

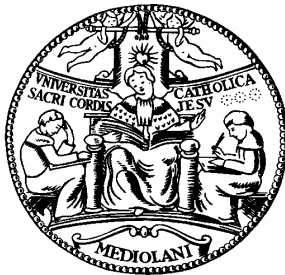
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a search theoretic perspective

Giuseppe Mastromatteo Luigi Ventura

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a search theoretic perspective

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## **Abstract**

Since the pioneering contributions by Stanley Jevons and Karl Menger, transaction costs have been classified among the most relevant forces capable of explaining the emergence of money to carry out transaction processes, partially or fully replacing barter. It was not, however, until the works by Jones and Oh that those ideas received a formal treatment, which subsequently led to a full-fledged theory of monetary equilibria, based on search theoretic paradigms. This survey aims at introducing readers to the main traits of search theoretic, monetary paradigms, as were proposed in the seminal papers by K. Iwai and Kiyotaki and Wright, and later on developed by the same and other authors. The perspective of the analysis will be critical and analytic, and will highlight the delicate trade-off between fundamentals and self-fulfilling beliefs which, in our view, drives the most interesting results in this branch of the literature.

Though this work should be considered as the outcome of a joint effort by the authors, sections 2,3,4,6,8 and 10 should be attributed to G. Mastromatteo, and sections 5,7 and 9 to L. Ventura

## 1. Introduction

Criticism to the walrasian approach has originated, in the last few decades, three different avenues of research, aiming at a more satisfactory explanation of the origin and the use of money: these alternative approaches are based upon the concepts of "transaction costs", "information asymmetries" and "legal restrictions".

If, at a first glance, these theories appear to be extremely diverse, they share the attempt to accurately describe the *transaction technology* available to agents engaging in trade. This way, the focus goes back to a more realistic kind of economy, with completely decentralised, and costly, exchanges. In such a perspective, of course, it has been necessary to acknowledge the existence of some kind of imperfection that inhibits the perfect functioning of existing markets.

The starting point of all the models which have explored these new ideas is given by the analysis of the transaction technology available in the economy; in fact, in a world operated by rational agents, a medium of exchange will only be adopted if it allows to obtain an efficiency gain with respect to pure barter.

However, within this approach it is possible to find remarkable differences as to the representations of such a technology. In particular, we may distinguish between a group of contributions concentrating on transaction costs involved in direct and indirect trade, and a second group emphasising the information requirements, and costs, of a decentralised trading system.

Both set of contributions, though, share the fundamental idea that money is essential in bringing about decentralisation.

Ostroy (1973) very clearly summarises the fundamentals of this approach: "To the standard theory of value, the phenomenon of monetary exchange is surprising and distressing; surprising because the phenomenon is inexplicable and distressing because the phenomenon would seem to be one of the most elemental conclusions to be derived from any theory of exchange. Once we give up the standard theory framework which allows the execution of exchange to be the province of a centralised agency and concentrate on the logistics of more disaggregated trading arrangements, monetary exchange becomes explicable as a matter of

course. It follows that these logistical considerations are worthy of attention by general equilibrium theorists”.

In the following paragraphs we will quite carefully analyse some very path-breaking contributions to the literature of the last few decades, which originated from the recognition of the difficulties arising in a world of bilateral trades, described by Jevons as "lack of double coincidence of wishes", and which focuses on the role of money in substantially reducing transaction costs.

In doing so, we will devote particular attention to search theoretic models of monetary equilibria, as we believe that such models have made a good job in capturing some essential aspects of the origin and functioning of money, which could not be accounted for by different and otherwise important earlier theorisation, such as the overlapping generation model, the cash in advance model and models with money in the utility function.

Though search theoretic paradigms will mainly constitute the focus of our analysis, we will try to carefully establish links to related approaches explored in earlier contributions. This is why, after a brief historical retrospective in section 2, and before discussing the details of Iwai's and Kiyotaki and Wright's models, in sections 5 and 6 respectively, we will introduce and sketch the main lines of Jones' and Oh's contributions, in sections 3 and 4. In discussing Iwai's and Kiyotaki's and Wright's models, an effort will be made to highlight the delicate balance between fundamentals and self-fulfilling beliefs. It is precisely this trade-off which drives many of the reported results, and therefore the illustration of its functioning constitutes one of the main motivations of our work.

Section 7 will deal with an interesting extension of Kiyotaki and Wright's economy, in which the structure of demand is not fixed. Section 8 introduces fiat money, while section 9 discusses the issue of relative price determination in a bargaining context. Section 10 covers, in a critical way, some normative aspects of search theoretic monetary equilibria. Section 11 concludes, with some final critical comments, and an eye to open questions.

All in all, as it should become clearer in the sequel, this work is meant as an attempt to a comprehensive, clear and somehow critical introduction to a large body of literature, without pretending, at the same time, to exhaust all historical references to the problems under

scrutiny. Moreover, it should be stressed that our main goal will be that of providing some tools which might prove useful in explaining the appearance and the role of money in an abstract economy, rather than explaining its actual emergence in a given historical context.

## **2. A brief historical retrospective**

Though the line of research that we are going to present is relatively new, in his full-fledged analytical developments, the underlying ideas are not (as is often the case, "nothing new under the sun"). Without pretending to be exhaustive in our brief historical retrospective, we may say that those ideas may be taken back to some influential work by the early and much distinguished theoreticians Karl Menger (1892) and Stanley Jevons (1875). To understand the importance of their respective theoretical research and the extent to which their research underpins the contemporary contributions that we are going to survey, it is useful to quote two very well known and influential sentences, which lie at the heart of modern treatment of monetary equilibria with transaction costs.

*"Even in the relatively simple and so often recurring case, where an economic unit, A, requires a commodity possessed by B, and B requires one possessed by C, while C wants one that is owned by A - even here, under a rule of mere barter, the exchange of the goods in question would as a rule be of necessity left undone."* (Menger, 1892; p. 242).

In turn, this makes clear reference to Stanley Jevons' idea that:

*"The earliest form of exchange must have consisted in giving what was not wanted directly for that which was wanted. This simple traffic we call barter [...] and distinguish it from sale and purchase in which one of the articles exchanged is intended to be held only for a short time until it is parted with in a second act of exchange. The object which thus temporarily intervenes in sale and purchase is money. [...] The first difficulty in barter is to find two persons whose disposable possessions mutually suit each others wants. There may be many people wanting, and many possessing those things wanted; but to allow of an act of barter, there must be a double coincidence\_which will rarely happen."* (Jevons (1876 [1910], pp. 3-4)).

In Menger's theory, money is generated by a selection process operated by market forces, whereby that commodity (or those commodities) which can be most easily traded is (are) selected to be used as an intermediary in exchanges. This is what Karl Menger defines as degree of "saleableness" of goods, an idea which has represented, as we will try to make clear in the following sections, an essential building block of modern theories on the origin and working of money.

What is also noteworthy is that, according to Menger, the degree of "saleableness" varied across commodities and in relation to changes in demand and supply conditions (as should be the case, in Menger's theory, with perishable commodities).

Money becomes, therefore, a commodity like all the others, and gets its value from the same sources as all the other goods, i.e. from their intrinsic properties; in the case of coins, for instance, from the intrinsic value of the constituent metals. *"No commodity - Menger argued - is by itself money, neither for its material, nor for its technical features, nor even for its external shape or the desire of its owner. Any good, however, that was hitherto used in consumption or production [...] - can always play the role of money, as soon as it is assigned, at a certain stage of the historical development of a given community, the role of an intermediary in trades. [...] On the same grounds any object of trade does not generically become money in abstract, but within precise limits of space and time, which are directly linked to the above mentioned role."*<sup>1</sup>.

This was in stark contrast with the standard view at that time, whereby money was taken to be a mere token of value, whose trading capability could only be attributed to law or social conventions. It seemed hard to Menger's contemporaries, in fact, to justify on theoretical grounds that money could be universally desired and accepted mainly for its value in facilitating trade, rather than for immediate utility due to its specific physical features.

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<sup>1</sup>Cfr. Menger (1976), pp. 433-434, our translation.



Since the contribution by Karl Menger (1892), money is thus no longer an "anomaly in economics": it can be singled out among other goods for its function as an intermediary in trades, which it gets first of all from being a commodity with a specific intrinsic value.<sup>2</sup>

An economy, according to Menger's theory, comes out of direct barter without any legal prescription. As trades get more and more frequent, and more and more intense, the most "saleable" commodity is selected to work as official intermediary in trades (over and above its utility-generating use as a normal commodity).

The Mengerian approach to the origin of money rests therefore on a process of progressive reduction of transaction costs, in a context of decentralised decisions of economic agents, who are only driven by their respective individual interest. It is exactly along this line of reasoning that some of the contributions appeared in the most recent literature, and which will be analyzed in this work, have sought to formally model this process, generating a stream of research that mainly centers on transaction costs.

### **3. Jones's model: on the origin and development of media of exchange**

Earlier models by Ostroy and Starr (1974) and Feldman (1973), though recognising in the possibility of reaching an efficient allocation in a decentralised economy the cause of the appearance of money, do not advance any hypothesis concerning the way in which a monetary economy starts and grows. Jürg Niehans (1969), on the other hand, focuses on transaction costs to show that money allows to minimise them, in the context of the reallocation of goods by final consumers. Money, therefore, enables an exchange economy to achieve an equilibrium, with the most appropriate structure of transaction costs. Also in this case, however, we cannot find any explanation concerning the origin and evolution of the trading instrument.

This is the main concern of the model put forward by Robert Jones (1976), who takes off from the mengerian hypothesis that individuals act only on the basis of their individual interest, and without any need for centralised decisions.

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<sup>2</sup> “*The theory of money necessarily presupposes a theory of the saleableness of goods*”. Menger (1892), p. 243.

The hypothesis underlying the model are quite similar to those in Ostroy and Starr: trade is only bilateral, and subject to a quid pro quo condition. Available information is limited, as individuals do not possess any knowledge about endowments, excess demands and trading plans of other agents prior to the moment they meet. Meetings occur randomly. Every agent selects his final consumption allocation in order to maximise his utility and to minimise transaction costs, at given prices. Trading strategies are fixed before entering the market and cannot be modified while implementing transactions.

In such a context, the appearance of a trading tool serving as money implies moving from a situation of direct trade to one of indirect trade, which is recognised by agents as a less costly system to run. This can only happen if the cost incurred to trade commodity  $i$  for commodity  $j$  is larger than the cost incurred to trade  $i$  for  $k$  and subsequently  $k$  for  $j$ . Let us denote by  $c_{ij}$  the cost incurred to directly exchange commodity  $i$  with commodity  $j$ .

If such a cost is additive, then it can be decomposed into the cost incurred to sell good  $i$  ( $c_i^v$ ) and the cost born to buy good  $j$  ( $c_j^c$ ); therefore  $c_{ij} = c_i^v + c_j^c$ .

If agents resort to indirect trade, using as an intermediary commodity  $k$ , the cost of trade will be given by:

$$c_{ik} + c_{kj} = (c_i^v + c_k^c) + (c_k^v + c_j^c) > c_{ij}.$$

Clearly enough, in this case direct trade is to be preferred, as least costly, to indirect exchange.

The assumption of additivity of trading costs, which is key to this result, makes reference to those costs - such as transportation, storage and inspection - which are directly related to the physical features of traded commodities. This kind of costs, therefore, does not explain the origin and the working of a means of exchange.

Jones, however, neglects this set of costs by making the assumption, which was is common to Ostroy and Starr, that commodities are virtually indistinguishable from a physical viewpoint. Therefore, he only considers non-additive costs, i.e. costs that are related to the search of a trading counterpart with complementary trading plans, and which can insure that the condition of double coincidence of wants may be fulfilled.

Search costs might be expressed in terms of the subjective estimate of the probability  $p_i$  that an agent be willing to buy or sell commodity  $i$ . By assuming away any correlation between the commodities sold and purchased by our agent,  $p_i p_j$  represents the probability of an exchange of  $i$  for  $j$ , and

$$\frac{1}{p_i p_j},$$

the expected number of meetings which are necessary to succeed in trading those two goods via direct barter.

If, on the other hand, our agent resorts to indirect barter, trading  $i$  for  $k$  and then  $k$  for  $j$ , the expected number of necessary meetings is:

$$\frac{1}{p_i p_k} + \frac{1}{p_k p_j},$$

which is minimised when the commodity with the highest  $p_k$  is selected as an intermediary.

Therefore, if  $\max \{p_k\} = p_n$ , then:

$$\min_k \left( \frac{1}{p_i p_k} + \frac{1}{p_k p_j} \right) = \frac{1}{p_i p_n} + \frac{1}{p_n p_j}.$$

Using commodity  $n$  as an intermediary in trade not only constitutes the best possible strategy among all those that might be played in the context of indirect barter, but also the best strategy in absolute terms<sup>22</sup>, i.e. also with respect to direct barter, if:

$$\frac{1}{p_i p_n} + \frac{1}{p_n p_j} < \frac{1}{p_i p_j},$$

i.e.

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<sup>3</sup>A strategy of multi-step barter is always worse than direct barter. To see this, it is sufficient to look at a barter strategy in three steps, which requires the use of two means of exchange,  $k$  e  $h$ . In this case we have:

$$\min_{k,h} \left( \frac{1}{p_i p_k} + \frac{1}{p_k p_h} + \frac{1}{p_h p_j} \right) = \frac{1}{p_i p_n} + \frac{1}{p_n p_n} + \frac{1}{p_n p_j}.$$

As is quite clear, the second trade is redundant.

$$p_n > p_i + p_j. \quad (1)$$

If condition (1) holds, it is in the agents' interest to implement indirect forms of trade which make use of commodity  $n$  as a means of exchange, and this will be even more the case, the more similar the expectations about  $p_n$  across agents<sup>3</sup>.

This does not necessarily mean, however, that the economy will be fully monetized, at least as long as condition (1) is not fulfilled for all pairs  $i, j$  of commodities. If that is not the case, we might get coexistence of direct and indirect trade in the same economy.

However, if agents resort to indirect trade even only for some of the exchanges, the utilisation of an intermediary commodity ( $n$ ) will turn out to be more common than it might be expected by only looking at individuals' final consumptions and condition (1). This is so because a sort of externality comes into the picture, whereby the utilisation of commodity ( $n$ ) as a means of exchange raises its supply and demand over and above what would be requested by direct consumption, which in turn induces an increase in  $p_n$  and a corresponding decrease in  $p_i$ , for  $i \neq n$ . This implies, in turn, that (1) holds for a larger number of goods compared to the initial situation, and that indirect barter increases more than one might initially expect, in a self-reinforcing process<sup>4</sup>.

Can we now get an idea as to how close to full monetization the economy will get, as an outcome of such a process? Jones' model suggests that this depends on the specific configurations of consumption functions and individual trading schemes, that might lead the economy towards a stable equilibrium with a) zero monetization (i.e. only direct barter), b) partial monetization (coexistence of direct and indirect barter) or c) full monetization (all trades are carried out using a means of exchange).

Jones' argument can be represented graphically as follows.

Let us denote the matrix  $\mathbf{U} = [u_{ij}]$ , representing the shares of individuals entering the market to trade good  $i$  for good  $j$ . Let us also assume that  $\mathbf{U}$  is time invariant and symmetric, i.e.:

$$u_i = \sum_{j=1}^n u_{ij} = \sum_{j=1}^n u_{ji}. \quad (2)$$

---

<sup>4</sup> Such a process very closely mimics the one envisaged by Karl Menger. See above, section 1.

Expression (2) insures that the fraction of agents demanding each good is just equal to the fraction of agents supplying the same good: it is therefore always possible to carry out exchanges by means of direct barter. This is very important, as it guarantees that the introduction of an intermediary does not simply fill up a structural gap in the system, but comes from free individual choices.

Let us denote by  $q = (q_1, \dots, q_n)$  the actual share of individuals demanding the corresponding goods  $(1, \dots, n)$  and  $s$  the share of those resorting to indirect trade<sup>5</sup>. The overall share of agents demanding  $n$  is:

$$q_n = \frac{(u_n + s)}{(1 + s)} \quad (3)$$

and the overall share of agents demanding  $i$  is:

$$q_i = \frac{u_i}{(1 + s)}. \quad (4)$$

Assuming rational expectations, we may set  $p = q$ , and condition (1) for indirect trade becomes:

$$q_n > q_i + q_j. \quad (5)$$

Consequently, the share  $s$  of final demand which is fulfilled by indirect trade is equal to:

$$s = s(\mathbf{U}, q) = \sum_{\substack{i, j \text{ tale che} \\ q_n > q_i + q_j}} u_{ij} = \sum_{\substack{i, j \text{ tale che} \\ u_n + s > u_i + u_j}} u_{ij} \quad (6)$$

Figure 1 conveniently illustrates expression (6).

< figure 1 >

It should be clear that nothing guarantees the achievement, at an equilibrium, of a value of  $s$  equal to  $(1 - u_n)$  (which is denoted by M in the graph); in fact, we may find other possible equilibria (denoted by A, B, C, in the picture) which correspond to values of  $s$  lower than  $(1 - u_n)$ .

---

<sup>5</sup> The variable  $s$  has a range comprised between 0 (pure barter) and  $(1 - u_n)$  (full monetization) as (1) cannot hold if  $s = 1 - u_n$  or  $s = 0$ .

In particular, we will obtain a (stable) state of zero monetization when expression (6) does not hold for any couple of goods, i.e.  $u_n < u_i + u_j, \forall n, i, j$ .

The situation of full monetization ( $s = 1 - u_n$ ), is always a stable equilibrium, however, for any possible value of final consumption functions  $U$ , even in the case when  $U$  or the expectations over  $p$  should vary following an external shock.

Jones' model, therefore, provides a theoretical underpinning for the way an economy may shift from a situation of direct barter, where the double coincidence of need and wants is essential, to another in which a commodity gets used as a means of exchange and brings about more efficient, indirect, trade, by reducing transaction costs. This outcome does not come out of legal prescriptions, or as a last resort, but directly from the optimizing behaviour of rational individuals, and from the market features of goods.

#### **4. Oh' s model, and the introduction of conditional trading strategies**

Jones' model, however, only used non conditional trade strategies, implying that an agent, who has decided to exchange good  $i$  for good  $k$  and, subsequently, good  $k$  for good  $j$ , will not modify his strategy even in the case in which he is confronted with the possibility of directly exchanging  $i$  for  $j$ . In other words, each individual decides over his trading strategy - in order to minimise transaction costs - before entering the market, and sticks to that strategy whatever happens next.

Seonghwan Oh's (1989) analysis is, in a sense, a reaction to this way of thinking the trading process, as he considers *conditional trading strategies*, allowing agents to implement trading strategies which are conditional on encounters on the market. This way, the optimal strategy for an agent is to trade one's own good for the desired good or, alternatively, with another good with a higher probability of being traded.

Oh's analysis shows that, under conditional trading strategies, even in the case of partial monetization there exists one good which is universally accepted as money; this is quite different from Jones' model, where it may well happen that the same individual sometimes rejects a given means of exchange on some occasions, and accepts it on others.

Oh claims that Jones' model must be recast in a way that considers a sequential behaviour of the following kind:

- a) there is a first round, in which an agent considers the expected cost (in the sense of Jones) of meeting both an agent owning commodity  $j$ , and an agent owning commodity  $k$  (which is the candidate for a money commodity). If our agent indeed succeeds in trading  $i$  for  $j$  in the first meeting round, then the sequence stops; otherwise, the agent goes on to a subsequent step, i.e.:
- b) in the second *round* our agent computes the expected cost of meeting another agent who is willing to trade good  $k$  for good  $j$ .

All in all, the total expected cost is equal to:

$$\frac{1}{p_j + p_k} \cdot \left( \frac{1}{p_i} + \frac{1}{p_j} \right) \cdot .$$

This expression can be compared to the one that we obtain if we only consider non conditional trading strategies, and which amounts to:

$$\frac{1}{p_i p_k} + \frac{1}{p_j p_k}$$

As can clearly be recognised, a conditional strategy is always superior to a non conditional one<sup>6</sup>.

By simple inspection of the following inequality:

$$\frac{1}{p_i p_j} > \frac{1}{p_i + p_n} \left( \frac{1}{p_i} + \frac{1}{p_j} \right) \quad \text{con } n \in \{1, \dots, k\}$$

---

<sup>6</sup> Stochastic independence between the variables  $\{p_i\}$  with  $i \in \{1, 2, \dots, n\}$ , was assumed. As Jones himself remarked, this is a simplifying assumption which should correspond to a sort of bounded rationality hypothesis.

it is also clear that a conditional strategy has to be preferred to direct barter if the following condition holds:

$$P_n > P_i$$

which is certainly a more interesting conclusion than the one which was discussed in the previous paragraph, i.e. that:  $P_n > P_i + P_j$ .

Moreover, Oh showed that the equilibrium configuration is unique, and the proof can be sketched as follows.

Let  $U = [u_{ij}]$  a matrix containing the fractions of individuals going to the market and willing to trade good  $i$  for good  $j$ .

As was the case in the previous section, this matrix is time invariant and symmetric.

Therefore:  $u_i = \sum u_{ij} = \sum u_{ji}$ .

Let us now denote by  $s$  the share of agents accepting indirect trade of commodity  $i$  for commodity  $j$ . This fraction can be computed as:

$$s = \sum_{i,j | P_n > P_i} u_{ij}$$

Defining now  $q$  as a vector expressing actual shares of agents' choices, and equating  $q = (q_1, q_2, \dots, q_n)$  to the vector  $p$  that was introduced above<sup>7</sup>, we obtain the share of agents demanding the "intermediate" commodity  $n$ , i.e.:

$$q_n = \frac{u_n + s}{1 + s}$$

---

<sup>7</sup> This is, on the other hand, a rather delicate assumption, in the spirit of a "rational expectations" framework, and which strongly conditions Oh's results.



The share of agents demanding commodity  $i$  can be computed, on the other hand, as

$$q_i = \frac{u_i}{1+s}.$$

In the general case, we would have  $s$  equal to:

$$s = \sum_{j \neq n} \frac{u_j q_n}{q_n + q_j} = \sum_{j \neq n} \frac{u_j (u_n + s)}{u_n + u_j + s}$$

using the notation developed in the previous paragraphs.

Oh shows that the equilibrium configuration tends to the upper limit  $1-u_n$ , that such an equilibrium exists and that it is unique (and this is remarkable, especially in view of the result we get in Jones' model, where the equilibrium is generically non unique, as can be seen from figure 1).

The main conclusion of Jones' and Oh's models is that a commodity may indeed emerge, at equilibrium, as a monetary instrument, based on its physical characteristics and agents' utilities. In other words, the intrinsic properties of a given commodity, which make it more frequently supplied and demanded in the economy, are crucial for the monetary role that it may eventually play.

A remarkable step forward was made by K. Iwai (1988,1996), who tried to show that intrinsic properties of commodities may not be essential in determining a monetary role, in as much as some self fulfilling mechanisms might be at work. This means, of course, that any commodity might play a monetary role, if certain conditions on agents' beliefs are met.

## **5. From fundamental to speculative monetary equilibria: Iwai's model of search and money**

Iwai's model is quite similar to Jones' and Oh's. It features a large but finite number of agents,  $N$  durable goods, and  $N(N-1)/2$  separate areas of bilateral meetings.

Agents are born with an endowment of a single good, and are in need of a different one. The frequency of agents born with an endowment of good  $i$  and a need for good  $j$  is denoted by

$\bar{e}_{ij}$ , which is one of the key variables in the model. This variable is what one might call the "fundamentals" of the model. It is in terms of this variable that the structure of the economy can be described. In particular, an important assumption made by the author is that the economy be *connected*, i.e. that for any  $i$  and  $j$  ( $\neq i$ ) there exists a connected sequence of positive "endowment-need" frequencies such that  $\bar{e}_{hi} > 0, \bar{e}_{gh} > 0, \dots, \bar{e}_{kl} > 0$  and  $\bar{e}_{jk} > 0$ . In other words, it must be the case that, with a sufficiently long chain of transactions, any individual can exchange the endowment he has with a unit of the desired good. This notion of connectedness bears a close resemblance to that of *irreducibility*, which is very well known in the domain of General Equilibrium, and which is due, in its simplest and original form, to McKenzie (1959, 1961) (as a matter of fact, the same notion was used, under the name of *resource relatedness*, by Arrow and Hahn (1971)). In plain words, *irreducibility* implies that, if we consider an agent  $h$  with his initial endowment  $w^h$ , there always exists another agent, say  $h'$ , who can be made better off by receiving part of agent  $h$ 's endowment.

Iwai underscores two particular forms of a connected economy, which he also exploits in the sequel of the analysis.

The first, and simplest one, is that of a doubly symmetric endowment-need distribution, which satisfies:

$$\bar{e}_{ij} = \bar{e}_{ji} = \frac{1}{N(N-1)} \text{ for any } i \text{ and } j (\neq i);$$

this is an economy in which it is particularly easy, for any agent, to find his counterpart in a barter process.

Not so in the second polar case he examines, and which can be defined as a "minimally connected" economy, satisfying the condition:

$$\bar{e}_{12} = \bar{e}_{23} = \dots = \bar{e}_{N-1,N} = \bar{e}_{N1} = 1/N; \quad \text{all the other } \bar{e}_{ij} = 0,$$

which is the most complex situation to deal with, in terms of co-ordination requirements to carry out a barter process. The minimal connected economy is one in which barter is precluded (as double coincidence of needs is absent) and precisely for this reason it will

constitute the focus of later analyses (in particular, the one by Kiyotaki and Wright (1989,1991,1993), that will be closely analysed in the following sections).

Agents meet in pairs, as each period they visit specialised trading zones in which commodities  $i$  and  $j$  can be exchanged. The probability of meeting another agent with whom to trade in such a trading zone is expressed by the frequency  $q_{ij}$ , the so called "supply-demand frequency", which stands for the frequency (relative to the whole population) of agents who supply commodity  $i$  and demand commodity  $j$ . This is another key parameter of the model, which will depend on the trading strategies decided over by agents, and which helps us in defining two important features of commodities, related to their "liquidity" properties. Iwai defines as "saleability" (quoting Menger) the relative ease with which a commodity can be sold, and which can be summarised, for commodity  $m$ , by the frequency  $q_{im}$ . A mirror property is that of "purchasability", which can be described, for good  $m$ , by the probability  $q_{mi}$ , describing the possibility for an agent to buy commodity  $m$ , by using a different commodity,  $i$ . Importantly, the fact that agents visit specialised trading zones instead of matching randomly in pairs is a truly distinctive feature of this model, which will bring about quite interesting results, different from those obtained in other models of search.

Agents choose a trading strategy (i.e. whether to engage in direct trade - barter-, or indirect trade) by solving a maximisation problem, whose objective function is the expected utility attached to the alternative trading strategies. More precisely, each agent tries to attain the maximum of the expected intertemporal utility which may be expressed by:

$$V_{ij} = \max_k (V_{kj} - b - c / q_{ik})$$

where  $b$  and  $c$  stand for the period costs of transaction and search, respectively, and  $q_{ij}$ , as was anticipated above, stands for the probability that the transaction  $(i, j)$  occurs in a given period.

The coefficients  $b$  and  $c$  are exogenous, independent of agents, commodities and time periods. When they meet and decide to trade, agents exchange goods on a one-to-one basis, which has the obvious consequence of ignoring the problem of determining terms of trade (relative prices).

With this scheme in mind and some simple algebra, Iwai shows that everyone chooses to barter if the following condition holds:

$$c/q_{ji} \leq b + c/q_{jk} + c/q_{ki},$$

indicating that the cost of directly exchanging  $i$  with  $j$  is less than the cost of exchanging  $i$  with  $k$  and subsequently  $k$  with  $j$ .

In a substantially similar manner, it is possible to show that every agent in the economy uses a commodity  $m$  as a medium of exchange, except those agents who are endowed with good  $m$  and those who need it, if the set of supply-demand frequencies satisfy all of the following conditions:

$$b + c/q_{jm} + c/q_{mi} < c/q_{ji} \quad \text{for any } i (\neq m) \text{ and } j (\neq m \text{ and } i)$$

$$b + c/q_{jm} + c/q_{mi} < b + c/q_{jk} + c/q_{ki} \quad \text{for any } i (\neq m), j (\neq m \text{ and } i), k (\neq m, i \text{ and } j);$$

$$c/q_{jm} \leq b + c/q_{jk} + c/q_{km} \quad \text{for any } j (\neq m) \text{ and } k (\neq m \text{ and } j);$$

$$c/q_{mi} \leq b + c/q_{mk} + c/q_{ki} \quad \text{for any } i (\neq m \text{ and } j) \text{ and } k (\neq m \text{ and } i).$$

In plain words, this set of conditions specify that 1) it is more convenient to use commodity  $m$  as a medium of exchange than to barter, 2) that commodity  $m$  is the least costly among all possible media of exchange, 3) that people already in possession of good  $m$  find more advantageous to directly exchange it with the good they need rather than follow an indirect trading strategy and 4) that the same is true for the agents who need commodity  $m$ .

The concept of equilibrium is quite natural, in this set-up: it consists of a set of trading strategies which maximise intertemporal expected utilities, and which are mutually compatible, in that the probability of occurrence of a given trade ( $q_{ij}$ ) coincides with the probability that agents use in solving their maximisation problems.

Iwai (1988) explores the conditions under which two different kinds of equilibria emerge:

- 1) barter equilibria, in which all agents follow a strategy of direct exchange, and that crucially hinge on the possibility that there may be a double coincidence of wants;
- 2) monetary equilibria, in which a given commodity is used to implement indirect trades by all agents, except those who consume such a commodity, and those who have it as an endowment.

It is useful to keep in mind the latter definition, as it will be used in the sequel.

Quite importantly, Iwai makes a distinction between short term equilibria, in which agents take the probability of a meeting as given, and do not consider the effect that other agents' trading strategies have on such probabilities, and long term, or "stationary" equilibria, in which all agents take those influences into account.

In the first case, he shows that a monetary equilibrium might emerge only if, for any non monetary good  $i$ , the share of agents endowed with (commodity) money and wishing to consume good  $i$  is positive. In other words, some fundamentals in the economy are required for the establishment of a monetary equilibrium.

For the long term, stationary case, however, this is no longer true, as the condition whereby the probability of agents holding money and desiring any good  $j$  is positive may always be fulfilled, regardless of the initial features of the economy. In Iwai's words, "money can support itself by its own bootstraps" (Iwai, 1996, pag.469). In particular, it is shown that a connected economy always has monetary equilibria, equal in number to the number of goods. This is a remarkable result, which should be contrasted to those obtained in Kiyotaki and Wright (1989), described below.

The author also shows that all equilibria are locally stable, which implies that if the economy was in a barter equilibrium, a relatively large shock, in terms of agents' beliefs, is needed to push it towards a monetary equilibrium.

Another relevant question addressed by the author is whether monetary equilibrium should be thought to emerge as a consequence of a co-operative process. In fact, Iwai shows that the effect of a change from a non monetary to a monetary equilibrium on the welfare of those agents who neither consume nor are endowed with a particular "monetary" commodity, is not necessarily positive. This is an extremely important finding, as it paves the way towards the development of a full fledged non co-operative theory for the origin of money, which constitutes the core of most recent search theoretic models. These models will be discussed at length in the following sections.

## **6. Search theory and monetary equilibria: Kiyotaki and Wright's model**

The analysis performed by Nobuhiro Kiyotaki and Randall Wright on money and its working as a means of exchange may be divided into two main parts, devoted respectively to commodity money and fiat money.

The economy they take into consideration is made up by individuals consuming a different good from the one they produce. All goods are storable, and the quantity  $c_{ij}$  represents the cost for individual  $i$  to store good  $j$ . In each period agents randomly meet in pairs, and they decide whether to exchange or not their endowments; exchange occurs on a quid pro quo basis, as credit does not exist, as the probability for two agents to meet again is zero.

It is worth stressing, at this point, a first important difference between this model and Iwai's, which will explain some fundamental differences in the results. Kiyotaki and Wright do not assume that trades are costly. In other words, agents only bear costs in terms of storing goods, whereas in Iwai (see section 4) they also incur into a cost  $b$  to engage in a transaction.

Based on this, each individual selects a trading strategy in order to maximise expected lifetime utility, net of production and storing costs, taking as given other agents' strategies and the distribution of potential meetings,  $(\mathbf{p}(t))$ , as we will see in the sequel).

The model shows that the economy will move towards a situation of non co-operative equilibrium that will determine the appearance of some goods serving as means of exchange, i.e. money. The equilibrium will be of two possible types, depending on the values taken by the parameters of the economy (in particular by storing costs,  $c_{ij}$ , and  $u_i$ , net utility of consumption).

In a *fundamental* equilibrium, agents prefer a low storage cost good to a high storage cost good, unless the latter is the good they consume.

In a *speculative* equilibrium, on the other hand, agents will exchange their endowed good with other goods which are not desired for direct consumption, but which are more apt to be exchanged in future trading rounds, even if they are more costly to store.

This also means that more "liquid" goods will be preferred in the exchange process, and this is very much reminiscent of the mengerian notion of "saleability", which is the true keystone of the theory of money by Karl Menger.

As was mentioned before, in this economy agents are also producers, and this represents another novelty with respect to models discussed in the previous sections, where production, even in this very simple and abstract form, was totally absent.

Agents choose strategies by looking at their (infinite) lifetime discounted utility, which can be defined as follows:

$$E \sum_{t=0}^{\infty} \left\{ B^t \left[ I_i^U(t) U_i - I_{i^*}^D(t) D_i - I_{ij}^C(t) c_j \right] \right\}$$

where:

$I_i^U$  is an indicator function taking value "1" if the agent  $i$  consumes commodity  $i$ , and "0" otherwise;

$I_{i^*}^D(t)$  is an indicator function taking value 1 if the agent produces a unit of commodity  $i$

(and therefore incurs into the corresponding production cost), and zero otherwise;

$I_{ij}^C(t)$  is an indicator function taking unit value if agent  $i$  holds commodity  $j$ , and zero otherwise;

$U_i$  represents the instantaneous utility associated to consuming good  $i$ , and  $D_i$  the instantaneous disutility associated to producing commodity  $j$ ;

$B$  indicates the discount factor.

In what follows we will adopt a simplification, which does not entail a great loss of generality, consisting in the fact that there are only three goods in the economy, and three types of agents, each one consisting of a unit mass of identical individuals. Agent  $i$  only consumes good  $i$  and produces good  $i^* = i + 1$  (modulo 3).

If this is the set-up, the possibility of barter is ruled out, as a very simple argument shows: agent 1 only wants to consume good 1, and only produces good 2. A situation of double coincidence would arise if agent 3, who produces good 1, desired good 2 as a consumption good; unfortunately, though, this is not the case, as agent 3 only consumes good 3. This is an example, with three goods, of a "minimal connected economy" illustrated in the work by Iwai (1988 and 1996).

We should notice, however, that ruling out direct barter as a possibility is not essential for the main conclusions of the model to hold, and could be dispensed with, (as the analysis of Iwai's model should have shown). We will keep that assumption, though, to be as close as possible to the original model.

The following assumption holds for all agents:

$$u_i = U_i - D_i > (c_{ii^*} - c_{ik}) \cdot (1 - B)^{-1}, \quad \forall k$$

This can be easily shown to be sufficient for agents not to be willing to drop out of the economy (for example, by deciding to hold some good  $k$  forever). For the latter assumption to hold, however, we need that the overall utility balance between consuming good  $i$  and instantaneously after producing good  $i^*$  be large enough, and that the disutility of storing good  $i^*$  not be too large relative to the disutility of storing any other good  $k$ .

The functioning of the model is extremely simple: each period, agents randomly meet in pairs and decide whether or not to trade with each other. If they do, they swap their respective holdings and leave. If they don't, they just leave and wait for the subsequent matching. Exchange occurs on a one for one basis, as agents hold one unit of the various goods, and cannot hold two goods at a time. The latter hypothesis, as will become clearer in the sequel, is likely to create difficulties in the interpretation of the prediction we get from the model, at least in the version with fiat money.

The probability of a given matching can be obtained from a probability distribution  $p(t)$ , where  $p(t) \equiv [\dots p_{ij}(t) \dots]$  and each  $p_{ij}(t)$  represents the proportion of type  $i$  agents holding good  $j$  at time period  $t$ . The authors only consider the time invariant case in which  $p(t) = p, \quad \forall t$ , which means that they are only looking at steady state equilibria.

Agents must decide over trading strategies, i.e., rules which will indicate the behaviour (trade, non trade) of agents when matched. As the economy is time invariant, these rules will only depend on the goods held by the two agents in a pair.

Strategies can therefore be denoted as follows:

$\tau_i(j,k) = 1$  if  $i$  wishes to trade commodity  $j$  for commodity  $k$ , and 0 otherwise.

$\tau_h(k,j) = 1$  if  $h$  wishes to trade commodity  $j$  with commodity  $k$ .



Trade between agents  $i$  and  $j$  occurs if and only if:

$$\tau_i(j,k) \tau_h(k,j) = 1.$$

Given an initial distribution of commodity holdings at a period  $t$ ,  $p(t)$ , the set of strategies played by agents will define a new probability distribution vector,  $p(t+1)$ . For any strategy vector  $(w_1, w_2, w_3)$ , where  $\{\tau_{ij}\} = w_i$ , we can define a steady state distribution of inventories a measure  $p$  such that  $p(t) = p, \forall t$ .

It is now possible to define the *steady state* Nash equilibrium, as a set of trade strategies,  $\{\tau_{ij}\} = w_i$  for any agent  $i$ , and a vector  $p$ , defined in the previous paragraph, such that all agents maximise their expected utility, given  $p$  and strategies by other agents. Moreover, expectations are rational (in terms of both strategies and  $p$ ).

Analytically, an equilibrium will be a vector of strategies  $(w_1^*, w_2^*, w_3^*)$ , a steady state distribution of holdings,  $p^*$  such that:

1.  $w_i^*$  maximises agent  $i$ 's expected discounted lifetime utility given others' strategies and  $p^*$ ;
2.  $p^*(w_1^*, w_2^*, w_3^*) = p^*$ .

In fact, computing an equilibrium in such a simple economy is quite straightforward: for a given vector of strategies,  $w^*$ , one computes the steady state distribution of inventories,  $p^*$ . Then one has to check that the assumed strategies are indeed optimal, in terms of agents' discounted utilities. Since the number of strategies in this simple economy is fairly small, this procedure can easily be carried out for each of them.

Kiyotaki and Wright (1989) consider the case in which  $c_{i3} > c_{i2} > c_{i1}$ , which introduces an element of asymmetry in the economy, and two possible situations, denoted Model A and Model B, respectively. In Model A agent  $i$  produces commodity  $i+1$ , whereas in Model B agent  $i$  produces commodity  $i+2$ . In view of the asymmetry induced by storage costs the two model will yield different results.

As anticipated in the introduction, two kinds of equilibria appear in such an economy: for some values of the parameters  $(c_{ij}, B, u_i)$  Model A features what is called a *fundamental* equilibrium, where agents prefer to hold a lower storage cost commodity than a higher storage cost one, unless the latter is their own consumption commodity. This is the case where agents 1 and 3 always hold their production goods (until they have the possibility of exchanging it with the commodity they consume, obviously), whereas agent 2 swap his production good with good 1 (which has lower storage costs) whenever possible. This implies that agent 2 uses good 1 as commodity money and that he acts as a *middleman* in the economy (the fact that *middlemen* are essential in a monetary economy has also been underscored by Alchian (1976)).

For different values of the parameters, a different kind of equilibrium, named *speculative*, appears, in which type 1 trades the lower storage good 2 for the higher storage cost 3, because good 3 has a higher *liquidity* than good 2, in that it can be more easily exchanged for good 1 than the others. Agents 2 and 3, however, keep on playing fundamental strategies, with type 2 using commodity 1 as commodity money (still acting as a middleman) and type 3 holding its produced good until he can trade it for his consumption good.

This *speculative equilibrium* is quite intriguing, as it shows that 1) more commodity monies can coexist and 2) liquidity can play a more basic role than storage costs in determining which commodity is used as money.

Finally, for some parameter intervals no equilibrium exists. It should be stressed, however, that Kiyotaki and Wright (1989) only looked at pure strategy Nash equilibria, where agents who are indifferent between holding any two commodities are assumed not to trade.

When the economy is symmetric, in that all storage costs are equal, it may be shown that one cannot find an equilibrium in pure strategies. For there to be an equilibrium, endogenous types formation should be allowed, as will be illustrated in the following sections. It is maybe worthwhile to spend a few words to explain why it is so.

Let us therefore consider the following candidate equilibrium:  $s = (1,0,0)$ , which indicates that agent 1 uses good three as a medium of exchange, and the other two agents - who either produce good three or need it - engage in direct barter (this corresponds, we may recall, to

the definition of a monetary equilibrium). The stationary holding frequencies corresponding to this candidate equilibrium are quite easy to compute, and they are given by ( $p_{12} = p_{13} = 0.5$ ;  $p_{23} = p_{31} = 1$ ). For the proposed strategy to constitute an equilibrium it must be the case, therefore, that types 2 and 3 are at least indifferent between holding the good they produce and holding the other good (respectively good three and one). Type 1, on the other hand, must find advantageous to hold good three, rather than good two. Analytically, it must be the case that  $V_{12} < V_{13}$ , whereas  $V_{23} > V_{21}$  and  $V_{31} > V_{32}$ . Unfortunately, though the first two conditions do hold, the third does not, because agent 3 finds it more convenient to hold good 2 (which would therefore work as a medium of exchange) rather than good 1. This is so in view of the relative larger number of agents wishing to exchange good 2 for good 3 than those who wish to exchange good 1 for good 3. The same reasoning, of course, applies to the other possible configurations of equilibria, in which one of the three goods alternatively serves as medium of exchange.

This interesting result should definitely be contrasted to the result contained in Proposition 7 in Iwai (1996), whereby there should exist, even in the absence of asymmetries, three speculative equilibria. The reason why proposition 7 does not apply here is twofold. On the one hand, transactions here are not costly. If they were, the relevant comparison should not be one between  $V_{31}$  and  $V_{32}$ , but one which also takes transaction costs into account. In other words, the indirect utility  $V_{32}$  should be diminished by the amount of the transaction costs (in fact, to hold commodity 2 instead of commodity 1 requires, for type 3 agents, one more transaction). If we denote transaction costs by  $c_T$ , it would not be difficult to show that for  $s = (1,0,0)$  to be an equilibrium, the condition  $c_T > \frac{b/2}{1-2b}u_3$  should be fulfilled (a similar condition would apply for showing existence of the other two, symmetric, equilibria). When transactions are costly, therefore, it is indeed possible that any commodity (commodities are treated symmetrically, as we may recall) may serve as money, even in the absence of (differentiated) storage costs. On the other hand, in this model the matching structure is random, whereas in Iwai's economy agents decide over visiting deterministic trading zones,

which prevents them from implementing sequential strategies. Once they get in such a trading zone randomness has to be taken into account, as supply and demand frequencies  $q_{ij}$  imply a certain waiting time for matching, but only a particular type of agents can be met in such a zone; here, on the converse, every type can meet with every other type. Which of the two models is more realistic, is definitely not easy to tell. In a sense, this would suggest that between Iwai's and Kiyotaki and Wright's search models we find more or less the same difference we could recognise in comparing Jones' and Oh's models of a monetary economy, but with a remarkable difference: in the latter, the inability to implement sequential strategies makes the introduction of money as a means of indirect exchange more difficult, whereas in the former quite the opposite is true. Given the bootstrap nature of money, the very fact of committing oneself to use it as a medium of exchange, rather than engage in barter, is sufficient for a monetary equilibrium to emerge.

There are at least three recent extensions of this model which are very much worth discussing, as they introduce and provide answers to some remarkable questions.

### **6.1 Could perishable goods work as commodity money?**

As was noticed by Einzig (1966) at times some perishable objects have come to play the role of commodity monies. This seems a bit counterintuitive, as one is normally tempted to fully endorse Jevons' (1875) idea that indestructibility belongs to the set of qualities that any object should feature to function as money.

However, the self-fulfilling mechanisms that underlie the appearance of money can convince us that this is not necessarily the case. This is the issue raised in a recent contribution by Cuadras-Morato' (1997), who used exactly the same framework of the previous section, but with a slight twist: commodity  $i$  can only be consumed for a given number of periods after it is produced,  $n_i$ , after which they become totally useless. To simplify matters and get

computable equilibria, the author considers good 1 and 2 as perfectly durable (i.e.  $n_1 = n_2 = \infty$ ) and good 3 only lasting for two period (i.e.  $n_3 = 2$ ).

Following the steps outlined in the previous paragraph, it is possible to check that, for certain values of the parameters, there exists a (pure strategy, Nash) equilibrium in which both good 1 (perfectly durable) and good 3 (perishable) are used as commodity money. For different regions of the parameter space, an equilibrium exists in which only good 1 is used as commodity money. The reason why commodity 3 is also used as a medium of exchange, albeit perishable, is that for some individuals the benefit of doing so, in terms of *liquidity*, outweighs the expected costs of accepting a perishable commodity (i.e. the possibility of being left with something completely worthless, in case one does not succeed in using it for trade in the following periods).

The author also shows that this result is robust to a number of changes, namely in the assumptions about the information available to the agents. If individuals possess imperfect information as to the age of commodities, the same kind of results can be obtained. Likewise, it can be shown that the value of holding money decreases over time, when money is perishable.

## **6.2 Commodity money when quality is heterogeneous**

The same model can be used to study a different problem, arising when commodities may feature different qualities, which can only be recognised when one gets the commodity itself. The question one might be willing to address is whether or not such uncertainty over the qualitative properties of a commodity may prevent it from being used as a medium of exchange. As we will see, this is not necessarily the case, as the marketability properties of a good may be more important than uncertainty over quality.

To effectively deal with this problem, Cuadras-Morato' (1994) modifies the standard search model in a relatively simple way: goods are produced in two possible quality, high and low, according to a stochastic process governed by a probability  $\alpha_i$ , which is the probability that

agent  $i-1$  produces good  $i$  as a high quality good. The qualitative property of goods, however, can only be recognised by those agents who consume them, and just after the goods have been traded. When a good is recognised as being of low quality (a *lemon*), its holder incurs a cost  $d_i$  which should be balanced against the benefit of holding such a good. For the rest, the basic model is virtually unchanged.

The author considers two versions of the model: a symmetric one, with no storage costs, and an asymmetric one, in which storage costs are positive and differ across goods.

In the first case, in which it is also assumed, for sake of tractability, that only good 3 can be produced in two possible qualities, it is shown that a pure Nash equilibrium exists in which both good 1 and 3 are used as mediums of exchange. The condition for that to be the case is shown to be that the expected cost of holding a differentiated good (which might turn out, on trading, to be a *lemon*) should be less than the advantage of using it as a medium of exchange (due to its marketability, or "saleability", in Menger's terminology). Once again, this is one more instance of a *speculative* equilibrium, in the sense of Kiyotaki and Wright (1989), which may also be implemented with commodities whose value, in terms of *fundamentals*, is lower than the value of other commodities.

When the advantage of good 3 in terms of liquidity is outweighed by the cost of holding it (in terms of qualitative uncertainty) only one pure strategy Nash equilibrium exists, where good 1 is the only commodity money.

When storage costs are positive and different across commodities, which makes the economy fundamentally asymmetric, it can still be shown that there exists equilibria in which differentiated commodities play the role of money, even in the presence of quality uncertainty.

### **6.3 Monetary equilibria and endogenous types**

An interesting question that we might be willing to ask in the context of the search model we are analysing is what happens when individual types are endogenous, i.e. are determined at equilibrium, along with the other variables (strategies and holding frequencies). This issue has been dealt with in a paper by Wright (1995), in several steps.

First of all, the author generalises the main results in Kiyotaki and Wright (1989) by allowing a non uniform distribution of types in the economy. In other words, the distribution of types in the economy is expressed by the vector  $(\theta_1, \theta_2, \theta_3)$ , whose entries are no longer all necessarily equal to  $\frac{1}{3}$ .

By so doing, it was possible to show that a new equilibrium emerges, in addition to the two presented in Kiyotaki and Wright (1989). In particular, a new, "speculative" equilibrium strategy vector  $w = (0,0,1)$  can be found, in which agent 3 uses good 2 as commodity money, and which is the exact mirror image of the speculative equilibrium strategy vector  $w = (1,1,0)$  which was found in Kiyotaki and Wright (1989). Interestingly, the new equilibrium can never exist for a uniform distribution of types and, provided this is not the case, it exists whenever its mirror image exists, though agents play in the exactly opposite way in the two equilibria. This also sharply contrasts with the fact that when the distribution of types is uniform, only one equilibrium at a time arises, if any.

As we may recall, in the original model it is possible to show no pure strategy equilibria survives, when storage costs,  $c_j$ , are all set equal to zero, which seems to indicate that *fundamentals* are somehow essential, even for showing the existence of a speculative equilibrium. In the new extension of the model, Wright clearly shows that when storage costs are set to zero it is still possible to get pure strategy Nash equilibria (both fundamental and speculative), provided the distribution of types is not even. For instance, for the strategy vector  $w = (0,1,0)$  to constitute an equilibrium, it must be the case that  $\theta_2 > \frac{1}{2}$ . This is

because the share of type 2 agents must be larger than  $\frac{1}{2}$  for type 1 to choose to hold the commodity he produces. If that were not the case, type 1 would find it convenient to trade good 2 for good 3 in order to achieve a larger discounted expected utility. A similar condition must hold for the strategy vector  $w = (0,0,1)$  to be an equilibrium.

The second part of Wright's (1995) contribution is devoted to endogenizing agents' type in the economy. The idea is that, somehow, the shares of the three types may change as a result of

some evolutionary process, which should lead to the survival of the fittest, or to the relative growth of the wealthiest group of agents.

As a steady state outcome of any sensible evolutionary process we should find that the payoff to the three types of agents (if they all survive, of course) be equal, i.e. that  $W_1 = W_2 = W_3$ , where  $W_i = p_{ii+1}V_{ii+1} + (1 - p_{ii+1})V_{ii+2}$  is the steady state payoff to agent  $i$ , which is not conditioned on the particular good he may be holding at a given point in time.

We may now introduce a new concept of equilibrium which fits this framework:

an equilibrium is a vector  $(w, p, \theta)$  of strategies, holding distributions and types' distributions such that:

- 1) holding shares  $p$  are stationary;
- 2) strategies  $w$  maximise agents' discounted expected payoffs;
- 3) (stationary) expected utilities  $W_i$  are equalised across agents.

If that is the definition of an equilibrium for the economy in which types are endogenous, Wright (1995) shows that there is only one equilibrium in which  $w = (1,1,0)$ . This is quite surprising in a sense, as this equilibrium is the one with a speculative component, one which is not solely driven by *fundamentals*. This, however, should also be contrasted to the results by Duffy and Ochs (1999), who ran an experiment in which search theoretic models of money were implemented. The main findings of such experiments were that people, who seem to be motivated mostly by past experience and less by "saleableness" considerations, had a strong tendency to play fundamental, rather than speculative, strategies, even when the latter had higher payoffs. In this respect, the predictions we obtain from most search theoretic models would seem to be disconfirmed, at least in an experimental set-up.

## 7. A search model with variable demand

A major departure from the original search model is also proposed in a recent contribution by Cuadras Morato' and Randall Wright (1997). The main tracts of the model are the same as before (three types of agents, a continuum of agents per type, three produced goods, random matching, etc...) but with an important difference. Although agents specialise in the



production of a single good (for example agent  $I$  specialises in producing commodity  $i + 1$ ), which implies that supply is given by the shares of types, they do not specialize in consumption. In other words, in each period all agents are hit by a taste shock (independent across periods and types), which determines the good they are going to consume.

This change in the structure of demand (together with the fact that commodities are taken to be completely homogeneous) has at least two important effects on the main conclusion we draw from this model.

On the one hand, a new vector of parameter appears, which was not necessary in the previous versions of the model: this is  $\delta_i$ , which stands for the percentage of agents desiring good  $i$ .

On the other hand, there might be agents who are lucky enough to exactly produce (or simply hold) the good they are going to demand. The number of such agents, who obviously do not need trading to consume, is equal to  $\sum_j \delta_j P_j$ , as for each good  $j$   $\delta_j P_j$  is the measure of

agents who happen to desire the good they are holding,  $P_j = \sum_i p_{ij}$  being the measure of agents holding good  $j$ . Therefore, the number of agents participating in the market is  $N = \sum_j (1 - \delta_j) P_j$ , which is also going to play a role in the analysis. With these premises, the

authors can prove that every agent chooses the same strategy vector: as the distribution of the shocks is the same for all types, in fact, agents evaluate goods in exactly the same way, conditional on the good they actually hold. The crucial parameter in the analysis is what the authors call  $\gamma_i$ , which they compute out of holding shares  $p$ , demand shares  $\delta$  and number of participants  $N$ . This parameter expresses the probability of consuming next period conditional on holding commodity  $i$  (because commodity  $i$  will be exchanged with the desired commodity or, even simpler, because commodity  $i$  will be desired next period). The authors show that the ranking of  $\gamma$ 's determine the ranking of indirect utilities, and therefore strategies. In other terms, we have that  $V_{ij} > V_{ik}$  if and only if  $\gamma_j > \gamma_k$  (which, we may notice, is independent of agents). This also means, of course, that commodities are ranked in terms of the

corresponding  $\gamma$ , and that there will always be a best one, in terms of capability to facilitate exchanges (and consumption).

The fact that agents have the same strategies, regardless of production, has a couple of noteworthy consequences. On the one side, this very much reduces the set of equilibria, compared to the case in which the three types of agent also differ in consumption; on the other, it implies that if one good is used as money, it is universally so (every agent uses it).

The authors do not have an existence proof in their paper; however, they do point at something essential as to the relative acceptability of commodities as media of exchange. In particular, they show that for a given good to be accepted as money, its demand must be sufficiently large. This has some relevance on equilibria with fiat money, of course, as fiat money is not demanded per se, but only for its services. For an otherwise worthless commodity to be (universally, in this case) accepted as money, therefore, it has to feature some advantages in terms of storability, production costs, and the like. In the version of the model we are analysing, which does not feature storability or production costs, therefore, there is no scope for equilibria with fiat money.

Finally, the authors show that supply conditions are not really crucial for the establishment of a monetary equilibrium. What really matters is demand and "liquidity value", as summarised by  $\gamma$ , as for any level of supply there is always a level of demand which makes a good  $i$  serve as money, and a different threshold below which this possibility is ruled out.

## **8. Fiat money and search equilibria**

The contributions that we have highlighted in the previous sections have been dealing with the emergence of commodity money, leaving aside the issue of whether an intrinsically useless good might have been adopted as a means of facilitating trade. It should be clear, however, especially from the reviewed analyses of Iwai and Kiyotaki and Wright, that there should exist some conditions, in terms of fundamentals and beliefs, which make this possible. In fact, Kiyotaki and Wright (1989) also discuss the possibility of an economy in which, besides the three commodities produced by the agents, another good, denoted by the subscript

$\theta$  and later on defined "fiat money", is available in quantity  $M$ , and has the following features: it does not generate utility as a consumption good, and, though this is not an essential assumption, its storage cost,  $c_{i\theta}$ , is set equal to zero. Agents cannot hold both a commodity and money, as it is assumed that, despite perfect storability, good  $\theta$  needs some space to be stored, and agents have only one unit of storage capacity.

Denoting by  $P$  the number of units of commodity  $\theta$  which are required to buy a unit of any other good (let us remember that in such simple models prices are fixed and equal for all goods),  $S=M/P$  denotes the amount of real balances in circulation.

In this new environment two things might happen. If, on the one hand, agent  $i$  believed that nobody would accept commodity  $\theta$  in future trading rounds, the indirect utility he will get from holding that commodity will be:

$$V_{i\theta} = 0 < V_{ij}, \forall j \neq \theta.$$

This implies, in view of the preceding discussion, that commodity  $\theta$  will not be accepted as an intermediary in trades.

If, on the other hand, agents believe that it *will* be accepted by all other agents in future rounds, this commodity will be preferred to all other commodities (as it has no storage costs), except in the case in which the proffered good is the one desired for direct consumption. In such a case, in fact, both fundamentals and "saleableness" favour a general acceptance of fiat money.

It should be clear that in this new context there is an additional difficulty to deal with: in particular, probability vectors  $p$  now depend on circulating real balances, and the authors define this dependence as a function  $\pi=\pi(S)$ , with the assumption  $\pi'>0$ .

By using this trick and with a remarkable amount of algebra, Kiyotaki e Wright show that the introduction of fiat money leads the economy towards an equilibrium in which the degree of utilisation of the various types of money (commodity moneys and fiat money) is determined by the value taken by real balances  $S$  (we may recall that the condition  $0<S<1$  holds). In the limit case where  $S = 0$ , equilibrium only features commodity money(s). At the opposite extreme, i.e.  $S = 1$ , the economy will only have fiat money. For all intermediate values of  $S$ , we will have coexistence of both kinds of money; in this case, however, fiat money will be

the only general means of exchange, i.e. the only object that, by definition, "which is habitually, and without hesitation, taken by anybody in exchange for any commodity" <sup>8</sup>.

If, as we claimed in the previous sections of this survey, the origin of commodity money can be traced to extrinsic beliefs over its acceptability, over and above its intrinsic properties, this is all the more true for fiat money: commodity  $0$  which, per se does not generate any utility, circulates only because people believe that other agents will accept it in their respective trades.

In a later contribution, dated 1993, the same authors introduce a different model, based on an economy featuring a large number of infinitely lived individuals, with a correspondingly large number of consumption goods, and where the extent to which real commodities and tastes are differentiated is captured by a parameter  $x$ , with  $0 < x < 1$ . This parameter turns out to play a crucial role in the model, as it can also be interpreted as the share of commodities that can be consumed by any given agent and/or the share of agents who can consume any given commodity. As all agents and all goods in this economy are treated symmetrically, we can also say that  $x$  represents the probability that a commodity trader is willing to accept any given commodity. Consumption commodities are produced by agents who consume, and the specific production is randomly drawn from the set of all commodities, following a Poisson process with constant arrival rate,  $\alpha$ .

The idea which underlies the new model is that agents, who randomly meet according to a Poisson process with constant arrival rate,  $\beta$ , decide strategically whether or not to accept trading the various goods or fiat money, and they do so by solving a problem of dynamic programming à la Bellman, in which they maximise their expected discounted utility, taking as given other agents' behaviour. The key variable, then, is  $\pi$ , which is the best response - in terms of the probability of accepting fiat money - to  $\Pi$ , the probability that a randomly selected commodity trader accepts money.

The authors finally solve the problem to find a symmetric, Nash equilibrium (though they clearly state that non symmetric equilibria might also exist).

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<sup>8</sup> Cfr. Wicksell K. (1967), *Lectures on political economy*, trad. di E. Classen, New York, pag. 17

To get a minimum insight into the mechanics of the problem, we may rapidly review the various Bellman's equations arising from the context under examination. The first, and fundamental one, is the following:

$$rV_0 = \alpha(V_1 - V_0) \quad [a]$$

where  $rV_0$  denotes the cash flow of the agent-producer, defined as the product of rate ( $\alpha$ ) by the difference between the quantity produced and exchanged.

In other words, expression [a] specifies a precise behaviour rule for producers, i.e. the maximisation of profits, by choosing an optimal *mix* production-trade.

It is now possible to introduce a second equation, i.e.:

$$rV_1 = \beta(1 - \mu)x^2(U - \varepsilon + V_0 - V_1) + \beta\mu x \max_{\pi} \pi(V_m - V_1) \quad [b]$$

where “0” indicates that the agent is a producer;

“1” is an agent wishing to trade commodities;

and “*m*” stands for an agent wishing to trade money for other commodities.

Expression (b), which is apparently quite complicated, can be decomposed in the following terms:

- $\beta(1 - \mu)$ , is the probability of meeting other traders;

- $x^2$  is the probability that they actually wish to trade;

- $U - \varepsilon + V_0 - V_1$ , represents the gain from trading, consuming and switching to production;  $V$  stands for the level of (indirect) utility, and  $\varepsilon$  denotes a welfare loss imputable to the implementation of trade;

-the second term, on the other hand, represents the optimal choice weighted by the probability that commodity traders accepts fiat money.

In addition to the above expressions, the authors introduce an equation [c], which specifies the profit flow to a money trader, as follows:

$$rV_m = \beta(1 - \mu)\Pi x(U - \varepsilon + V_0 - V_m) \quad [c]$$

This relationship is a function of the rate at which a money trader meets commodity traders,  $[\beta(1 - \mu)]$ , times the probability that both agents wish to trade,  $(\Pi x)$ , times the utility derived from exchange, consumption, and switching to production,  $U - \varepsilon + V_0 - V_m$ .

It is quite obvious that the outcomes of such programs depend mutually on the actual strategies envisaged by commodity traders,  $\Pi$ , and on the proportion of traders holding money, denoted by  $\mu$ . The latter may be determined by computing the steady state distribution of types, taking into account the initial proportions of types in the whole population, and the variable  $\Pi$ .

Once a steady state value for  $\mu$  has been computed, it can be plugged back into equations [a] to [c], which yields a function from  $\Pi$  to  $\pi$ , i.e. a best response correspondence. Finding the fixed points of this function amounts to computing the equilibria of this economy, as illustrated in figure 2, borrowed from Kyiotaki and Wright (1993).

< figure 2 >

Where  $\Pi$  is the optimal response relative to money acceptance on the side of commodity traders and commodity acceptance on the side of money holders.

In figure 2 we can see three possible equilibrium levels:  $\Pi = 0$ ;  $\Pi = 1$  e  $\Pi = x$ .

At the equilibrium with  $\Pi = 0$  money cannot circulate, as all agents think money does not have any value. This is the case in which  $\Pi < x$ , which implies that money gets accepted with a lower probability that a commodity offer, implying  $V_m < V_1$ .

The equilibrium with  $\Pi = 1$  is a purely monetary equilibrium, in the sense that all agents expect money to be universally accepted, and corresponds to the case where money is accepted more often than a barter offer, which implies  $V_m > V_1$ .

When  $\Pi = x$  we have a mixed monetary equilibrium, as agents are indifferent between accepting and rejecting money, in implementing their trades.

One of the main draw-backs of the first search theoretic models of a monetary economy lies in the fact that relative and nominal prices are fixed, i.e. the purchasing power of money is given. This is due to the fundamental indivisibilities which are imposed onto agents' endowments of goods (and fiat money, if present) and implies that there is no scope, in those models, for such phenomena as indeterminacy and economic fluctuations, which were accounted for, though with some limitations, by the overlapping generations model, the cash in advance model, and models with money in the utility functions.

## 9. Determining relative prices in search theoretic models

In the contributions by Shi (1995) and Trejos and Wright (1993, 1995), a bargaining model was independently adapted to a Kiyotaki and Wright type of an economy, in which a seller and a buyer bargain over the quantity of good to be (produced and) exchanged against a unit of money. Once an offer has been submitted, if it is accepted consumption occurs; if it is not, either the agents part and look for a different match, or they simply wait for a certain time (which will be denoted by  $\Delta$  in the sequel) to start bargaining again.

In the latter case, if it is the seller's turn to submit a bid he will propose a quantity  $q_s$  which makes the buyer just indifferent between accepting and rejecting the offer, i.e. solving the following expression:

$$V_s + u(q_s) = \frac{1}{1+r\Delta} [V_s + \frac{1}{2}u(q_s) + \frac{1}{2}u(q_b)]. \quad (10.1)$$

The left hand side in (10.1) represents the value, for the buyer, of accepting the seller's offer; this is given by the summation of the utility from consuming  $q_s$ , plus the expected utility of becoming a seller,  $V_s$ . This has to be equated to the expected discounted value of rejecting the offer, which is the discounted (for one period at the rate  $r$ ) utility of consuming  $q_s$  and becoming a seller, or consuming  $q_b$  and again becoming a seller, with equal probabilities.

If the buyer makes the proposal, on the other hand, the equation to solve is the following:

$$V_b + u(q_b) = \frac{1}{1+r\Delta} [V_b + \frac{1}{2}u(q_s) + \frac{1}{2}u(q_b)], \quad (10.2)$$

which has an analogous interpretation.

By multiplying out and rearranging we get:

$$2\Delta r[V_s + u(q_s)] = u(q_b) - u(q_s) \quad \text{and} \quad (10.3)$$

$$2\Delta r[V_b + u(q_b)] = c(q_b) - c(q_s), \quad (10.4)$$

which, for  $\Delta \rightarrow 0$ , implies  $q_s = q_b = q$ . Taking the ratio of (10.3) to (10.4) and computing the

limit for  $\Delta \rightarrow 0$ , we may obtain:  $\frac{V_s + u(q)}{V_b - c(q)} = \frac{u'(q)}{c'(q)}$ , which can also be considered as the

first order condition of the maximisation problem:

$q = \arg \max[V_s + u(q)][V_b - c(q)]$ , which we may easily recognise as a standard problem of Nash bargaining, in which reservation utilities have been set at zero<sup>9</sup>.

The notion of equilibrium should therefore be suitably modified, to account for the bargaining portion of the game, in quite a natural way: an equilibrium is a vector  $(q^*, V_s^*, V_b^*)$  such that:

a)  $q^*$  solves the bargaining problem, for given  $V_s^*$  and  $V_b^*$ , and  $V_s^*$  and  $V_b^*$  solve problems (10.1) and (10.2), given  $q^*$ .

From this model Trejos and Wright (1993, 1995) obtain some interesting results. Firstly, they show that in the absence of barter, a monetary equilibrium exists for all values of the parameters, and is unique.

When barter is allowed, on the other hand, and when agents do not look for other matches in the time interval between two offers, the authors show that there exists a discount rate  $R$  such that: if  $r < R$ , there exists at least one monetary equilibrium; on the other hand, if  $r > R$  there is no monetary equilibrium exists.

Shi (1995), on the other hand, gets two purely monetary equilibria, where purchasing power of money interestingly differs across equilibria, although the bilateral bargaining problem has only one solution. We understand, from this result, that self-fulfilling beliefs can indeed play a role at equilibrium. This way it is also possible to obtain sunspot equilibria, i.e. equilibrium allocations which are different across states of extrinsic uncertainty (which is not related to the fundamentals of the economy). The appearance of sunspot equilibria, which is not brought about by far-fetched preferences or technologies, is quite interesting, as it broadens the scope for indeterminacy of monetary equilibria, and for the quest of a "superior" equilibrium in welfare terms.

It is now to this delicate issue of assessing the welfare properties of monetary equilibria that we turn. Unfortunately, as we are going to make clear in the following section, it is not very easy to discuss about the welfare properties of monetary and non monetary equilibria in

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<sup>9</sup> In the case in which agents can meet other potential trading partners during the bargaining break after rejecting an offer (a case which has been named of "exogenous breakdown"), the reservation utilities - threat points - are



search models like those we have been exploring so far; this is so for various reasons, which we will try to make clearer in what follows.

### **10. Some remarks about welfare in models with money and search**

The first model we discuss under a welfare perspective is related to Iwai's economy, where we should distinguish between the case in which barter is possible and the case in which it is not possible. In the latter case it is evident that the adoption of a monetary instrument is essential in bringing about a Pareto improvement. For example, in the case of a minimally connected economy, which is the one adopted by the subsequent search literature, agents are doomed to autarchy, if commodity money cannot be used. Everybody is therefore better off in the case of a monetary agreement.

The situation gets less clear when barter remains a possibility, besides monetary trade. To discuss this issue, Iwai distinguishes between three types of agents. The first group is made up by agents born with a good which also serves as money; the second by those in need of such a monetized commodity, and the third by all the other agents. It is clearly shown that when passing from a barter economy to a monetary one the first two groups of people obtain an advantage, and this comes from the fact that when a commodity is also used as a means of payment it can be found (and used in an exchange) in relatively more trading posts than it is the case with barter. It is not possible, however, to reach the same unambiguous conclusion when dealing with the third group of agents. In this case a welfare improvement occurs when the number of types of agents (and thus the number of commodities) is large enough. Intuitively, the more difficult becomes the double coincidence of wants and needs (as the number of goods increases), the larger the welfare gains from the introduction of a monetary arrangement.

Iwai (1996) concludes, then, that "even though a monetary equilibrium is not always Pareto superior to a barter equilibrium, it is likely to be so in a wide variety of economies, especially when the number of goods is sufficiently large".

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not equal to zero.

In the standard search literature on monetary equilibria, initiated by Kiyotaki and Wright (1989), the quest for an answer to these welfare questions becomes more difficult, as the question itself gets more complex. In other words, as we only move in the context of a no barter economy, the question we may ask is whether or not an optimal quantity of money exists. In so doing, we have to operate a further distinction, between an economy with commodity money and an economy with fiat money.

In both cases the answer is not clear, and the difficulty stems from the specific features of the models under examination.

Let us consider, first, the set-up of an economy with real commodities, only, and more precisely the economy considered in Wright (1995), where the proportions of agents are allowed to vary.

We know from section (5.3) that there exist three monetary equilibria, according to the values of the parameters. The first two constitute a generalisation of some equilibria found in Kiyotaki and Wright (1989), in which one and, respectively, two commodities are used as means of exchange.

Taking into examination the first equilibrium (in which agent 1 and 3 do not use commodity money, whereas agent 2 does, acting as a middleman), for instance, we might try and find out the effects on agents' welfare of increasing the proportion of agents "producing" money (i.e. increasing the number of units of commodity money in circulation). To do this, we compute a measure of steady state individual  $i$ 's welfare, as a weighted average of the *flow* payoffs to agent  $i$  in the various circumstances. This is very simple for agents 1 and 3, who only engage in direct trading, as the welfare measure coincides with the flow payoffs  $rV_{12}$ , which yields agent 1's payoff in holding his produced good all the time, and  $rV_{31}$ , which expresses the equivalent measure for agent type 3.

For agent 2, who uses commodity 1 to implement indirect trade, the steady state welfare measure, using Wright (1995)'s terminology, will be:

$$W_2 = p_2 rV_{23} + (1 - p_2) rV_{21}$$

where  $p_2$  stands for the (steady state) probability of agent 2 holding the good he produces.

It can be shown that, in the range of the parameters which allow for the existence of an equilibrium, steady state utility is always increasing in the quantity of money for agent 1 and always decreasing for agent 3; agent 1 is, in fact, the one who needs the commodity money to live, and is benefited by its larger availability, whereas agent 3 is the one who produces it, and needs commodity 3, which becomes more difficult to find, if the percentage of agents of type 2 holding commodity 3 decreases (and, indeed, it is fairly straightforward to show that such percentage,  $p_2$ , is decreasing in  $\theta_3$ ). On the other hand, agent 2's welfare attains a maximum for a particular value of  $\theta_3$ , the share of agent type 3 in the population, defined as follows:

$$\theta_3 = \frac{(c_3 - c_1) + u(1 - \theta_2)}{u}.$$

If storage costs were equal ( $c_3 = c_1$ ), the optimal share of type 3 agents (who produce the commodity money) would be just equal to its maximum possible, given the share of agents of type 2. If money is more costly to store, however, its share must be reduced by a quantity  $(c_3 - c_1)/u$ . If it is less costly to store, the optimal quantity, from the standpoint of middlemen, would even be larger than the maximum feasible share. It is therefore not possible to identify an optimal quantity of (commodity) money, at least in a paretian sense, in view of these considerations.

Also in the case of the second, speculative equilibrium, it can be shown that steady state welfare levels are not monotone in the quantity of money.

When the economy features fiat money the difficulty becomes even bigger, as it is usually assumed that agents holding (fiat) money cannot at the same time hold (or produce) any commodity.

In their analyses, Kiyotaki and Wright computed the welfare level ( $W$ ) which can be obtained by introducing fiat money in the economy, and this is shown to be a function of real balances, ( $S$ ). The authors show that

$$\frac{\partial W_i}{\partial S} > 0, \quad \forall i$$

provided net utilities,  $u_i$  are not too large<sup>10</sup>. The positive relationship between  $W_i$  and  $S$  is rather intuitive, once we realize that fiat money reduces the inefficiency deriving from the other commodities' larger storage costs.

However, since the introduction of fiat money (commodity  $\theta$ ) also reduces the quantity of real commodities, and therefore aggregate consumption, in order to obtain a net increase in overall welfare it has to be the case that utilities  $u_i$  is not too large compared to  $c_{ij}$ <sup>11</sup>.

Fiat money, therefore, might allow the achievement of a higher level of welfare, which is, in a sense, in sharp contrast with Menger's position, who strongly opposed the introduction of money by legal measures, and who deeply believed that worthless commodities might have never played a monetary role.

Moreover, with fiat money it is more frequent to find multiple equilibria (see Kiyotaki and Wright, 1993), and a legitimate, and related, question is whether one equilibrium may dominate another in a Paretian sense, given a fixed quantity  $M$  of fiat money in the economy. In their later contributions on monetary equilibria (1991, 1993), Kiyotaki and Wright showed that the welfare functions of the various types of agents (the welfare of agents of type  $\theta$  is not graphed, as it is collinear with type 1's welfare) in the economy may be represented by the following graphs:

< figure 3 >

Where subscripts  $l, m$  respectively denote agents with commodities and money, and  $W = N_0V_0 + N_1V_1 + N_mV_m$  represents ex-ante expected utility of all agents before the actual distribution of output and money.

As the graph makes clear, the quantity of fiat money which maximizes ex-ante expected utility does not coincide with the levels of money maximising different types's welfare (those

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<sup>10</sup> For the proof of this result cfr. Kiyotaki-Wright (1989), op. cit., Appendix, p. 952.

<sup>11</sup> The fundamental role of fiat money should be remarked. Welfare depends, in fact, only on real balances,  $S$ , and not on nominal quantity of money,  $M$ .

levels being, let us note in passing, generically different). To sum up, it seems safe to say that, also in this case, there does not exist an ex-ante optimal quantity of fiat money.

From a normative standpoint, therefore, money does not play in these models the crucial role it has in alternative models of a monetary economy, which is why authors observe, rightfully, that “ [...] *there are many unanswered questions and much work remains to be done* , we think that these search-theoretic models have definitely enhanced our understanding of the exchange process, in general, and of money, in particular” (Kiyotaki e Right, 1993, p. 75).

This remark is not particularly new, however, as it was, in its essence, already present in Keynes' General Theory (and beyond) (Garretsen H., year, pp. 135 ss.).

## **11. Concluding remarks**

In the previous sections we tried to provide a detailed and critical account of some of the most relevant contributions to the theory of monetary equilibria which have appeared in the literature of the last few decades, and have focussed upon the importance of transaction costs in determining the emergence of money as a means of exchange.

Our account, however, is only a part of the story, albeit an important one. There are still many aspects of the process leading to monetary economies which cannot really be accounted for by only looking at transaction or searching costs. In particular, in the models we have examined it is completely impossible to introduce any form of endogenously generated money, such as the one constituted by debit-credit relationships. The introduction of inside money requires some form of dynamics, which was essentially absent from the models we analysed. Once we explicitly allow for an intertemporal structure with sequential exchanges, the concepts of opportunity costs and information costs are bound to play an important role, and have to be properly included.

One more important limitation of the approach we have reviewed is the *de facto* absence of production. The relationships between money and the organization of productive processes are not covered at any depths, and will possibly require complex extensions of the search theoretic paradigms, or totally different formalizations altogether. Equally absent is the

analysis of the role of financial intermediaries, such as banks, in determining monetary equilibria. Monetary policy effects, therefore, cannot be properly and thoroughly analysed in such contexts which, let us stress once more, turned out extremely helpful in developing a few building blocks of monetary theory, namely those related to the concepts of liquidity and saleability.

Figure 1

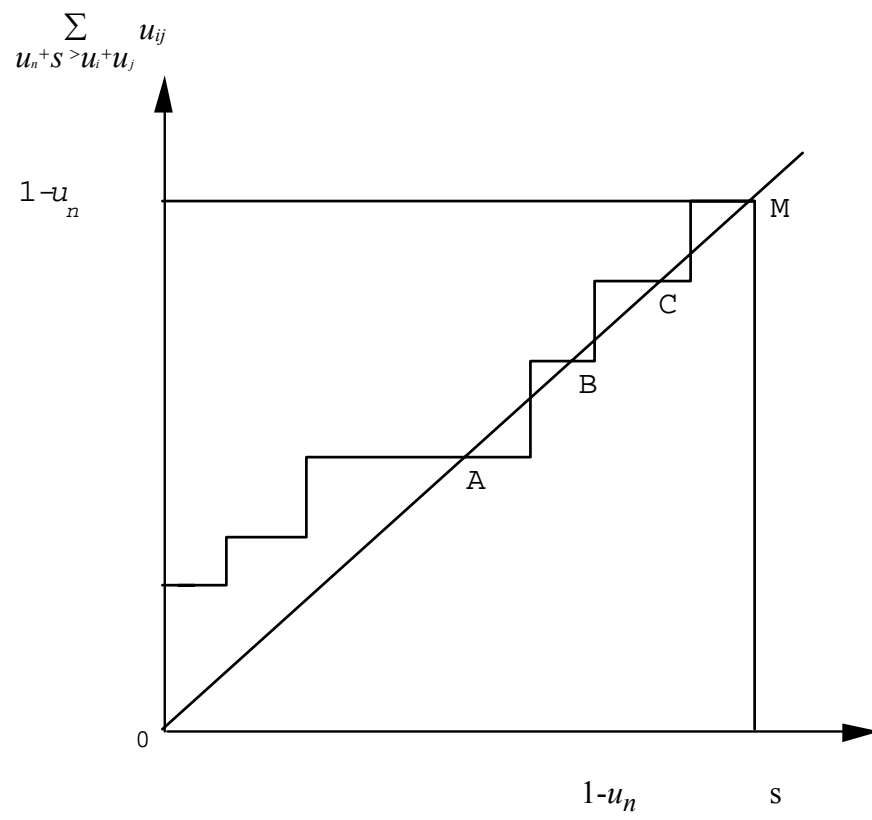


Figure 2

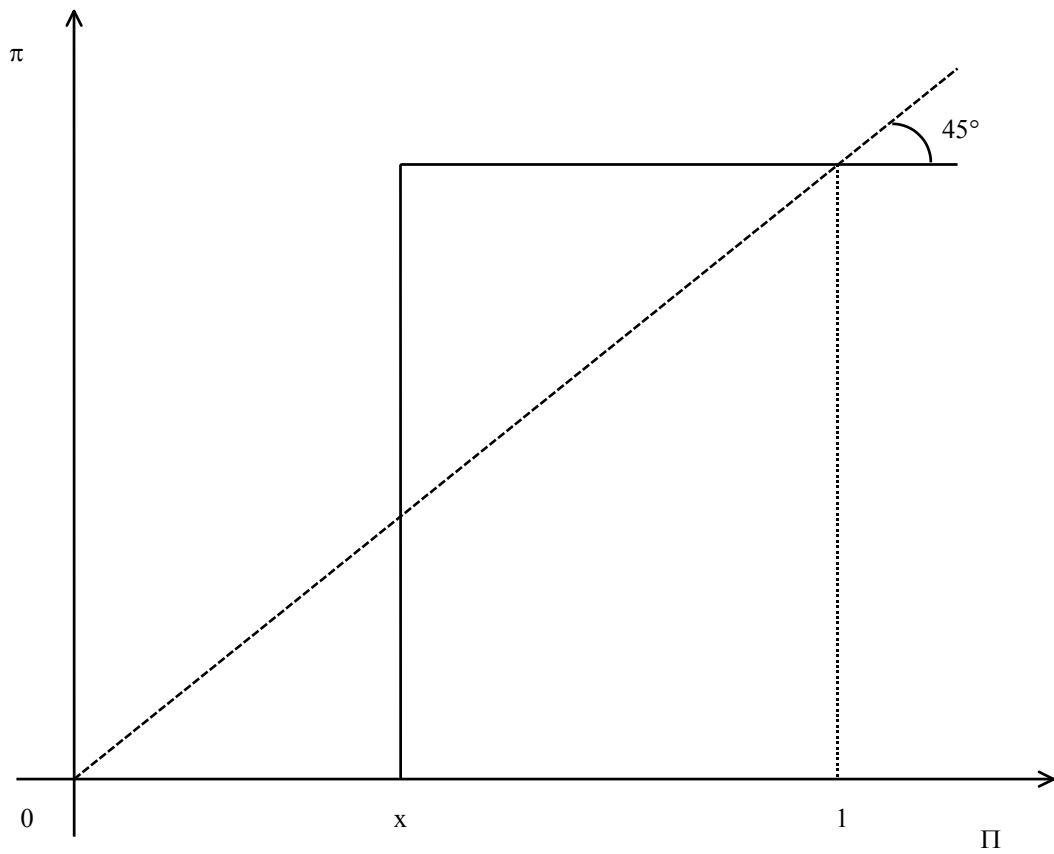
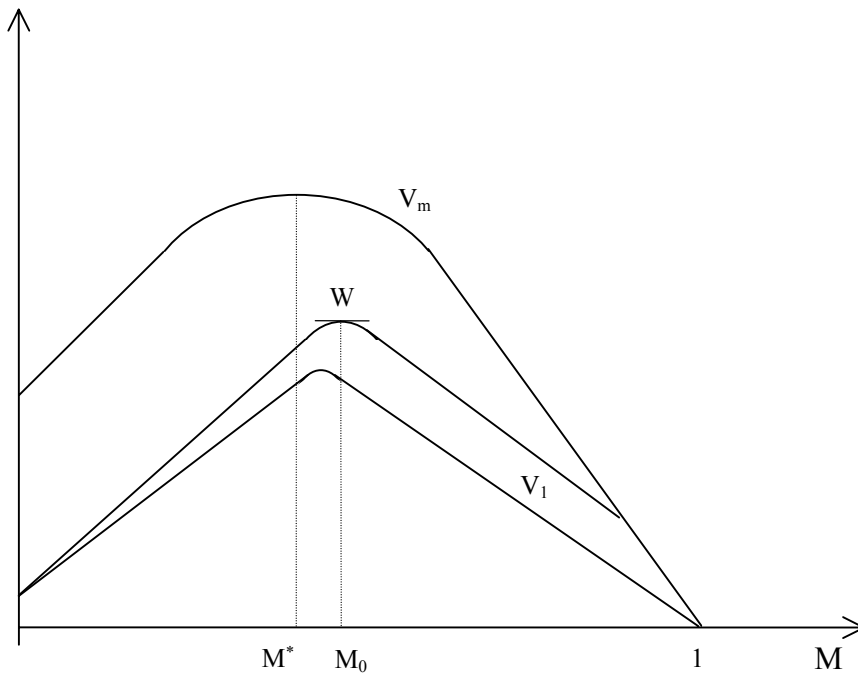




Figure 3



(Kyiotaki and Wright, 1993) .

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