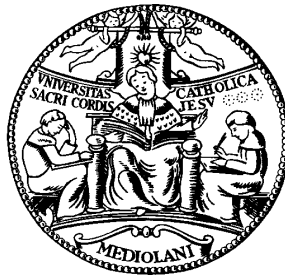UNIVERSITA' CATTOLICA DEL SACRO CUORE
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Collusion when the Number of Firms is Large

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Abstract

In antitrust analysis it is generally agreed that a small number of firms operating in the industry is an essential precondition for collusive behavior to be sustainable. However, the Italian Competition Authority (AGCM) challenged this view in the recent case *RCA* (2000), when an information exchange among forty-four firms in the car insurance market was assessed as having an anticompetitive object. The AGCM's basic argument was that an information exchange facilitates collusion because it changes the market environment in such a way as to relax the incentive compatibility constraint for collusion, thus circumventing the decrease in the critical discount factor when the number of firms in the industry increases.

In this paper we model collusive behavior in a "dispersed" oligopoly. We prove that, when the technology exhibits decreasing returns to scale, collusion can always be sustained, regardless of the number of firms, provided the marginal cost function is sufficiently steep. Moreover, we show how an information exchange can sustain collusive behavior when the number of firms is "large" independently of the assumptions on technology.

JEL codes: L41, L13, L11, K21

Keywords: Collusion; Industry structure; Facilitating practices

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1   Introduction

In antitrust analysis, it is standard view to link the likelihood of collusive behavior to a "structural" index, such as the number of firms in the industry. Such presumption is well-rooted in the theory of industrial organization, as the analysis of the incentive compatibility constraint for collusion shows that the "critical discount factor" always becomes smaller as the number of firms in the industry increases. In accordance with this analytical result, Motta (2004) lists the number of firms as the first, and perhaps the most important, among the "factors that facilitate collusion" though the author is careful in specifying "other things being equal".

In the antitrust case law, a concentrated oligopoly has been usually held to be a necessary condition whenever an explicit hard-core agreement is absent and the antitrust agency has to evaluate the circumstantial evidence of a collusive behavior. A typical instance is information exchange.[1] When assessing as an anticompetitive agreement the information exchange in *UK Tractors* (1992), the European Commission explicitly took account of the high concentration in the market of agricultural tractors in the UK,[2] a view which was upheld by both the Tribunal of First Instance and the European Court of Justice.[3] An even sharper view was expressed in *Wirtschaftsvereinigung Stahl* (1998) whereby the

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[1]For an economic assessment of information exchanges in the antitrust perspective, see Kühn (2001).

[2]The eight companies that participated in the agreement held 88% of the UK tractor market. The first four companies shared 77% of the market, 80% after Ford New Holland was taken over by Fiat.

[3]See European Commission, *UK Agricultural Tractor Registration Exchange*, 1992; Tribunal of First Instance, Judgements of 27 October 1994 in case T-35/92 *John Deere* and T-34/92 *Fiatagri and Ford New Holland*; European Court of Justice, Cases C-7/95 *John Deere Ltd. V. EC Commission*, 1998.

Commission stated that "*. . . the assessment of the exchange is directly linked with the degree of concentration of the market. . .*". In the same vein, the U.S. Supreme Court's *Container Corporation* decision (1969) found the exchange of information among competitors in a highly concentrated industry to be unlawful.[4]

In the Italian case *RCA*,[5] which assessed as anticompetitive an information exchange in the car insurance market, firms resisted by pointing out that, in contrast with *UK Tractors*, the relevant market could not be defined as a "concentrated oligopoly", since a "large" number of firms, namely forty-four, was involved in the contested agreement. In contrast with the conventional view, the Italian Competition Authority argued however that the information agreement had an anticompetitive object because it allowed the incentive constraint being satisfied in the indefinitely repeated game in which, absent the information exchange, collusive behavior would not have arisen as a noncompetitive (Nash) equilibrium, given the structural condition of "dispersed oligopoly". In fact, the information exchange relaxes the incentive constraint, by making punishment more effective, since the time interval for firms' reaction to a deviation from a collusive behavior becomes shorter. This is equivalent to saying that, for given values of firms' discount factor, the shorter the reaction time, the larger the number of firms for which the incentive constraint is satisfied.[6]

In this paper we provide a theoretical analysis of how collusion can be sustained, in equilibrium, in industries with a large number of firms. We show first that there always exist conditions under which the incentive constraint for collusion can be satisfied in an oligopoly, whatever the number of firms. We then

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[4] The U.S. approach to competitor communications is thoroughly reviewed by DeSanti and Nagata (1994).

[5] Autorità Garante della Concorrenza e del Mercato, *RC Auto*, 2000.

[6] The decision of the Italian Competition Authority was upheld both in the first and in the second instance judgements, whereby the above mentioned argument was explicitly confirmed.

argue that, in order to secure themselves that (tacit) collusion is attainable independently of the "structure" of the market, firms can exploit those conditions by means of a number of arrangements. We show that to organize an information exchange in order to "control" the reaction time to a deviation is just one of such arrangements. In a novel perspective, we characterize a simple setting whereby firms may "control" their marginal cost function with the purpose of raising its slope. Therefore we conclude that, provided that a finite upper bound can be set to the number of firms that make non-negative profits in the collusive solution (a condition for which a small sunk cost suffices), the likelihood of collusion in oligopoly can never be ignored by relying on the large number of firms only (as the structuralist approach would suggest).

The possibility of arrangements (like information exchanges and the control of the cost function) aimed at supporting collusive behavior, can be taken as a foundation for the notion of *facilitating practices* in antitrust law. A facilitating practice is a social "artifice", i.e., a mechanism that firms *artificially* design to change the market environment in such a way as to relax the incentive constraint for every firm to collude (see Grillo, 2002). In fact, in the US antitrust law a facilitating practice can be either used as circumstantial evidence from which a *per se* unlawful price fixing agreement may be inferred, or found to be unlawful under the *rule of reason* if the evidence shows that its anti-competitive effect outweighs any pro-competitive justification. In contrast, in the European antitrust law, a facilitating practice can be challenged as anticompetitive for its *object* whenever it is the result of an agreement.

We claim that the European perspective can generally be grounded on our analytical model. In contrast with the European case law (according to which a concentrated oligopoly is necessary for collusive behavior) but in accordance with the Italian case law, we also suggest that agreements on facilitating practices are

to be viewed as particularly relevant in dispersed oligopolies, as the anticompetitive object of a facilitating practice precisely is, to help circumvent the decrease in the critical discount factor when the number of firms in the industry increases.

The paper is organized as follows. In Section 2, simply elaborating on a textbook model, we show that, contrary to the conventional view, collusive behavior may also occur in oligopolies with a quite large number of firms. In Section 3, we study an industry with decreasing returns and prove that collusion can always be sustained, regardless of the number of firms, provided the marginal cost function is sufficiently steep. In Section 4, we show how information exchanges can sustain collusive behavior in a dispersed oligopoly independently of the assumptions on technology. Section 5 concludes.

2 Collusion can be sustainable in "dispersed" oligopolies even under a textbook approach

In Antitrust case Law, it is common to presume that a "concentrated" oligopoly is a necessary condition for collusion. It is not however clear what has to be meant by "concentrated" (or, in contrast, "dispersed") oligopoly when the likelihood of collusive behavior is at stake. For instance, Scherer and Ross (1990) state (at page 277) that "*As a very crude and general rule, if evenly matched firms supply homogeneous products in a well-defined market, they are likely to begin ignoring their influence on price when their number exceeds ten or twelve*".

However, Scherer and Ross' "crude rule", and the more general presumption in Antitrust Law, are in a sense unwarranted. This can easily be seen by analyzing the simple relationship between the discount rate, r, and the maximum number of firms, n^*, for which the incentive compatibility constraint is satisfied when firms

supply a homogeneous product under linear demand and linear cost functions.[7]

For plausible values of the discount rate under the hypothesis of a one-year detection period $(0,025 \leq r \leq 0,05)$, Table 1 shows that the maximum number, n^*, of firms will range between respectively twenty-one and forty-one, under price (i.e., Bertrand) competition and seventy-seven and one hundred and fifty-seven under quantity (i.e., Cournot) competition.[8]

Table 1

	n^* under Cournot competition	n^* under Bertrand competition
$r = 0,025$	157	41
$r = 0,05$	77	21

In the following sections we turn to a theoretical analysis of how collusion can be sustained in dispersed oligopolies. Section 3 analyzes a simple model of an industry where a large number of firms adopt a technology with decreasing returns, to show that collusion can be sustained provided the marginal cost function is

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[7]Under linear demand and cost functions, the incentive compatibility constraint is independent of both the demand and cost parameters, thus resulting in a simple relation that links n^* to r. Observe also that cost linearity is quite an unfavorable hypothesis to collusion as it implies that a firm intending to deviate from a collusive behavior can produce whichever quantity at the same marginal cost.

[8]Note that under Cournot competition, the incentive compatibility constraint for collusion binds when

$$r = 4n^*/\left(n^* + 1\right)^2,$$

whereas under Bertrand competition it binds when

$$r = 1/\left(n^* - 1\right).$$

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sufficiently steep. Section 4 considers a more general framework (that allows also for constant returns to scale) in which collusive behavior in a dispersed oligopoly can be sustained by means of an information exchange.

3 Collusion in 'dispersed' oligopolies: the case of decreasing returns

Consider an industry where a large number of identical firms i, $i \in \{1, ..., n\}$, produce a homogeneous good, using a technology with decreasing returns. In order to gain access to the industry, a firm has to sustain a sunk cost $A > 0$.

We assume a linear inverse market demand function

$$p = a - bQ, \tag{1}$$

$a > 0$, $b > 0$ and $Q = \sum_{i=1}^{n} q_i$, where Q denotes aggregate production and q_i is the quantity produced by the generic firm i.

We also assume that each firm faces a cost function of the type

$$C(q_i) = \alpha q_i^2, \tag{2}$$

with $\alpha > 0$.

To start with, we look for the existence of collusive equilibria in the industry when the indefinitely repeated game is built on the following trigger strategy: at stage zero each firm plays the collusion quantity q^C (as the game is symmetric q^C is equal to $1/n$-th of the quantity that a multi-plant monopolist, facing the cost function (2) at each plant, would produce) and then continues to play it provided that in the preceding stage all other firms have played the collusion quantity. If, in any period, a firm deviates from the collusive strategy — by playing a quantity q^D that is the best response to the other players producing q^C — a punishment phase in which all firms play the Cournot quantity q^P forever follows.

Let us calculate the quantity produced and the profits under collusion, deviation and punishment, respectively.

The collusion quantity q^C can be obtained as the solution of the following profit maximization problem

$$\max_{q_1,\ldots,q_n} \left(a - b \sum_{j=1}^{n} q_j \right) (q_1 + \ldots + q_n) - \left(\alpha \left(q_1^2 + \ldots q_n^2 \right) \right). \tag{3}$$

As the inverse market demand function is linear and each cost function strictly convex, first order conditions are necessary and sufficient for a maximum. From the first order condition the generic q_i is:

$$q_i^C \equiv q^C := \frac{a}{2 \left(\alpha + nb \right)}, \tag{4}$$

By substituting (4) into (1) and (2), it is immediate to see that each firm's profits under the collusive agreement (gross of the sunk cost A) are

$$\Pi_i^C = \frac{a^2}{4 \left(\alpha + nb \right)}. \tag{5}$$

Observe that $\partial \Pi^C / \partial n < 0$ and $\partial \Pi^C / \partial \alpha < 0$.

It is also straightforward to see that there is a maximal number of firms in the industry, $\bar{n}$, that earn a stream of collusion profits the present value of which net of the sunk cost A is non-negative, i.e.

$$\frac{1}{1 - \delta} \Pi_i^C \left(\alpha, \bar{n} \right) = A,$$

which implies,

$$\bar{n} = \frac{1}{b} \left(\frac{a^2}{4A \left(1 - \delta \right)} - \alpha \right).$$

The quantity q_i^D produced by firm i that deviates from the collusive equilibrium when the other $n - 1$ firms stick to it can be obtained as the solution of the following problem

$$\max_{q_i} \left(a - b q_i - \frac{ab \left(n - 1 \right)}{2 \left(\alpha + nb \right)} \right) q_i - \alpha q_i^2. \tag{6}$$

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From the first order condition and after some algebra

$$q_i^D = \frac{a\left(2\alpha + (n+1)\, b\right)}{4\left(\alpha + b\right)\left(\alpha + nb\right)} = q_i^C \cdot \frac{2\alpha + (n+1)\, b}{2\left(\alpha + b\right)} \tag{7}$$

and

$$\Pi_i^D = \frac{a^2\left(2\alpha + (n+1)\, b\right)^2}{16\left(\alpha + b\right)\left(\alpha + nb\right)^2}, \tag{8}$$

where Π_i^D are the deviation profits gross of the sunk cost A. Notice that $\partial \Pi^D/\partial n < 0$ and $\partial \Pi^D/\partial \alpha < 0$. Notice also that Π_i^D can be written as

$$\Pi_i^D = \Pi_i^C \cdot H\left(n; \alpha\right), \tag{9}$$

where

$$H\left(n; \alpha\right) := \frac{\left(2\alpha + b\left(n+1\right)\right)^2}{4\left(\alpha + nb\right)\left(\alpha + b\right)}. \tag{10}$$

Simple algebra shows that, for all α, $H\left(n; \alpha\right) > 1$ for $n \geq 2$. Moreover, for $n \geq 2$, $\partial H\left(n; \alpha\right)/\partial \alpha < 0$ and $\partial H\left(n; \alpha\right)/\partial n > 0$. As $\partial \Pi_i^C/\partial \alpha < 0$, the deviation profits Π_i^D decrease faster than Π_i^C when α increases. By using (4) and (7) it is also easy to see that the difference between the quantity produced by the deviating firm and the one supplied under the collusive agreement is increasing in the number of firms in the market, i.e. $\partial \left(q_i^D - q_i^C\right)/\partial n > 0$.

In the (permanent) punishment phase following deviation, each firm reverts to the Nash equilibrium strategy of the constituent game, producing the Cournot quantity q^P, which can be obtained as the solution of the problem

$$\max_{q_i} \quad \left(a - bq_i - b\sum_{j \neq i} q_j\right) q_i - \alpha q_i^2, \tag{11}$$

implying that

$$q_i^P = \frac{a}{2\left(\alpha + b\right) + \left(n-1\right) b} \tag{12}$$

and

$$\Pi_i^P = \frac{a^2\left(\alpha + b\right)}{\left(2\alpha + b\left(n+1\right)\right)^2}, \tag{13}$$

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where Π_i^P are the Cournot profits gross of the sunk cost A.

By comparing (5), (8) and (13) one can immediately see that, for $n \geq 2$ and for $\alpha > 0$, $\Pi_i^D > \Pi_i^C > \Pi_i^P$.

For a given discount factor δ, the collusive agreement can be sustained if and only if the *incentive compatibility constraint* (IC) faced by each firm is satisfied. Focusing again on firm i, IC can be written as

$$\frac{1}{1-\delta}\Pi_i^C - A \geq \Pi_i^D + \frac{\delta}{1-\delta}\Pi_i^P - A, \tag{14}$$

and, rearranging,

$$\delta \geq \frac{\Pi_i^D - \Pi_i^C}{\Pi_i^D - \Pi_i^P}.$$

Recalling that $\delta = \frac{1}{1+r}$, where r denotes an appropriate discount rate, the above inequality can also be written as

$$r \leq \frac{\Pi_i^C - \Pi_i^P}{\Pi_i^D - \Pi_i^C}. \tag{15}$$

Observe that, within the specific setting of this paper, $H(n;\alpha)$ has a nice interpretation, as it equals not only Π_i^D/Π_i^C (see Expression (9) above), but also Π_i^C/Π_i^P. Thus, the incentive compatibility constraint (15) simply reduces to

$$H(n;\alpha) \cdot r \leq 1. \tag{16}$$

As $H(n;\alpha) > 1$ for all α and $n \geq 2$ it straightforwardly follows from (16) that

Lemma 1 $r < 1$ *is necessary for collusion.*

We are now ready to state the following Proposition, which is qualitatively illustrated in Figure 1.

Proposition 1

Let $r \in (0,1)$. For all n, $2 \leq n \leq \bar{n}$, define $\bar{H}(n) := H(n;0)$.

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1. *If $r \leq 1/\bar{H}(n)$ then $H(n;\alpha) \cdot r < 1$ for all $\alpha > 0$;*

2. *if $1/\bar{H}(n) < r < 1$, then there exists a unique $\alpha^*(n) > 0$ such that, for all $\alpha \in [\alpha^*(n), +\infty)$ it is $H(n;\alpha) \cdot r \leq 1$ and $H(n;\alpha^*(n)) \cdot r = 1$.*

3. *$\alpha^*(n)$ is increasing in n.*

Proof. By inspection of Equation (10), one can see that $\bar{H}(n) > 1$ for all $n \geq 2$. The first claim follows immediately since $H(n;\alpha)$ is a continuous and monotonically decreasing function in α. To show the second claim consider also that $\lim_{\alpha \to +\infty} H(n;\alpha) = 1$ for all n.

To show that $\alpha^*(n)$ is increasing in n, let $\Phi(n;\alpha^*,r) = H(n;\alpha^*) \cdot r - 1$. By applying the implicit function theorem

$$\frac{d\alpha^*}{dn} = -\frac{\partial\Phi/\partial n}{\partial\Phi/\partial\alpha} = -\frac{\partial H/\partial n}{\partial H/\partial\alpha}.$$

As $\partial H(n;\alpha)/\partial\alpha < 0$ and $\partial H(n;\alpha)/\partial n > 0$, it is $d\alpha^*/dn > 0$. ∎
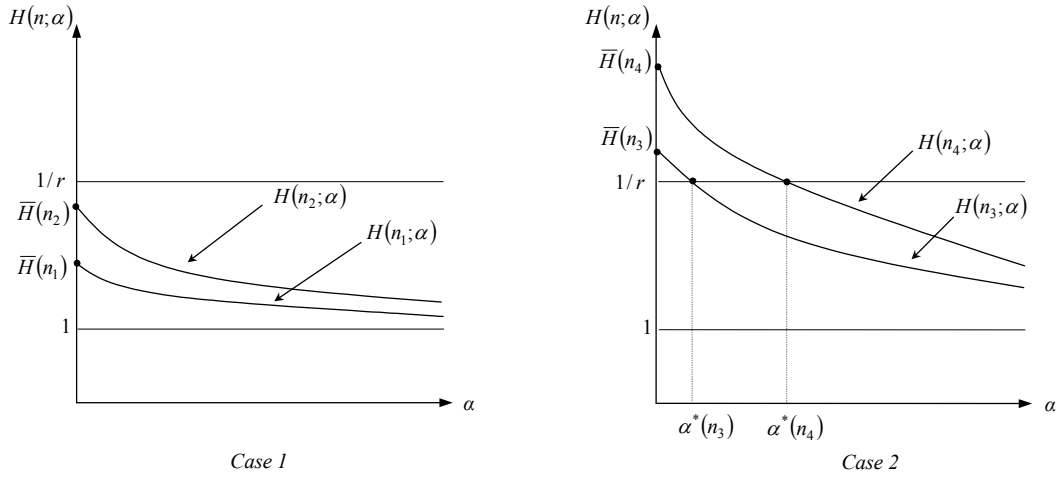

Figure 1: An illustration of Proposition 1, where $n_4 > n_3 > n_2 > n_1$

For all finite number of firms that earn non-negative collusive profits (net of the sunk cost A to gain access to the market), Proposition 1 says on the one

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hand that the incentive compatibility constraint is met for all α and the collusive agreement is always implementable, provided r is sufficiently low (*Case 1* in Figure 1). On the other hand, for higher values of r (*Case 2* in Figure 1), there always exists a cost parameter α, increasing in the number of firms operating in the market, such that the collusive agreement is sustainable.[9]

Proposition 1 has been derived from a model whereby the punishment phase is based on infinite reversion to the Cournot-Nash equilibrium. The results obtained are, however, of greater generality, as they hold for "optimal" punishment strategies as well.[10]

To better illustrate the point, write the incentive compatibility constraint (14) as

$$\frac{1}{1-\delta}\Pi_i^C - A \geq \Pi_i^D + V_i^P - A, \tag{17}$$

where $V_i^P \geq 0$ denotes the present value of the stream of profits in the punishment phase. As the cost function exhibits decreasing returns, when the punishment phase is based on infinite reversal to the Cournot-Nash equilibrium, $V_i^P > 0$ independently of n. Under optimal punishment strategies, the most severe admissible punishment implies $V_i^P = 0$ (any punishment phase implying $V_i^P < 0$ would violate the individual rationality constraints of firms, as they always have the outside option of leaving the market in order to cut losses). Under the limit case $V_i^P = 0$, the incentive compatibility constraint (15) becomes

$$r \leq \frac{\Pi_i^C}{\Pi_i^D - \Pi_i^C},$$

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[9]For $n \geq 2$, note that $d\bar{H}(n)/dn > 0$. Hence, for given r, the second case in the proposition is more likely to be the relevant one when the number of firms in the industry becomes larger.

[10]Following Abreu (1986), "optimal" punishment schemes yield the smallest critical discount factor, and hence maximize the scope for collusive behavior.

which, recalling the definition of $H(n;\alpha)$, can in turn be written as[11]

$$(H(n;\alpha) - 1) \cdot r \leq 1. \tag{18}$$

It is straightforward to see, first that Inequality (18) always holds for all pairs (n, α) for which Proposition 1 is true. Second, there always exists a pair (n, α) such that (18) holds for all r.[12] Finally, for all r such that $H(n;0) - 1 > 1/r$, let $\breve{\alpha}(n)$ solve $(H(n; \breve{\alpha}(n)) - 1) \cdot r = 1$. Then, by applying the implicit function theorem, one obtains

$$\frac{d\breve{\alpha}}{dn} = -\frac{\partial H/\partial n}{\partial H/\partial \alpha} > 0,$$

which is exactly the same expression obtained when a punishment scheme based on the Cournot-Nash reversion is adopted.

Quite naturally, whether firms resort to a trigger strategy or to an optimal penal code has an impact on the maximum number of firms at which collusion can be sustained. In particular, given r and α, since $H(n;\alpha)$ is increasing in n, the incentive compatibility constraint under the (harshest) optimal penal code binds at $(H(n;\alpha) - 1) \cdot r = 1$ for a larger number of firms than that at which the incentive compatibility constraint under the Cournot-Nash reversion is binding (i.e. $H(n;\alpha) \cdot r = 1$). This means that, for given r and α, the maximum number of firms for which a collusive behavior can be sustained is always lower under the Cournot-Nash reversion than under an optimal penal code. Hence, in the proof

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[11]When $V_i^P = 0$, recalling that $H(n;\alpha) = \Pi_i^D/\Pi_i^C$, the incentive compatibility constraint (17) can be written as

$$\frac{1}{1-\delta}\Pi_i^C \geq H(n;\alpha) \cdot \Pi_i^C$$

from which, since $\delta = \frac{1}{1+r}$, we obtain

$$r \leq \frac{1}{H(n;\alpha) - 1}.$$

[12]This follows from $\lim_{\alpha \to +\infty} H(n;\alpha) = 1$ for $n \geq 2$.

of Proposition 1, we are put in the weakest position to show how collusion can be sustained with a large number of firms.

Proposition 1 has a clear interpretation in terms of the firms' ability to sustain collusion. When, given r and α, the number of firms in an oligopoly appears to be too "high" for the incentive compatibility constraint to be met, then an "artifice" — in our model, a distortion of the cost function — is always open to the firms in order to facilitate collusion. The possibility of "artifices" aimed at supporting collusive behavior can be taken as a foundation for the notion of *facilitating practices* in antitrust law. A facilitating practice is a mechanism that firms *artificially* design to change the market environment in such a way as to relax the incentive compatibility constraint for every firm to collude.

The control of the cost function is just one of such arrangements. As already known from competition policy literature, information exchanges play an analogous role (see Kühn, 2001). An information exchange can be easily accommodated in our analytical setting as follows. Consider that the discount rate r crucially depends on the length of the time interval that elapses before all (other) firms in the industry can react to a firm's deviation from the collusive behavior. An information exchange critically reduces this time interval. As can be easily seen from (16), for any n and α there always exists a value of r, $r > 0$, such that the incentive compatibility constraint is met. In fact, the possible role of an information exchange as a practice that facilitates collusion is independent of any hypothesis about the technology. Hence, we briefly discuss it in Section 4 in a more general setting, where we also turn to continuos time to better illustrate our argument.

4 Information Exchange

Consider an industry with n firms. In a more general perspective than the one adopted in Section 3, we impose no restrictions on the demand function and the firms' technology, and denote generically with Π_i^C, Π_i^D and Π_i^P, respectively, the collusion, deviation and punishment profits of firm i in every single period of conventionally fixed length 1. In addition, we assume that a firm can deviate from the collusive behavior, without being detected by the other firms, for T periods, where T can take any positive real value. To develop our analysis in continuous time, we then rewrite the profits made by a firm in every single period, under the three different contexts, in terms of the implicit "instantaneous" profits and the instantaneous discount rate $\rho > 0$. Looking without loss of generality at the profits under collusion, Π_i^C, we have

$$\Pi_i^C = \int_0^1 \omega_i^C e^{-\rho t} dt,$$

where ω_i^C denotes the instantaneous collusion profits of firm i. Solving the integral for ω_i^C, we obtain

$$\omega_i^C = \frac{\rho}{1 - e^{-\rho}} \Pi_i^C.$$

In the same way, we can write:

$$\omega_i^D = \frac{\rho}{1 - e^{-\rho}} \Pi_i^D$$

and

$$\omega_i^P = \frac{\rho}{1 - e^{-\rho}} \Pi_i^P.$$

The sustainability of collusive behavior requires the following continuos time version of the incentive compatibility constraint for collusion (14) in Section 3 to be met

$$\int_{t=0}^{+\infty} \omega_i^C e^{-\rho t} dt \geq \int_{t=0}^{T} \omega_i^D e^{-\rho t} dt + \int_T^{+\infty} \omega_i^P e^{-\rho t} dt. \tag{19}$$

By solving the integrals, (19) can be rewritten as

$$e^{\rho T} \leq \frac{\Pi_i^D - \Pi_i^P}{\Pi_i^D - \Pi_i^C}. \tag{20}$$

We can now state the following proposition.

Proposition 2 *For all $n \geq 2$, there exists $T^* > 0$ such that the incentive compatibility constraint for collusion (20) holds with inequality for $0 < T < T^*$ and with equality for $T = T^*$.*

Proof. For all $n \geq 2$, $\Pi_i^D > \Pi_i^C > \Pi_i^P$. Therefore, the right hand side of Inequality (20) is always greater than one. The claim straigthforwardly follows from the fact that $e^{\rho T}$ is continuous and monotonic in T and $e^{\rho T} \longrightarrow 1$ for $T \longrightarrow 0$. ∎

5 Concluding Remarks

It is a tenet in competition policy that a small number of firms is an essential pre-requisite to sustain collusive behavior in an industry.

In this paper we contrast this view, showing that collusion can be sustained in "dispersed" oligopolies as well. When the technology exhibits decreasing returns to scale, collusion can always be sustained, regardless of the number of firms, provided the marginal cost function is sufficiently steep. Moreover, an information exchange can sustain collusive behavior when the number of firms is "large" independently of the assumptions on technology.

We interpret our results by arguing that, when the number of firms is too high for the incentive compatibility constraint to be met, then an "artifice" — such as a distortion of the cost function or an information exchange — is always open to the firms in order to facilitate collusion. Artifices aimed at supporting

collusive behavior pave the way for a theoretical analysis of the notion of *facilitating practices* in antitrust law: namely, mechanisms that firms *artificially* design to change the market environment in such a way as to relax the incentive compatibility constraint for collusion.

References

Abreu D.J. (1986). "Extremal Equilibria of Oligopolistic Supergames", *Journal of Economic Theory*, 39, 191-225.

DeSanti S. and E. Nagata (1994). "Competitor Communications: Facilitating Practices or Invitations to Collude? An Application of Theories to Proposed Horizontal Agreements Submitted for Antitrust Review", *Antitrust Law Journal*, 63, 93-131.

Grillo M. (2002). "Collusion and Facilitating Practices: A New Perspective in Antitrust Analysis", *European Journal of Law and Economics*, 14, 151-169.

Kühn K. (2001). "Fighting Collusion by Regulating Communication Between Firms", *Economic Policy*, 32, 169-204.

Motta M. (2004). *Competition Policy*, Cambridge University Press, Cambridge.

Scherer F. and Ross D. (1990). *Industrial Market Structure and Economic Performance*, Houghton Mifflin Company, Boston.

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