregator.[20] The consumption choices of agent $i$ depend on the price of the product she produces $k_i$, on the aggregate price level $K$ and on the price dispersion $\sigma_k^2$. Each individual $i$ provides labor services that are used in the production of a final good $Y_i$, of which she is the unique producer. The disutility of labor (and hence labor supply) $V(Y_i)$ is affected by a shock $\theta$. Our setup is close to those of Hellwig (2005) and Adam (2007), but for the fact that we assume that each consumer $i$ is the sole producer of good $i$. This latter assumption aligns individual utilities and social welfare (defined as in Section 5). The influence of monetary policy on output is summarized by the quantity equation $KY = \bar{M}$, which can also be interpreted as a simple nominal GDP targeting.[21]

[20] A description and characterization of the economy is given in Appendix B.

[21] $Y$ indicates the aggregate production level, and $\bar{M}$ denotes a fixed stock of nominal money. Although $\bar{M}$ is taken as exogenously given, it would be possible to incorporate in the quantity equation

The resulting utility function of each consumer-producer – net of costs of information precision – is as follows (see Appendix B for the derivation of Equation 25):

$$\begin{aligned} U\left(k_i, K, \sigma_k^2, \theta\right) &= J\left(C_i\right) - \theta V\left(Y_i\right) = \bar{U}\left(k_i, K, \theta\right) + \frac{J_C\left(\bar{Y}\right)\bar{Y}}{2}\left(-v^2\sigma_k^2 + \right. \\ &\left. -\omega\left(\left(v-1\right)K - vk_i\right)^2 - 2\frac{\theta}{\bar{\theta}}\left(\left(v-1\right)K - vk_i\right)\right) - c_{pr}p_x - c_{pb}p_y. \end{aligned} \tag{25}$$

$\bar{U}\left(k_i, K, \theta\right)$ collects all the terms that are constant or linear in $\theta$, $K$ and $k_i$, $\bar{Y}$ denotes the non-stochastic equilibrium level of output and consumption,[22] and

$$\omega \equiv -\frac{\left(J_{CC}\left(\bar{Y}\right) - \bar{\theta}V_{YY}\left(\bar{Y}\right)\right)\bar{Y}}{J_C\left(\bar{Y}\right)} > 0$$

captures both the curvature of the marginal utility of consumption, and the sensitivity of producers' prices to the output gap. $v > 1$ is the elasticity of substitution between goods characterizing the Dixit-Stiglitz aggregator. Note that, as it is standard in the literature focusing on aggregate shocks (see, e.g., Hellwig, 2005; Roca, 2006), the variance of individual prices has a negative impact on utility due to the Dixit-Stiglitz's setup.[23]

Letting $\Psi\left(\bar{Y}\right) = J_C\left(\bar{Y}\right)\bar{Y}$, it is easy to obtain: $U_{kk} = -\Psi\left(\bar{Y}\right)\omega v^2$, $U_{kK} = \Psi\left(\bar{Y}\right)\omega v\left(v-1\right)$, $U_{KK} = -\Psi\left(\bar{Y}\right)\omega\left(v-1\right)^2$, $U_{k\theta} = \Psi\left(\bar{Y}\right)\frac{v}{\bar{\theta}}$, $U_{K\theta} = -\Psi\left(\bar{Y}\right)\frac{v-1}{\bar{\theta}}$. Moreover, it is $U_{\sigma\sigma} = -\Psi(\bar{Y})v^2$, which highlights the presence of a distaste for dispersion. It is also immediate to verify that pricing decisions are complementary, since $\alpha = \frac{v-1}{v} > 0$. Furthermore, it is $\alpha^* = \frac{v^2(1+\omega)-\omega}{v^2(1+\omega)} > \alpha$, indicating that the optimal degree of coordination is higher than the privately perceived one. The privately perceived degree of coordination falls short of the optimal one for two reasons. First, each price setter $i$ disregards the contribution to $\sigma_k^2$ of the dis-alignment in her individual price $k_i$ from the average; second, each producer does not consider the positive effect of an increase in her specific price on the average price, and therefore on other agents' marginal utility. Notice also that $\kappa_1 = \kappa_1^* = 1/\left(\omega\bar{\theta}\right)$. Hence, under perfect information the price setters' reaction to the fundamental is aligned with the socially optimal one. Since $W_{KK} = -\Psi\left(\bar{Y}\right)\omega$, $W' = -\Psi\left(\bar{Y}\right)\omega/2$, and $W_{\sigma\sigma} = -\Psi\left(\bar{Y}\right)v^2\left(1+\omega\right) < 0$, Assumptions 2.1 and 6.2 hold whenever $v > 1$ and $\omega > 0$.

Observe that, since $\alpha = \frac{v-1}{v}$, the strategic complementarity in agents' actions (e.g., price setting) depends on the elasticity parameter $v$.[24] Because the demand function of

nominal velocity or money shocks (as in Hellwig, 2005).

[22]The non stochastic equilibrium is defined by assuming that the variance of the labor supply shock $\theta$ is equal to 0; i.e. $\sigma_\theta^2$.

[23]Hellwig (2005) assumes that agents are uncertain about monetary policy shocks but do not face idiosyncratic productivity shocks. Hence, an increase in information transparency reduces price dispersion. Conversely, Lorenzoni (2010) focuses on disaggregated shocks. In his setup, a more precise public information increases price dispersion, which raises welfare since an increase in public information helps setting relative prices more aligned to productivity differentials.

[24]In Roca (2006), the degree of complementarity in prices is affected both by $v$ and by $\omega$. Conversely, in Adam (2007) and Baeriswyl and Cornand (2007), $v$ does not play any role in, as in these papers individual prices are directly proportional to the aggregate price level.

the good produced by the generic agent $i$ is negatively sloped, it emerges immediately that an increase in the price level $(K)$ raises the demand of the product supplied by agent $i$, which in turn induces her to increase her own price $k_i$.

Given the assumption of linear private and public costs of information precisions, the thresholds in Proposition 6.1 become $c'' = \frac{v(1+2\omega)-\omega}{\omega} > 1$, and $c' = 1 < c''$. It is interesting to note that, due to the complementarity in pricing, the private perception of the coordination motive falls short of the socially optimal one, which causes the threshold $c''$ to be always larger than one since $v > 1$.

The threshold $c''$ discriminating between private and public information (see Proposition 6.1, case 2) is illustrated in Figure 3 for different values of $\alpha$.[25]

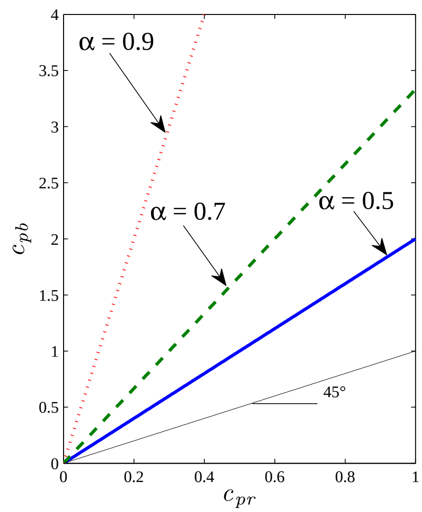


Figure 3: The cost threshold $c''$ with price complementarities for different values of $\alpha$

Note that, even when $\alpha = 0.5$ (corresponding to a cross-elasticity parameter of 2), the acquisition of private information is dominated by the provision of public information for a very large range of parameters values. As it is to be expected, this effect becomes stronger for larger values of $v$ and therefore of $\alpha$. This is due to the fact that, in addition to the cost saving effect already discussed for the beauty contest, the optimal degree of coordination of agents' actions $(\alpha^*)$ is always larger than the equilibrium one $(\alpha)$, meaning that in equilibrium agents systematically underweight the importance of the public signal with respect to what it would be optimal. This implies that an increase in the precision of public information, by augmenting the weight agents assign to it, favors the alignment of their actions, which is welfare improving unless the cost of public information precision is far larger than that of private information.[26]

These results suggest that business cycles are unlikely to be driven by the dispersion of information across agents. When the economy is hit by an aggregate shock, the

[25]Our numerical exercises are based on the normalization $\bar{\theta} = 1$ and on the assumption that $\omega = 4$, which is plausible given that $\omega$ reflects both the curvature of the marginal utility of consumption and the elasticity of the real cost with respect to output.

[26]Note that the emphasis on public information with respect to private information is consistent with the empirical findings reported, e.g., by Romer and Romer (2000).

possibility of agents' acquiring private information does not have a significant impact on the effects of monetary policy on the cycle. This finding follows from the fact that the dispersion of private information across agents is offset by an optimal provision of public information. In a recent paper, Angeletos and La'O (2010) show that the persistence of the business cycle stems both from the effects of idiosyncratic shocks and from the dispersion of agents' information. We argue that the latter effect is unlikely to be the dominant one.

### 7.3 Negative production externalities

We consider a simple consumer-producer competitive economy, populated by a continuum of identical agents. Each agent produces $k_i$ units of a non-differentiated private good using labor as the only input. She derives the utility $\theta J(k_i)$ from consuming the good, where $\theta$ denotes an aggregate shock. Her utility function can be written as

$$U(k_i, K, \theta) = \theta J(k_i) - V(k_i, K). \tag{26}$$

Note that aggregate production $K$ determines negative externalities, such as those due to pollution or congestion.[27] Labor disutility and the negative production externality are embedded in the term $V(k_i, K)$ of the individual utility function (26). By taking into account also the cost of information precision, the objective function of the consumer-producer can therefore be written as

$$U(k_i, K, \theta) = \theta J(k_i) - V(k_i, K) - c_{pr} p_x - c_{pb} p_y. \tag{27}$$

The privately perceived marginal utility of the final good is assumed being positive and decreasing; i.e., $J_k(k_i) > 0$, $J_{kk}(k_i) < 0$. Furthermore, both the marginal disutility of producing the final good, $V_k(k_i, K)$, and the marginal impact on utility of the negative externality associated to aggregate production, $V_K(k_i, K)$, are assumed being positive and increasing; i.e., $V_k(k_i, K) > 0$, $V_K(k_i, K) > 0$, $V_{kk}(k_i, K) > 0$, $V_{KK}(k_i, K) > 0$. Finally, it is natural to assume that the marginal disutility of production increases with the negative externality, which implies $V_{kK}(k_i, K) > 0$.

By taking a quadratic expansion of Equation (27), we can conveniently write

$$\begin{aligned} U(k_i, K, \theta) &= \bar{U}(k_i, K, \theta) + J_k(\bar{K}) k_i \theta + \\ &+ \frac{1}{2} k_i^2 \left(\bar{\theta} J_{kk}(\bar{K}) - V_{kk}(\bar{K}, \bar{K})\right) - V_{kK}(\bar{K}, \bar{K}) k_i K + \\ &- \frac{V_{KK}(\bar{K}, \bar{K})}{2} K^2 - c_{pr} p_x - c_{pb} p_y, \end{aligned} \tag{28}$$

where $\bar{K}$ denotes the non stochastic equilibrium level of aggregate production, and $\bar{\theta}$ the expected value of the utility shock $\theta$ (i.e., $\theta_{-1}$).

Note that $\bar{U}(k_i, K, \theta)$ collects all the terms that are constant or linear in $\theta$, $K$, and $k_j$.[28] We define $\omega \equiv -\frac{\left(\bar{\theta} J_{kk}(\bar{K}) - V_{kk}(\bar{K}, \bar{K})\right)\bar{K}}{\bar{\theta} J_k(\bar{K})} > 0$ as the sum of consumption and

[27] Classical examples are Tybout (1972) and Rothemberg (1970). Models having the same qualitative features can be obtained in entirely different frameworks, such as those related to the exploitation of natural resources (as in Scott Gordon, 1954, and Baumol and Oates, 1988), or to the private provision of public goods (as in Bergstrom, Blume and Varian, 1986).

[28] Recall that $\bar{k} = \bar{K}$ follows since agents have unit mass, and by symmetry.

production elasticities, and we let $\chi \equiv V_{KK}\left(\bar{K},\bar{K}\right)/V_{kK}\left(\bar{K},\bar{K}\right) > 0$. Accordingly, Equation (28) becomes:

$$\begin{aligned} U\left(k_i,K,\theta\right) &= \bar{U}\left(k_i,K,\theta\right) - \left(\bar{\theta}J_{kk}\left(\bar{K}\right) - V_{kk}\left(\bar{K},\bar{K}\right)\right)\left(\frac{1}{\omega\bar{\theta}}k_i\theta - \frac{1}{2}k_i^2\right) + \\ & -V_{kK}\left(\bar{K},\bar{K}\right)\left(k_iK + \frac{\chi}{2}K^2\right) - c_{pr}p_x - c_{pb}p_y. \end{aligned} \tag{29}$$

By letting $\bar{\theta} = 1$ without loss of generality, it is immediate to see that $U_{kk} = J_{kk}\left(\bar{K}\right) - V_{kk}\left(\bar{K},\bar{K}\right)$, $U_{kK} = -V_{kK}\left(\bar{K},\bar{K}\right)$, $U_{KK} = \chi U_{kK}$, $U_{k\theta} = -\frac{J_{kk}\left(\bar{K}\right)-V_{kk}\left(\bar{K},\bar{K}\right)}{\omega}$, $U_{K\theta} = U_{\theta\theta} = 0$. Since $U_{\sigma\sigma} = 0$, there is neither a taste nor a distaste for dispersion. It is important to stress that production decisions are strategic substitutes since $\alpha = \frac{V_{kK}\left(\bar{K},\bar{K}\right)}{J_{kk}\left(\bar{K}\right)-V_{kk}\left(\bar{K},\bar{K}\right)} < 0$. By exploiting the definition of $\alpha$, we obtain that $\kappa_1 = \frac{1}{\omega(1-\alpha)}$. Moreover, it is $W_{KK} = \left(1-\left(2+\chi\right)\alpha\right)U_{kk}$, so that the optimal response to a change in the fundamental is $\kappa_1^* = \frac{1}{\omega(1-\alpha(2+\chi))}$.[29] Notice also that our assumptions concerning the utility function guarantee that $\kappa_1 > \kappa_1^* > 0$. Since $W_{\sigma\sigma} = U_{kk}$ and $\alpha^* = 1 - \frac{W_{KK}}{W_{\sigma\sigma}}$, it is $\alpha^* = \left(\chi+2\right)\alpha < \alpha$. Therefore, the optimal degree of coordination falls short of the privately perceived one. This follows directly from the fact that individual producers ignore their contribution to the aggregate externality. Finally, since $W' = \frac{1+\alpha\chi}{2}U_{kk}$, Assumption 6.2 holds if and only if $\alpha\chi > -1$.

With the linear private and public costs of information precisions given in Assumptions 4.1 and 6.1, respectively, the thresholds in Proposition 6.1 become

$$c' = c'' = \frac{1+\alpha\chi}{\left(1-\alpha\right)^2} < 1.$$

Observe that in the presence of strategic substitutability the relevant cost thresholds are smaller than one.[30] This implies that the scope for public information provision is reduced since – if $c_{pb}/c_{pr} > c'$ – there is no need to provide a more precise public signal even when private information is more costly than public information. To understand why, recall that in the presence of strategic substitutability firms demand a more precise private information (see Equation 10), as the latter carries a large weight on firms' choices. Furthermore, since $\alpha^* < \alpha$, a larger precision of private information is socially beneficial because it reduces the disproportionately large weight of the public signal.

The cost thresholds $c' = c''$ discriminating between public and private information for different degrees of strategic substitutability are illustrated in Figure 4.[31]

As expected, and contrary to the findings for the price setting framework discussed in Section 7.2 entailing strategic complementarity, under strategic substitutability the acquisition of private information dominates that of public information for a very large range of parameter values even for relatively small levels of strategic substitutability (i.e., for $\alpha = -0.3$), unless the cost of private information acquisition significantly exceeds that of public information provision. As already noted, this follows from the fact

[29] Observe that $W_{KK} < 0$, which guarantees that Assumption 2.1 (iii) is satisfied.

[30] The fact that $c'' = c'$ follows since $U_{\sigma\sigma} = U_{K\theta} = 0$.

[31] The figure is drawn under the assumption that $\chi = 0$, which is the most unfavorable case for us given that $\frac{\partial c'}{\partial \chi} < 0$.

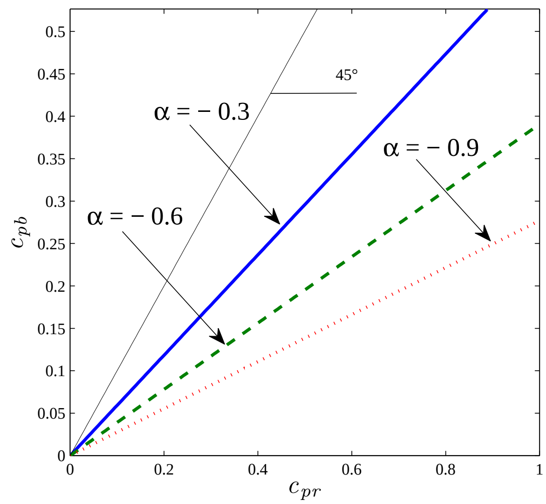


Figure 4: Cost thresholds for different values of $\alpha$ in the presence of negative production externalities

that agents systematically overweight public information in equilibrium since $\alpha^* < \alpha$. Hence, an increase in the precision of private information, by increasing the dispersion of agents' actions, is welfare improving; an effect that (as shown in Figure 4) becomes stronger the larger is the equilibrium degree of strategic substitutability.[32]

From the analysis above it follows that in a framework characterized by strategic substitutability, the scope for more precise public information is greatly reduced. Therefore, the pro-transparency recipes that should be used in the presence of strategic complementarities lose most of their appeal, sending a word of caution to policy makers as for the value of transparency in environments where negative externalities are likely to be relevant.

# 8 Concluding remarks

This paper investigates the equilibrium and welfare implications of the interaction between the use and collection of information. Its main theoretical contribution is the analysis of the mechanisms behind the choice of public information provision by a policy maker, and that of private information acquisition by heterogeneous agents.

Our analytical framework rests on a number of assumptions that are needed to

[32]Models involving positive production externalities would lead to opposite results. For instance, Angeletos and Pavan (2004) consider a partial equilibrium model of investment complementarities that entails a positive externality in production inducing strategic complementarities in agents' actions (consistently with, e.g., Cooper and John, 1988; and Acemoglu, 1993). By encompassing their model in our framework with costly information, it is easy to show that the private perception of the coordination motive falls short of the socially optimal one; i.e., $\alpha < \alpha^*$. Therefore, it is optimal to increase the coordination of agents' actions. This goal can be achieved by increasing the precision of public information, since in taking their decisions private agents assign a lower weight to their private information than to the public signal (as it is evident from Equation (10)), which would render an increase in private information an inefficient and costly tool to enhance the coordination of agents' actions.

guarantee the analytical tractability of the model. In continuity with most of the literature, we assume a quadratic specification of agents' utility functions and a Gaussian information structure. These two assumptions, however, do not seem crucial for our results, and they may indeed prove a good benchmark also for more general environments (as noted also by Angeletos and Pavan, 2007).

Specific to our setup is the assumption of linear costs of information precision. This hypothesis, besides being useful to guarantee the analytical tractability of the model, has entirely obvious implications. All our main insights are confirmed when focusing on isoelastic convex cost functions (see Appendix C). In this case, however, very little can be proved analytically, requiring us to resort to a computational analysis in order to disentangle the different effects at play.

The concavity of payoffs and the uniqueness of equilibrium that are essential in Angeletos and Pavan's (2007) setup remain crucial in the derivation of our results as well. As stressed in their paper, whenever aggregate welfare is convex over some region, a lottery may be socially preferable to the complete information equilibrium. In this case, the analysis of the implications of costly information acquisition/provision is constrained by the negative value associated with more precise private and public signals.

Generalizing the nature of the shocks we study in this paper is a promising avenue for future research. All the shocks we consider here share the property that the response of the economy to a shock under perfect information is efficient. Recent papers (e.g., Paciello and Wiederholt, 2010) consider also disturbances – such as the mark-up shocks in the New-Keyenesian monopolistic competition model – having the property that the response of the private sector to the shock under perfect information is inefficient. It would be interesting to investigate whether the pro-transparency results we obtain for environments with strategic complementarity continue to hold, and to what extent, when considering shocks with such 'inefficiency property'.

A second interesting extension is to explore the welfare implications of different assumptions on the interplay between the provision of public information and the collection of private information by heterogeneous agents. A deeper understanding of the ways in which public signals are used by individuals and their implications in terms of beliefs correlation, as well as of the mechanisms presiding the aggregation of information, may reveal additional aspects of the interplay between public and private information that remain unexplored in the current setup.

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# Appendix A. Proofs

**Proof of Proposition 3.1**. Consider any agent $i$. Solving problem (2), we find that the individual best response function of agent $i$ to other agents' actions is

$$k(x,y) = -\frac{U_k}{U_{kk}} - \frac{U_{kK}}{U_{kk}} E[K \mid x,y] - \frac{U_{k\theta}}{U_{kk}} E[\theta \mid x,y] \tag{A.1}$$

that, by using the definitions of $\alpha$, $\kappa_1$ and $\kappa_0$, can be rewritten as in (3).

(ii) Following the equilibrium reasoning in Morris and Shin (2002), suppose that

$$k(x,y) = a + bx + cz,$$

for some $\{a,b,c\} \in \mathbb{R}$. Then $K(\theta,y) = a + b\theta + cz$. By substituting $k(x,y)$ and $K(\theta,y)$ into (3) we obtain: $a = \kappa_0$, $b = \kappa_1(1-\gamma)$, and $c = \kappa_1\gamma$ with $\gamma = \frac{\delta}{1-\alpha(1-\delta)}$, which proves the claim. ■

**Proof of Proposition 3.2**. Observe that agent $j$ has zero-measure. The equilibrium value of $\gamma$ when all agents $i$ other than $j$ choose a precision $p_{x_i} = p_x$ is given by (4). Therefore, by exploiting the definition of $k(x,y)$ in Proposition 3.1, we obtain

$$K(\theta,y) = \int_x k(x,y)\,dP(x \mid \theta,y) = \kappa_0 + \kappa_1(\gamma z + (1-\gamma)\theta). \tag{A.2}$$

By substituting Equation (A.2) into the first order condition of agent $j$'s utility maximization problem, we obtain her best response to other agents' actions; i.e.,

$$k_j(x_j,y) = -\frac{U_k}{U_{kk}} + \alpha(\kappa_0 + \kappa_1(\gamma z + (1-\gamma)E[\theta \mid x_j,y])) + \kappa_1(1-\alpha)E[\theta \mid x_j,y].$$

Recalling that $-\frac{U_k}{U_{kk}} = \kappa_0(1-\alpha)$, we can write

$$k_j(x_j,y) = \kappa_0 + \kappa_1(1-\alpha\gamma)E[\theta \mid x_j,y] + \kappa_1\alpha\gamma z, \tag{A.3}$$

in which

$$E[\theta \mid x_j,y] = \delta_j z + (1-\delta_j)x_j. \tag{A.4}$$

Rearranging, (A.3) reduces to

$$k_j(x_j,y) = \kappa_0 + \kappa_1(\gamma_j z + (1-\gamma_j)x_j),$$

where

$$\gamma_j = (1-\alpha\gamma)\delta_j + \alpha\gamma. \tag{A.5}$$

By substituting into Equation (A.5) the expression for $\gamma$ given in Equation (4), we obtain

$$\gamma_j = \frac{(1-\alpha)\delta_j + \alpha\delta}{1-\alpha(1-\delta)},$$

which proves the claim. ■

**Derivation of individual ex-ante expected utility (Equation 8).** The ex-ante utility of agent $j$ is given by the expectation of Equation (1) in period $-1$; i.e.,

$$E\left[U\left(k_j, K, \sigma_k^2, \theta\left|\theta_{-1}, p_{x_j}, p_x, p_z\right.\right)\right] = U_0 + E\left[U_k k_j + U_K K + U_\theta \theta\right] +$$
$$+\frac{1}{2}E\left[U_{kk}k_j^2 + 2U_{kK}k_j K + 2U_{k\theta}k_j\theta + U_{KK}K^2 + 2U_{K\theta}K\theta + U_{\theta\theta}\theta^2\right] +$$
$$+\frac{1}{2}U_{\sigma\sigma}\sigma_k^2 - C\left(p_{x_j}\right) - T_j. \tag{A.6}$$

In period $-1$, agent $j$ knows that in period 0 all agents $i \in [0,1]$, $i \neq j$, will play the linear equilibrium strategy (5), so that the average action $K$ will be given by (A.2). Moreover, agent $j$ anticipates that she will play her optimal best reply (6) when all other agents play the linear equilibrium strategy described in Proposition (3.1).

Exploiting Equations (A.2) and (6) to substitute for $K$ and $k_j$, respectively, into the terms of degree 1 in Equation (A.6), we obtain

$$E\left[U_k k_j + U_K K + U_\theta\theta\right] = U_k\left(\kappa_0 + \kappa_1\theta_{-1}\right) + U_K\left(\kappa_0 + \kappa_1\theta_{-1}\right) + U_\theta\theta_{-1}. \tag{A.7}$$

Because we consider the period $-1$ expectation, the linear terms in (A.7) are independent of the signals, and hence of their variances and of the weights $\gamma$ and $\gamma_j$.

As for the quadratic terms in (A.6), consider first $E\left[U_{kk}k_j^2\right] = U_{kk}E\left[k_j^2\right]$. By using (6) to substitute for $k_j$, we obtain

$$E\left[U_{kk}k_j^2\right] = U_{kk}\left(\kappa_0^2 + \kappa_1^2 E\left[\left(\gamma_j z + \left(1-\gamma_j\right)x_j\right)^2\right] + 2\kappa_0\kappa_1\theta_{-1}\right).$$

Defining $\lambda \equiv \frac{p_y}{p_\theta + p_y}$, we have that $z = E\left[\theta|\, y\right] = (1-\lambda)\,\theta_{-1} + \lambda y$. Recalling that $y = \theta_{-1} + \varphi + \varepsilon$ and that $x_j = \theta_{-1} + \varphi + \xi_j$, we can reformulate the equation above as

$$U_{kk}\left(\kappa_0^2 + \kappa_1^2 E\left[\left(\gamma_j\left(\theta_{-1} + \lambda\left(\varphi + \varepsilon\right)\right) + \left(1-\gamma_j\right)\left(\theta_{-1} + \varphi + \xi_j\right)\right)^2\right] + 2\kappa_0\kappa_1\theta_{-1}\right),$$

which can be rewritten as

$$U_{kk}\left(\kappa_0^2 + \kappa_1^2\left(\theta_{-1}^2 + E\left[\left(\varphi + \gamma_j\left(\lambda\varepsilon - \left(1-\lambda\right)\varphi\right) + \left(1-\gamma_j\right)\xi_j\right)^2\right]\right) + 2\kappa_0\kappa_1\theta_{-1}\right).$$

Taking the unconditional expectation, exploiting the definition of $\lambda$, and recalling that $p_z \equiv p_\theta + p_y$, we obtain

$$U_{kk}\left(\kappa_0^2 + \kappa_1^2\left(\theta_{-1}^2 + \left(\frac{1}{p_\theta} + \frac{\gamma_j^2}{p_z} + \frac{\left(1-\gamma_j\right)^2}{p_{x_j}} - 2\frac{\gamma_j}{p_z}\right)\right) + 2\kappa_0\kappa_1\theta_{-1}\right),$$

which rearranging the terms becomes

$$U_{kk}\left(\kappa_0^2 + \kappa_1^2\left(\theta_{-1}^2 + \frac{1}{p_\theta}\right) + 2\kappa_0\kappa_1\theta_{-1}\right) + U_{kk}\kappa_1^2\left(\frac{\gamma_j^2}{p_z} + \frac{\left(1-\gamma_j\right)^2}{p_{x_j}} - 2\frac{\gamma_j}{p_z}\right). \tag{A.8}$$

Focus now on $E\left[U_{kK}k_j K\right] = U_{kK}E\left[k_j K\right]$. By exploiting Equations (A.2) and (6) to substitute for $K$ and $k_j$, respectively, we obtain

$$U_{kK}E\left[\left(\kappa_0 + \kappa_1\left(\gamma_j z + \left(1-\gamma_j\right)x_j\right)\right)\left(\kappa_0 + \kappa_1\left(\gamma z + \left(1-\gamma\right)\theta\right)\right)\right].$$

Substituting for $z$ and $x_j$, and collecting the terms involving $\theta_{-1}$, yields

$$U_{kK}\left(\kappa_0^2+2\kappa_0\kappa_1\theta_{-1}+\kappa_1^2\theta_{-1}^2+\right.$$
$$\left.+\kappa_1^2 E\left[\left(\varphi+\gamma_j\left(\lambda\varepsilon-(1-\lambda)\varphi\right)+(1-\gamma_j)\,\xi_j\right)\left(\varphi+\gamma\left(\lambda\varepsilon-(1-\lambda)\varphi\right)\right)\right]\right)$$

that, by recalling the definition of $\lambda$, can be written as

$$U_{kK}\left(\kappa_0^2+\kappa_1^2\left(\theta_{-1}^2+\frac{1}{p_\theta}\right)+2\kappa_0\kappa_1\theta_{-1}\right)+U_{kK}\kappa_1^2\left(\frac{\gamma_j\gamma}{p_z}-\frac{\gamma_j+\gamma}{p_z}\right). \tag{A.9}$$

The computation of $E\left[U_{KK}K^2\right]$, $E\left[U_{k\theta}k_j\theta\right]$, and $E\left[U_{K\theta}K\theta\right]$ requires only marginal variations of the above procedure. We start with $E\left[U_{KK}K^2\right]$ that, by substituting for $K$, can be written as

$$U_{KK}E\left[\left(\kappa_0+\kappa_1\left(\gamma z+(1-\gamma)\,\theta\right)\right)^2\right]=$$
$$=U_{KK}\left(\kappa_0^2+2\kappa_0\kappa_1\theta_{-1}+\kappa_1^2E\left[\left(\gamma\left(\theta_{-1}+\lambda\left(\varphi+\varepsilon\right)\right)+(1-\gamma)\left(\theta_{-1}+\varphi\right)\right)^2\right]\right)=$$
$$=U_{KK}\left(\kappa_0^2+2\kappa_0\kappa_1\theta_{-1}+\kappa_1^2\theta_{-1}^2+\kappa_1^2E\left[\left(\varphi+\gamma\left(\lambda\varepsilon-(1-\lambda)\varphi\right)\right)^2\right]\right)=$$
$$=U_{KK}\left(\kappa_0^2+\kappa_1^2\left(\theta_{-1}^2+\frac{1}{p_\theta}\right)+2\kappa_0\kappa_1\theta_{-1}\right)+U_{KK}\kappa_1^2\left(\frac{\gamma^2}{p_z}-2\frac{\gamma}{p_z}\right). \tag{A.10}$$

As for $E\left[U_{k\theta}k_j\theta\right]$, we have

$$U_{k\theta}E\left[\left(\kappa_0+\kappa_1\left(\gamma_j z+(1-\gamma_j)\,x_j\right)\right)\theta\right]=$$
$$=U_{k\theta}\left(\kappa_0\theta_{-1}+\kappa_1E\left[\left(\gamma_j\left(\theta_{-1}+\lambda\left(\varphi+\varepsilon\right)\right)+(1-\gamma_j)\left(\theta_{-1}+\varphi+\xi_j\right)\right)\left(\theta_{-1}+\varphi\right)\right]\right)=$$
$$=U_{k\theta}\left(\kappa_0\theta_{-1}+\kappa_1\theta_{-1}^2+\kappa_1E\left[\left(\varphi+\gamma_j(\lambda\varepsilon-(1-\lambda)\varphi)+(1-\gamma_j)\,\xi_j\right)\varphi\right]\right)=$$
$$=U_{k\theta}\left(\kappa_0\theta_{-1}+\kappa_1\left(\theta_{-1}^2+\frac{1}{p_\theta}\right)\right)-U_{k\theta}\kappa_1\left(\frac{\gamma_j}{p_z}\right). \tag{A.11}$$

Finally, for $E\left[U_{K\theta}K\theta\right]$, we obtain

$$U_{K\theta}E\left[\left(\kappa_0+\kappa_1\left(\gamma z+(1-\gamma)\,\theta\right)\right)\theta\right]=$$
$$=U_{K\theta}\left(\kappa_0\theta_{-1}+\kappa_1E\left[\left(\gamma\left(\theta_{-1}+\lambda\left(\varphi+\varepsilon\right)\right)+(1-\gamma)\left(\theta_{-1}+\varphi\right)\right)\left(\theta_{-1}+\varphi\right)\right]\right)=$$
$$=U_{K\theta}\left(\kappa_0\theta_{-1}+\kappa_1\theta_{-1}^2+\kappa_1E\left[\left(\varphi+\gamma\left(\lambda\varepsilon-(1-\lambda)\varphi\right)\right)\varphi\right]\right)=$$
$$=U_{K\theta}\left(\kappa_0\theta_{-1}+\kappa_1\left(\theta_{-1}^2+\frac{1}{p_\theta}\right)\right)-U_{K\theta}\kappa_1\left(\frac{\gamma}{p_z}\right). \tag{A.12}$$

Using the fact that $\theta=\theta_{-1}+\varphi$, the term $E\left[U_{\theta\theta}\theta^2\right]=U_{\theta\theta}E\left[\theta^2\right]$ can be written as

$$U_{\theta\theta}\left(\theta_{-1}^2+E\left[\varphi^2\right]\right);$$

i.e., $U_{\theta\theta}\left(\theta_{-1}^2+\frac{1}{p_\theta}\right)$.

Because agent $j$ has zero-measure, exploiting Equations (A.2) and (5), the dispersion of individual actions is given by

$$\sigma_k^2 = \int_i [k_i - K]^2 \, di = \int_i [\kappa_1 (1-\gamma)(x_i - \theta)]^2 \, di.$$

Since $x_i = \theta + \xi_i$ and $p_{x_i} = p_x$ for all $i \in [0,1]$, we obtain

$$\sigma_k^2 = \frac{\kappa_1^2 (1-\gamma)^2}{p_x}.$$

We collect all the terms that are independent of the variances of signals and of the weights $\gamma$ and $\gamma_j$ in the term

$$\begin{aligned} \tilde{\kappa} \equiv{} & U_0 + (U_k + U_K)\kappa_0 + ((U_k + U_K)\kappa_1 + U_\theta)\theta_{-1} + \\ & + \frac{1}{2}(U_{kk} + 2U_{kK} + U_{KK})\left(\kappa_0^2 + \kappa_1^2\left(\theta_{-1}^2 + \frac{1}{p_\theta}\right) + 2\kappa_0\kappa_1\theta_{-1}\right) + \\ & + (U_{k\theta} + U_{K\theta})\kappa_0\theta_{-1} + ((U_{k\theta} + U_{K\theta})\kappa_1 + U_{\theta\theta})\left(\theta_{-1}^2 + \frac{1}{p_\theta}\right). \end{aligned}$$

By exploiting Equations (A.8) - (A.12) and using the definition of $\sigma_k^2$, we can rewrite (A.6) as

$$\begin{aligned} E\left[U\left(k_j, K, \sigma_k^2, \theta \left|\theta_{-1}, p_{x_j}, p_x, p_z\right.\right)\right] ={} & \tilde{\kappa} + \frac{U_{kk}}{2}\kappa_1^2\left(\frac{\gamma_j^2}{p_z} + \frac{(1-\gamma_j)^2}{p_{x_j}} - 2\frac{\gamma_j}{p_z}\right) + \\ & + U_{kK}\kappa_1^2\left(\frac{\gamma_j\gamma}{p_z} - \frac{\gamma_j + \gamma}{p_z}\right) + \frac{U_{KK}}{2}\kappa_1^2\left(\frac{\gamma^2}{p_z} - 2\frac{\gamma}{p_z}\right) + \\ & - U_{k\theta}\kappa_1\frac{\gamma_j}{p_z} - U_{K\theta}\kappa_1\frac{\gamma}{p_z} + \frac{1}{2}U_{\sigma\sigma}\frac{\kappa_1^2(1-\gamma)^2}{p_x} - C\left(p_{x_j}\right) - T_j, \end{aligned}$$

Finally, recalling that $-U_{k\theta} - \kappa_1(U_{kk} + U_{kK}) = 0$ by definition of $\kappa_1$, the ex ante utility becomes

$$\begin{aligned} E\left[U\left(k_j, K, \sigma_k^2, \theta \left|\theta_{-1}, p_{x_j}, p_x, p_z\right.\right)\right] ={} & \tilde{\kappa} + \frac{U_{kk}}{2}\kappa_1^2\left(\frac{\gamma_j^2}{p_z} + \frac{(1-\gamma_j)^2}{p_{x_j}}\right) + U_{kK}\kappa_1^2(\gamma_j - 1)\frac{\gamma}{p_z} + \\ & + \frac{U_{KK}}{2}\kappa_1^2(\gamma - 2)\frac{\gamma}{p_z} - U_{K\theta}\kappa_1\frac{\gamma}{p_z} + \frac{1}{2}U_{\sigma\sigma}\frac{\kappa_1^2(1-\gamma)^2}{p_x} - C\left(p_{x_j}\right) - T_j, \end{aligned}$$

which is Equation (8) in the main text. ■

**Proof of Proposition 4.1**. (i) We need to show that

$$\partial E\left[U\left(k_j, K, \sigma_k^2, \theta \left|\theta_{-1}, p_{x_j}, p_x, p_z\right.\right)\right] / \partial\gamma_j = 0$$

when $\gamma_j$ takes the value given in Equation (7). By equating to zero the derivative of (8) with respect to $\gamma_j$, we obtain

$$U_{kk}\kappa_1^2\left(\frac{\gamma_j}{p_z} - \frac{1-\gamma_j}{p_{x_j}}\right) + U_{kK}\kappa_1^2\frac{\gamma}{p_z} = 0$$

that, by making use of the fact that $\alpha \equiv -\frac{U_{kK}}{U_{kk}}$ and rearranging terms, becomes

$$\gamma_j = \frac{p_z}{p_{x_j} + p_z} + \alpha\gamma \frac{p_{x_j}}{p_{x_j} + p_z}.$$

Recalling that $\delta_j \equiv \frac{p_z}{p_z + p_{x_j}}$ and $\gamma = \frac{\delta}{1-\alpha(1-\delta)}$, the above equation can be rewritten as

$$\gamma_j = \frac{(1-\alpha)\,\delta_j + \alpha\delta}{1-\alpha\,(1-\delta)},$$

which proves the claim.

(ii) By maximizing (8) with respect to $p_{x_j}$ and using Assumption 4.1, we obtain the first order condition

$$-\frac{U_{kk}\kappa_1^2}{2} \frac{(1-\gamma_j)^2}{p_{x_j}^2} - c_{pr} = 0.$$

Using (7), we have that $1-\gamma_j = \frac{(1-\alpha)(1-\delta_j)}{1-\alpha(1-\delta)}$, which – by exploiting the definition of $\delta_j$ and recalling that $\delta \equiv \frac{p_z}{p_z+p_x}$ – can be rewritten as $1-\gamma_j = p_{x_j} \frac{1-\alpha}{p_z+p_{x_j}} \frac{p_z+p_x}{p_z+(1-\alpha)p_x}$. It is therefore immediate to obtain that

$$p_{x_j} = \sqrt{-\frac{U_{kk}\kappa_1^2}{2c_{pr}} \frac{(p_x+p_z)\,(1-\alpha)}{p_z + (1-\alpha)\,p_x}} - p_z,$$

which proves the claim. ■

**Proof of Lemma 5.1**. The result is immediate from the first order condition with respect to $K$ of the maximization of the welfare (12); i.e.,

$$\frac{\partial W\left(K, \sigma_k^2, \theta\right)}{\partial K} = W_K + W_{KK}K + W_{K\theta}\theta.$$

Recall that the second order condition of the maximization problem holds by Assumption 2.1. ■

**Proof of Proposition 5.1**. The Lagrangian corresponding to the maximization problem in Definition 5.1 is:

$$\begin{aligned}\mathcal{L} = &\int_\theta W\left(K(\theta,y), \sigma_k^2(\theta,y), \theta\right) dP(\theta \mid y) \\ &+ \int_\theta \lambda(\theta,y)\left(K(\theta,y) - \int_x k(x,y)\, dP(x \mid \theta,y)\right) dP(\theta \mid y) + \\ &+ \int_\theta \eta(\theta,y)\left(\sigma_k^2(\theta,y) - \int_x [k(x,y) - K(\theta,y)]^2\, dP(x \mid \theta,y)\right) dP(\theta \mid y).\end{aligned}$$

By Assumption 2.1 it is $W_{KK} < 0$ and $W_{\sigma\sigma} < 0$, so that the program is concave. The solution is given by the first-order conditions for $k(x,y)$, $K(\theta,y)$, and $\sigma_k^2(\theta,y)$ that, by making use of Definition (11), can be written respectively as:

$$\begin{aligned}\int_\theta &[U_k + U_{kk}k(x,y) + U_{kK}K(\theta,y) + U_{k\theta}\theta - \lambda(\theta,y) + \\ &-2\eta(\theta,y)\,(k(x,y) - K(\theta,y))]\, dP(\theta \mid x,y) = 0\end{aligned} \tag{A.13}$$

$$\int_{\theta}\left[\int_{x}\left[U_{K}+U_{KK}K(\theta,y)+U_{kK}k(x,y)+U_{K\theta}\theta\right]dP(x\mid\theta,y)\right. \tag{A.14}$$
$$\left.+\lambda(\theta,y)+2\eta(\theta,y)\int_{x}\left[k(x,y)-K(\theta,y)\right]dP(x\mid\theta,y)\right]dP(\theta\mid y)=0$$

$$\int_{\theta}\left[\int_{x}\frac{U_{\sigma\sigma}}{2}dP(x\mid\theta,y)+\eta(\theta,y)\right]dP(\theta\mid y)=0, \tag{A.15}$$

where in formulating Equation (A.13) we make use of the fundamental lemma on conditional probability (see, e.g., Grimmet and Stirzaker, 1992).[33] By focusing on (A.14), it is immediate to see that $\int_{x}\left[k(x,y)-K(\theta,y)\right]dP(x\mid\theta,y)=0$. Accordingly, Equation (A.14) can be rewritten as

$$\int_{\theta}\left[U_{K}+(U_{KK}+U_{kK})K(\theta,y)+U_{K\theta}\theta+\lambda(\theta,y)\right]dP(\theta\mid y)=0. \tag{A.16}$$

Noting that by Equation (A.15) it must be $U_{\sigma\sigma}=-2\eta(\theta,y)$ for all $\theta$ and $y$, and substituting this expression together with (A.16) into (A.13), we obtain

$$\int_{\theta}\left[U_{k}+U_{K}+U_{kk}k(x,y)+(U_{KK}+2U_{kK})K(\theta,y)+(U_{k\theta}+U_{K\theta})\theta+\right. \tag{A.17}$$
$$\left.+U_{\sigma\sigma}(k(x,y)-K(\theta,y))\right]dP(\theta\mid x,y)=0.$$

Since the program in Definition 5.1 is quadratic, it is natural to assume that $k(x,y)$ and $K(\theta,y)$ are linear in their arguments; i.e.,

$$k(x,y)=a+bx+cz, \tag{A.18}$$

$$K(\theta,y)=\int_{x}k(x,y)dP(x\mid\theta,y)=a+b\theta+cz, \tag{A.19}$$

where, as discussed in Section 2, $z=\frac{p_{\theta}\theta_{-1}+p_{y}y}{p_{\theta}+p_{y}}$. By substituting (A.18) and (A.19) into (A.17), and recalling that $\int_{\theta}\theta dP(\theta\mid x,y)=\delta z+(1-\delta)x$, after some algebra we have

$$U_{k}+U_{K}+U_{kk}(a+bx+cz)+(U_{KK}+2U_{kK})(a+b(\delta z+(1-\delta)x)+cz)+$$
$$+(U_{k\theta}+U_{K\theta})(\delta z+(1-\delta)x)+U_{\sigma\sigma}b\delta(x-z)=0.$$

By matching the coefficients for all terms of degrees zero and one, we easily obtain

$$\begin{aligned}
a &= -\frac{U_{k}+U_{K}}{W_{KK}}=\kappa_{0}^{*},\\
b &= -\frac{W_{K\theta}(1-\delta)}{W_{KK}(1-\delta)+\delta W_{\sigma\sigma}},\\
c &= -\frac{W_{K\theta}}{W_{KK}}\delta+\left(\frac{W_{\sigma\sigma}}{W_{KK}}-1\right)\delta b,
\end{aligned}$$

[33] By repeatedly applying the fundamental lemma, we obtain:

$$\begin{aligned}
dP(x\mid\theta,y)\,dP(\theta\mid y) &= \frac{dP(x,\theta,y)}{dP(\theta,y)}dP(\theta\mid y)=\frac{dP(x,\theta,y)}{dP(\theta,y)}\frac{dP(\theta,\ y)}{dP(y)}=\frac{dP(\theta\mid x,y)\,dP(x,y)}{dP(y)}=\\
&= dP(\theta\mid x,y)\,dP(x\mid y).
\end{aligned}$$

in which $W_{KK} \equiv U_{kk} + 2U_{kK} + U_{KK}$, $W_{K\theta} \equiv U_{k\theta} + U_{K\theta}$, and $W_{\sigma\sigma} \equiv U_{kk} + U_{\sigma\sigma}$. Dividing both the numerator and the denominator in the expression of $b$ by $W_{KK}$, and exploiting the definition of $\kappa_1^*$ given in Lemma 5.1, yields

$$b = \kappa_1^* \frac{(1-\delta)}{(1-\delta) + \delta \frac{W_{\sigma\sigma}}{W_{KK}}}.$$

Finally, using Definitions (15) and (17), we can write

$$b = \kappa_1^* \frac{(1-\alpha^*)(1-\delta)}{1-\alpha^*(1-\delta)} = \kappa_1^* (1-\gamma^*). \tag{A.20}$$

By exploiting (A.20), and the definition of $\kappa_1^*$, we have

$$c = \kappa_1^* \delta \left(1 + \left(\frac{W_{\sigma\sigma}}{W_{KK}} - 1\right) \frac{(1-\alpha^*)(1-\delta)}{1-\alpha^*(1-\delta)}\right).$$

Since the definition of $\alpha^*$ guarantees that $\frac{W_{\sigma\sigma}}{W_{KK}} - 1 = \frac{\alpha^*}{1-\alpha^*}$, the expression above reduces to $c = \kappa_1^* \gamma^*$. Therefore:

$$k^*(x,y) = \kappa_0^* + \kappa_1^* ((1-\gamma^*) x + \gamma^* z).$$

Accordingly, the allocation (16) is efficient and unique within the class of linear allocations. Since this allocation coincides with the solution obtained in Angeletos and Pavan (2007), it is globally unique. ■

**Derivation of the ex-ante welfare criterion (Equation 19).** Substituting Equation (12) in the welfare criterion (18), and exploiting both the expression of $K(\theta, y)$ in Definition 5.1 and the agents' equilibrium strategies (5), we obtain:

$$\begin{aligned} W(p_x, p_z, \theta_{-1}) = {} & U_0 + W_K E[\kappa_0 + \kappa_1 (\gamma z + (1-\gamma)\theta) | \theta_{-1}] + U_\theta E[\theta | \theta_{-1}] + \\ & + \frac{1}{2} W_{KK} E\left[(\kappa_0 + \kappa_1 (\gamma z + (1-\gamma)\theta))^2 | \theta_{-1}\right] + \frac{1}{2} W_{\sigma\sigma} \kappa_1^2 (1-\gamma)^2 E\left[(x-\theta)^2 | \theta_{-1}\right] + \\ & + W_{K\theta} E[(\kappa_0 \theta + \kappa_1 (\gamma z + (1-\gamma)\theta)\theta) | \theta_{-1}] + U_{\theta\theta} E\left[\theta^2 | \theta_{-1}\right] - \int_i C(p_{x_i})\, di - C(p_z), \end{aligned} \tag{A.21}$$

Notice that $E\left[z^2|\theta_{-1}\right] = E\left[\left(\frac{p_\theta \theta_{-1} + p_y y}{p_\theta + p_y}\right)^2 \middle| \theta_{-1}\right] = E\left[(\theta_{-1} + \lambda(\varphi + \varepsilon))^2 \middle| \theta_{-1}\right] = \theta_{-1}^2 + \lambda^2 \left(p_\theta^{-1} + p_y^{-1}\right)$, where $\lambda$ is defined as $p_y/(p_y + p_\theta) = p_y/p_z$. The information structure detailed in Section (2) implies that $E[z\theta|\theta_{-1}] = \theta_{-1}^2 + \lambda p_\theta^{-1}$, and that $E[x\theta|\theta_{-1}] = \theta_{-1}^2 + p_\theta^{-1}$. Moreover, we have by definition that $E\left[(x-\theta)^2 | \theta_{-1}\right] = p_x^{-1}$ and $E\left[\theta^2|\theta_{-1}\right] = \theta_{-1}^2 + p_\theta^{-1}$. Accordingly, Equation (A.21) becomes

$$\begin{aligned} W(p_x, p_z, \theta_{-1}) = {} & U_0 + W_K (\kappa_0 + \kappa_1 \theta_{-1}) + U_\theta \theta_{-1} + \frac{1}{2} W_{KK} \left(\kappa_0^2 + 2\kappa_0 \kappa_1 \theta_{-1} + \right. \\ & \left. + \kappa_1^2 \left(\gamma^2 \left(\left(\theta_{-1}^2 + \lambda^2 \left(p_\theta^{-1} + p_y^{-1}\right)\right) + (1-\gamma)^2 \left(\theta_{-1}^2 + p_\theta^{-1}\right) + 2\gamma(1-\gamma)\left(\theta_{-1}^2 + \lambda p_\theta^{-1}\right)\right)\right)\right) + \\ & + \frac{1}{2} W_{\sigma\sigma} \kappa_1^2 (1-\gamma)^2 p_x^{-1} + W_{K\theta} \left(\kappa_0 \theta_{-1} + \kappa_1 \left(\gamma \left(\theta_{-1}^2 + \lambda p_\theta^{-1}\right) + (1-\gamma)\left(\theta_{-1}^2 + p_\theta^{-1}\right)\right)\right) + \\ & + U_{\theta\theta} \left(\theta_{-1}^2 + p_\theta^{-1}\right) - \int_i C(p_{x_i})\, di - C(p_z). \end{aligned}$$

Collecting all the terms that are independent of signals precisions, and letting

$$\begin{aligned}\bar{W} \equiv\;& U_0 + W_K(\kappa_0 + \kappa_1\theta_{-1}) + U_\theta\theta_{-1} + \frac{1}{2}W_{KK}\left(\kappa_0^2 + 2\kappa_0\kappa_1\theta_{-1} + \kappa_1^2\theta_{-1}^2\right) + \\ & + W_{K\theta}\left(\kappa_0\theta_{-1} + \kappa_1\theta_{-1}^2\right) + U_{\theta\theta}\left(\theta_{-1}^2 + p_\theta^{-1}\right),\end{aligned}$$

we obtain

$$\begin{aligned}W(p_x, p_z, \theta_{-1}) = \bar{W} &+ \frac{1}{2}W_{KK}\kappa_1^2\left(\gamma^2\lambda^2\left(\frac{1}{p_\theta} + \frac{1}{p_y}\right) + \frac{(1-\gamma)^2}{p_\theta} + 2\frac{\gamma(1-\gamma)\lambda}{p_\theta}\right) + \\ &+ \frac{1}{2}W_{\sigma\sigma}\kappa_1^2\frac{(1-\gamma)^2}{p_x} + W_{K\theta}\kappa_1\left(\frac{\gamma\lambda}{p_\theta} + \frac{1-\gamma}{p_\theta}\right) + \\ &- \int_i C(p_{x_i})\,di - C(p_z).\end{aligned} \tag{A.22}$$

By separating the addenda involving $p_\theta$ and $p_y$, the term multiplied by $\frac{1}{2}W_{KK}\kappa_1^2$ in the first line of (A.22) can be written as

$$\left(\frac{\gamma^2\lambda^2}{p_y} + \frac{\gamma^2\lambda^2 + 1 + \gamma^2 - 2\gamma + 2\gamma\lambda - 2\gamma^2\lambda}{p_\theta}\right);$$

i.e.,

$$\left(\frac{\gamma^2\lambda^2}{p_y} + \frac{1 + \gamma^2(1-\lambda)^2 - 2\gamma(1-\lambda)}{p_\theta}\right).$$

Recalling that $\lambda \equiv \frac{p_y}{p_z}$, we have

$$\left(\frac{\gamma^2 p_y}{p_z^2} + \frac{1}{p_\theta} + \frac{\gamma^2 p_\theta}{p_z^2} - \frac{2\gamma}{p_z}\right),$$

and hence

$$\left(\frac{1}{p_\theta} + \frac{\gamma^2 - 2\gamma}{p_z}\right).$$

Moreover, by recalling that $W_{K\theta} = -W_{KK}\kappa_1^*$ and by exploiting the definition of $\lambda$, we can write

$$W_{K\theta}\kappa_1\left(\frac{\gamma\lambda}{p_\theta} + \frac{1-\gamma}{p_\theta}\right) = -W_{KK}\frac{\kappa_1^*}{\kappa_1}\kappa_1^2\left(\frac{1}{p_\theta} - \frac{\gamma}{p_z}\right).$$

Accordingly, Equation (A.22) becomes

$$\begin{aligned}W(\theta_{-1}, p_x, p_z) = \tilde{W} &+ W_{KK}\kappa_1^2\left(\frac{1}{2}\frac{\gamma(\gamma-1)}{p_z} + \left(\frac{\kappa_1^*}{\kappa_1} - \frac{1}{2}\right)\frac{\gamma}{p_z}\right) + \frac{1}{2}W_{\sigma\sigma}\kappa_1^2\frac{(1-\gamma)^2}{p_x} + \\ &- \int_i C(p_{x_i})\,di - C(p_z),\end{aligned} \tag{A.23}$$

where $\tilde{W} \equiv \bar{W} + \frac{W_{KK}}{p_\theta}\kappa_1^2\left(\frac{1}{2} - \frac{\kappa_1^*}{\kappa_1}\right)$ collects all the addenda that are independent of public and private precisions. Equation (19) in the main text follows immediately. ■

**Proof of Proposition 6.1**. To ease the analysis of the two cases in the proposition, we first study the problem of optimal public information provision when no private information is acquired.

By inspection of Equation (10) it is immediate to see that $p_x^* = 0$ if and only if $p_z \geq (1-\alpha)\sqrt{-\frac{U_{kk}\kappa_1^2}{2c_{pr}}}$ or, recalling that $p_z = p_y + p_\theta$, if and only if

$$p_y \geq \bar{p}_y \equiv (1-\alpha)\sqrt{-\frac{U_{kk}\kappa_1^2}{2c_{pr}}} - p_\theta,$$

which is non-negative by Assumption 6.3. Since in this case it is $p_x^* = 0$, the policy maker chooses $p_y$ by maximizing (20) under the constraint $p_y \geq \bar{p}_y$. Denoting with $\psi$ the Lagrange multiplier associated to this constraint, we obtain the first order condition

$$-\kappa_1^2 W' \frac{1}{(p_y + p_\theta)^2} - c_{pb} + \psi = 0,$$

from which it follows immediately that

$$\hat{p}_y = \begin{cases} \sqrt{-\frac{\kappa_1^2 W'}{c_{pb}}} - p_\theta > \bar{p}_y & \text{when } c_{pb}/c_{pr} < c' \\ \bar{p}_y & \text{when } c_{pb}/c_{pr} \geq c' \end{cases}, \tag{A.24}$$

where we denote with $\hat{p}_y$ the optimum of the policy maker's problem in the interval $p_y \in [\bar{p}_y, \infty)$. Note that $\hat{p}_y > 0$ by Assumption 6.3.

Turning to the case of private information acquisition of positive precision, whenever $0 \leq p_y \leq \bar{p}_y$, from Equation (10) we obtain

$$p_x^* = \sqrt{-\frac{U_{kk}\kappa_1^2}{2c_{pr}}} - \frac{p_y + p_\theta}{1-\alpha} \geq 0.$$

Given private agents' behavior, substituting for $p_x^*$ and after some algebra, the social welfare function (22) can be written as

$$\begin{aligned} W(\theta_{-1}, p_x^*, p_\theta, p_y) &= \tilde{W} + \kappa_1^2 W' \frac{\sqrt{2c_{pr}}}{(1-\alpha)\sqrt{-U_{kk}\kappa_1^2}} + \\ &\quad - \left(\sqrt{-\frac{U_{kk}\kappa_1^2}{2c_{pr}}} - \frac{p_y + p_\theta}{1-\alpha}\right)\left(\frac{(\alpha^* - \alpha) W_{\sigma\sigma}}{(1-\alpha) U_{kk}} + 1\right) c_{pr} - c_{pb} p_y. \end{aligned} \tag{A.25}$$

Hence, the policy maker's maximization problem becomes

$$\max_{0 \leq p_y \leq \bar{p}_y} W(\theta_{-1}, p_x^*, p_\theta, p_y).$$

Denote with $\zeta$ and $\mu$ the Lagrange multipliers corresponding to the two constraints $p_y \geq 0$ and $p_y \leq \bar{p}_y$, respectively. Taking the first order condition of the policy maker's maximization problem, we get

$$\frac{1}{1-\alpha}\left(1 + \frac{(\alpha^* - \alpha) W_{\sigma\sigma}}{(1-\alpha) U_{kk}}\right) c_{pr} - c_{pb} + \zeta - \mu \geq 0,$$

which can immediately be rewritten as

$$c''c_{pr} - c_{pb} + \zeta - \mu \geq 0,$$

where $c'' \equiv \frac{1}{1-\alpha}\left(1 + \frac{\alpha^*-\alpha}{1-\alpha}\frac{W\sigma\sigma}{U_{kk}}\right)$. Denote by $\check{p}_y$ the optimum of the policy maker's problem in $p_y \in [0,\ \bar{p}_y]$, and suppose that $\zeta > 0$ and $\mu = 0$. Then, it must be $c_{pb}/c_{pr} > c''$, so that $\check{p}_y = 0$ and $p_x^* = \sqrt{-\frac{U_{kk}\kappa_1^2}{2c_{pr}}} - \frac{p_\theta}{1-\alpha}$. Conversely, suppose that $\zeta = 0$ and $\mu > 0$, then it must be $c_{pb}/c_{pr} \leq c''$ so that $\check{p}_y = \bar{p}_y$, and hence $p_x^* = 0$.

We can now solve for a global maximum of the policy maker's problem. We focus first on Case 1; i.e., $c'' < c'$. The proof of the claim is in three steps.

Step 1. Suppose that $c_{pb}/c_{pr} < c''$, a case which is relevant only when $c'' > 0$. Both the relevant social welfare function in the interval $p_y \in [0, \bar{p}_y]$ – given in Equation (A.25) – and that in the interval $p_y \in [\bar{p}_y, \hat{p}_y]$ – given in Equation (20) – are increasing in $p_y$ (see Panel (a) in Figure A.1). The fact that $\hat{p}_y > \bar{p}_y$ follows from $c_{pb}/c_{pr} < c'' < c'$. Note that at $p_y = \bar{p}_y$ the social welfare functions (A.25) and (20) take the same value because for both functions it is $p_x^* = 0$. Accordingly, the welfare-maximizing level of public information precision is given by $p_y^* = \hat{p}_y$, and agents do not acquire any private information, i.e. $p_x^* = 0$.

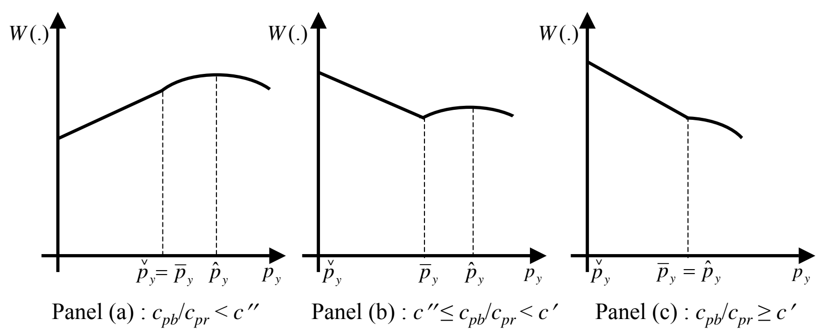


Figure A.1: Alternative behaviors of the welfare function

Step 2. Consider now the case $c'' \leq c_{pb}/c_{pr} < c'$. When $c_{pb}/c_{pr} = c''$, the maximum of the social welfare function (A.25) is lower than that of the welfare function (20). This result follows since the function (A.25) is constant in $p_y$ ($\zeta = \mu = 0$), while (20) has a maximum at $\hat{p}_y > \bar{p}_y$ since $c_{pb}/c_{pr} < c'$ and the values of the two social welfare functions match at $p_y = \bar{p}_y$. Conversely, in the limit case in which $c_{pb}/c_{pr} = c'$, the maximum welfare is attained at $p_y^* = \check{p}_y = 0$. This follows directly from the observations that the social welfare function (A.25) is decreasing in $p_y$ for $p_y \in [0, \bar{p}_y]$ – because $c_{pb}/c_{pr} > c''$ – while (20) is non-increasing in $p_y$ for $p_y \in [\bar{p}_y, \infty)$, and that the values of the two social welfare functions match at $p_y = \bar{p}_y$.
The reasoning above ensures that for $c'' \leq c_{pb}/c_{pr} < c'$ we only need to compare the value of the social welfare function (A.25) computed at $\check{p}_y = 0$, with that of the welfare function (20) computed at $p_y = \hat{p}_y$. These two local maxima are depicted in Panel

(b) of Figure A.1. Substituting for (A.24), we obtain that the maximum of the social welfare (20) in $p_y \in [\bar{p}_y, \infty)$ is

$$W(\theta_{-1}, 0, p_\theta, \hat{p}_y) = \tilde{W} - 2\sqrt{-\kappa_1^2 W' c_{pb}} + c_{pb} p_\theta,$$

which implies that $\frac{\partial W(\theta_{-1},0,p_\theta,\hat{p}_y)}{\partial c_{pb}} < 0$ if Assumption 6.3 holds. Note that, because $c_{pb}/c_{pr} > c''$, the maximum value of the social welfare function (A.25) in $p_y \in [0, \bar{p}_y]$ is given by $\check{p}_y = 0$, so that

$$\begin{aligned} W(\theta_{-1}, p_x^*, 0, p_\theta) &= \tilde{W} + \kappa_1^2 W' \frac{\sqrt{2c_{pr}}}{(1-\alpha)\sqrt{-U_{kk}\kappa_1^2}} + \\ &\quad - \left(\sqrt{-\frac{U_{kk}\kappa_1^2}{2c_{pr}}} - \frac{p_\theta}{1-\alpha}\right)\left(\frac{(\alpha^* - \alpha) W_{\sigma\sigma}}{(1-\alpha) U_{kk}} + 1\right) c_{pr}. \end{aligned}$$

Observe that $\frac{\partial W(\theta_{-1},p_x^*,0,p_\theta)}{\partial c_{pb}} = 0$. Therefore, there exists a threshold $\tilde{c}(c_{pb})$, $c'' \leq \tilde{c}(c_{pb}) < c'$, such that the optimal choice of precision by the policy maker switches from $p_y^* = \hat{p}_y$ to $p_y^* = \check{p}_y = 0$ at $\tilde{c}(c_{pb})$. Note also that, for $c'' < 0$, we have $\tilde{c}(c_{pb}) \geq 0$ if $\kappa_1^* \geq \kappa_1$; a condition that is stronger than the one required by Assumption 6.2. In fact, a necessary condition for $c''$ to be negative is $\alpha^* < \alpha$, which discourages the provision of public information.

Step 3. Finally, suppose that $c_{pb}/c_{pr} \geq c'$. In this case, the maximum of the social welfare function (A.25) lies above that of the social welfare function (20). To prove it, observe that the social welfare (A.25) is decreasing in $p_y$ for $p_y \in [0, \bar{p}_y]$ – since $c_{pb}/c_{pr} > c''$ – and (20) is non-increasing in $p_y$ for $p_y > \bar{p}_y$ – as $c_{pb}/c_{pr} \geq c'$. Because the two welfare functions take the same value at $p_y = \bar{p}_y$, it must be that the welfare function (A.25) takes values larger than the maximum of (20), as shown in Panel (c) of Figure A.1. Thus, the policy maker does not provide a public signal of positive precision (i.e., $p_y^* = \check{p}_y = 0$), and the precision of the private information acquired by agents is $p_x^* = \sqrt{-\frac{U_{kk}\kappa_1^2}{2c_{pr}}} - \frac{p_\theta}{1-\alpha}$.

Finally, we study Case 2 (i.e., $c' \leq c''$), relying on the three steps above. We consider first the case $c_{pb}/c_{pr} < c' \leq c''$, which is essentially equivalent to that studied in Step 1 above, so that $p_x^* = 0$ and $p_y^* = \hat{p}_y$. Second, consider $c' \leq c_{pb}/c_{pr} < c''$. In this case, the social welfare function (A.25) is increasing in $p_y$ for $p_y < \bar{p}_y$ since $c_{pb}/c_{pr} < c''$, while the social welfare function (20) is non-increasing in $p_y$ for $p_y > \bar{p}_y$, as $c_{pb}/c_{pr} \geq c'$. Hence, the precision of the public signal is given by the constrained optimum $p_y^* = \bar{p}_y$, and $p_x^* = 0$. Finally, we assume that $c' \leq c'' \leq c_{pb}/c_{pr}$, which coincides with Step 3 in the proof of Case 1. The social welfare function (A.25) is non-increasing in $p_y$ as $c_{pb}/c_{pr} \geq c''$, and the social welfare function (20) is non-increasing in $p_y$ since $c_{pb}/c_{pr} \geq c'$. As the two functions take the same value at $p_y = \bar{p}_y$, it must be that $p_y^* = \check{p}_y = 0$ and $p_x^* = \sqrt{-\frac{U_{kk}\kappa_1^2}{2c_{pr}}} - \frac{p_\theta}{1-\alpha}$. ■

## Appendix B. A model of price setting complementarities

We study an economy inhabited by a continuum of agents $i \in [0,1]$, each of whom consumes a basket $C_i$ and is the unique producer of a final good $Y_i$. Being a monopolist in the production of $Y_i$, agent $i$ chooses its price $p_i$. Abstracting from the cost of information acquisition, the consumer-producer utility function can be written as

$$U(\theta, C, Y_i) = J(C_i) - \theta V(Y_i), \tag{B.1}$$

where $J(C_i)$ denotes the utility of consumption and $V(Y_i)$ the disutility of labor.[34] Note that $J_C(C_i) > 0$, $J_{CC}(C_i) < 0$, $\lim_{C_i \to \infty} J(C_i) = 0$, $V_Y(Y_i) > 0$ and $V_{YY}(Y_i) > 0$. The expected value of the labor supply shock, $\theta_{-1}$, is denoted for notational convenience with $\bar{\theta} > 0$. Our setup is close to those of Hellwig (2005) and Adam (2007), but for the fact that we assume that each consumer $i$ is the sole producer of good $i$. This assumption aligns individual utilities and social welfare, defined as the integral sum of individual utilities as in Angeletos and Pavan (2007).[35]

Prices are set at the beginning of each period, after agents have received both the public and the private signal, but before production and consumption choices are made. Agent $i$'s consumption basket is composed of a continuum of goods indexed by $h \in [0,1]$

$$C_i = \left( \int_{[0,1]} c_{hi}^{\frac{v-1}{v}} dh \right)^{\frac{v}{v-1}}, \tag{B.2}$$

obtained by means of the standard Dixit-Stiglitz aggregator, where $c_{hi}$ denotes the consumption of good $h$ by consumer $i$, and $v > 1$ denotes the elasticity of substitution between goods. This specification implies the price index

$$P = \left( \int_{[0,1]} p_i^{1-v} di \right)^{\frac{1}{1-v}}. \tag{B.3}$$

Given that consumer $i$ observes relative prices in the economy, her demand for any good $h$ is

$$c_{hi} = \left( \frac{P}{p_h} \right)^{v} C_i. \tag{B.4}$$

She chooses her consumption basket under the budget constraint

$$C_i \leq (1+\tau) \frac{p_i}{P} Y_i - T, \tag{B.5}$$

in which $\tau$ denotes an output subsidy aimed at neutralizing the monopolistic distortion, and $T$ is a lump-sum tax balancing the subsidy.

[34] More precisely, we should write $V(L_i)$ rather than $V(Y_i)$. The formulation in (B.1) is implied by the assumption of a linear production function (i.e., $Y_i = L_i$), which is standard in this literature.

[35] Roca (2006), following Woodford (2003), assumes that each agent supplies a specific type of labor that is used in every sector of the economy. This hypothesis could be accommodated for in our framework as well.

We define by $Y$ the economy-wide production level obtained as the integral sum of the individual consumption levels of the aggregate good; i.e.,

$$Y = \left( \int_{[0,1]} C_i di \right). \tag{B.6}$$

By aggregating the demand functions of the generic good $h$ (Equation B.4) over all agents, and exploiting Equation (B.6), we obtain the aggregate demand

$$Y_h = \left( \frac{P}{p_h} \right)^v Y. \tag{B.7}$$

By substituting Equation (B.7) into the budget constraint (B.5) taken with equality, and the latter into the demand for good $h$ (Equation B.4), we get

$$c_{hi} = \left( \frac{P}{p_h} \right)^v \left( (1+\tau) \left( \frac{P}{p_i} \right)^{v-1} Y - T \right), \tag{B.8}$$

which allows us to get the following first order approximation of (B.2),

$$C_i - \bar{Y} = v\left(P - p\right) \frac{\bar{Y}}{\bar{P}} + v\left(P - p_i\right) \frac{\bar{Y}}{\bar{P}} + Y - \bar{Y},$$

where $\bar{Y}$ denotes the non stochastic aggregate production level in equilibrium, and the production subsidy has been used to offset the monopolistic distortion (i.e., $\tau = \frac{1}{v-1}$), and $p = \int_{[0,1]} p_i di$ denotes the average price. Notice that – by using the price aggregator in (B.3) – we have that $P = p - v\sigma_p^2 \bar{P}/2$, where $\sigma_p^2 = \int_{[0,1]} \left(\frac{p_i - p}{\bar{P}}\right)^2 di$ is the variance of individual prices, normalized using the non-stochastic aggregate price level in equilibrium (i.e., $\bar{P}$). Accordingly, we can write

$$C_i - \bar{Y} = v\left(p - p_i - \frac{v}{2}\sigma_p^2 \bar{P}\right) \frac{\bar{Y}}{\bar{P}} + Y - \bar{Y} - \frac{v^2}{2}\sigma_p^2 \bar{Y}. \tag{B.9}$$

We summarize the influence of the monetary policy on output by the quantity equation $PY = \bar{M}$ that implies $Y - \bar{Y} = -\left(P - \bar{P}\right) \bar{Y}/\bar{P}$, which can be rewritten as $Y - \bar{Y} = -\left(p - v\sigma_p^2 \bar{P}/2 - \bar{P}\right) \bar{Y}/\bar{P}$. Therefore, Equation (B.9) becomes

$$C_i - \bar{Y} = v\left(p - p_i - \frac{v}{2}\sigma_p^2 \bar{P}\right) \frac{\bar{Y}}{\bar{P}} - \left(p - \frac{v}{2}\sigma_p^2 \bar{P} - \bar{P}\right) \frac{\bar{Y}}{\bar{P}} - \frac{v^2}{2}\sigma_p^2 \bar{Y}. \tag{B.10}$$

Using similar arguments, it is easy to derive from Equation (B.7) the first order approximation

$$Y_i - \bar{Y} = v\left(p - p_i\right) \frac{\bar{Y}}{\bar{P}} - \left(p - \frac{v}{2}\sigma_p^2 \bar{P} - \bar{P}\right) \frac{\bar{Y}}{\bar{P}} - \frac{v^2}{2}\sigma_p^2 \bar{Y}. \tag{B.11}$$

Following the literature (see, e.g., Woodford, 2003), we focus on a second order approximation of the utility function (B.1) around the non stochastic equilibrium $\{\bar{Y}, \bar{P}, \bar{\theta}\}$; i.e.,

$$J\left(C_i\right)-\theta V\left(Y_i\right)\cong J\left(\bar{Y}\right)-\bar{\theta}V\left(\bar{Y}\right)+J_C\left(\bar{Y}\right)\left(C_i-\bar{Y}\right)+$$
$$-\bar{\theta}V_Y\left(\bar{Y}\right)\left(Y_i-\bar{Y}\right)-V\left(\bar{Y}\right)\left(\theta-\bar{\theta}\right)+$$
$$+\frac{J_{CC}\left(\bar{Y}\right)\left(C_i-\bar{Y}\right)^2-\bar{\theta}V_{YY}\left(\bar{Y}\right)\left(Y_i-\bar{Y}\right)^2}{2}-V_Y\left(\bar{Y}\right)\left(\theta-\bar{\theta}\right)\left(Y_i-\bar{Y}\right).$$

By exploiting the fact that $J_C\left(\bar{Y}\right)=\bar{\theta}V_Y\left(\bar{Y}\right)$ due to the effect of the production subsidy $\tau=\frac{1}{v-1}$, and using Equations (B.10) and (B.11), we can write

$$J\left(C_i\right)-\theta V\left(Y_i\right)\cong J\left(\bar{Y}\right)-\bar{\theta}V\left(\bar{Y}\right)-J_C\left(\bar{Y}\right)\frac{v^2}{2}\sigma_p^2\bar{Y}-V\left(\bar{Y}\right)\left(\theta-\bar{\theta}\right)+$$
$$+\frac{J_{CC}\left(\bar{Y}\right)-\bar{\theta}V_{YY}\left(\bar{Y}\right)}{2}\left(\left(v-1\right)\frac{p}{\bar{P}}-v\frac{p_i}{\bar{P}}+1\right)^2\bar{Y}^2+$$
$$-J_C\left(\bar{Y}\right)\frac{\theta-\bar{\theta}}{\bar{\theta}}\left(\left(v-1\right)\frac{p}{\bar{P}}-v\frac{p_i}{\bar{P}}+1\right)\bar{Y},$$

where all the terms of order higher than one have been omitted.
By collecting in $\bar{U}\left(\theta,\frac{p}{\bar{P}},\frac{p_i}{\bar{P}}\right)$ all terms that are constant or linear in $\theta$, $p$, and $p_i$, we obtain

$$J\left(C_i\right)-\theta V\left(Y_i\right)\cong\bar{U}\left(\theta,\frac{p}{\bar{P}},\frac{p_j}{\bar{P}}\right)-J_C\left(\bar{Y}\right)\frac{v^2}{2}\sigma_p^2\bar{Y}+$$
$$+\frac{J_{CC}\left(\bar{Y}\right)-\bar{\theta}V_{YY}\left(\bar{Y}\right)}{2}\left(\left(v-1\right)\frac{p}{\bar{P}}-v\frac{p_i}{\bar{P}}\right)^2\bar{Y}^2-J_C\left(\bar{Y}\right)\frac{\theta}{\bar{\theta}}\left(\left(v-1\right)\frac{p}{\bar{P}}-v\frac{p_i}{\bar{P}}\right)\bar{Y},$$

which can be rewritten as

$$J\left(C_i\right)-\theta V\left(Y_i\right)\cong\bar{U}\left(\theta,\frac{p}{\bar{P}},\frac{p_i}{\bar{P}}\right)+\frac{J_C\left(\bar{Y}\right)\bar{Y}}{2}\left(-v^2\sigma_p^2-\omega\left(\left(v-1\right)\frac{p}{\bar{P}}-v\frac{p_i}{\bar{P}}\right)^2+\right.$$
$$\left.-2\frac{\theta}{\bar{\theta}}\left(\left(v-1\right)\frac{p}{\bar{P}}-v\frac{p_i}{\bar{P}}\right)\right),\tag{B.12}$$

with $\omega\equiv-\frac{\left(J_{CC}\left(\bar{Y}\right)-\bar{\theta}V_{YY}\left(\bar{Y}\right)\right)\bar{Y}}{J_C\left(\bar{Y}\right)}>0$. Note that, as it is standard in this literature (see, e.g., Hellwig, 2005; Roca, 2006), the variance of individual prices has a negative impact on utility due to the Dixit-Stiglitz's setup.

By rewriting the utility (B.12) using the notation adopted in the main text (i.e., $p/\bar{P}=K$, $p_i/\bar{P}=k_i$, and $\sigma_p^2=\sigma_k^2$) and by taking into account information acquisition costs, we obtain Equation (25).

## Appendix C. Cost convexities

The result of Proposition 6.1 – that public and private signals of positive precision do not coexist – depends crucially on the linearity of precision costs. Although this assumption captures theoretically interesting cases, and makes the model analytically tractable, it is undeniably quite restrictive. It is easy to see that, under more general convex cost functions, public and private signals of positive precisions typically coexist. In fact, as in this case the marginal costs of information precisions are increasing, it is never optimal for the policy maker to provide a level of precision so large that private agents acquire no information at all. For the same reason, it is always optimal for the policy maker to provide public information of strictly positive precision, as not providing any public signal would impose a too high marginal cost on private agents.

The main drawback of assuming strictly convex cost functions is that it becomes impossible to investigate the implications of information provision/acquisition by means of analytical tools only. In this Appendix we reconsider – by resorting to numerical simulations – beauty contests, price setting complementarities and negative externalities, showing that the results obtained under convex cost specifications are qualitatively analogous to the analytic results derived in the main text for the case of linear costs.

In particular, in all applications below we consider the isoelastic cost functions

$$C(p_y) = c_{pb}\frac{p_y^{\eta}}{\eta}; \quad C(p_x) = c_{pr}\frac{p_x^{\eta}}{\eta}, \tag{C.1}$$

with $\eta > 1$.

## 'Beauty Contests'

We adopt the same setup of Section 7.1 but for the fact that we substitute the linear costs in Equation (24) with the convex cost functions (C.1). We investigate by means of numerical simulations the effects of different degrees of coordination ($\alpha$) on the ratio between the optimal precisions of public and private information (i.e. $p_y/p_x$). We also explore the implications for this ratio of different cost functions convexities ($\eta$) and of different relative costs of information precision (as stemming from different ratios $c_{pb}/c_{pr}$). Figure C.1 illustrates the results of our analysis.[36]

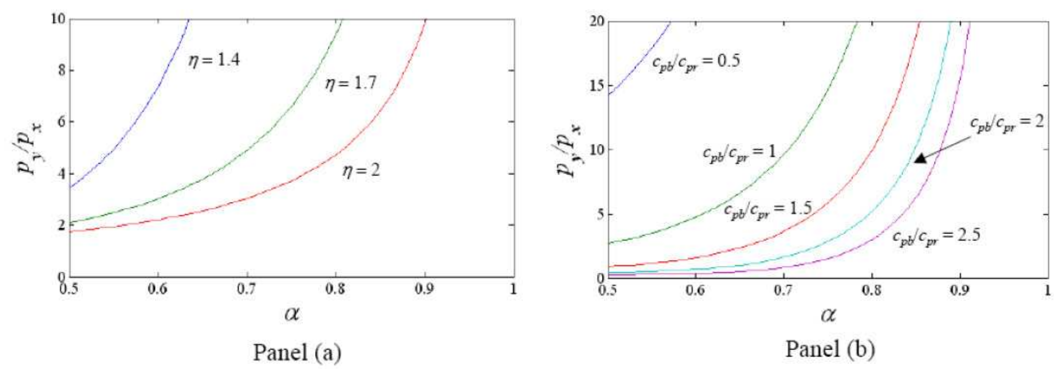


Figure C.1: Morris and Shin's beauty contest

In all simulations the precision of the fundamental, $p_\theta$, is exogenously given and set to a level of 20 so that the variance of the fundamental is 5%.[37] Panel (a) of Figure C.1

[36] As it is impossible to fully characterize the first and second order conditions of the optimization problems under investigation, our routines compute the welfare levels for a wide range of parameters, subsequently choosing the largest one.

[37] This assumption bears no implications on the qualitative features of the simulations results.

shows the optimal ratio of public vs. private information precisions as a function of $\alpha$, under the assumption that $c_{pb} = c_{pr}$ and letting $\eta \in \{1.4, 1.7, 2\}$. It is immediate to see that the ratio $p_y/p_x$ increases with the privately perceived degree of coordination, and hence also with the difference between $\alpha$ and the optimal degree of coordination $\alpha^*$ (that is equal to zero). This happens because each agent's evaluation of her private information is decreasing in $\alpha$. In the limiting case in which $\alpha = 1$, private information is useless as it plays no role in the estimation of the fundamental.

To understand why the two types of information typically coexist, recall first that with linear costs private and public information precisions are linearly substitutable (as shown in Equation (10)). This substitutability relationship is preserved for the cost functions (C.1), becoming however a convex one. The shape of the cost function implies in fact that the private cost savings induced by an increase in the precision of the public signal are smaller the larger is the precision of public information. Hence, from an individual standpoint, the benefit of acquiring a less precise private information decreases with the precision of public information. Focusing now on the behavior of the policy maker, it is easy to see that she would never provide a public signal sufficiently precise to entirely offset the acquisition of private information, as this would turn out to be too costly. At the same time, however, she always provides a public signal of some positive precision, since for low levels of $p_y$ (large $p_x$), a reduction of $p_x$ would entail substantial cost savings for the agents due to cost convexity.

Panel (a) of Figure C.1 also shows that the ratio $p_y/p_x$ decreases in $\eta$. To see why this is the case, recall that in a beauty contest private information is less 'valuable' than public information, which together with $c_{pb} = c_{pr}$ explains why in all our simulations we obtain $p_y > p_x$. Since $p_y > p_x$, an increase in the convexity of the cost functions reduces the optimal amount of public information provision because the marginal cost of $p_y$ becomes larger than that of $p_x$.

Panel (b) of Figure C.1 shows the effects of different values of the $c_{pb}/c_{pr}$ ratio (i.e., $c_{pb}/c_{pr} \in \{0.5, 1, 1.5, 2, 2.5\}$) for given $\eta$ ($\eta = 1.5$). From a qualitative point of view, the results of the exercise are unsurprising: the lower the relative cost, the higher the ratio $p_y/p_x$. However, in a quantitative perspective, the results we obtain are quite strong. Even when the precision of public information is much costlier than that of private information (e.g., $c_{pb}/c_{pr} = 2.5$), the level of public information provision remains larger than that of private information also for relatively small values of $\alpha$ (i.e., $\alpha \geq 0.71$).[38] In this perspective, the results of the numerical analysis support the strong pro-transparency message already drawn for the linear costs case.

## Price setting complementarity and transparency

We study the same model of Section 7.2, but for the substitution of the linear costs used in Equation (25) with the isoelastic cost functions (C.1). Our numerical exercises are based on the same assumptions about the cost ratio $c_{pb}/c_{pr}$ and $\eta$ that we made in the beauty contest case discussed above. As for the parametrization of the utility function, consistently with the assumption made when illustrating the model in Section

[38]Obviously, a reduction of the cost ratio below 1 induces a remarkable increase in the acquisition of public information precision. Note also that when $c_{pb}/c_{pr} = 2.5$ and $\alpha = r = 0.5$, the ratio $p_y/p_x$ reaches its lowest level of 0.34.

7.2 for the case of linear costs, we set $\omega = 4$ and we normalize $\bar{\theta} = 1$.

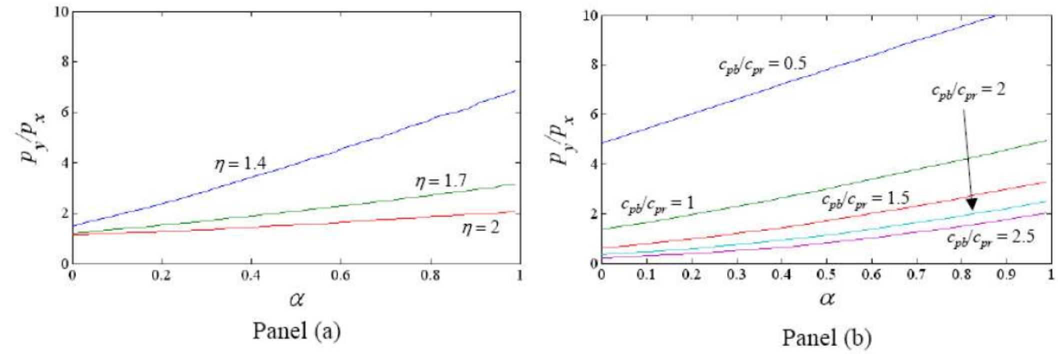


Figure C.2: Information precision and price coordination

Panel (a) of Figure C.2 plots the optimal ratio $p_y/p_x$ for $\alpha \in (0, 1)$ – i.e., $v \in (1, \infty)$ – showing that it is increasing in the degree of price coordination. As in the beauty contest case, this finding follows from the fact that private information becomes less valuable in the estimation of the fundamental the larger is $\alpha$.[39] Moreover, analogously to what already discussed in the previous sub-section, it is immediate from Panel (a) that the 'convexity effect' in the privately perceived rate of substitution between the accuracies of private and public information reduces the optimal precision of public information. Finally, Panel (b), drawn for $\eta = 1.5$, illustrates the same effect of the relative cost on the precisions of public and private information already highlighted for the beauty contest case, confirming once more the social value of a more precise public information in the presence of strategic complementarities.

## Negative Production Externalities

Figure C.3 illustrates the results of the numerical analysis on the model of Section 7.3 with the isoelastic cost functions (C.1). The simulations are based on the same assumptions about the cost ratio $c_{pb}/c_{pr}$ and $\eta$ that have been made in the previous two applications. Furthermore, consistently with the hypothesis in Section 7.3 for the case of linear costs, we have set $\omega = 4$ and $\chi = 0$, and normalized $\bar{\theta}$ to one.

Panel (a) of Figure C.3 plots the optimal ratio $p_y/p_x$ for $\alpha \in [-1, 0]$, showing that $p_y/p_x$ is (significantly) smaller than 1 for all values of $\alpha < 0$. This result is strikingly different with respect to that obtained in the two applications above dealing with strategic complementarities. The presence of a negative externality induces strategic substitutability between agents' actions, which implies that private information carries a larger weight on agents' choices than public information. As it is evident from panel (a) in the Figure, this effect holds for all values of $\alpha$ smaller than zero, and it is

[39]Similarly to the case of investment complementarities studied by Angeletos and Pavan (2004), this effect adds to the one implied by the wedge between the privately perceived and the optimal motives for coordination.

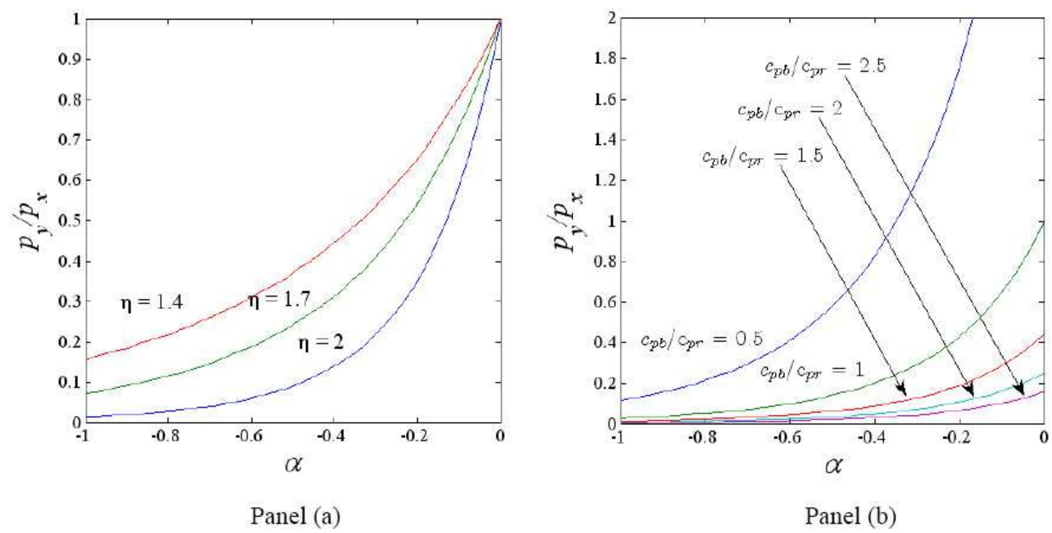


Figure C.3: Information precision and negative externalities

quantitatively relevant also for small values of the cost elasticity $(\eta - 1)$. Moreover, $p_y/p_x$ is increasing in the degree of substitutability of agents' actions. This follows as usual from the fact that private information becomes less valuable in the estimation of the fundamental the larger is $\alpha$. As it is to be expected, when $\alpha = 0$, we have that $p_y = p_x$ for any $\eta$ since $c_{pb} = c_{pr}$. In this case, in fact, the externality has no impact on utility and welfare.

Panel (b) of Figure C.3 illustrates once more the well understood effect of the relative cost on the precisions of public and private information. Note that the results illustrated in Figure C.3 confirm, for the case of cost convexities, the policy implications discussed in Section 7.3 under the assumption of linear cost functions; namely that in the presence of strategic substitutability a more precise public information may often be welfare reducing.

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