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The Social Value of Public Information
with Costly Information Acquisition

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# The Social Value of Public Information with Costly Information Acquisition*

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**Abstract**

In a beauty contest framework, we consider a Stackelberg game in which public authorities decide the accuracy of public information taking into account how it affects the acquisition of private information and the choice of individual actions in equilibrium. We find that, irrespective of the strength of the beauty contest motive, an increase in the precision of public information increases welfare when its marginal cost is not higher than that of private information. In this case, a more precise public information, by reducing the incentives for acquisition of private information, induces socially valuable savings of private resources.

**Keywords:** Public information, private information, coordination, welfare

**JEL classification:** C70, D82, E10

## 1 Introduction

In a highly debated paper, Morris and Shin (2002) have shown that public information may have a detrimental effect on welfare in a beauty contest framework. In the same

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context, Cornand and Heinemann (2007) argue that social welfare rises when more precise public information reaches only a fraction of market participants, while Svensson (2006) questions the empirical plausibility of Morris and Shin's result. Angeletos and Pavan (2004) show that public information is welfare improving in presence of positive investment spillovers. In a monopolistic competition framework, Hellwig (2005) finds that it can be socially valuable to disregard some private information, as firms partly neglect their contribution to aggregate risk. Angeletos and Pavan (2007) develop a unifying framework where the impact of public information on welfare depends on the degree of complementarity or substitutability among agents' actions. Finally Hellwig and Veldkamp (2006), focusing on optimal individual information choices, show that strategic complementarities in actions induce coordination motives in private information acquisition.

In this note we investigate the welfare effects of public information when both private and public information are endogenous. We consider the beauty contest model examined in Morris and Shin (2002), but we assume that private agents choose the precision of their private information after observing the precision of public information set by a public authority. Such modeling of the information acquisition process is consistent with the widespread idea that private agents are typically faster than public authorities in adjusting the accuracy of their information.

The public authority acts as a Stackelberg leader, who optimally exploits the fact that an increase in the precision of public information reduces the incentives for acquisition of private information, thereby inducing socially valuable savings of private resources. The negative welfare effect of the reduction in the precision of private information is, in fact, more than compensated by the corresponding cost-saving effect. Our main result shows that a more precise public information is welfare enhancing when the marginal cost of public information is not higher than that of private information.

## 2 The Setup

The economy is populated by a continuum of agents indexed by the unit interval $[0, 1]$. Every agent observes noisy private and public signals about the fundamental $\theta \in \mathbb{R}$, so that information is incomplete. We consider a two periods setting. In period $-1$, agents observe the state of the economy ($\theta_{-1}$), then a public authority acting as a benevolent social planner chooses the precision of next period public information by maximizing the sum of individual expected utilities, and finally private agents decide how much to invest in the precision of their private signals. In period 0, every agent receives her signals (public and private), and chooses an action affecting her utility as well as that of the other individuals.

The state of the economy evolves according to the stochastic process

$$\theta = \theta_{-1} + \chi.$$

The shock $\chi$ (normally distributed with mean zero, variance $\sigma_\theta^2$, and hence precision $\alpha_\theta \equiv \sigma_\theta^{-2}$) occurs at the beginning of period 0. All agents – private and public – have a common *ex ante* expectation $\theta_{-1}$ of the state variable $\theta$, which allows us to compute ex ante expected utilities, and hence welfare, on a common stand-point.

At the beginning of period 0, every agent $i$ receives a public signal, $y$, and a private signal, $x_i$:

$$y = \theta + \eta,$$

$$x_i = \theta + \varepsilon_i,$$

where $\eta$ is normally distributed, independent of $\theta$, with mean zero and precision $\alpha_y$, and the noise terms $\varepsilon_i$ are normally distributed, independent of $\theta$, $\eta$, and $\varepsilon_j$ $(j \neq i)$, with mean zero and precision $\beta_i$. Note that $y$ is common knowledge to all agents, whereas $x_i$ is specific to each agent $i$ and not observable by the others. The precision of the private signal may vary across agents.

The common posterior for $\theta$, given public information, is normally distributed with mean $E(\theta|\, y) = \frac{\alpha_\theta \theta_{-1} + \alpha_y y}{\alpha_\theta + \alpha_y}$, and precision $P(\theta|\, y) = \alpha_\theta + \alpha_y$. We define $z \equiv E(\theta|\, y)$, and $\alpha \equiv \alpha_\theta + \alpha_y$. Private posteriors are normally distributed, with mean $E(\theta|\, y, x_i) = \frac{\alpha z + \beta_i x_i}{\alpha + \beta_i}$ and precision $P(\theta|\, y, x_i) = \alpha + \beta_i$. It is easy to show that in our setting the definition of precision coincides with the notion of 'accuracy' of agents' forecasts introduced by Angeletos and Pavan (2007).

Agent $i$'s payoff function is

$$u_i\,(a, \theta, \beta_i) \equiv -\,(1-r)\,(a_i - \theta)^2 - r\left(L_i - \bar{L}\right) - C\,(\beta_i) - T_i, \tag{1}$$

where $a$ is the action profile over all agents, $a_i \in \mathbb{R}$ denotes agent $i$'s action, $\theta$ represents the state of the economy, $r \in (0,1)$,

$$L_i \equiv \int_0^1 (a_j - a_i)^2 \, dj, \qquad \bar{L} \equiv \int_0^1 L_j dj,$$

$C(\beta_i)$ is the cost of choosing precision $\beta_i$, and $T_i$ is a lump-sum tax used by the public authority to finance the acquisition of public information. Agents acquire independent signals in a market where the price for a unit of precision is $p$.[1] Hence:

$$C(\beta_i) \equiv p \cdot \beta_i.$$

Observe that the first two terms in Equation (1) coincide with agent $i$'s payoff function in Morris and Shin (2002), while the third and fourth terms capture the costs of information acquisition. Note also that the loss $L_i$ increases in the distance between agent $i$'s action and the action profile of the whole population. Hence, $r$ captures the weight of a beauty contest externality: the larger $r$ is, the more important the second guessing motive for each agent is.

# 3 The Equilibrium

The equilibrium is solved by backward induction. In period 0, for given signal precisions, agent $i$ chooses action $a_i$ by maximizing $E(u_i(a, \theta, \beta_i) \mid y, x_i, \theta_{-1})$, which gives

$$a_i = (1-r) E_i(\theta) + r E_i \left( \int_0^1 a_j \cdot dj \right). \tag{2}$$

As in Morris and Shin (2002) and Angeletos and Pavan (2004), we construct a linear equilibrium in which

$$a_j = \gamma z + (1-\gamma) x_j, \tag{3}$$

where $\gamma$ is a parameter being determined in equilibrium.

[1] The assumption of a constant price per unit of precision seems natural. With Gaussian disturbances, agents who have access to several independent signals can combine them in a sufficient statistic, the precision of which is the sum of those of the original signals. Accordingly, the cost of the precision for the 'aggregate' signal is the sum of the prices of the signals, and the relation between precision and its cost is linear. This induces the information providers to opt for a pricing policy that is linear in precision, i.e. for a constant unit price of precision.

Assuming that all agents other than $i$ follow the strategy given in (3), and that they all choose the same precision of information $\beta = \beta_j$ at $t = -1$, we then have that

$$\int_0^1 a_j \cdot dj = \gamma z + (1-\gamma)\,\theta,$$

in which case agent $i$'s best response reduces to

$$\begin{aligned} a_i &= (1-r)\,E_i(\theta) + r\,(\gamma z + (1-\gamma)\,E_i(\theta)) = \\ &\quad (1-r\gamma)\,E_i(\theta) + r\gamma z, \end{aligned} \tag{4}$$

where

$$E_i(\theta) = \delta_i z + (1-\delta_i)\,x_i, \tag{5}$$

with $\delta_i = \frac{\alpha}{\alpha+\beta_i}$.

Rearranging, (4) reduces to

$$a_i = \gamma_i z + (1-\gamma_i)\,x_i, \tag{6}$$

where

$$\gamma_i = (1-r\gamma)\,\delta_i + r\gamma. \tag{7}$$

Because agent $i$ has zero-measure, the equilibrium value of $\gamma$ when all agents $j$ other than $i$ choose a precision $\beta_j = \beta$ is the same as in Morris and Shin (2002), and given by

$$\gamma = \frac{\delta}{1-r\,(1-\delta)}, \tag{8}$$

with $\delta = \frac{\alpha}{\alpha+\beta}$. Simple algebra shows that $\delta$ is the correlation across agents' forecasting errors on $\theta$; hence it corresponds to the notion of 'commonality' in Angeletos and Pavan (2007).

Substituting for $\gamma$ from (8), Equation (7) can now be rewritten as

$$\gamma_i = \frac{(1-r)\,\delta_i + r\delta}{1-r\,(1-\delta)}. \tag{9}$$

Note that $\gamma_i = \gamma$ if $\delta_i = \delta$ (equivalently if $\beta_i = \beta$), and that $\gamma_i < \gamma$ if $\delta_i < \delta$: a higher precision of agent $i$'s private signal (i. e. a lower $\delta_i$) implies a smaller weight on the public signal.

Having characterized the equilibrium at time 0, we can now move backward studying agent $i$'s choice of $\beta_i$. From Equation (1), her expected utility at time $-1$ can be written as

$$E\left[u_i\left(a,\theta,\beta_i\right)\mid\theta_{-1}\right]=-\left(1-r\right)E\left[\left(a_i-\theta\right)^2\mid\theta_{-1}\right]- \tag{10}$$
$$+\,rE\left[\int_0^1\left(a_j-a_i\right)^2dj\mid\theta_{-1}\right]+rE\left[\int_0^1\int_0^1\left(a_h-a_j\right)^2dh\;dj\mid\theta_{-1}\right]-p\cdot\beta_i-T_i.$$

Using the results for period 0 obtained above, after some tedious algebra, (10) becomes:

$$E\left[u_i\left(a,\theta,\beta_i\right)\mid\theta_{-1}\right]=-\frac{\left(1-r\right)\left(\alpha+\beta\right)^2}{\left(\alpha+\left(1-r\right)\beta\right)^2}\left(\frac{1-r}{\alpha+\beta_i}+\frac{r\left(\alpha+\left(r-1\right)\beta\right)}{\left(\alpha+\beta\right)^2}\right)-p\cdot\beta_i-T_i. \tag{11}$$

Agent $i$ chooses $\beta_i\geq0$ so as to maximize (11). Because $E\left[u_i\left(a,\theta,\beta_i\right)\mid\theta_{-1}\right]$ is concave in $\beta_i$, the solution is given by the first order condition

$$\frac{\left(1-r\right)^2\left(\alpha+\beta\right)^2}{\left(\alpha+\left(1-r\right)\beta\right)^2\left(\alpha+\beta_i\right)^2}-p+\mu=0, \tag{12}$$

where $\mu\geq0$ is the Lagrange multiplier associated to the constraint $\beta_i\geq0$.

We need to consider two cases. When $p\geq\left(\frac{1-r}{\alpha}\right)^2$, the unique symmetric equilibrium is given by $\beta_i=0$ for all $i$, so that the expected utility reduces to

$$E\left[u_i\left(a,\theta,\beta_i\right)\mid\theta_{-1}\right]=-\frac{1-r}{\alpha}. \tag{13}$$

When instead $p<\left(\frac{1-r}{\alpha}\right)^2$, the unique symmetric equilibrium is

$$\beta_i=\beta=\frac{1}{\sqrt{p}}-\frac{\alpha}{1-r} \tag{14}$$

for all $i$.

Focusing on Equation (12), it is apparent that individual decisions to invest in private information acquisition are strategic complements, as in Hellwig and Veldkamp (2006). In fact, an increase in $\beta$ implies a higher accuracy but a lower commonality, since the precision of agents' forecasts on $\theta$ improves but the correlation of forecast errors across agents is reduced. Due to the lower commonality, private agents have less incentives to coordinate, as they give more prominence to their private information in forecasting other agents' decisions (one immediately sees from Equation (8) that $\gamma$ is decreasing in $\delta$ ). In a beauty contest framework, an agent not increasing the precision of her private signal would suffer from the reduction in coordination, without gaining anything in terms of improved accuracy. Hence, for such an agent a more precise private information becomes more valuable.

It is important to notice that, from (12), it also follows that the precisions of private and public information are strategic substitutes. An increase in $\alpha_y$ (and therefore in $\alpha$, since $\alpha = \alpha_\theta + \alpha_y$) implies both higher accuracy, and higher commonality. The increase in commonality induces stronger coordination, which in a beauty contest framework benefits every private agent. At the same time, due to the decreasing marginal utility of precision, the increase in accuracy makes individual precision less valuable. By inspection of (14), it follows that the crowding out effect of the precision of public information on the acquisition of private information becomes larger the stronger the beauty contest motive is (i. e. the larger $r$). In fact, an increase in public information has a coordinating power which is stronger the larger is the beauty contest motive. Such an increase reduces more than proportionally agents' willingness to purchase more precise private information.

# 4 Aggregate welfare

Having determined how the equilibrium depends on public information, we now turn to the impact of $\alpha_y$ on social welfare.

We assume that the cost of the precision of public information is linear and equal to $\tilde{p}$; moreover, to rule out the case in which it would be optimal not to acquire neither public nor private information, we assume that $p < [(1-r)/\alpha_\theta]^2$. The precision of the private information chosen by the agents in the first stage of the game is endogenous in that of the public signal set by the welfare-maximizing social planner in the second stage of the game. Several cases need to be investigated, depending on the choice of the precision of public information set by the planner, which in turn is affected by the comparison of the prices of public and private information.

Suppose first that the choice of the social planner in the second stage of the game implies a precision of the public signal sufficiently low to induce agents to acquire private information in the first stage of the game; i. e. $\alpha \leq (1-r)/\sqrt{p}$, so that $\beta \geq 0$

(see Eq. (14)). In this case, the ex ante utility is given by

$$E\left[u_i\left(a,\theta,\beta\right)\mid\theta_{-1}\right]=-\frac{(1-r)\left(\alpha+(1-r)^2\beta\right)}{\left(\alpha+(1-r)\beta\right)^2}-p\cdot\beta-\tilde{p}\cdot\alpha_y. \tag{15}$$

The social planner maximizes (15) subject to $\beta\geq 0$ and to (14), defining the optimal precision level chosen by private agents. The derivative of (15) with respect to $\alpha_y$ is given by:

$$\frac{dE\left[u_i\left(a,\theta,\beta\right)\mid\theta_{-1}\right]}{d\alpha_y}=\frac{(1-r)}{\left[\alpha+\beta(1-r)\right]^3}\cdot$$
$$\cdot\left\{\alpha+\beta(1-r)(1-2r)+\frac{d\beta}{d\alpha_y}(1-r)\left[(1+r)\alpha+(1-r)^2\beta\right]\right\}-p\frac{d\beta}{d\alpha_y}-\tilde{p}$$

and, using the fact that $\frac{d\beta}{d\alpha_y}=-\frac{1}{1-r}$, we obtain

$$\frac{dE\left[u_i\left(a,\theta,\beta\right)\mid\theta_{-1}\right]}{d\alpha_y}=-\frac{r(1-r)}{\left[\alpha+\beta(1-r)\right]^2}+p\frac{1}{1-r}-\tilde{p}. \tag{16}$$

Equation (16) is best discussed in terms of accuracy and commonality of information. When $\beta$ is strictly positive, an increase in the precision of public information leads to an increase in commonality, and to a reduction in accuracy (because of the reduction of $\beta$). As shown by Angeletos and Pavan (2007), this contributes to lower welfare when one does not take the cost of information acquisition into account. This effect is highlighted by the first addendum in (16), while the second one shows the cost-saving effect induced by the reduction of the individual incentives to invest in the precision of private information.

By substitution of (14), Eq. (16) reduces to:

$$\frac{dE\left[u_i\left(a,\theta,\beta\right)\mid\theta_{-1}\right]}{d\alpha_y}=p-\tilde{p},$$

which is positive whenever $p>\tilde{p}$. We thus established the following result:

**Proposition 1** *With non negative investments and linear costs for both private and public information, welfare increases in the precision of public information if $p>\tilde{p}$.*

Note also that the constraint $\beta\geq 0$ implies $\alpha_y\leq\bar{\alpha}_y=\left(\frac{1-r}{\sqrt{p}}\right)-\alpha_\theta$. It is immediate to observe that, when $p>\tilde{p}$, the precisions of private and public information are set equal to $\beta = 0$ and $\alpha_y = \bar{\alpha}_y$, respectively; for $p < \tilde{p}$, these precisions are $\beta = \frac{1}{\sqrt{p}} - \frac{\alpha_\theta}{1-r}$ and $\alpha_y = 0$; while, for $p = \tilde{p}$, the solution is indeterminate, i.e. $\beta \in \left[0, \frac{1}{\sqrt{p}} - \frac{\alpha_\theta}{1-r}\right]$, and $\alpha_y \in [0, \bar{\alpha}_y]$.

Consider now the case $\alpha \geq (1-r)/\sqrt{p}$, so that private agents do not acquire private information and the corresponding ex ante utility is equal to $E\left[u_i\left(a, \theta, \beta_i\right) \mid \theta_{-1}\right] = -\frac{1-r}{\alpha} - \tilde{p} \cdot \alpha_y$. It is easy to see that in this case the marginal value of the precision of public information is always positive.

The social planner maximizes $E\left[u_i\left(a, \theta, \beta_i\right) \mid \theta_{-1}\right]$ under the constraint $\alpha \geq (1-r)/\sqrt{p}$, which implies $\alpha_y \geq \bar{\alpha}_y$. Hence, we immediately obtain that

$$\alpha_y^* = \max\left\{\sqrt{\frac{1-r}{\tilde{p}}} - \alpha_\theta, \bar{\alpha}_y\right\}.$$

Accordingly, to characterize the solution of the planner problem, we must distinguish two cases:

*Ai)* $\quad p \geq (1-r)\tilde{p}$, which implies $\alpha_y^* = \sqrt{\frac{1-r}{\tilde{p}}} - \alpha_\theta$, [2] and hence

$$E\left[u_i\left(a, \theta, \beta_i\right) \mid \theta_{-1}\right] = \tilde{p} \cdot \alpha_\theta - 2\sqrt{\tilde{p}(1-r)}. \tag{17}$$

When the cost of the precision of public information is low, it is set at a high level by the social planner knowing and exploiting the fact that private agents have no incentives to invest in the precision of private information.

*Aii)* $\quad p < (1-r)\tilde{p}$ so that $\alpha_y^* = \bar{\alpha}_y$, and

$$E\left[u_i\left(a, \theta, \beta_i\right) \mid \theta_{-1}\right] = \tilde{p} \cdot \alpha_\theta - \sqrt{p} - \frac{\tilde{p}}{\sqrt{p}}(1-r) \tag{18}$$

In this case, the cost of private information precision is low in comparison to that of public precision. Accordingly, a social planner who aims at inducing no private investment in information sets the precision of public information at the lowest level consistent with no information acquisition by private agents.

To fully characterize the social planner's policy, we need to compare the social welfare obtained in cases *Ai)* and *Aii)* above with that attained when the chosen precision

[2] It is *not* necessary to explicitly assume $\tilde{p} \leq \left(\frac{1-r}{\alpha_\theta^2}\right)$ (so that $\alpha_y^* \geq 0$), because this is already implicit in the constraints $p < \left(\frac{1-r}{\alpha_\theta}\right)^2$ and $p \geq (1-r)\tilde{p}$.

of public information induces private agents to set the precision of private information either to zero or to the level (14), depending on $p$ and $\tilde{p}$. As for the second term in the comparison, from the analysis above it follows that:

*Bi*) when $p > \tilde{p}$ and $p < [(1-r)/\alpha_\theta]^2$, the equilibrium is $\beta = 0$ and $\alpha_y = \bar{\alpha}_y$, so that the social welfare is

$$E\left[u_i\left(a, \theta, \beta_i\right) \mid \theta_{-1}\right] = \tilde{p} \cdot \alpha_\theta - \sqrt{p} - \frac{\tilde{p}}{\sqrt{p}}(1-r); \tag{19}$$

*Bii*) when $p \leq \tilde{p}$ and $p < [(1-r)/\alpha_\theta]^2$, we have $\beta = \frac{1}{\sqrt{p}} - \frac{\alpha_\theta}{1-r}$ and $\alpha_y = 0$, which imply the following social welfare

$$E\left[u_i\left(a, \theta, \beta_i\right) \mid \theta_{-1}\right] = p \cdot \alpha_\theta - (2-r)\sqrt{p}. \tag{20}$$

Cases *Ai*) and *Aii*) must therefore be compared with *Bi*) and *Bii*).

When $p > \tilde{p}$ and $p < [(1-r)/\alpha_\theta]^2$, the planner must compare (17) with (19). As the private cost is higher than the public one, it is immediate to conclude that the optimal choice is to set $\alpha_y^* = \sqrt{\frac{1-r}{\tilde{p}}} - \alpha_\theta$, inducing $\beta = 0$.

For $(1-r)\tilde{p} \leq p \leq \tilde{p}$ and $p < [(1-r)/\alpha_\theta]^2$, the social planner needs to compare (17) with (20). To do so, observe that the inequality

$$\tilde{p} \cdot \alpha_\theta - 2\sqrt{\tilde{p}(1-r)} \geq p \cdot \alpha_\theta - (2-r)\sqrt{p},$$

is verified at $p = \tilde{p}$, for $r \in [0,1)$, with the equality applying for $r = 0$. Moreover, when $p = (1-r)\tilde{p}$, the above inequality is satisfied for $\tilde{p} > (1-r)/\alpha_\theta^2$, which however is outside the portion of the parameters space for this sub-case. Observe also that the second derivative of the right hand side of the inequality is always positive. Hence, there is a unique $p^* \in [(1-r)\tilde{p}, \tilde{p}]$, such that the planner's optimal choice is $\alpha_y^* = \sqrt{\frac{1-r}{\tilde{p}}} - \alpha_\theta$ (and hence $\beta = 0$) for all $p > p^*$.

Finally, when $p < (1-r)\tilde{p}$ and $p < [(1-r)/\alpha_\theta]^2$, the planner compares (18) with (20). In this case, it is immediate to check that $p$ is low enough for the central planner being optimal to let the private agents to provide all the information, so that $\beta = \frac{1}{\sqrt{p}} - \frac{\alpha_\theta}{1-r}$, and $\alpha_y = 0$.

The above discussion is summarized by the following proposition:

**Proposition 2** *For all $p > p^*$, $p^* \in [(1-r)\tilde{p}, \tilde{p}]$, the social planner provides public information of precision $\alpha_y^* = \sqrt{\frac{1-r}{\tilde{p}}} - \alpha_\theta$, and private agents do not acquire private information.*

To understand why it is optimal to provide only public information even when its cost exceeds the price of private information precision, observe that $\alpha_y^* > \bar{\alpha}_y$, for $r \in (0, 1)$. This follows from the fact that, since $\frac{d\beta}{d\alpha_y} = -\frac{1}{1-r}$, when the social planner aims at inducing an equilibrium in which $\beta \geq 0$, he under-invests with respect to the case in which private agents have no incentives to invest in the precision of private information.

Summarizing, when the costs of information acquisition are taken into account, not only the Morris and Shin non-monotonic effect of increasing the precision of public information disappears, but also the social planner's optimal choices call for the provision of public information only for a larger set of prices than in the absence of the beauty contest externality. This occurs because an increase in the precision of public information – increasing coordination among private agents – reduces more than proportionally their incentives to improve the precision of the available information.

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## Appendix

To obtain the single-individual utility function (11), we concentrate on the first three terms of Eq. (10), i. e.

$$-(1-r)\,E\left[(a_i-\theta)^2\right]-rE\left[\int_0^1 (a_j-a_i)^2\,dj\right]+rE\left[\int_0^1\int_0^1 (a_h-a_j)^2\,dh\,dj\right]. \quad \text{(A1)}$$

In the following, we take as understood that the expectation is computed from the standpoint of the period $-1$ information set. First, we focus on the first addendum in Eq. (A1). Exploiting Eq. (6) we obtain:

$$-(1-r)\,E\left[(a_i-\theta)^2\right]=-(1-r)\,E\left[(\gamma_i z+(1-\gamma_i)x_i-\theta)^2\right].$$

Recalling that $z\equiv E(\theta|\,y)=\frac{\alpha_\theta\theta_{-1}+\alpha_y y}{\alpha_\theta+\alpha_y}$, and substituting it in the expression above yields:

$$-(1-r)\,E\left[\left(\gamma_i\frac{\alpha_\theta\theta_{-1}+\alpha_y y}{\alpha_\theta+\alpha_y}+(1-\gamma_i)x_i-\theta\right)^2\right].$$

Taking into account the structure of the signals and the evolution of the fundamental we obtain

$$-(1-r)\,E\left[\left(\gamma_i\frac{\alpha_\theta\theta_{-1}+\alpha_y(\theta_{-1}+\chi+\eta)}{\alpha_\theta+\alpha_y}+(1-\gamma_i)(\theta_{-1}+\chi+\varepsilon_i)-(\theta_{-1}+\chi)\right)^2\right],$$

which gives:

$$-(1-r)\,E\left[\left(\gamma_i\frac{\alpha_y(\chi+\eta)}{\alpha_\theta+\alpha_y}+(1-\gamma_i)\varepsilon_i-\gamma_i\chi\right)^2\right],$$

and hence:

$$-(1-r)\left[\frac{\gamma_i^2}{\alpha}+\frac{(1-\gamma_i)^2}{\beta_i}\right].$$

Substituting for $\gamma_i$ (Eq. (9)), we obtain

$$-(1-r)\left\{\frac{\left[(1-r)\delta_i+r\delta\right]^2/\alpha+\left[(1-r)(1-\delta_i)\right]^2/\beta_i}{\left[1-r(1-\delta)\right]^2}\right\},$$

and, after developing the terms in square brackets:

$$-\frac{(1-r)}{[1-r(1-\delta)]^2}\left\{(1-r)^2\left[\frac{\delta_i^2}{\alpha}+\frac{(1-\delta_i)^2}{\beta_i}\right]+\frac{2r(1-r)\delta\delta_i}{\alpha}+\frac{r^2\delta^2}{\alpha}\right\}.$$

Because $\delta_i = \frac{\alpha}{\alpha+\beta_i}$, and $(1-\delta_i) = \frac{\beta_i}{\alpha+\beta_i}$, we have that $\frac{\delta_i^2}{\alpha}+\frac{(1-\delta_i)^2}{\beta_i} = \frac{1}{\alpha+\beta_i} = \frac{\delta_i}{\alpha}$. Hence, we obtain:

$$-\frac{(1-r)}{[1-r(1-\delta)]^2}\left\{\frac{(1-r)^2\delta_i}{\alpha}+\frac{2r(1-r)\delta\delta_i}{\alpha}+\frac{r^2\delta^2}{\alpha}\right\} \tag{A2}$$

We now consider the second addendum in Eq. (A1). Exploiting Eqs. (3) and (6), we get:

$$-rE\left[\int_0^1 (a_j-a_i)^2\,dj\right] = -rE\left[\int_0^1 ((\gamma-\gamma_i)z+(1-\gamma)x_j-(1-\gamma_i)x_i)^2\,dj\right].$$

Substitution of $z$, $x_i$ and $x_j$ yields:

$$-rE\left[\left((\gamma-\gamma_i)\frac{\alpha_y(\chi+\eta)}{\alpha_\theta+\alpha_y}+(1-\gamma)(\chi+\varepsilon_j)-(1-\gamma_i)(\chi+\varepsilon_i)\right)^2\right],$$

Because $\alpha \equiv \alpha_\theta+\alpha_y$, $\alpha_\theta \equiv \sigma_\theta^{-2}$, and $\alpha_y \equiv \sigma_\eta^{-2}$ the above equation gives:

$$-r\left[\frac{(\gamma-\gamma_i)^2}{\alpha}+\frac{(1-\gamma)^2}{\beta}+\frac{(1-\gamma_i)^2}{\beta_i}\right]. \tag{A3}$$

Finally, as for the third addendum in Eq. (A1), we simply observe that, from the standpoint of agent $i$, all signals perceived by the other agents are identical, so that

$$rE\left[\int_0^1\int_0^1 (a_h-a_j)^2\,dh\,dj\right] = \frac{2r(1-\gamma)^2}{\beta}. \tag{A4}$$

Summing Eqs. (A3) and (A4), we readily obtain

$$-r\left[\frac{(\gamma-\gamma_i)^2}{\alpha}+\frac{(1-\gamma_i)^2}{\beta_i}-\frac{(1-\gamma)^2}{\beta}\right],$$

which – by substituting for $\gamma$ and $\gamma_i$ from Eqs. (8) and (9), respectively – becomes:

$$-\frac{r}{[1-r(1-\delta)]^2}\left[\frac{(1-r)^2(\delta-\delta_i)^2}{\alpha}+\frac{(1-r)^2(1-\delta_i)^2}{\beta_i}-\frac{(1-r)^2(1-\delta)^2}{\beta}\right].$$

Collecting the common $(1-r)^2$ term, developing the square terms and rearranging, we get:

$$-\frac{r(1-r)^2}{[1-r(1-\delta)]^2}\left[\delta^2\left(\frac{\beta-\alpha}{\alpha\beta}\right)+\delta_i^2\left(\frac{\beta_i+\alpha}{\alpha\beta_i}\right)-\frac{2\delta\delta_i}{\alpha}+\frac{(1-2\delta_i)}{\beta_i}-\frac{(1-2\delta)}{\beta}\right].$$

Notice that, from the definition of $\delta_i$, we have: $\delta_i^2(\alpha+\beta_i)/(\alpha\beta_i)=\delta_i/\beta_i$. Hence:

$$-\frac{r(1-r)^2}{[1-r(1-\delta)]^2}\left[\delta^2\left(\frac{\beta-\alpha}{\alpha\beta}\right)-\frac{2\delta\delta_i}{\alpha}+\frac{(1-\delta_i)}{\beta_i}-\frac{(1-2\delta)}{\beta}\right].$$

Substituting $\delta_i$ in the third addendum in the big square brackets, and $\delta$ in the fourth, we obtain

$$-\frac{r(1-r)^2}{[1-r(1-\delta)]^2}\left[\delta^2\left(\frac{\beta-\alpha}{\alpha\beta}\right)-\frac{2\delta\delta_i}{\alpha}+\frac{\delta_i}{\alpha}-\delta\left(\frac{\beta-\alpha}{\alpha\beta}\right)\right],$$

and rearranging:

$$-\frac{r(1-r)^2}{[1-r(1-\delta)]^2}\left[\delta(\delta-1)\left(\frac{\beta-\alpha}{\alpha\beta}\right)+\delta_i\frac{(1-2\delta)}{\alpha}\right].$$

Recall that $(1-2\delta)=\frac{\beta-\alpha}{\alpha+\beta}$, then $\frac{(1-2\delta)}{\alpha}=\frac{\beta-\alpha}{\alpha\beta}\frac{\beta}{\alpha+\beta}=\frac{\beta-\alpha}{\alpha\beta}(1-\delta)$. Therefore, we can write:

$$-\frac{r(1-r)^2}{[1-r(1-\delta)]^2}\left(\frac{\beta-\alpha}{\alpha\beta}\right)(1-\delta)(\delta_i-\delta) \tag{A5}$$

We now consider the sum of (A2) and (A5), that is:

$$-\frac{(1-r)}{(1-r(1-\delta))^2}\left[\frac{(1-r)^2\delta_i}{\alpha}+\frac{2(1-r)r\delta\delta_i}{\alpha}+\right.$$
$$\left.+\frac{r^2\delta^2}{\alpha}+r(1-r)\left(\frac{\beta-\alpha}{\alpha\beta}\right)(1-\delta)(\delta_i-\delta)\right].$$

Focus on the term inside the square brackets in the equation above, and collect the $\delta_i\alpha^{-1}$ terms that can be found in the first, in the second, and in the last addendum. This yields:

$$-\frac{(1-r)}{(1-r(1-\delta))^2}\left\{\left[(1-r)^2+2(1-r)r\delta+r(1-r)(1-\delta)\right]\frac{\delta_i}{\alpha}+\right. \qquad \text{(A6)}$$
$$\left.+\frac{r^2\delta^2}{\alpha}-\frac{r(1-r)(1-\delta)\delta_i}{\beta}-r(1-r)(1-\delta)\delta\left(\frac{\beta-\alpha}{\alpha\beta}\right)\right\}.$$

Collecting $r\delta$, the second and the fourth addenda in the curly brackets of the last equation become $r\delta\left[r\delta\alpha^{-1}-(1-r)(1-\delta)\left(\alpha^{-1}-\beta^{-1}\right)\right]$, that is: $r\delta[(-1+r+\delta)\alpha^{-1}+$ $+(1-r)(1-\delta)\beta^{-1}]$. Because $(1-\delta)\beta^{-1}=\delta\alpha^{-1}$, the last expression can be written as $r\delta\left[\delta-(1-r)(1-\delta)\right]\alpha^{-1}$. Using again $(1-\delta)\beta^{-1}=\delta\alpha^{-1}$ in the third addendum gives $-r(1-r)\delta\delta_i\alpha^{-1}$; finally, adding this expression to the first addendum and simplifying we obtain $(1-r)\delta_i\alpha^{-1}$. Accordingly, Eq. (A6) can be rewritten as:

$$-\frac{(1-r)}{\alpha(1-r(1-\delta))^2}\left\{(1-r)\delta_i+r\delta\left[\delta-(1-r)(1-\delta)\right]\right\}.$$

Substituting for $\delta$ and $\delta_i$ in the last expression, we obtain

$$-\frac{(1-r)}{\alpha\left[1-r\left(1-\frac{\alpha}{\alpha+\beta}\right)\right]^2}\left\{(1-r)\frac{\alpha}{\alpha+\beta_i}+r\frac{\alpha}{\alpha+\beta}\left[\frac{\alpha}{\alpha+\beta}-(1-r)\left(\frac{\beta}{\alpha+\beta}\right)\right]\right\},$$

and simplifying:

$$-\frac{(1-r)(\alpha+\beta)^2}{[\alpha+(1-r)\beta]^2}\left[\frac{1-r}{\alpha+\beta_i}+\frac{r}{(\alpha+\beta)^2}(\alpha-(1-r)\beta)\right],$$

from which Eq. (11) in the main text follows immediately. Notice that, when $\beta_i=\beta$, we obtain an expression that is equivalent to social welfare as defined by Morris and Shin (2002).

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