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Fabrizio Mattesini Lorenza Rossi

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Fabrizio Mattesini (*) Lorenza Rossi (^)

(*)Università Tor Vergata, Roma

(^)Istituto di Economia e Finanza, Università Cattolica del Sacro Cuore,
Largo Gemelli 1 – 20123 Milano, e-mail: lorenza.rossi@unicatt.it

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Optimal Monetary Policy in Economies with Dual Labor Markets

Fabrizio Mattesini* Lorenza Rossi†

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Abstract

We analyze, in this paper, a DSGE New Keynesian model with indivisible labor where firms may belong to two different final goods producing sectors: one where wages and employment are determined in competitive labor markets and the orther where wages and employment are the result of a contractual process between unions and firms. Bargaining between firms and monopoly unions implies real wage rigidity in the model and, in turn, an endogenous trade-off between output stabilization and inflation stabilization. We show that the negative effect of a productivity shock on inflation and the positive effect of a cost-push shock is crucially determined by the proportion of firms that belong to the competitive sector. The larger is this number, the smaller are these effects. We derive a welfare based objective function as a second order Taylor approximation of the expected utility of the economy's representative agent and we analyze optimal monetary policy. We show that the larger is the number of firms that belong to the competitive sector, the smaller should be the response of the nominal interest rate to exogenous productivity and cost-push shocks. If we consider, however, an instrument rule where the interest rate must react to inflationary expectations, the rule is not affected by the structure of the labor market. The results of the model are consistent with a well known empirical regularity in macroeconomics, i.e. that employment volatility is larger than real wage volatility.

JEL codes: E24, E32, E50, J23, J51

1 Introduction

One of the most striking changes in the structure of the American and British labor markets over the last fourty years is the decrease in the role of trade unions. In the US, trade union membership in the private sector declined from

* *Università di Roma "Tor Vergata" Via Columbia 2, 00133 Rome, Italy*

† Corresponding author. *Istituto di Economia e Finanza, Università Cattolica del Sacro Cuore. Largo Gemelli 1, 20123 Milano, Italy. Phone*: +39 02 72342696. E-mail Adress: lorenza.rossi@unicatt.it

37% in 1970 to 26% in 1978 and is now only 7.40%. In the UK, in 1979, the proportion of employees who were trade union members was over 50%; today this number is below 30% and, in the private sector, below 20%. If, instead of union density, we consider union coverage, we find that in the US less than 15% of workers are covered by collective contract agreements[1], and in the UK less than 36%. What are the consequences of these dramatic structural changes for monetary policy? Should central banks operating in economies characterized by competitive labor markets behave differently from those operating in economies where unions play a major role?

Some recent empirical evidence, although not focused directly on this issue, indicates for the US and the UK a considerable difference in the response of monetary policy to productivity shocks before and after 1979. Galì et al. [22] find that in the pre-Volcker period the FED responded to a positive technology shock with a significant decrease in the nominal interest rate, while in the Volcker-Greespan period the response was small and positive. Similarly, Francis et al. [20] estimate that for the US the response of the nominal interest rate to a positive technology shock was negative in both periods but larger before 1979. For the UK, the same response was negative before 1979 and smaller and positive after 1979. Although the stance of monetary policy is certainly the result of many possible considerations, it is quite likely that central banks, in setting interest rates, play close attention to the evolution of labor market conditions.

In this paper we try to shed some light on these issues by studying monetary policy in dual labor markets. Firms may belong to two different final-goods producing sectors: one where wages and employment are determined under perfect competition, and the other where wages and employment are the result of a contractual process between unions and firms. We focus on the normative aspect, and we investigate what monetary policy should look like in these markets. The model provides a benchmark against which we can evaluate not only the consequences, for monetary policy, of the structural changes in the US and the UK labor markets, but also of the large differences that still exist between these countries and other European countries such as Italy, France or Sweden. [2]

We propose of a Dynamic Stochastic General Equilibrium New Keynesian (DSGE-NK henceforth) where, as in Hansen [30] and Rogerson and Wright [41], labor supply is indivisible and workers face a positive probability to remain unemployed; as in Maffezzoli [33] and Zanetti [54], we assume that unions set wages according to the popular monopoly-union model introduced by Dunlop [18] and Oswald [36]. By doing this we depart from the recent literature, that has recently analyzed search and matching frictions à la Mortensen-Pissarides

[1] Union coverage is defined as the number of workers covered by collective contracts as a percentage of total employment. This concept is different than the concept of union density, i.e. the percentage of workers that belong to a union, since in many countries collective contracts signed by unions and representative of firms are binding also for non-members.

[2] In these countries union coverage is above 84%. More precisely, the number of persons covered by collective agreeements over total employment was 94.5% in France in 2003, 84.1% in Italy in the year 2000 and 85.1% in Sweden in the year 2000. For a complete set of data on various countries see Lawrence and Ishikawa [32].

([34])[3] in DSGE-NK models and we concentrate on the consequences of collective bargaining between unions and firms. [4].

In this model union bargaining gives rise to real wage rigidity and this, as was recently shown by Blanchard and Galì [6], has very important consequences for monetary policy. What these authors define as the "divine coincidence" does not generally hold: for a central bank stabilizing output around the level that would prevail under flexible prices (natural output) is not equivalent to pursuing the efficient level of output and a trade-off arises between output stabilization and inflation stabilization. An important aspect of our model is that we do not simply assume real wage rigidity as in Christoffel and Linzert [13] and Blanchard and Galì [6], [7], but we derive it as a consequence of the structure of the labor market. Another important aspect of our framework with labor indivisibilities we don't have the problem that is usually found when wage rigidities are introduced in models with search and matching frictions (see for example [47]) i.e. the fact that wage rigidities apply only to new hirings and not to ongoing relationships.

A first major result of our model is that the trade-off between inflation stabilization and the level of output (and unemployment) depends on the relative weight of the unionized and the competitive sectors: the larger is the fraction of firms that are able to set wages in a competitive labor market, the smaller is the trade-off they face in response to productivity shocks. This has significant consequences for optimal monetary policy that we derive, as in Woodford [52], from the maximization by the central bank of a second order approximation of agents'utility function. Differently from the model recently proposed by Thomas [47] where wage rigidity is the result of wage norms, inflation targeting is not optimal.

A second result is that in an economy where unions are not very important the nominal interest rate should change much less in response to a productivity shock than in an economy where wages are largely set by collective bargaining between unions and firms. The larger is the fraction of firms that set wages in competitive labor markets, the smaller is the effect of productivity shocks on inflation, and therefore the smaller the need to increase interest rates to prevent an increase in the rate of inflation. In our model, therefore, the behavior of central banks is consistent with the different volatilities of the interest rate in the US and the UK that are found before 1979 (a period in which trade unions still had an important role on wage determination) and after 1979 (a period in which the role of trade unions declined drammatically).

The fact that the response of monetary policy to technology shocks should depend on the relative weight of trade unions, however, does not mean that central banks should behave differently in response to inflationary expectations.

[3] Among these papers we find Chéron and Langot [11], Walsh [50] [51], Trigari [48], [49], Moyen and Sahuc [35] and Andres et al. [2] and, more recently by Christoffel and Linzert [13] and Blanchard and Galì [6], [7].

[4] Also Christoffel and Linzert [13] and Blanchard and Galì [6], [7] consider explicitly real wage rigidity, but in their models real wage rigidity is simply assumed and it is not derived as a consequences of the institutional structure that characterizes the labor market.

In general, the Taylor principle should apply, but the more than proportional increase of nominal interest rates with respect to inflation should be independent of the structure of labor market. Therefore, if we consider two countries hit by the same shocks and where the central bank behaves optimally, we will observe that in the country where the number of "walrasian" firms is larger the interest rate will vary much less than in the other country. This, however, will not be the consequence of differences in the reaction functions of the two central banks to a unit change in expected inflation; rather it will be caused by the fact that the economy where the labor market is more competitive will experience smaller inflationary tensions.

Our model provides also a convenient framework to address important normative issues such as, for example, the optimal behavior of central banks in periods characterized by labor market turmoil and exogenous wage shocks. In the framework we propose here a policy trade-off for the central bank arises also in response to exogenous changes in the unions' reservation wage, that we interpret as cost push shocks. If the unions' reservation wage is subject to exogenous changes, and these changes tend to be persistent over time, then a welfare maximizing central bank must again face the problem of whether to accommodate these shocks with a easier monetary policy. As in the case of technology shocks, also in this case optimal monetary policy requires only partial accommodation, and the response of the central bank is crucially determined by the fraction of firms that, in the economy, set wages competitively. Since both the technology shock and the reservation wage shock, in our model, affect the Phillips curve, we are able to analyze optimal monetary policy when the economy is hit by these two shocks at the same time. One last important result is that the model is able to account for a well known stylized fact in macroeconomics, i.e. the relatively smooth behavior of wages and the relatively volatile behavior of unemployment over the business cycle.[5]

We finally calibrate the model and we analyze the differences between an economy where the central bank follows a standard Taylor rule, as the one estimated by Smets and Wouters [53] for Europe, and an economy where the central bank follows the optimal rule. We show that, under a Taylor rule, a unionized economy tends to have larger responses to productivity shocks than an economy where competitive labor markets prevail. The difference in the impulse response function between these two types of economies becomes much larger, however, under an optimal monetary policy. Optimality implies also that monetary policy be much more accomodating when wages are the result of bargaining between unions and firms.

The paper is organized as follows. In Section 2 we develop a DSGE-NK model with indivisible labor and two-sector labor market while in Section 3 we study optimal monetary policy. In Section 4 we discuss the calibration of the model.

[5] Also Gertler and Trigari [26] propose a model where wages and unemployment move consistently with the observed data. They achieve this result, however, by introducing exogenous multiperiod wage contracts.

2 The model

2.1 The Representative Household

We consider an economy populated by many identical, infinitely lived worker-households each of measure zero. Households demand a Dixit, Stiglitz [17] composite consumption bundle produced by a continuum of monopolistically competitive firms. In each period households sell labor services to the firms and each firm is endowed with a pool of households from which it can hire. As a matter of fact firms hire workers from a pool composed of infinitely many households so that the individual household member is again of measure zero. Since each household supplies its labor only to one firm, which can be clearly identified, workers try to extract some producer surplus by organizing themselves into firm specific trade unions. As organizing a union is costly, we assume that workers, each time, succeed in organizing a union only in $1-q$ firms, while in the remaining q firms they do not succeed and labor markets remain competitive. Given the structure of the economy, q not only represents the number of firms that face a walrasian labor market but also the probability that a worker is assigned to the walrasian sector. Once a household is assigned to a firm specific sector, as in Hansen [30], Rogerson [41] and Rogerson and Wright [42], it has the alternative between working a fixed number of hours and not working at all. For the sake of simplicity we assume that q is constant.

As we show in Appendix A1, we are able to write the life-time expected intertemporal utility function of a representative household as:

$$U_t = E_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} \frac{1}{1-\sigma} \left[C_t \phi\left(N_t\right)\right]^{1-\sigma}, \tag{1}$$

where $0<\beta<1$ is the subjective discount rate. We define

$$\begin{aligned}\phi\left(N_t\right) &= \left[\left(qN_t^w + (1-q)\,N_t^u\right) \upsilon_0^{\frac{1-\sigma}{\sigma}} + \left(qN_t^w + (1-q)\,N_t^u\right) \upsilon_1^{\frac{1-\sigma}{\sigma}}\right]^{\frac{\sigma}{1-\sigma}} \\ &= \left[N_t \upsilon_0^{\frac{1-\sigma}{\sigma}} + \left(1-N_t\right) \upsilon_1^{\frac{1-\sigma}{\sigma}}\right]^{\frac{\sigma}{1-\sigma}}\end{aligned}$$

where N_t^w and N_t^u are respectively the probability to be employed in the walrasian and in the unionized sector and $N_t = qN_t^w + (1-q)\,N_t^u$ is the probability to be employed. The flow budget constraint of the representative household is given by:

$$P_t C_t + R_t^{-1} B_t \leq qW_t^w N_t^w + (1-q)\,W_t^u N_t^u + B_{t-1} + \Pi_t - T_t \tag{2}$$

where W_t^h $(h=w,u)$ is the wage rate in the two sectors. Total consumption C_t is a geometric average of consumption of the good produced in the walrasian sector, $C_{w,t}$, and of the good produced in the unionized sector, $C_{u,t}$. Then,

$$C_t = \frac{\left(C_{w,t}\right)^q \left(C_{u,t}\right)^{1-q}}{q^q \left(1-q\right)^{1-q}}. \tag{3}$$

and

$$P_t = (P_t^w)^q (P_t^u)^{1-q} \tag{4}$$

is the corresponding consumption price index (CPI) which is derived in Appendix A3, and P_t^w and P_t^u are respectively the price index of goods produced in the walrasian and the unionized sectors. The purchase of consumption goods, is financed by labor income, profit income Π_t, and a lump-sum transfers T_t from the Government. We assume that agents can also have access to a financial market where nominal bonds are exchanged. We denote by B_t the holdings of a nominal bond carried over from period t that pays one unit of currency in period $t+1$. Its price is R_t^{-1}, where R_t denotes the gross nominal yield.

In solving the maximization of (1) subject to (2) we should remember that the worker chooses the levels of consumption C_t and C_{t+1} and the supply of labor N_t^w, while N_t^u is taken as given, as it is determined by the union together with the firm. The first order conditions imply

$$1 = \beta R_t E_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \left(\frac{\phi(N_t)}{\phi(N_{t+1})} \right)^{1-\sigma} \frac{P_t}{P_{t+1}} \right] \tag{5}$$

$$\frac{W_t^w}{P_t} = -C_t \frac{\phi_{N^w}(N_t)}{\phi(N_t)} = -C_t q \frac{\phi_N(N_t)}{\phi(N_t)} \tag{6}$$

where equation (5) is the standard consumption Euler equation[6]. Equation (6) holds only for households employed in the walrasian sector. Optimality requires that the no-Ponzi game condition on wealth is also satisfied.

2.2 The Two Representative Final Goods-Producing Firms

In each sector ($h = w, u$) a perfectly competitive final good producer purchases a $Y_t^h(j)$ units of each intermediate good $j \in [0,1]$ at a nominal price $P_t^h(j)$ to produce Y_t^h units of the final good with the following constant returns to scale technology:

$$Y_t^w = \left[\int_0^q \frac{1}{q} Y_t^w(j)^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}} \qquad \text{and} \qquad Y_t^u = \left[\int_q^1 \frac{1}{1-q} Y_t^u(j)^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}} \tag{7}$$

where $\theta > 1$ is the elasticity of substitution across intermediate goods, which is equal for the two sectors. Profit maximization yields the following set of demands for intermediate goods:

$$Y_t^w(j) = \left(\frac{P_t^w(j)}{P_t^w} \right)^{-\theta} Y_t^w \qquad \text{and} \qquad Y_t^u(j) = \left(\frac{P_t^u(j)}{P_t^u} \right)^{-\theta} Y_t^u \tag{8}$$

for all i. In Appendix A2 we show that

$$P_t^w = \left[\int_0^q \frac{1}{q} P_t^w(j)^{1-\theta} dj \right]^{\frac{1}{1-\theta}} \qquad \text{and} \qquad P_t^u = \left[\int_q^1 \frac{1}{1-q} P_t^u(j)^{1-\theta} dj \right]^{\frac{1}{1-\theta}}. \tag{9}$$

[6] See Appendix A2 for derivation

are the price indexes of the walrasian and unionized sectors.

2.3 The Two Representative Intermediate Goods-Producing Firms

We abstract from capital accumulation and assume that the representative intermediate good-producing firm j in sector h, hires N_t^h units of labor from the household and produces $Y_t^h(j)$ units of the intermediate good using the following technology:

$$Y_t^h(j) = A_t N_t^h(j)^\alpha \tag{10}$$

where A_t is an exogenous productivity shock common to all firms. We assume that the $\ln A_t \equiv a_t$ follows the autoregressive process

$$a_t = \rho_a a_{t-1} + \hat{a}_t \tag{11}$$

where $\rho_a < 1$ and $\hat{a}_t$ is a normally distributed serially uncorrelated innovation with zero mean and standard deviation σ_a. The assumption of decreasing returns to scale, which is in line with a non-competitive intermediate good sector, has important implications on the optimal price-setting rule, and then on the derivation of the traditional Phillips curve.[7]

Before choosing the price of its goods, a firm chooses the level of $N_t^h(j)$ which minimizes its total costs, obtaining the following labor demand,

$$\left(1-\tau^h\right)\frac{W_t^h(j)}{P_t^h(j)} = \frac{MC_t^{n,h}(j)}{P_t^h}\alpha\frac{Y_t^h(j)}{N_t^h(j)}, \tag{12}$$

where $MC_t^{n,h}(j)$ represents the nominal marginal costs of firm j in sector h and where τ^h represents an employment subsidy to the sector h firm, which is set so that the steady state equilibrium in both sectors coincides with the efficient one.[8] Aggregating across firms j, sector h real marginal costs are:

$$\frac{MC_t^{n,h}}{P_t^h} = \frac{\left(1-\tau^h\right)}{\alpha}\frac{W_t^h}{P_t^h}\frac{N_t^h}{Y_t^h}. \tag{13}$$

2.4 Unions' Wage Setting

For households hired by firms in the unionized sector, unions negotiate wages on behalf of their members. Since each household supplies its labor to only one firm, which can be clearly identified, workers try to extract some producer surplus by organizing themselves into a firm-specific trade union. The economy is populated by decentralized trade unions, so that each intermediate goods-producing firm negotiates with a single union $i \in (0,1)$ which is too small to

[7] In fact, as shown in Sbordone [44] and in Galì et al. [25], it should be taken into account that marginal costs are no longer common across firms (see appendix A5 or).

[8] We assume that the subsidy is covered by a lump sum tax in that the Government runs always a balanced budget.

influence the outcome of the market.[9] Unions negotiate the wage on behalf of their members.

Once unions are introduced in the analysis, two important issues arise: what is the objective function of the union and what are the variables of the bargaining process. Both these questions have been extensively investigated by the literature, although no conclusive agreement has been reached on the issue.[10] The problem of identifying an appropriate maximand for the union dates back to Dunlop [18] and Ross [43]; since then the debate has revolved over the relative importance of economic considerations (basically how employers respond to wage bargaining) and political considerations in the determination of union wage policy. For political considerations we intend how the preferences of workers, the preferences of union leaders and market constraints interact in determining a union's objective.

One approach often followed in the literature is the "utilitarian" approach pioneered by Oswald [36] which consists on assuming that all workers are equal and that the union simply maximizes the sum of workers' utility, defined over wages. Although simple and appealing because coherent with a standard economic approach, this formulation of unions' utility does not allow for political considerations. An alternative approach, initially proposed by Dertouzos and Pencavel [16] and Pencavel [37] and, more recently, reproposed by De la Croix et al. [15] and Raurich and Sorolla [40], is to assume that unions maximize a modified Stone-Geary utility function of the form[11]:

$$V\left(\frac{W_t^u}{P_t}\right) = \left(\frac{W_t^u\left(i\right)}{P_t} - \frac{W_t^r}{P_t}\right)^{\gamma} N_t\left(i\right)^{\varsigma} \tag{14}$$

The relative value of γ and ς is an indicator of the relative importance of wages and employment in the union's objective function.[12] The reservation wage W_t^r is the absolute minimum wage the union can tolerate. This reservation wage has many possible interpretations. One possible interpretation is that W_t^r is the opportunity wage of the workers (Pencavel 1984) since it is unlikely that a union can survive if it negotiates a wage below such level. Another possible interpretation of W_t^r is what Blanchard and Katz [5] define an "aspiration wage",

[9] For tractability, we consider atomistic unions and we abstract in this paper from the issue of strategic interaction between unions and central banks, which as recently considered by Gnocchi [?] in the context of a DSGE-NK model.

[10] For an extensive survey of unions model see Farber [19], and, more recently, Kaufman [27].

[11] As it can be easily verified, if unions set wage to simply maximize agents' utility, the wage schedule would be similar to the labor supply in the indivisible labor model, with the difference that the wage would be a constant mark-up over the marginal rate of substitution. In this case, wages would fully respond to technology shocks and no significant trade-off between inflation and unemployment (output gap) would emerge. Therefore, assuming that the union leader has this type of objective function is a very simple and realistic way to obtain endogenous real wage rigidities.

[12] The objective function we consider is closed to the one suggested by the Ross tradition. In fact, for different parameters values, the union's objective function is almost equivalent to the one of a union which maximizes his income or his membership, as for example in Skatun [45] and in Booth [8].

i.e. a wage that workers have come to regard as "fair". The unions' reservation wage is generally unobservable and therefore hard to model. As in De la Croix[13] [15], however, we assume that:

$$\frac{W_t^r}{P_t} = \varpi e^{\varepsilon_t^w} \tag{15}$$

with

$$\varepsilon_t^w = \rho_w \varepsilon_{t-1}^w + \hat{\varepsilon}_t^w \tag{16}$$

where $\varpi > 0$, $\rho_w < 1$ and ε_t^w is a normally distributed serially uncorrelated innovation with zero mean and standard deviation σ_w. If the real reservation wage is constant, $\hat{\varepsilon}_t^w = 0$. The fact that the reservation wage is subject to persistent shocks is meant to capture the exogenous wage shocks that have often characterized industrialized economies, especially in Europe.[14] Notice that the Stone-Geary utility function is appealing not only for its ability to approximate the actual behavior of unions and for its flexibility and tractability, but also for its generality. If we set for example $\gamma = 1$, $\varsigma = 1$ and $\hat{\varepsilon}_t^w = 0$, maximizing (14) is equivalent to maximize the unions' objective function assumed by Maffezzoli [33] and Zanetti [54] in their recent papers.

The bargaining process we consider here is in the tradition of the "right to manage" models. In particular, we follow the popular "monopoly union" model first proposed by Dunlop [18] and Oswald [36], where the employment rate and the wage rate are determined in a non-cooperative dynamic game between unions and firms. We restrict the attention to Markov strategies, so that in each period unions and firms solve a sequence of independent static games. Each union behaves as a Stackelberg leader and each firm as a Stackelberg follower. Once the wage has been chosen, each firm decides the employment rate along its labor demand function. Even if unions are large at the firm level, they are small at the economy level, and therefore they take the aggregate wage as given. The ex-ante probability of being employed is equal to the aggregate employment rate and the allocation of union members to work or leisure is completely random and independent over time. Finally, we assume that workers are able to perfectly insure themselves against the possibility of being unemployed. Insurance is supplied under a zero-profit condition and is therefore actuarially fair.

Unions maximize (14) subject to the labor demand of firms, (13), in sector

[13] In the model of De la Croix et al. [15] the real reservation wage is a weighted sum of a constant term and of the past real wage. In order not to add further ad-hockery to the model, we chose not to include past real wages. Nevertheless adding these to equation (15) would leave the results unchanged. A technical appendix is avalable upon request.

[14] We consider both these two alternative in order to show that the our results on the endogenous inflation unemployment (output) trade-off is not qualitatively influenced by the fact that the reservation wage shock is an exogenous shock. Moreover, to our knowledge this is the first attempt to study how the optimal interest rate rule should react in response to more than one supply shock.

u. The first order condition with respect to $W_t^u(i)$ is,

$$\frac{W_t^u(i)}{P_t} = \frac{\varsigma}{\varsigma - \gamma(1-\alpha)} \frac{W_t^r}{P_t}. \tag{17}$$

Notice that the technology shock has no effect on the real wage rate chosen by the monopoly union. Since $\frac{1}{1-\alpha} > 1$, we see that the real wage rate is always set above the reservation wage.

2.5 Market Clearing

Equilibrium in the goods market requires that the production of the final good be allocated to expenditure, as follows:

$$Y_t^h(j) = C_{h,t}(j) \qquad h = w, u \tag{18}$$

where $C_{h,t}$ is the total consumption of the good produced by sector h. The market clearing conditions for the two final consumption goods therefore are given by:

$$Y_t^w = \int_0^q \frac{1}{q} C_{w,t}(j)\, dj \qquad \text{and} \qquad Y_t^u = \int_q^1 \frac{1}{1-q} C_{w,t}(j)\, dj \tag{19}$$

which given (7) imply

$$Y_t = C_t. \tag{20}$$

Equation (20) represents the aggregate economy's resource constraint. Since the net supply of bonds, in equilibrium is zero, equilibrium in the bonds market, implies

$$B_t = 0. \tag{21}$$

From equations (8) and (10) we have

$$D_t^w Y_t^w = A_t (N_t^w)^\alpha \qquad \text{and} \qquad D_t^u Y_t^u = A_t (N_t^u)^\alpha \tag{22}$$

where

$$D_t^w = \left[\int_0^q \frac{1}{q} \left(\frac{P_t^w(j)}{P_t^w}\right)^{-\frac{\theta}{\alpha}} dj\right]^\alpha \qquad \text{and} \qquad D_t^u = \left[\int_q^1 \frac{1}{1-q} \left(\frac{P_t^u(j)}{P_t^u}\right)^{-\frac{\theta}{\alpha}} dj\right]^\alpha \tag{23}$$

are measures of price dispersion. Given the market clearing conditions and given equation (3) we have that,

$$Y_t = \left(\frac{Y_t^w}{q}\right)^q \left(\frac{Y_t^u}{1-q}\right)^{1-q}. \tag{24}$$

Since in a neighborhood of a symmetric equilibrium and up to a first order approximation $D_t^h \simeq 1$, the total amount of goods produced by the economy is a geometric average of the aggregate production of the two sectors.

2.6 The First Best Level of Output

The efficient level of output can be obtained by solving the problem of a benevolent planner that maximizes the intertemporal utility of the representative household, subject to the resource constraint and the production function. This problem is analyzed in the Appendix A4 where we show that the efficient level of output is given by:

$$y_t^{Eff} = a_t. \tag{25}$$

2.7 The Two Sectors Labor Market Equilibrium

Labor market equilibrium in the walrasian sector is obtained equating labor demand (13) to labor supply (6), so that

$$-C_t \frac{\phi_{N^w}(N_t)}{\phi(N_t)} = MC_t^w \frac{P_t^w}{P_t} \frac{\alpha}{(1-\tau^w)} \frac{Y_t^w}{N_t^w}. \tag{26}$$

From the households' intertemporal problem (derived in Appendix A2) we have $P_t^w C_{w,t} = qP_tC_t$ and, since the market clearing condition implies $C_{w,t} = Y_t^w$,

$$-\frac{\phi_{N^w}(N_t^w, N_t^u)}{\phi(N_t^w, N_t^u)} N_t^w = \frac{\alpha q}{(1-\tau^w)} MC_t^w. \tag{27}$$

Similarly, in the unionized sector, considering the wage schedule (14) and the labor demand (13), equilibrium in the labor market is given by:

$$\frac{1}{\alpha}\frac{W_t^r}{P_t} = MC_t^u \left(\frac{P_t^u}{P_t^w}\right)^q \frac{\alpha}{(1-\tau^u)} \frac{Y_t^u}{N_t^u}. \tag{28}$$

Notice that, differently from what happens in the walrasian sector, equation (28) contains the relative price between goods produced in the walrasian and in the unionized sectors. In the walrasian labor market the relative price does not affect equilibrium, since movements in the relative price are corrected by movements in the relative wage. In the unionized sector instead, because of real wage rigidity, a change in the relative price has a significant effect on equilibrium.

Since, from the intertemporal household problem, (Appendix A2) we have $P_t^u C_{u,t} = (1-q) C_t P_t$ and given equation (24) we have

$$\frac{1}{\alpha}\frac{W_t^r}{P_t} = \frac{\alpha}{(1-\tau^w)} MC_t^u \left(\frac{1-q}{q}\right)^q \frac{Y_t}{N_t^u}. \tag{29}$$

2.8 The Flexible Price Equilibrium Output in the Walrasian and in the Unionized Sectors

The log-linearization of (27) which is shown in Appendix A5 implies

$$mc_t^w = \frac{1}{\alpha} y_t^w - \frac{\sigma - \alpha(\sigma-1)}{\alpha\sigma} a_t - \frac{(\sigma-1)}{\sigma} y_t. \tag{30}$$

Notice that real marginal costs in the walrasian sector are increasing in the output of the walrasian sector and decreasing in the aggregate output. Considering that in the flexible price equilibrium we must have $mc_t^w = 0$, from the aggregate production function we find

$$y_t^{wf} = \frac{\sigma - \alpha(\sigma - 1)}{\sigma} a_t + \frac{\alpha(\sigma - 1)}{\sigma} y_t, \tag{31}$$

which implies that flexible price equilibrium output in the walrasian sector is an increasing function of the productivity shock and of the aggregate output. Notice that when $q = 1$, (31) can be rewritten as $y_t^{wf} = a_t$, i.e., the flexible price equilibrium output coincides with the efficient one.

In the unionized sector the log-linearizzation of (29) implies:

$$mc_t^u = \frac{1}{\alpha} y_t^u - y_t - \frac{1}{\alpha} a_t + w_t^r. \tag{32}$$

As in the walrasian sector, real marginal costs are increasing in the output of the unionized sector and decreasing in the aggregate output. When $mc_t^u = 0$ then

$$y_t^{uf} = \alpha y_t + a_t - \alpha w_t^r \tag{33}$$

which implies that the flexible price equilibrium output in the unionized sector is increasing in the productivity shock and aggregate output.

Given equations (30), (31), (32) and (33), in both sectors real marginal costs can be rewritten in terms of the gap between actual output and flexible price output:

$$mc_t^h = \frac{1}{\alpha}\left(y_t^h - y_t^{hf}\right). \tag{34}$$

2.9 The Natural Output

We define the natural output of the economy y_t^f as the weighted sum of flexible price equilibrium output of the walrasian and unionized sectors (equations 31 and 33), where the weight is given by the fraction of firms in each sector. Therefore

$$y_t^f = \frac{\sigma - q\alpha(\sigma - 1)}{\sigma(1 - \alpha) + \alpha q} a_t - \frac{\sigma(1 - q)\alpha}{\sigma(1 - \alpha) + \alpha q} w_t^r. \tag{35}$$

Given (25), the diffence between natural and efficient equilibrium output is:

$$y_t^f - y_t^{Eff} = \frac{\alpha\sigma(1 - q)}{\sigma(1 - \alpha) + \alpha q} a_t - \frac{\sigma(1 - q)\alpha}{\sigma(1 - \alpha) + \alpha q} w_t^r. \tag{36}$$

What is important to notice, here, is that, unlike what happens in the walrasian model, the difference between flexible equilibrium output (natural output) and efficient equilibrium output is not constant, but is a function of the relevant shocks that hit the economy. In this model therefore, as in Blanchard and Galì [6] stabilizing the output gap - the difference between actual and natural output - is not equivalent to stabilizing the *welfare relevant* output gap - the gap between actual and efficient output. In other words, what Blanchard and Galì call "the divine coincidence" does not hold, since any policy that brings the economy to its natural level is not necessarily an optimal policy.

Defining by $\Upsilon = \frac{\sigma(1-q)}{\sigma(1-\alpha)+\alpha q}$, the response of the welfare relevant output gap to the relevant shocks (notice that the response to a technology shock is identical, but with the opposit sign, to the response to a reservation wage shock), we immediately observe that

$$\frac{d\Upsilon}{dq} < 0. \tag{37}$$

As the number of walrasian firms increases, the difference between natural output and efficient output decreases, i.e. the natural output tends to the efficient output. The reason is quite intuitive: the smaller is the population of unionized firms, the smaller is the importance of real wage rigidity in the economy and both the technology and reservation wage shocks become less and less relevant.

2.10 The Reduced Dynamic System

We assume that firms choose $P_t^h(j)$ in a staggered price setting à la Calvo-Yun [9]. As shown in Appendix A6, the solution of the firm's problem in the case of a production function with decreasing returns to scale, is given by:

$$\pi_t^h = \beta E_t \pi_{t+1}^h + \lambda_\alpha \frac{1}{\alpha}\left(y_t^h - y_t^{hf}\right) \tag{38}$$

where $\lambda_\alpha = \frac{(1-\psi)(1-\beta\psi)}{\psi}\frac{\alpha}{\alpha+\theta(1-\alpha)}$ and ψ is the probability with which firms reset prices. Since aggregate inflation is $\pi_t = q\pi_t^w + (1-q)\pi_t^u$, and considering the welfare relevant output gap, given by the difference between actual and efficient output $x_t = y_t - y_t^{Eff}$, the Phillips curve for the aggregate equation can be written as,

$$\pi_t = \beta E_t \pi_{t+1} + \lambda_a \frac{1}{\alpha} x_t - \lambda_a \Upsilon a_t + \lambda_a \Upsilon \hat{\varepsilon}_t^w \tag{39}$$

where we have considered that in log-linear terms equation (15) implies that $w_t^r = \hat{\varepsilon}_t^w$ and where $\Upsilon = \frac{\sigma(1-q)}{\sigma(1-\alpha)+\alpha q}$. From equation (39) is quite clear that, for a central bank, achieving $x_t = 0$ does not imply obtaining $\pi_t = 0$. We therefore have:

Result 1. *In a two sector labor market economy, because of the presence of unions, the "divine coincidence" does not hold, i.e., stabilizing inflation is not equivalent to stabilizing the welfare relevant output gap. A negative (positive) productivity shock has a positive (negative) effect on inflation, while a cost push shock has an effect on inflation of the same size but with the opposite sign.*

This result depends on the existence of a real distortion in the economy, beside the one induced by monopolistic competition, and the nominal distortion caused by firms' staggered price setting. When a productivity shock hits the economy, efficient output, given by equation (25), increases by the same amount. Natural output instead (i.e., the level of output that would prevail in a flexible price equilibrium) increases more than proportionally so that the difference between efficient output and natural output decreases. This is due to the fact that in a unionized sector, following a productivity shock, real wages remain constant and therefore do not offset the effects of the shock on real marginal cost. Therefore, the natural level of output differs from the efficient level and this difference is not constant. As it is evident from equation (40), if the Central Bank stabilizes output around the efficient level, inflation will be completely vulnerable to productivity and cost-push shocks; in other words,the output gap is no longer a sufficient statistics for the effect of real activity on inflation.

Given (37), we immediately observe that the response of inflation to technology and exogenous wage shocks decreases as the fraction of walrasian firms in the market increases. We can therefore state,

Result 2. *The response of inflation to a negative productivity shock and to a positive reservation wage shock decreases as the number q of walrasian firms increases.*

Another interesting aspect of this model is that we are able to express the Phillips curve in its more traditional form, i.e. in terms of unemployment. Let $U_t = 1 - N_t$ be the rate of unemployment. From the log-linearization of the aggregate production function and from equations (25), (36) and (39) we obtain

$$\pi_t = \beta E_t \pi_{t+1} - \frac{\lambda_a}{\eta} u_t - \lambda_a \frac{\sigma(1-q)}{\sigma(1-\alpha)+\alpha q} a_t + \lambda_a \frac{\sigma(1-q)}{\sigma(1-\alpha)+\alpha q} \hat{\varepsilon}_t^w. \tag{40}$$

where $\eta = \frac{N}{1-N}$. The relationship between unemployment and the output gap allows us to consider, indifferently, the output gap and the unemployment rate as policy objectives for the central bank.

In order to obtain the IS curve we start by log-linearizing[15] the Euler equation (5) around the steady state. Considering the optimal subsidy setting, which implies $\frac{\phi_N(N)N}{\phi(N)} = -\alpha$, the resource constraint (20), the aggregate production function and the definition of the welfare relevant output gap, we obtain,

$$x_t = E_t x_{t+1} - \left(\hat{r}_t - E_t\{\pi_{t+1}\} - \hat{r}_t^e\right). \tag{41}$$

The interest rate is defined as $\hat{r}_t = r_t - \varrho$, where $r_t = \ln R_t$ and $\varrho = -\ln\beta$ is the steady state interest rate; $\hat{r}_t^e$ is the interest rate supporting the efficient

[15] In order to loglinearize $\phi(N_t)^{1-\sigma}$ we first log-linearize the term N_t obtaining $\phi[N(1+n_t)]^{1-\sigma}$ .Applying a first order Taylor expansion, we obtain

$$\phi[N(1+n_t)]^{1-\sigma} = \phi(N)^{1-\sigma} + (1-\sigma)\phi(N)^{-\sigma}\phi_N(N)Nn_t$$

equilibrium and is given by:

$$\hat{r}_t^e = \sigma E_t \{\Delta a_{t+1}\} = \sigma E_t \left\{\Delta y_{t+1}^{Eff}\right\} = -\sigma (1-\rho_a) a_t. \tag{42}$$

The flexible price (natural or Wicksellian) rate of interest is instead defined as[16]:

$$\begin{aligned} r_t^n &= r_t^e - \frac{\alpha\sigma(1-q)}{\sigma(1-\alpha)+\alpha q}\left[(1-\rho_a) a_t - (1-\rho_w)\hat{\varepsilon}_t^w\right] \\ &= r_t^e - \Upsilon\left[(1-\rho_a) a_t - (1-\rho_w)\hat{\varepsilon}_t^w\right] \end{aligned} \tag{43}$$

Suppose that the economy is hit at the same time by a 1 standard deviation technology shock and by a 1 standard deviation reservation wage shock. If the persistence of former is greater (lower) than the persistence of the latter, then, the natural rate of output is greater (lower) than the efficient one. When the shocks have the same persistence the natural and the efficient interest rate coincide. Note that, the difference between the natural and the efficient rate of interest is decreasing in the number of walrasian firms and, in particular, for $q = 1$, the endogenous trade-off cancels out and the natural rate of interest is equal to the efficient interest rate.

3 Monetary Policy

In Appendix A7 we show that also for the non-separable preferences assumed in our framework, consumers' utility can be approximated up to the second order by the quadratic equation:

$$W_t = E_t \sum_{t=0}^{\infty} \beta^t \tilde{U}_{t+k} = -\frac{U_{Y,t}}{2} E_t \sum_{t=0}^{\infty} \beta^t \left[\pi_{t+k}^2 + \frac{\lambda_a}{\theta\sigma} x_{t+k}^2\right] + \bigcirc\left(\|\alpha\|^3\right) \tag{44}$$

where $\tilde{U}_{t+k} = U_{t+k} - \bar{U}_{t+k}$ is the deviation of consumers' utility from the level achievable in the efficient equilibrium, and θ is the elasticity of substitution between intermediate goods, which are used as inputs in the final good sectors. Notice that the relative weights assigned to inflation and to the output gap are linked to the structural parameters reflecting preferences and technology.

If the Central Bank cannot credibly commit in advance to a future policy action or a sequence of future policy actions, then the optimal monetary policy is discretionary, in the sense that the policy makers choose in each period the value to assign to the policy instrument, that here we assume to be the short-term nominal interest rate $\hat{r}_t$. In order to do so, the Central Bank maximizes the welfare-based loss function (44), subject to the economy's Phillips curve (39) and to the IS curve, (41), taking all expectations as given.

The first order conditions imply:

$$x_t = -\frac{\theta\sigma}{\alpha}\pi_t. \tag{45}$$

[16] The equation of the natural interest rate is obtained combining (42), (36) and the IS (of the flexible price equilibrium) as in Woodford [52].

Substituting into (39), iterating forward, and considering the law of motion of (11) and (16), we obtain,

$$\pi_t = -\frac{\Upsilon\lambda_a}{\Omega - \beta\rho_a} a_t + \frac{\Upsilon\lambda_a}{\Omega - \beta\rho_w} \hat{\varepsilon}_t^w \tag{46}$$

and

$$E_t\pi_{t+1} = -\frac{\rho_a\Upsilon\lambda_a}{\Omega - \beta\rho_a} a_t + \frac{\rho_w\Upsilon\lambda_a}{\Omega - \beta\rho_w} \hat{\varepsilon}_t^w \tag{47}$$

where $\Omega = 1 + \lambda_a \frac{\theta\sigma}{\alpha^2}$. Notice that we can express current inflation as a function of the relevant shocks a_t and $\hat{\varepsilon}_t^w$. A positive productivity shock requires a decrease in inflation and a positive cost push shock requires an increase in inflation. Using (46), (47) and the definition of the efficient interest rate (42) we have:

$$E_t\pi_{t+1} = \rho_w\pi_t + \frac{(\rho_a - \rho_w)}{\sigma(1-\rho_a)} \frac{\lambda}{\Omega - \beta\rho_a} r_t^e. \tag{48}$$

Expected inflation can be written as a function of actual inflation and of the efficient rate of interest.[17]

Given equation (39) and (46) we can write the expression of the output gap as a function of the exogenous shocks

$$x_t = \frac{\theta\sigma\Upsilon\lambda_a}{\alpha(\Omega - \beta\rho_a)} a_t - \frac{\theta\sigma\Upsilon\lambda_a}{\alpha(\Omega - \beta\rho_w)} \hat{\varepsilon}_t^w. \tag{49}$$

The optimal level of inflation can be implemented by the Central Bank by setting the nominal interest rate. The interest rate rule can be obtained by substituting (45), (46) and (47) into the IS curve (41), in which case we obtain:

$$\begin{aligned} \hat{r}_t^* = &- \left[\left[1 + \left(\frac{1-\rho_a}{\rho_a} \right) \frac{\theta\sigma}{\alpha} \right] \left(\frac{\Upsilon\lambda_a\rho_a}{\Omega - \beta\rho_a} \right) + \sigma(1-\rho_a) \right] a_t + \\ &+ \left[1 + \left(\frac{1-\rho_w}{\rho_w} \right) \frac{\theta\sigma}{\alpha} \right] \left(\frac{\Upsilon\lambda_a\rho_w}{\Omega - \beta\rho_w} \right) \hat{\varepsilon}_t^w. \end{aligned} \tag{50}$$

We can therefore state

Result 3. *Under discretion an optimal monetary policy requires a decrease in the nominal interest rate following a positive productivity shock and an increase in the nominal interest rate following a positive reservation wage shock. The response of the nominal interest rate to both shocks decreases as the fraction of walrasian firms q increases.*

Equations (46) and (49) can also be rewritten in terms of standard deviations, which allows us to derive the output-gap inflation volatility frontier. Since

[17] This result holds only when we consider that the economy is contemporary hit by the two shocks. When considering only one shock it holds that as in Clarida et al. expexted inflation depends only on actual inflation, i.e. $E_t\pi_{t+1} = \rho\pi_t$.

by assumption both shocks are iid. and therefore $\sigma_{aw} = 0$, we can express the volatility of inflation and the volatility of the output gap as a function of the volatility of the technology and reservation wage shocks. In particular we have:

$$\sigma_\pi = \left(\frac{\Upsilon \lambda_a}{\Omega - \beta \rho_a} \right) \sigma_a + \left(\frac{\Upsilon \lambda_a}{\Omega - \beta \rho_w} \right) \sigma_w \tag{51}$$

and

$$\sigma_x = \left(\frac{\theta \sigma \Upsilon \lambda_a}{\alpha \left(\Omega - \beta \rho_a \right)} \right) \sigma_a + \left(\frac{\theta \sigma \Upsilon \lambda}{\alpha \left(\Omega - \beta \rho_a \right)} \right) \sigma_w . \tag{52}$$

Notice that, as $q \longrightarrow 1$ then $\Upsilon \equiv \frac{\sigma(1-q)}{\sigma(1-\alpha)+\alpha q} = 0$ and therefore $\sigma_x = \sigma_\pi = 0$. When instead $q \longrightarrow 0$, then $\Upsilon = \frac{\sigma}{\sigma(1-\alpha)+\alpha}$ and both σ_x and σ_π reach their maximum possible values.

Let us now turn to the instrument rule that implements the optimal monetary policy. We consider the case in which the economy is hit only by a single technology shock, this means that we assume that $\sigma_w = 0$, i.e. the reservation wage is always equal to its steady state ($w_t^r = 0$). In this case from equation (46), (47) and the equation of the IS curve we obtain:

$$\hat{r}_t^* = \hat{r}_t^e + \left[1 + \left(\frac{1-\rho_a}{\rho_a} \right) \frac{\theta \sigma}{\alpha} \right] E_t \pi_{t+1} \tag{53}$$

Analogously, when the economy is hit by a single reservation shock, then the instrument rule becomes:

$$\hat{r}_t^* = \hat{r}_t^e + \left[1 + \left(\frac{1-\rho_w}{\rho_w} \right) \frac{\theta \sigma}{\alpha} \right] E_t \pi_{t+1} \tag{54}$$

We can now state:

Result 4. *Optimal monetary policy under discretion requires a more than proportional increase in the nominal interest rate following an increase in the expected rate of inflation. An increase in the fraction of walrasian firms does not affect the response of the nominal interest rate to expected inflation but affects the response of the nominal interest rate to its efficient level.*

Also in our model therefore, as in the standard New Keynesian model, optimality requires that the Central Bank respond to increasing inflationary expectations by raising nominal interest rates more than proportionally. In other words, also in a dualistic economy where part of the labor market is unionized, the Taylor principle applies. The optimal response of the nominal interest rate to an increase in the efficient rate of interest, instead, is different from the one that is usually obtained in the "standard" DSGE New Keynesian model. While the ratio between walrasian and unionized firms is crucial in determining the effect of technology and cost-push shocks on inflation, it does not affect the amount by which the interest rate must be raised in response to a unit increase in expected inflation. In other words, the extent of real wage rigidity in the economy does not affect the response coefficient to expected inflation.

In order to study analycally the unemployment and the real wage volatility we derive an expression for unemployment and real wage standard deviation. In particular, log-linearizing the demand for labor (13) in both sectors, we have that, for $w = u, h$,

$$w_t^h = mc_t^h + y_t^h - \frac{1}{\alpha} y_t^h + \frac{1}{\alpha} a_t.$$

Since the aggregate wage is given by the weighted sum of wages in the two sectors, it can be expressed as:

$$w_t = qmc_t^w + (1-q)mc_t^u - \frac{1-\alpha}{\alpha} y_t + \frac{1}{\alpha} a_t,$$

and given equations (30), (31), (33) and (34), we obtain

$$w_t = x_t - (\Upsilon - 1)\, a_t + \Upsilon \hat{\varepsilon}_t^w$$

which, considering (49) becomes

$$w_t = \left[1 + \Upsilon \left(\frac{\theta\sigma\lambda_a}{\alpha\left(\Omega - \beta\rho_a\right)} - 1\right)\right] a_t + \Upsilon \left[1 - \frac{\theta\sigma\lambda_a}{\alpha\left(\Omega - \beta\rho_w\right)}\right] \hat{\varepsilon}_t^w. \tag{55}$$

Moreover given the relationship between the output gap and unemployment, i.e., $x_t = -\frac{\alpha}{\eta} u_t$, given equation (49), we get

$$u_t = -\eta \frac{\theta\sigma\Upsilon\lambda_a}{\Omega - \beta\rho_a} a_t + \eta \frac{\theta\sigma\Upsilon\lambda_a}{\Omega - \beta\rho_w} \hat{\varepsilon}_t^w. \tag{56}$$

Equation (55) and (56) imply that the standard deviation of real wage and unemployment with respect to technology shock a are given by

$$\sigma_w = \left[1 + \Upsilon \left(\frac{\theta\sigma\lambda_a}{\alpha\left(\Omega - \beta\rho_a\right)} - 1\right)\right] \sigma_a \tag{57}$$

$$\sigma_u = \eta\Upsilon \frac{\theta\sigma\lambda_a}{\Omega - \beta\rho_a} \sigma_a. \tag{58}$$

Notice that $\frac{\partial\sigma_w}{\partial q} > 0$ (in fact $\frac{\theta\sigma\lambda_a}{\alpha(\Omega-\beta\rho_a)} - 1 < 0$) and $\frac{\partial\sigma_u}{\partial q} < 0$, and therefore $\frac{\partial(\sigma_u - \sigma_w)}{\partial q} < 0$. We can therefore state

Result 6. *Under an optimal discretionary policy and in response to a productivity shock, the difference between unemployment volatiliy and real wage volatility decreases as the number of walrasian firms increases.*

4 Calibration

Since we are interested in the impact of the degree of unionization in the labor market on monetary policy, we calibrate the model under different values of the parameter q which represents the fraction of walrasian firms in the market. We follow closely the calibration of Zanetti [54], who studies, with a similar framework,[18] the response of monetary policy to technology shocks[19]. However, while Zanetti considers capital accumulation and assumes that the central bank follows a Taylor rule as the one estimated for Europe by Smets and Wouters, [53] our simpler framework allows us to study optimal monetary policy. Since we introduce a dual labor market, we are also able to compare economies with different degree of unionization.

The model is calibrated on quarterly frequencies. For the parameters describing preferences, we set the elasticity of intertemporal substitution at $\sigma = 2$. The output elasticity of labor, $\alpha = 0.72$, is based on the estimate of Christoffel et al. [14]. The discount factor β, the Calvo parameter φ, and the elasticity of substitution among intermediate goods θ, are set at values commonly found in the literature. In particular we set $\beta = 0.99$, $\varphi = 0.75$, which implies an average price duration of one year, and finally $\theta = 6$, which is consistent with a 10% markup in the steady state. The persistence of the technology shock ρ_a is set as in Zanetti [54] i.e. $\rho_a = 0.8476$. As discussed in Zanetti [54] $N = 0.61$.

The exercise we perform in this section is to first consider the impulse response functions (IRFs henceforth) under a Taylor rule similar to the one considered by Zanetti [54], and then compare it with the IRFs obtained under the optimal monetary policy. In the first case the Central Bank is assumed to follow the following Taylor-type monetary policy rule:

$$\hat{r}_t = \phi_r \hat{r}_{t-1} + (1 - \phi_r)\left[\phi_\pi \pi_{t-1} + \phi_x x_{t-1}\right] \tag{59}$$

As in Smets and Wouters [53], the degree of interest rate smoothing is set at $\phi_r = 0.9$, the response of the nominal interest rate to inflation is set at $\phi_\pi = 1.658$ and the response to output at $\phi_x = 0.148$.[20] We first analyze an economy where the labor market is fully unionized, i.e $q = 0$, and study the IRFs to a one standard deviation productivity shock, under the Taylor type rule (59) and under the optimal monetary policy.[21] Then we repeat our simulation with an economy in which the percentage of firms belonging to a unionized labor market is low, i.e. $q = 0.85$.

In figure 1 we plot the response of the nominal interest rate, employment, inflation, real interest rate, output and output gap to a one unit standard deviation technology shock under the Taylor rule (59), with $q = 0$ (dotted line)

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[18] He consider a DSGE NK model with phisycal and human capital accumulation, where all firms are unionized.

[19] Zanetti ([54]) studies also the response of the economy to monetary policy shock.

[20] As in Zanetti,[54] who follows the suggestion of Carlstrom and Fuerst [10], we employ lagged values for output and inflation because it can be considered consistent with the information set of the Central Bank at time t.

[21] Assuming that $q = 0$ labor market is completely unionized as in Zanetti [54].

and with $q = 0.85$ (continuous line). Under the first assumption, which implies full unionization, our simple model behaves, from a qualitative point of view, like the model proposed by Zanetti [54]. This suggests that adding physical and human capital accumulation does not change the dynamics of the model. When unions have a small weight in the economy and most of the labor market is competitive, i.e. the case where $q = 0.85$, the response of the main economic variables to productivity shocks is smaller and shocks are less persistent.

In figure 2 we consider the impulse response functions of the same variables under the optimal rule. We find that the behavior of the nominal interest rate, inflation, real interest rate and output gap is qualitatively similar to the one found under the Taylor rule (59). An optimal policy, however, implies that the response of these variables to a productivity shock is much larger when $q = 0$ than in the case where $q = 0.85$. The behavior of the nominal and real interest rates under an optimal rule, indicates that monetary policy must be much more accomodating when unions play a large role. Differently from the case where the central bank follows a Taylor rule, after a positive technology shock employment increases in both the unionized and competitive cases. This can be explained by the fact that, under the optimal rule, monetary policy is much more accomodating than under the Taylor rule (59) and this allows a larger increase output.

5 Conclusions

We have considered in this paper a DSGE New Keynesian model where labor is indivisible and where there are, at the same time, two types of labor markets: one where wages are set competitively and one where wages are the result of the bargaining between firms and monopoly unions. We found that, with respect to the standard DSGE-NK framework, our model gives a more satisfactory description of the reality of modern industrialized economies, since it is able to account for the existence of significant trade-offs between stabilizing inflation and stabilizing unemployment, in response to technology and exogenous wage shocks. Because of real wage rigidity which is induced by the presence of unions, an optimizing central bank must respond to negative (positive) technology shocks by increasing (decreasing) the interest rate and, similarly, must respond with an interest rate increase to exogenous increases in unions' reservation wage.

The effect of these shocks on inflation and the necessary interest rate movements set by an optimizing central bank depend on the size of the walrasian sector relative to the unionized sector. If a large part of wages are set in a competititve market, technology and cost-push shocks will have little effect on inflation and will induce small interest rate movements, while an economy where large part of wages are set in unionized markets will experience larger inflation and interest rate movements. If we consider however an optimal instrument rule where the central bank reacts to expected inflation, the response of the nominal interest rate to an increase in expected inflation is not influenced by the dualistic structure of the labor market. The model is also capable of accounting for the greater volatility of unemployment relative to the wage volatility that is usually found in the data.

Even though, for the sake of simplicity, we concentrate on a rigid dualistic structure of the labor market and we abstract from other market imperfections like search and matching and firing costs we are able to single out, with this model, some of the challenges provided to monetary policy by different institutional settings in the labor market. The model, in particular, captures a relevant structural change in the US and UK before and after 1979 and its consequences for monetary policy. At the same time it single out an important difference between Anglo-Saxon economies and continental Europe providing, therefore, a useful benchmark to evaluate and compare the monetary policies enacted by the Fed, the Bank of England and the ECB.

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Appendix

A1 Derivation of the Representative Agent's Utility Function

Let us first consider the problem of an agent that supplies his labor to a firm in the walrasian sector, i.e. to a firm that faces a competitive labor market where firms and workers act as a price taker. We assume that households enter employment lotteries, i.e. sign with a firm a contract that commits them to work a fixed number of hours, that we normalize to one, with probability N_t^w. Since all households are identical, they will all choose the same contract, i.e. the same N_t^w. However, although households are ex-ante identical, they will differ ex-post depending on the outcome of the lottery: a fraction N_t^w of the continuum of households will work and the rest $1 - N_t^w$ will remain unemployed. Lottery outcomes are independent over time. Before the lottery draw, the expected intratemporal utility function is:

$$N_t^w \left[C_{0,t}^w \upsilon\left(0\right)\right]^{1-\sigma} + \left(1 - N_t^w\right)\left[C_{1,t}^w \upsilon\left(1\right)\right]^{1-\sigma} \tag{A1.1}$$

where $C_{0,t}^w$ is the consumption level of employed individuals. We denote by $\upsilon\left(\cdot\right)$ the utility of leisure. Since the utility of leisure of employed individuals $\upsilon\left(0\right)$ and the utility of leisure of unemployed individuals $\upsilon\left(1\right)$ are positive constants, we assume $\upsilon\left(0\right) = \upsilon_0$ and $\upsilon\left(1\right) = \upsilon_1$. As in King and Rebelo[22] [29], we assume $\upsilon_0 < \upsilon_1$.

Since they face a probability $1 - N_t^w$ of not working at all, workers will try to acquire insurance against the risk of remaining unemployed. We assume that asset markets are complete, so that employed and unemployed individuals are able to achieve perfect risk sharing, equating the marginal utility of consuption accross states.

Let us now consider the case of a household that works in a unionized labor market. The unionized sector is populated by decentralized trade unions, so that each intermediate goods-producing firm negotiate with a single union $i \in \left(0, 1\right)$, which is too small to influence the outcome of the market. Unions negotiate the wage on behalf of their members. Once the wage rate is defined, firms chose the amount of labor that maximize their profits. Similarly to what happens in the competitive case, labor is indivisible and workers participate to employment lotteries. As in the previous case, therefore, before the lottery draw, the expected intratemporal utility function of workers, who happens to belong to the unionized sector is

$$N_t^u \left[C_{0,t}^u \upsilon\left(0\right)\right]^{1-\sigma} + \left(1 - N_t^u\right)\left[C_{1,t}^u \upsilon\left(1\right)\right]^{1-\sigma} \tag{A1.2}$$

where $C_{0,t}^u$ is the consumption level of employed individuals. Again, we assume $\upsilon\left(0\right) = \upsilon_0$ and $\upsilon\left(1\right) = \upsilon_1$.

[22] This depends on the fact that the utility of leisure $\phi\left(1 - N_t\right)$ as usual, is an increasing function of the time spend in leisure. Given that the time spend in leisure is greater for unemployed agent than for employed agent this means that $\upsilon\left(1\right) > \upsilon\left(0\right)$.

Since they face a positive probability of being unemployed, risk averse workers will try to obtain insurance against the risk of being unemployed; access to complete asset markets will allow individuals to achieve perfect risk sharing. It is important to observe that, beside the risk of remaining unemployed, workers in this model face also another type of uncertainty since they do not know, a priori, whether they will participate to a competitive labor market or to a unionized one. We assume that, through complete asset markets, agents can also acquire insurance against the income fluctuations implied by this type of uncertainty. Recalling that q is the probability of belonging to the walrasian sector and $1-q$ is the probability of belonging to the unionized sector, before the lotteries are drawn and before learning in what sector they will happen to work, given (A1.1) and (A1.2) the expected intratemporal utility function of an household is:

$$\frac{1}{1-\sigma}\left\{\begin{array}{c} qN_t^w\left[C_{0,t}^w\upsilon_0\right]^{1-\sigma}+q(1-N_t^w)\left[C_{1,t}^w\upsilon_1\right]^{1-\sigma}+ \\ +(1-q)N_t^u\left[C_{0,t}^u\upsilon_0\right]^{1-\sigma}+(1-q)(1-N_t^u)\left[C_{1,t}^u\upsilon_1\right]^{1-\sigma} \end{array}\right\} \tag{A1.3}$$

Perfect risk sharing implies,

$$\begin{array}{rcl} (C_{0,t}^w)^{-\sigma}\upsilon\left(0\right)^{1-\sigma} & = & (C_{1,t}^w)^{-\sigma}\upsilon\left(1\right)^{1-\sigma} \\ (C_{0,t}^u)^{-\sigma}\upsilon\left(0\right)^{1-\sigma} & = & (C_{1,t}^u)^{-\sigma}\upsilon\left(1\right)^{1-\sigma} \\ & & \\ (C_{0,t}^w)^{-\sigma}\upsilon\left(0\right)^{1-\sigma} & = & (C_{0,t}^u)^{-\sigma}\upsilon\left(0\right)^{1-\sigma} \\ (C_{1,t}^w)^{-\sigma}\upsilon\left(1\right)^{1-\sigma} & = & (C_{1,t}^u)^{-\sigma}\upsilon\left(1\right)^{1-\sigma} \end{array} \tag{A1.4}$$

which imply

$$C_{0,t}^u = C_{0,t}^w = C_{0,t} \qquad \text{and} \qquad C_{1,t}^u = C_{1,t}^w = C_{1,t}$$

The average consumption level can be then rewritten as:

$$C_t = \left[qN_t^w + (1-q)N_t^u\right]C_{0,t} + \left[q(1-N_t^w)+(1-q)(1-N_t^u)\right]C_{1,t} \tag{A1.5}$$

the first two equations of the perfect risk sharing conditions can also be rewritten in a more compact way as

$$C_{0,t}^{-\sigma}\upsilon_0^{1-\sigma} = C_{1,t}^{-\sigma}\upsilon_1^{1-\sigma} \tag{A1.6}$$

Solving (A1.6) for $C_{1,t}$ we get

$$C_{0,t} = C_{1,t}\left(\frac{\upsilon_0}{\upsilon_1}\right)^{\frac{1-\sigma}{\sigma}} \tag{A1.7}$$

Substituting (A1.7) in (A1.5) and solving for $C_{0,t}$

$$C_t = \left[qN_t^w + (1-q)N_t^u\right]C_{1,t}\left(\frac{\upsilon_0}{\upsilon_1}\right)^{\frac{1-\sigma}{\sigma}} + \left[q(1-N_t^w)+(1-q)(1-N_t^u)\right]C_{1,t} \tag{A1.8}$$

Solving (A1.8) for $C_{1,t}$

$$C_{1,t} = \frac{C_t}{[qN_t^w + (1-q)N_t^u]\left(\frac{\upsilon_0}{\upsilon_1}\right)^{\frac{1-\sigma}{\sigma}} + [q(1-N_t^w) + (1-q)(1-N_t^u)]} \tag{A1.9}$$

substituting (A1.9) in (A1.5)

$$C_{0,t} = \frac{C_t}{[qN_t^w + (1-q)N_t^u]\left(\frac{\upsilon_0}{\upsilon_1}\right)^{\frac{1-\sigma}{\sigma}} + [q(1-N_t^w) + (1-q)(1-N_t^u)]}\left(\frac{\upsilon 0}{\upsilon 1}\right)^{\frac{1-\sigma}{\sigma}} \tag{A1.10}$$

substituting (A1.9) and (A1.10) in (A1.3)

$$\begin{aligned} &C_t^{1-\sigma}\left\{[qN_t^w + (1-q)N_t^u]\left(\frac{\upsilon_0}{\upsilon_1}\right)^{\frac{1-\sigma}{\sigma}} + [q(1-N_t^w) + (1-q)(1-N_t^u)]\right\}^{\sigma-1} \cdot \\ &\cdot\left\{[qN_t^w + (1-q)N_t^u]\left(\frac{\upsilon_0}{\upsilon_1}\right)^{\frac{1-\sigma}{\sigma}} + [q(1-N_t^w) + (1-q)(1-N_t^u)]\right\} \end{aligned} \tag{A1.11}$$

equation (A1.11) can be rewritten as:

$$C_t^{1-\sigma}\left\{[qN_t^w + (1-q)N_t^u]\left(\frac{\upsilon_0}{\upsilon_1}\right)^{\frac{1-\sigma}{\sigma}} + [q(1-N_t^w) + (1-q)(1-N_t^u)]\right\}^{\sigma} \tag{A1.12}$$

defining phi as...we can write the agents' intertemporal utility function as equation (6) in the text.

A2 Intertemporal Allocation

Given the representative agent utility function (num) in the text, the equation of the aggregate consumption $C_t = \left(\frac{C_{u,t}}{q}\right)^q \left(\frac{C_{w,t}}{1-q}\right)^{1-q}$ and the budget constraint, which can be rewritten as follows

$$P_{u,t}C_{u,t}+P_{w,t}C_{w,t}+E_tD_{t,t+1}B_{t+1} \leq W_{u,t}N_{u,t}+W_{w,t}N_{w,t}+B_t+\Pi_t-T_t \tag{A2.1}$$

The first order conditions with respect to $C_{u,t}$ and $C_{w,t}$, B_t and B_{t+1} imply,

$$\begin{aligned} qC_t^{-\sigma}\left(\frac{C_{w,t}}{q}\right)^{q-1}\left(\frac{C_{u,t}}{1-q}\right)^{1-q}\phi\left(N_t\right)^{1-\sigma} &= \lambda_t P_t^w \\ qC_t^{-\sigma}\frac{C_t}{C_{w,t}}\phi\left(N_t\right)^{1-\sigma} &= \lambda_t P_t^w \end{aligned} \tag{A2.2}$$

and

$$(1-q)\,C_t^{-\sigma}\frac{C_t}{C_{u,t}}\phi\left(N_t\right)^{1-\sigma} = \lambda_t P_{u,t}. \tag{A2.3}$$

Moreover we have:

$$\lambda_t = E_t \left(\beta \lambda_{t+1} R_t \frac{P_t}{P_{t+1}} \right) \tag{A2.4}$$

Taking a geometric average of (A2.2) and (A2.3) with weights q and $(1-q)$ we have,

$$\frac{C_t^{-\sigma} \phi \left(N_t\right)^{1-\sigma}}{P_t} = \lambda_t \tag{A2.5}$$

and substituting in (A2.2) and (A2.3) we obtain:

$$P_{w,t} C_{w,t} = q P_t C_t \tag{A2.6}$$

and

$$P_{u,t} C_{u,t} = (1-q) P_t C_t \tag{A2.7}$$

Summing (A2.7) and (A2.6) it is easy to verify that:

$$P_t C_t = P_{u,t} C_{u,t} + P_{w,t} C_{w,t} \tag{A2.8}$$

Finally, combining (A2.2), (A2.3) and (A2.4) we find the consuption euler equation in the text.

A3 Derivation of the CPI

For a given consumption index C^h $(h \in w, u)$ let P^w be the price of the goods produced in the Walrasian sector that solves:

$$\min_{C^w} \int_0^q p^w(j) C^w(j)\, dj$$

s.t to

$$C^w = \left[\left(\frac{1}{q} \right)^{\frac{1}{\theta}} \int_0^q C^w(j)^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}} = 1$$

From the first order condition we obtain

$$\int_0^q C^w(j)^{\frac{\theta-1}{\theta}} dj = \lambda p^w(j)^{-1} C^w(j)^{-\frac{1}{\theta}} \tag{A3.1}$$

Given the budget constraint, this implies,

$$\left(\frac{1}{q} \right)^{\frac{1}{\theta}} \lambda p^w(j)^{-1} C^w(j)^{-\frac{1}{\theta}} = 1 \tag{A3.2}$$

which, in turn, implies

$$C^w(j) = \left(\frac{1}{q} \right) \lambda^\theta p^w(j)^{-\theta}. \tag{A3.3}$$

We can now write

$$P^w C^w = P^w = \int_0^q p^w(j) C^w(j)\, dj = \int_0^q p^w(j) \left(\frac{1}{q}\right) \lambda^\theta p^w(j)^\theta dj \tag{A3.4}$$

which implies

$$\lambda = (P^w)^{\frac{1}{\theta}} \left[\int_0^q \left(\frac{1}{q}\right) p^w(j)^{1-\theta} dj\right]^{-\frac{1}{\theta}}. \tag{A3.5}$$

Notice now that $C^w = 1$ implies

$$\left(\frac{1}{q}\right)^{\frac{1}{\theta}} \int_0^q C^w(j)^{\frac{\theta-1}{\theta}} dj = \int_0^q \left(\frac{1}{q}\right)^{\frac{1}{\theta}} \lambda^{\theta-1} p^w(j)^{1-\theta} dj = 1 \tag{A3.6}$$

Combining these last two equations we obtain

$$P^w = \left[\frac{1}{q}\int_0^q p^w(j)^{1-\theta} dj\right]^{\frac{1}{1-\theta}}. \tag{A3.7}$$

Analogously, P^u is the price of the goods produced in the unionized sector that solves:

$$\min_{C^u} \int_0^q p^u(j) C^u(j)\, dj$$

subject to

$$C^u = \left[\left(\frac{1}{1-q}\right)^{\frac{1}{\theta}} \int_q^1 C^u(j)^{\frac{\theta-1}{\theta}} dj\right]^{\frac{\theta}{\theta-1}} = 1.$$

and obtaining

$$P^u = \left[\frac{1}{1-q}\int_q^1 p^u(j)^{1-\theta} dj\right]^{\frac{1}{1-\theta}}. \tag{A3.8}$$

The consumption based price index solves the problem of minimizing $qC_{w,t} + (1-q)\, C$ subject to

$$C_t = \frac{1}{q^q (1-q)^{1-q}} (C_{w,t})^q (C_{u,t})^{1-q} = 1$$

hence, substituting the optimal demands (A2.6) and (A2.7) in the previous equation we obtain,

$$C_t = \frac{1}{q^q (1-q)^{1-q}} \left(q \frac{P_t}{P_t^w} C_t\right)^q \left((1-q) \frac{P_t}{P_t^u} C_t\right)^{1-q} = 1$$

and simplifying we have:

$$P_t = (P_t^w)^q (P_t^u)^{1-q}.$$

A4 The Ramsey Problem

We consider a social planner which maximizes the representative household utility subject to the economy resource constraint and production function as follows:

$$\max_{N_t} U(C_t, N_t) = \frac{1}{1-\sigma} C_t^{1-\sigma} \phi(N_t)^{1-\sigma}$$
$$s.t.$$
$$C_t = Y_t$$
$$Y_t = A_t N_t^{\alpha}$$

Substituting the constraint into the utility function the problem is:

$$\max_{N} \frac{1}{1-\sigma} (A_t N_t^{\alpha})^{1-\sigma} \phi(N_t)^{1-\sigma} \tag{A4.1}$$

the first order condition requires

$$(A_t N_t^{\alpha})^{-\sigma} \alpha \frac{Y_t}{N_t} \phi(N_t)^{1-\sigma} = -(A_t N_t^{\alpha})^{1-\sigma} \phi(N_t)^{-\sigma} \phi_N(N_t) \tag{A4.2}$$

simplifying

$$Y_t \frac{\phi_N(N_t)}{\phi(N_t)} = -\alpha \frac{Y_t}{N_t} \tag{A4.3}$$

Multiplying both sides of equation for $\frac{N_t}{Y_t}$ we find

$$\frac{\phi_N(N_t)}{\phi(N_t)} N_t = -\alpha. \tag{A4.4}$$

In order to find an equation for the efficient output we first log-linearizing the previous equation around the steady state as follows,

$$[\phi_N(N) + \phi_{NN}(N) N n_t] N (1 + n_t) = -\alpha (\phi(N) + \phi_N(N) N n_t) \tag{A4.5}$$

which can be rewritten as

$$\phi_N(N) N + \phi_N(N) N n_t + \phi_{NN}(N) N^2 n_t = -\alpha (\phi(N) + \phi_N(N) N n_t) \tag{A4.6}$$

considering the steady state equation $\phi_N(N) N_t = -\alpha \phi(N)$ and collecting terms in n_t we obtain,

$$\left(1 + \frac{\phi_N(N) N_t}{\phi(N)} + \frac{\phi_{NN}(N) N^2}{\phi(N)} \left(\frac{\phi_N(N) N_t}{\phi(N)}\right)^{-1}\right) n_t = 0 \tag{A4.7}$$

given that $1 + \frac{\phi_N(N) N_t}{\phi(N)} + \frac{\phi_{NN}(N) N^2}{\alpha \phi(N)} \left(\frac{\phi_N(N) N_t}{\phi(N)}\right)^{-1} \neq 0$ we require,

$$n_t = 0 \tag{A4.8}$$

and then from the aggregate production function we obtain equation (25) in the text.

A5 Derivation of the Flexible Price Equilibrium Output in the Walrasian Sector

Consider the equation of labor market equilbrium of the walrasian economy:

$$\phi_{N^w}\left(N_t\right)N_t^w = -\frac{\alpha q}{(1-\tau)}MC_t^w\phi\left(N_t\right) \tag{A5.1}$$

and consider that

$$\phi_{N^w}\left(N_t\right) = q\phi_N\left(N_t\right). \tag{A5.2}$$

At the steady state we have,

$$\phi_N\left(N\right)N = -\frac{\alpha}{(1-\tau)}MC^w\phi\left(N\right). \tag{A5.3}$$

Since the utility of leisure, $\phi\left(N_t\right)$, can is given by:

$$\phi\left(N_t\right) = \left\{\begin{array}{c} [qN_t^w + (1-q)N_t^u]\left(\frac{\upsilon_0}{\upsilon_1}\right)^{\frac{1-\sigma}{\sigma}} + \\ +[q(1-N_t^w)+(1-q)(1-N_t^u)] \end{array}\right\}^{\frac{\sigma}{1-\sigma}} \tag{A5.4}$$

we have the following derivatives:

$$\begin{aligned} \phi_{N^w}\left(N_t^w, N_t^u\right) &= \frac{\sigma}{1-\sigma}\left\{\begin{array}{c} [qN_t^w + (1-q)N_t^u]\left(\frac{\upsilon_0}{\upsilon_1}\right)^{\frac{1-\sigma}{\sigma}} + \\ +[q(1-N_t^w)+(1-q)(1-N_t^u)] \end{array}\right\}^{\frac{\sigma}{1-\sigma}-1}. \\ q\left(\left(\frac{\upsilon_0}{\upsilon_1}\right)^{\frac{1-\sigma}{\sigma}}-1\right) &= q\phi_N\left(N\right) \end{aligned} \tag{A5.5}$$

$$\begin{aligned} \phi_{N^u}\left(N_t^w, N_t^u\right) &= \frac{\sigma}{1-\sigma}\left\{\begin{array}{c} [qN_t^w + (1-q)N_t^u]\left(\frac{\upsilon_0}{\upsilon_1}\right)^{\frac{1-\sigma}{\sigma}} + \\ +[q(1-N_t^w)+(1-q)(1-N_t^u)] \end{array}\right\}^{\frac{\sigma}{1-\sigma}-1}(1-q)\left(\left(\frac{\upsilon_0}{\upsilon_1}\right)^{\frac{1-\sigma}{\sigma}}-1\right) \\ &= (1-q)\,\phi_N\left(N\right) \end{aligned} \tag{A5.6}$$

$$\begin{aligned} \phi_{N^wN^w}\left(N_t^w, N_t^u\right) &= \left(\frac{1-2\sigma}{1-\sigma}\right)\frac{\sigma}{1-\sigma}\left\{\begin{array}{c} [qN_t^w + (1-q)N_t^u]\left(\frac{\upsilon_0}{\upsilon_1}\right)^{\frac{1-\sigma}{\sigma}} + \\ +[q(1-N_t^w)+(1-q)(1-N_t^u)] \end{array}\right\}^{\frac{\sigma}{1-\sigma}-2}. \\ &\quad q^2\left(\left(\frac{\upsilon_0}{\upsilon_1}\right)^{\frac{1-\sigma}{\sigma}}-1\right)^2 \end{aligned} \tag{A5.7}$$

and

$$\phi_{N^w N^u}\left(N_t^w, N_t^u\right) = \left(\frac{1-2\sigma}{1-\sigma}\right)\frac{\sigma}{1-\sigma}\left\{\begin{array}{c}\left[qN_t^w + (1-q)N_t^u\right]\left(\frac{\upsilon_0}{\upsilon_1}\right)^{\frac{1-\sigma}{\sigma}} + \\ +\left[q(1-N_t^w) + (1-q)(1-N_t^u)\right]\end{array}\right\}^{\frac{\sigma}{1-\sigma}-2} \cdot$$

$$q\left(\left(\frac{\upsilon_0}{\upsilon_1}\right)^{\frac{1-\sigma}{\sigma}} - 1\right)^2 (1-q) \tag{A5.8}$$

The last two derivatives at the steady state gives:

$$\phi_{N^w N^w}\left(N^w, N^u\right) = \left(\frac{1-2\sigma}{1-\sigma}\right)\frac{\sigma}{1-\sigma}\left\{N\left(\frac{\upsilon_0}{\upsilon_1}\right)^{\frac{1-\sigma}{\sigma}} + [1-N]\right\}^{\frac{\sigma}{1-\sigma}-2} \cdot$$

$$q^2\left(\left(\frac{\upsilon_0}{\upsilon_1}\right)^{\frac{1-\sigma}{\sigma}} - 1\right)^2 = \phi_{NN}(N)\, q^2 \tag{A5.9}$$

and

$$\phi_{N^w N^u}\left(N^w, N^u\right) = \left(\frac{1-2\sigma}{1-\sigma}\right)\frac{\sigma}{1-\sigma}\left\{N\left(\frac{\upsilon_0}{\upsilon_1}\right)^{\frac{1-\sigma}{\sigma}} + [1-N]\right\}^{\frac{\sigma}{1-\sigma}-2} \cdot$$

$$q\left(\left(\frac{\upsilon_0}{\upsilon_1}\right)^{\frac{1-\sigma}{\sigma}} - 1\right)^2 (1-q) = \phi_{NN}(N)\, q\,(1-q) \tag{A5.10}$$

since the optimal subsidy is set such that in steady state $N^w = N^u = N$. Then log-linearizing (A5.1) we obtain,

$$\begin{aligned}&\left\{\begin{array}{c}\phi_{N^w}\left(N^w, N^u\right) + \phi_{N^w N^w}\left(N^w, N^u\right) N^w n_t^w \\ +\phi_{N^w N^u}\left(N^w, N^u\right) N^u n_t^u\end{array}\right\} N^w\left(1+n_t^w\right) + \\ &+\frac{q\alpha}{(1-\tau)} MC\left(1+mc_t^w\right)\left\{\begin{array}{c}\phi\left(N^w, N^u\right) + \phi_{N^w}\left(N^w, N^u\right) N^w n_t^w \\ +\phi_{N^u}\left(N^w, N^u\right) N^u n_t^u\end{array}\right\} \\ = \; & 0\end{aligned} \tag{A5.11}$$

which can be rewritten as:

$$\begin{aligned}&\phi_N(N)\,N + \phi_N(N)\,N n_t^w + \left\{q\phi_{NN}(N)\,N^2 n_t^w + (1-q)\,\phi_{NN}\left(N^w, N^u\right) N^2 n_t^u\right\} + \\ &+\frac{\alpha}{(1-\tau)} MC\left(1+mc_t^w\right)\left\{\begin{array}{c}\phi(N) + q\phi_N(N)\,N n_t^w \\ +(1-q)\,\phi_N(N)\,N n_t^u\end{array}\right\} \\ = \; & 0\end{aligned} \tag{A5.12}$$

Considering now that in steady state the optimal subsidy is set in such a way that $\frac{\phi_N(N)}{\phi(N)}N = -\alpha$ and that $\frac{\phi_{NN}(N)}{\phi(N)}N^2 = \left(\frac{\phi_N(N)}{\phi(N)}N\right)^2 \frac{2\sigma-1}{\sigma}$, then solving for mc_t^w and collecting terms in n_t^w and n_t^u we obtain,

$$mc_t^w = \left[1 - \alpha q\frac{\sigma-1}{\sigma}\right] n_t^w - (1-q)\,\alpha\frac{\sigma-1}{\sigma} n_t^u \tag{A5.13}$$

which the equation of real marginal costs in the text.

A6 Derivation of the Phillips Curve

Following Calvo ([9]), we assume that each firm of each sector (may reset its price with probability $1-\varphi$ each period, indipendently from the time elapsed since the last adjustment. This means that each period a measure $1-\varphi$ of firms reset their price, while a fraction φ of them keep their price unchanged. The law of motion of the aggregate price is given by:

$$\ln P_t^h = \varphi \ln P_{t-1}^h + (1-\varphi) \ln P_t^{*h} \tag{A6.1}$$

which implies

$$\pi_t^h = (1-\varphi) \ln \left(\frac{p_t^{*h}}{P_{t-1}^h} \right) \tag{A6.2}$$

where $\ln P_t^*$ denotes the (log) price set by a firm i adjusting its price in period t. Under Calvo price-setting structure $p_{t+k}(i) = p_t^*$ with probability φ^k for $k = 0, 1, 2, ...$, hence firms have to be forward-looking.

Given that the individual firm technology is characterized by decreasing return to scale, the optimal price setting rule should take into account that marginal cost is no longer common across firms. In particular, in the neighborhood of the zero inflation steady state, we have the following price-setting rule:

$$\ln P_t^{*h}(i) = \mu^P + (1-\beta\varphi) \sum_{k=0}^{\infty} (\beta\varphi)^k E_t \left\{ mc_{t,t+k}^{nh} \right\} \tag{A6.3}$$

where $mc_{t,t+k}^n$ is the log-linearized nominal marginal cost in period $t+k$ of a firm which last set its price in period t. Considering the equation of real marginal cost and the one of the aggregate production function,

$$\begin{aligned} MC_{t,t+k}^h &= (1-\tau) \frac{\left(W_{t+k}^h / P_{t+k}^h\right)}{\alpha \left(Y_{t,t+k}^h / N_{t,t+k}^h\right)} \\ &= MC_{t+k}^h \frac{\left(Y_{t+k}^h / N_{t+k}^h\right)}{\left(Y_{t,t+k}^h / N_{t,t+k}^h\right)} \\ &= MC_{t+k}^h \left(\frac{Y_{t+k}^h}{Y_{t,t+k}^h} \right)^{\frac{1-\alpha}{\alpha}} \\ &= MC_{t+k}^h \left(\frac{P_t^{*h}}{P_{t+k}^h} \right)^{\theta \frac{1-\alpha}{\alpha}} \end{aligned} \tag{A6.4}$$

taking the logs

$$\ln MC_{t,t+k}^h = \ln MC_{t+k}^h - \theta \frac{1-\alpha}{\alpha} \ln \left(\frac{P_t^*}{P_{t+k}} \right) \tag{A6.5}$$

Considering that all firms resetting prices in period t will choose the same price P_t^+ we can rewrite equation (A6.3) as,

$$\begin{aligned}
\ln P_t^{*h}(i) - \ln P_{t-1}^h &= \mu^P + (1-\beta\varphi)\sum_{k=0}^{\infty}(\beta\varphi)^k E_t\left\{\ln MC_{t,t+k}^{nh} - \ln P_{t-1}^h\right\} \\
&= \mu^P + (1-\beta\varphi)\sum_{k=0}^{\infty}(\beta\varphi)^k E_t\left\{\ln MC_{t,t+k}^{nh}\right\} + \sum_{k=0}^{\infty}(\beta\varphi)^k\left\{\pi_{t+k}^h\right\} \qquad \text{(A6.6)}
\end{aligned}$$

substituting equation (A6.6) which can be rewritten as

$$\begin{aligned}
\ln P_t^{*h}(i) - \ln P_{t-1}^h &= \mu^P + (1-\beta\varphi)\sum_{k=0}^{\infty}(\beta\varphi)^k E_t\left\{\ln MC_{t,t+k}^{nh} - \theta\frac{1-\alpha}{\alpha}\ln\left(\frac{P_t^{*h}}{P_{t+k}^h}\right)\right\} \\
&\quad + \sum_{k=0}^{\infty}(\beta\varphi)^k\left\{\pi_{t+k}^h\right\} \qquad \text{(A6.7)}
\end{aligned}$$

then

$$\ln P_t^{*h}(i) - \ln P_{t-1}^h = \mu^P + \beta\varphi E_t\left\{\ln P_{t+1}^{*h} - \ln P_t^h\right\} + (1-\beta\varphi)\ln MC_t^h \qquad \text{(A6.8)}$$

Combining (A6.8) with (A6.1) we obtain

$$\pi_t^h = \beta E_t \pi_{t+1}^h + \lambda_\alpha mc_t^h \qquad \text{(A6.9)}$$

as in the text.

A8 The Welfare-Based Loss Function

A second-order Taylor expansion of the period utility around the efficient equilibrium yields,

$$U_t = \bar{U}_t + \bar{U}_{\bar{C},t}\bar{C}_t\tilde{C}_t + \frac{1}{2}\bar{U}_{\bar{C}\bar{C},t}\bar{C}_t^2\tilde{C}_t^2 + \bar{U}_{\bar{N},t}\bar{N}_t\tilde{N}_t + \frac{1}{2}\bar{U}_{\bar{N}\bar{N},t}\bar{N}_t^2\tilde{N}_t^2 + \\ + \bar{U}_{\bar{C}\bar{N},t}\bar{C}_t\bar{N}_t\tilde{C}_t\tilde{N}_t + \bigcirc\left(\|\alpha\|^3\right) \quad \text{(A7.1)}$$

where the generic $\tilde{X} = \ln\left(X/\bar{X}_t\right)$ denotes log-deviations from the efficient equilibrium and $\bar{X}_t$ denotes the value of the variable under efficient equilibrium. Moreover, we denote as $\bar{x}_t = \ln\left(\frac{\bar{X}_t}{X}\right)$.

Considering the flexible prices economy resource constraint,

$$\begin{aligned} U_t &= \bar{U}_t + \bar{U}_{\bar{Y},t}\bar{Y}_t\tilde{Y}_t + \frac{1}{2}\bar{U}_{\bar{Y}\bar{Y},t}\bar{Y}_t^2\tilde{Y}_t^2 + \bar{U}_{\bar{N},t}\bar{N}_t\tilde{N}_t + \frac{1}{2}\bar{U}_{\bar{N}\bar{N},t}\bar{N}_t^2\tilde{N}_t^2 + \\ &\quad + \bar{U}_{\bar{Y}\bar{N},t}\bar{Y}_t\bar{N}_t\tilde{Y}_t\tilde{N}_t + \bigcirc\left(\|\alpha\|^3\right) \end{aligned} \quad \text{(A7.2)}$$

Collecting terms yields

$$U_t = \bar{U}_t + \bar{U}_{\bar{Y},t}\bar{Y}_t\begin{bmatrix} \tilde{Y}_t + \frac{\bar{U}_{\bar{N},t}\bar{N}_t}{\bar{U}_{\bar{Y},t}\bar{Y}_t}\tilde{N}_t + \frac{1}{2}\frac{\bar{U}_{\bar{Y}\bar{Y},t}}{\bar{U}_{\bar{Y},t}}\bar{Y}_t\tilde{Y}_t^2 + \\ + \frac{1}{2}\frac{\bar{U}_{\bar{N}\bar{N},t}\bar{N}_t^2}{\bar{U}_{\bar{Y},t}\bar{Y}_t}\tilde{N}_t^2 + \frac{\bar{U}_{\bar{Y}\bar{N},t}\bar{N}_t}{\bar{U}_{\bar{Y},t}}\tilde{Y}_t\tilde{N}_t \end{bmatrix} + \bigcirc\left(\|\alpha\|^3\right) \quad \text{(A7.3)}$$

Considering that, $\frac{\bar{U}_{\bar{Y}\bar{N},t}\bar{N}_t}{\bar{U}_{\bar{Y},t}} = \frac{\phi_{\bar{N},t}(\bar{N}_t)\bar{N}_t}{\phi(\bar{N}_t)} = -(1-\sigma)\alpha$, we have,

$$U_t = \bar{U}_t + \bar{U}_{\bar{Y},t}\bar{Y}_t\begin{bmatrix} \tilde{Y}_t - \alpha\tilde{N}_t - \frac{\sigma}{2}\tilde{Y}_t^2 + (1-\sigma)\frac{\phi_N(\bar{N}_t)}{\phi(\bar{N}_t)}\bar{N}_t\tilde{Y}_t\tilde{N}_t \\ +\frac{1}{2}\left[\frac{\phi_{NN}(\bar{N}_t)}{\phi(\bar{N}_t)} - \sigma\left(\frac{\phi_N(\bar{N}_t)}{\phi(\bar{N}_t)}\right)^2\right]\bar{N}_t^2\tilde{N}_t^2 \end{bmatrix} + \bigcirc\left(\|\alpha\|^3\right) \quad \text{(A7.4)}$$

It can be shown that $\frac{\phi_{NN,t}(\bar{N}_t)}{\phi(\bar{N}_t)} = \frac{2\sigma-1}{\sigma}\left(\frac{\phi_N(\bar{N}_t)}{\phi(\bar{N}_t)}\right)^2$, hence

$$U_t = \bar{U}_t + \bar{U}_{\bar{Y},t}\bar{Y}_t\begin{bmatrix} \tilde{Y}_t - \alpha\tilde{N}_t - \frac{\sigma}{2}\tilde{Y}_t^2 + (1-\sigma)\frac{\phi_N(\bar{N}_t)}{\phi(\bar{N}_t)}\bar{N}_t\tilde{Y}_t\tilde{N}_t \\ +\frac{1}{2}\left[\left(\frac{2\sigma-1}{\sigma} - \sigma\right)\left(\frac{\phi_N(\bar{N}_t)}{\phi(\bar{N}_t)}\right)^2\right]\bar{N}_t^2\tilde{N}_t^2 \end{bmatrix} + \bigcirc\left(\|\alpha\|^3\right) \quad \text{(A7.5)}$$

$$U_t = \bar{U}_t + \bar{U}_{\bar{Y},t}\bar{Y}_t\left[\tilde{Y}_t - \alpha\tilde{N}_t - \frac{\sigma}{2}\tilde{Y}_t^2 - (1-\sigma)\alpha\tilde{Y}_t\tilde{N}_t + \frac{1}{2}\left(\frac{2\sigma-1}{\sigma} - \sigma\right)\alpha^2\tilde{N}_t^2\right] + \bigcirc\left(\|\alpha\|^3\right) \quad \text{(A7.6)}$$

We now take a first-order expansion of the term $\bar{U}_{\bar{Y},t}\bar{Y}_t$ around the steady state.

$$\begin{aligned} \bar{U}_{\bar{Y},t}\bar{Y}_t &= U_Y\left(1 + (1-\sigma)\bar{y}_t + (1-\sigma)\frac{\phi_N(N)}{\phi(N)}N\bar{n}_t\right) + \bigcirc\left(\|\alpha\|^2\right) \\ &= U_Y\left(1 + (1-\sigma)\bar{y}_t - (1-\sigma)\alpha\bar{n}_t\right) + \bigcirc\left(\|\alpha\|^2\right) \end{aligned} \quad \text{(A7.7)}$$

$$\frac{\phi_N\left(\bar{N}_t\right)}{\phi\left(\bar{N}_t\right)}\bar{N}_t = \frac{\phi_N\left(N\right)}{\phi\left(N\right)}N + \Gamma_n \bar{n}_t + + \bigcirc\left(\|\alpha\|^2\right) \tag{A7.8}$$

where $\Gamma_n = \left(\frac{\phi_N(N)N}{\phi(N)} + \frac{\phi_{NN}(N)N^2}{\phi(N)} - \frac{\phi_N(N)^2 N^2}{\phi(N)^2}\right)$

$$\left(\frac{\phi_N\left(\bar{N}_t\right)\bar{N}_t}{\phi\left(\bar{N}_t\right)}\right)^2 = \left(\frac{\phi_N\left(N\right)N^2}{\phi\left(N\right)}\right)^2 + \Lambda_n \bar{n}_t + \bigcirc\left(\|\alpha\|^2\right) \tag{A7.9}$$

where $\Lambda_n = 2\left(\frac{\phi_N(N)\phi_{NN}(N)N}{\phi(N)^2} + \left(\frac{\phi_N(N)N}{\phi(N)}\right)^2 - \left(\frac{\phi_N(N)}{\phi(N)}\right)^3 N\right)$

given that $\bar{n}_t = 0$, and that $\frac{\phi_N(N)N}{\phi(N)} = -\alpha$, substituting into the Welfare function,

$$U_t = \bar{U}_t + U_Y\left(1 + (1-\sigma)\bar{y}_t\right)\left[\begin{array}{c} \tilde{Y}_t - \alpha\tilde{N}_t - \frac{\sigma}{2}\tilde{Y}_t^2 - \alpha(1-\sigma)\tilde{Y}_t\tilde{N}_t \\ +\frac{1}{2}\left(\frac{2\sigma-1}{\sigma} - \sigma\right)\alpha^2\tilde{N}_t^2 \end{array}\right] + \bigcirc\left(\|\alpha\|^3\right) \tag{a7.10}$$

Given the aggregate production function and that the log-deviations of the price dispersion index $d_t = \tilde{Y}_t - \alpha\tilde{N}_t$ are of second-order, and that:

$$\tilde{Y}_t^2 = \alpha^2\tilde{N}_t^2 \qquad n_t\alpha\tilde{N}_t = n_t\tilde{Y}_t \qquad y_t\alpha\tilde{N}_t = y_t\tilde{Y}_t \qquad \tilde{Y}_t\alpha\tilde{N}_t = \tilde{Y}_t^2$$

considering only terms up to the second-order we have:

$$U_t = \bar{U}_t + U_Y\left[\begin{array}{c} \tilde{Y}_t - \tilde{N}_t - \frac{\sigma}{2}\tilde{Y}_t^2 - (1-\sigma)\tilde{Y}_t^2 \\ +\frac{1}{2}\left(\frac{2\sigma-1}{\sigma} - \sigma\right)\tilde{Y}_t^2 \end{array}\right] + \bigcirc\left(\|\alpha\|^3\right) \tag{A7.11}$$

$$\begin{aligned} \tilde{U}_t &\equiv U_t - \bar{U}_t = -U_Y\left\{d_t + \frac{1}{2}\left(\frac{2\sigma-1}{\sigma} - 2\right)\tilde{Y}_t^2\right\} + \bigcirc\left(\|\alpha\|^3\right) \\ &= U_t - \bar{U}_t = -U_Y\left\{d_t - \frac{1}{2\sigma}\tilde{Y}_t^2\right\} + \bigcirc\left(\|\alpha\|^3\right) \end{aligned} \tag{A7.12}$$

As proven by Galì and Monacelli [24], the log-index of the relative-price distortion is of second-order and proportional to the variance of prices across firms, which implies that:

$$d_t = \ln\left(\int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{-\frac{\theta}{\alpha}} di\right)^{\alpha} = \frac{\theta}{2}var_i\left\{p_t(i) + \bigcirc\left(\|\alpha\|^3\right)\right\} \tag{A7.13}$$

proof Galì and Monacelli [24].

As shown in Woodford [52], this means that

$$\sum_{t=0}^{\infty}\beta^t var_i\left\{p_t(i)\right\} = \frac{1}{\lambda_a}\sum_{t=0}^{\infty}\beta^t\pi_t^2 \tag{A7.14}$$

where $\lambda = (1-\psi)(1-\psi\beta)/\psi$.

Finally, denoting the output gap $\tilde{Y}_t$ as in the standard way x_t, the Welfare-Based loss-function can be written as,

$$W_t = E_t \sum_{k=0}^{\infty} \beta^t \tilde{U}_{t+k} = -\frac{U_Y}{2} E_t \sum_{k=0}^{\infty} \left\{ \frac{\theta}{\lambda_a} \pi_{t+k}^2 + \frac{1}{\sigma} x_{t+k}^2 \right\} + \bigcirc \left(\|\alpha\|^3 \right) \quad \text{(A7.15)}$$

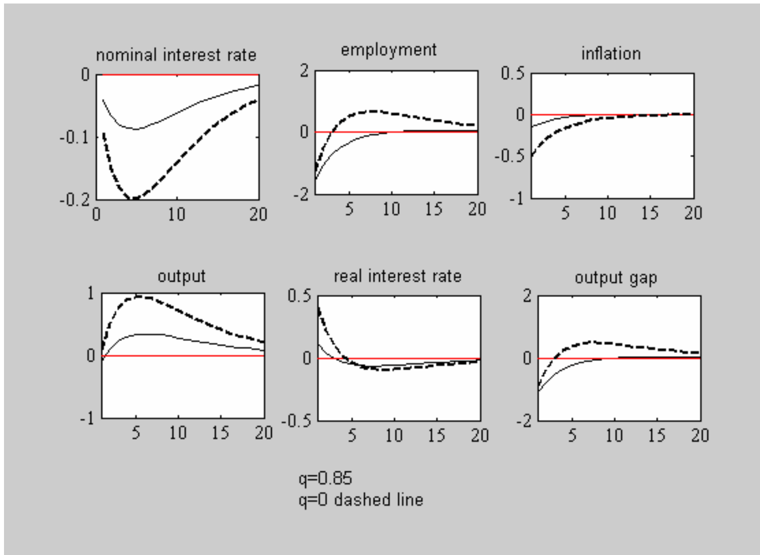

Figure 1: IRFs to a 1 unit standard deviation productivity shock under the EU estimated Taylor rule.

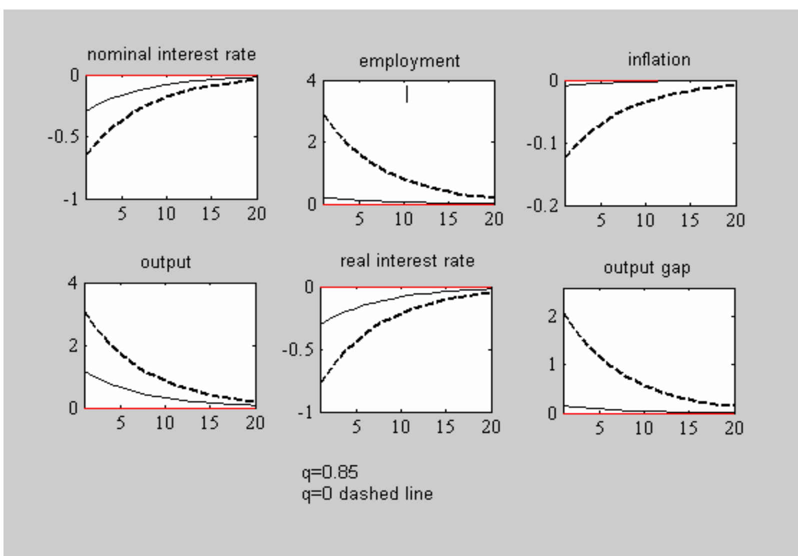

Figure 2: IRFs to a 1 unit standard deviation productivity shock under the optimal monetary policy

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