UNIVERSITA' CATTOLICA DEL SACRO CUORE

WORKING PAPER

DISCE

Dipartimenti e Istituti di Scienze Economiche

Product differentiation, price discrimination and collusion

Stefano Colombo

IEF0085 - March - 2009

UNIVERSITA' CATTOLICA DEL SACRO CUORE
- Milano -

QUADERNI DELL'ISTITUTO DI ECONOMIA E FINANZA

Product differentiation, price discrimination and collusion

Stefano Colombo

n. 85 - marzo 2009

Quaderni dell’Istituto di Economia e Finanza
numero 85 marzo 2009

Product differentiation, price discrimination and collusion

Stefano Colombo (*)

(*)Istituto di Economia e Finanza, Università Cattolica del Sacro Cuore, Largo Gemelli 1 – 20123 Milano, e-mail: stefano.colombo@unicatt.it

Redazione

Istituto di Economia e Finanza
Università Cattolica del S. Cuore
Largo Gemelli 1
20123 Milano
tel.: 0039.02.7234.2976
fax: 0039.02.7234.2781
e-mail: ist.ef@unicatt.it

* Esemplare fuori commercio per il deposito legale agli effetti della Legge n. 106 del 15 aprile 2004.

* La Redazione ottempera agli obblighi previsti dalla Legge n. 106 del 15.04.2006, Decreto del Presidente della Repubblica del 03.05.2006 n. 252 pubblicato nella G.U. del 18.08.2006 n. 191.

* I quaderni sono disponibili on-line all'indirizzo dell'Istituto http://www.unicatt.it/istituti/EconomiaFinanza

* I *Quaderni dell'Istituto di Economia e Finanza* costituiscono un servizio atto a fornire la tempestiva divulgazione di ricerche scientifiche originali, siano esse in forma definitiva o provvisoria.
L'accesso alla collana è approvato dal Comitato Scientifico, sentito il parere di un referee.

Product differentiation, price discrimination and collusion

Stefano Colombo*

Abstract

The existing literature which analyses the relationship between the product differentiation degree and the sustainability of a collusive agreement on price assumes that firms cannot price discriminate, and concludes that there is a negative relationship between the product differentiation degree and the critical discount factor. This paper, in contrast, assumes that firms are able to price discriminate. Within the Hotelling framework, three different collusive schemes are studied: optimal collusion on discriminatory prices; optimal collusion on a uniform price; collusion not to discriminate. We obtain that the critical discount factor of the first and the third collusive scheme does not depend on the product differentiation degree, while the critical discount factor of the second collusive scheme depends positively on the product differentiation degree. Moreover, we show that suboptimal collusion is more difficult to sustain than optimal collusion.

.

JEL codes: D43; L11; L41

Keywords: Horizontal differentiation; Price discrimination; Tacit collusion.

*Università Cattolica del Sacro Cuore – Milano, Largo A. Gemelli 1, I-20123, Milano. E-mail: stefano.colombo@unicatt.it. I greatly benefited from discussions with Michele Grillo. All remaining errors are my own.

1 Introduction

Product differentiation affects both the way in which the firms compete and the way in which they collude. When the firms produce differentiated goods, their pricing decisions depend on the substitutability between the products: if the products are good substitute no firm can command a high price for its product. Therefore, the lower is the product differentiation degree the lower is the non-cooperative equilibrium price. For this reason, the firms may try to coordinate their pricing decisions in order to jointly raise the price above the competitive level. However, a low product differentiation degree not only increases the opportunity for collusion, but it also increases the incentive to cheat from the collusive agreement. Indeed, with highly substitutable products, a cheating firm can capture a large fraction of the market and obtain large short-term profits by slightly lowering the price unilaterally. Therefore, the impact of the product differentiation degree on the sustainability of a collusive agreement is not *a priori* an obvious issue.

The relationship between the product differentiation degree and the ability of the firms to collude has been studied by, among others, Chang (1991), Chang (1992) and Hackner (1995). Chang (1991) employs the spatial competition framework of Hotelling (1929) with quadratic transportation costs. He assumes fixed and symmetric locations of the firms. The sustainability of the cartel agreement is measured by the minimum discount factor supporting the joint maximum profits as a sub-game perfect equilibrium of an infinitely repeated game. Chang (1991) shows that collusion is easier to sustain the more differentiated are the firms. In fact, the critical discount factor monotonically increases as the product differentiation decreases. A similar result is found in Chang (1992), where the initial degree of differentiation is exogenous, but firms can relocate once the collusive agreement has been broken. Chang (1992) concludes that a higher initial product differentiation degree makes collusion easier to sustain. Hackner (1995) instead considers the possibility that firms collude not only with respect to the price but also with respect to the location. When the market discount factor is high enough, firms collude to locate at 1/4 and 3/4. The lower is the market discount factor the more the firms collude on a higher product differentiation degree in order to keep collusion from breaking down. Hackner (1995) concludes that there is "*a fairly general tendency within the Hotelling framework for differentiation to facilitate collusion*" (pag. 293).

All these articles are characterized by the assumption that firms cannot price discriminate. In this paper we remove this hypothesis, and we study how the product differentiation degree affects the sustainability of a collusive agreement *when firms can perfectly price discriminate*. At our knowledge, the only article that studies the sustainability of collusion taking into account price discrimination is Liu and Serfes (2007). They assume that firms are maximally differentiated on the Hotelling segment, while they allow for different customer-specific information quality (see also Liu and Serfes, 2004). Firms have access to information of a given quality which allows them to partition consumers into different groups and charge each group with a different price. Higher information quality is modelled as a refinement of the partition. At the limit, firms know the position of each consumer in the market and can charge each consumer with a different price (perfect price discrimination). Liu and Serfes (2007) show that collusion becomes more difficult to sustain as the quality of consumer-specific information improves. Better information allows for higher collusive profits and harsher punishment, but at the same time makes deviation more profitable: this last effect dominates, and the critical discount factor is a positive function of the quality of information.

On the one hand, our analysis is less general than Liu and Serfes (2007), since we consider only the case of perfect price discrimination[1]. On the other hand, our analysis is more general, since we do not limit the analysis to the case of maximally differentiated firms, but we allow for different product differentiation degrees. As in Liu and Serfes (2007), we study three different collusive schemes: 1) collusion on discriminatory prices; 2) collusion on a uniform price; 3) collusion not to discriminate. In the first collusive scheme firms coordinate on the price to be applied to each consumer, without the constraint that the price must be equal for all consumers. Clearly, this collusive scheme yields the highest collusive profits, since it allows the colluding firms to perfectly target the price on the willingness to pay of each consumer. However, such collusive scheme may be very difficult to implement, since it requires negotiating on a huge number of prices (one for each consumer). A less "extreme" collusion is

[1] In the appendix, however, we extend the analysis of two of the three collusive schemes we analyzed to the case of third-degree price discrimination.

represented by the second collusive scheme: here firms try to coordinate on a uniform price. This scheme is less profitable, but it is easier to implement, because it requires firms to agree only on one price. Finally, in the third collusive scheme firms do not agree directly on the price(s), but simply agree not to price discriminate. Since in the spatial competition framework price discrimination causes lower equilibrium profits[2], firms have the incentive to coordinate in order to compete less fiercely: an agreement not to discriminate has precisely this purpose[3].

For each collusive scheme we search the minimum discount factor which is needed to sustain the joint maximum profits. We are mainly interested in the following question: how does the easiness of collusion change with the product differentiation degree? A linked question is the following: which collusive scheme is easier to sustain in equilibrium for any given product differentiation degree? We obtain the following results. The sustainability of the first and the third collusive scheme does not depend on the product differentiation degree. The sustainability of the second collusive scheme instead depends *negatively* on the product differentiation degree. This result contrasts with the findings by Chang (1991), Chang (1992) and Hackner (1995): the hypothesis of price discrimination reverses the relationship between the sustainability of collusion and the product differentiation degree. Moreover, in contrast with Chang (1991), the sustainability of collusion depends *negatively* on the transportation costs. We obtain also that, independently on the product differentiation degree, the first collusive scheme is easier to sustain than the second collusive scheme, which in turn is easier to sustain than the third collusive scheme. In addition, we consider the possibility that firms collude on a suboptimal discriminatory price schedule and on a suboptimal uniform price. In both cases we obtain that if optimal collusion is not sustainable, suboptimal collusion is not sustainable too. Finally, we extend in the appendix the analysis of the second and third collusive scheme to a third-degree price discrimination framework *a la* Liu and Serfes (2004, 2007), and we show that the results do not change.

[2] See for example Thisse and Vives (1988).

[3] Each of the collusive schemes we study in this paper is well documented in European antitrust cases. Examples of the first collusive scheme are: *Cast Iron and Steel* (D. Comm., Oct. 17, 1983) and *Pre-insulated Pipes* (D. Comm., Oct. 21, 1998); examples of the second collusive scheme are: *Austrian Banks* (D. Comm., June 12, 2002) and *Specialty Graphite* (D. Comm., Dec. 17, 2002); examples of the third collusive scheme are: *IFTRA Glass* (D. Comm., May 15, 1974), *IFTRA Aluminium* (D. Comm., July 15, 1975) and *Far East Trade Tariff Charges and Surcharges Agreement* (*FETTCSA*) (D. Comm., May 16, 2000).

This paper is structured as follows. In section 2 the model is introduced. In section 3 we describe the infinitely repeated game. In section 4 the sustainability of each collusive scheme is studied, while in section 5 the model is extended to include the possibility of suboptimal collusion. Section 6 summarizes. The appendix generalizes the second and the third collusive scheme results to the case of third-degree price discrimination.

2. The model of differentiated firms

Assume a linear market of length 1. Consumers are uniformly distributed along the market. Define with $x \in [0,1]$ the location of each consumer. Each point in the linear market represents a certain variety of a given good. For a consumer positioned at a certain point, the preferred variety is represented by the point in which the consumer is located: the more the variety is far from the point in which the consumer is located, the less it is appreciated by the consumer. Each consumer consumes no more than 1 unit of the good. Define with *v* the maximum price that a consumer is willing to pay for buying his preferred variety.

There are two firms, *A* and *B*, competing in the market. Marginal costs are zero. Following Chang (1991), we consider symmetric firms. Firm *A* produces the variety $a \in [0,1/2]$ and firm *B*, given the symmetry assumption, produces the variety $1-a$. The parameter *a* measures the product differentiation: when $a=0$, firms are maximally differentiated; when $a=1/2$ firms are identical. Finally, define with p_x^J the price charged by firm $J = A, B$ to the consumer *x*: clearly, when firm *J* sets a uniform price, it must be $p_x^J = p_{x'}^J$ for every $x, x' \in [0,1]$.

The utility of a consumer depends on *v*, on the price set by the firm from which he buys, and on the distance between his preferred variety and the variety produced by the firm. Following D'Aspremont *et al.* (1979), we assume quadratic transportation costs. Define with *t*, equal for all consumers, the importance attributed by the consumer to the distance between his preferred variety and the variety offered by the firm. The utility of a consumer located at *x* when he buys from firm *A* is given by: $u_x^A = v - p_x^A - t(x-a)^2$, while the utility of a consumer located at *x* when he buys from firm *B* is given by:

$u_x^B = v - p_x^B - t(x-1+a)^2$. As in Chang (1991), Chang (1992) and Hackner (1995) we assume $w \equiv v/t \geq 5/4$: this assumption is sufficient to guarantee that under any optimal collusive agreement the entire market is served.

3. The infinitely repeated game

Suppose that firms interact repeatedly in an infinite horizon setting. As in Chang (1991), Chang (1992) and Hackner (1995), a *grim strategy* is assumed (Friedman, 1971)[4]. Moreover, there is perfect monitoring. Define Π^C, Π^D and Π^N respectively as the one-shot collusive profits, the one-shot deviation profits and the one-shot punishment (or Nash) profits for each firm: obviously, $\Pi^D > \Pi^C > \Pi^N$. Define δ as the market discount factor, which is assumed to be exogenous and common for each firm. It is well known that collusion is sustainable as a sub-game perfect Nash equilibrium if and only if the discounted value of the profits that each firm obtains under collusion exceeds the discounted value of the profits that each firm obtains deviating from the tacit agreement. Formally, the following incentive-compatibility constraint must be satisfied:

$$\sum_{t=0}^{\infty} \delta^t \Pi^C \geq \Pi^D + \sum_{t=1}^{\infty} \delta^t \Pi^N ,$$

After some manipulations, the incentive-compatibility constraint can be rewritten as follows:

$$\delta \geq \delta^* = \frac{\Pi^D - \Pi^C}{\Pi^D - \Pi^N} \tag{1}$$

[4] The *grim strategy* implies that firms start by charging the collusive price schedule, p^C. The firms continue to set p^C until one firm has played p^D in the previous period, where p^D is the price schedule set by a firm which deviates from the collusive agreement. If a firm sets p^D at time *t*, from $t+1$ onward both firms play p^N, where p^N is the equilibrium price schedule emerging in the non-cooperative constituent game (Nash price).

Define δ^* as the critical discount factor. Equation (1) says that if the market discount factor is greater than the critical discount factor collusion is sustainable, otherwise it is not sustainable. Then, the critical discount factor measures the sustainability of the agreement: the greater is δ^* the smaller is the set of market discount factors which support collusion.

4. Sustainability of the collusive schemes

The stage game

Given the varieties produced by the firms and given the price set by each of them, each consumer buys from the firm which gives him the higher utility. If the utility of a consumer is the same when he buys from firm *A* and when he buys from firm *B*, we assume that he buys from the nearer firm[5]. The following proposition defines the Nash prices for any *a*:

Proposition 1: *when the firms can perfectly price discriminate between the consumers, the equilibrium prices during the punishment stage are the following*:

$$p^{A,N} = \begin{cases} t(1-2x-2a+4ax) & \text{if} \quad x \le 1/2 \\ 0 & \text{if} \quad x \ge 1/2 \end{cases} \tag{2}$$

$$p^{B,N} = \begin{cases} 0 & \text{if} \quad x \le 1/2 \\ -t(1-2x-2a+4ax) & \text{if} \quad x \ge 1/2 \end{cases} \tag{3}$$

Proof. Suppose $x < 1/2$. Consider firm *B*. First, we show that $p_x^B > 0$ cannot be an equilibrium price. When $p_x^B > 0$, the highest price firm *A* can set in order to serve consumer x is: $p_x^A = p_x^B + t(1-2x-2a+4ax)$. But now firm *B* has convenience to

[5] This assumption is very common in spatial models, since it allows avoiding the technicality of ε-equilibria. For more details about this assumption, see among the others Hurter and Lederer (1985), Lederer and Hurter (1986), Thisse and Vives (1988), Hamilton *et al.* (1989), Hamilton and Thisse (1992).

undercut p_x^A in order to serve consumer *x*. Therefore $p_x^B > 0$ cannot be an equilibrium price. Second, we show that $p_x^B = 0$ is an equilibrium price. When $p_x^B = 0$, the highest price firm *A* can set in order to serve consumer *x* is: $p_x^A = t(1-2x-2a+4ax)$. With such a price firm *B* obtains zero profits from consumer *x*, which buys from firm *A*, but it has no incentive to change its price, because increasing the price it would continue to obtain zero profits, and setting a price lower than the marginal costs would entail a loss. On the other hand, firm *A* is setting the highest possible price which guarantees it to serve consumer *x*: therefore, it has no incentive to change its price. It follows that $p_x^A = t(1-2x-2a+4ax)$ and $p_x^B = 0$ represents the (unique) price equilibrium. The proof for $x > 1/2$ is symmetric. Finally, when $x = 1/2$, the standard Bertrand's result holds: the unique price equilibrium when two undifferentiated firms compete on price is represented by both firms setting a price equal to the marginal cost. ■

The punishment profits of each firm follow directly from Proposition 1:

$$\Pi^N = \frac{t}{4}(1-2a) \tag{4}$$

Optimal collusion on discriminatory prices

Consider the first collusive scheme. Define p^* as the optimal collusive discriminatory price schedule. Since the individual price can be perfectly targeted to each consumer, the optimal collusive price is the highest price which satisfies the participation constraint of the consumer. Therefore:

$$p^* = \begin{cases} v - t(x-a)^2 & \text{if} \quad x \leq 1/2 \\ v - t(x-1+a)^2 & \text{if} \quad x \geq 1/2 \end{cases} \tag{5}$$

Given the assumption on *w*, p^* is strictly positive for any *a* and *x* and the whole market is served. It is immediate to note that consumers located at $x \in [0, 1/2]$ buy from firm *A*, while consumers located at $x \in [1/2, 1]$ buy from firm *B*. Consumer surplus is totally transferred to the firms, and the collusive profits of each firm are equal to:

$$\Pi^C = \frac{v}{2} - \frac{t}{2}(a^2 - \frac{a}{2} + \frac{1}{12}) \tag{6}$$

Suppose that firm *A* deviates from the collusive agreement. Define with $\hat{p}_1$ the price that makes the consumer located in *x* indifferent between buying from firm *A* and from firm *B*, which is setting the collusive price p^*. Solving $u_x^A(\hat{p}_1) = u_x^B(p^*)$ with respect to $\hat{p}_1$ we get: $\hat{p}_1 = p^* + t(1 - 2x - 2a + 4ax)$. We assume that when the consumer is indifferent between the deviating firm and the colluding firm, he buys from the deviating firm[6]. Therefore $\hat{p}_1$ is the highest price which allows firm *A* to steal a consumer from firm *B*. However, firm *A* may be impeded from setting $\hat{p}_1$: this occurs when $\hat{p}_1$ is too high for the participation constraint of the consumer or when it is too low for the participation constraint of the firm. Since p^* extracts the whole consumer surplus, it represents a natural upper bound for the deviation price, while the marginal cost (equal to zero) is the lower bound for the deviation price. Then:

$$p^D = \max\{0; \min[p^*; \hat{p}_1]\}, \tag{7}$$

where p^D is the deviation price schedule. The following lemma fully characterizes the deviation price schedule:

Lemma 1. *When firms collude on* p^* *the deviation price schedule is*:

$$p^D = v - t(x - a)^2 \tag{8}$$

[6] This assumption can be rationalized noting that the deviating firm can always offer to the consumer a utility which is strictly larger than the utility he receives from the colluding firm by setting a price equal to $\hat{p}_1 - \varepsilon$, where ε is a positive small number.

Proof. First, consider the consumers located at $x \leq 1/2$. Since $t(1-2x-2a+4ax) \geq 0$ $\forall x \leq 1/2$, it follows that $\hat{p}_1 \geq p^*$ (i.e. the participation constraint of the consumer binds). Moreover, given the assumption on *w*, $p^* \geq 0$ (i.e. the participation constraint of the deviating firm does not bind). Then, by equation (7), it must be: $p^D = p^*$. Finally, using equation (5) it follows that: $p^D = v - t(x-a)^2, \forall x \leq 1/2$. Next, consider the consumers located at $x \geq 1/2$. Since $t(1-2x-2a+4ax) \leq 0$ $\forall x \geq 1/2$, it follows that $\hat{p}_1 \leq p^*$ (i.e. the participation constraint of the consumer does not bind). Moreover, given the assumption on *w*, $\hat{p}_1 \geq 0$ for $x \geq 1/2$ (i.e. the participation constraint of the firm does not bind). By equation (7) it must be: $p^D = \hat{p}_1$. Substituting equation (5) into $\hat{p}_1$ we get: $p^D = v - t(x-a)^2$, $\forall x \geq 1/2$. ■

The deviation profits are the following:

$$\Pi^D = v - \frac{t}{3} - ta^2 + ta \tag{9}$$

By substituting equations (4), (6) and (9) into equation (1) we obtain the critical discount factor:

$$\delta_1^* = \frac{1}{2} \tag{10}$$

Then, the sustainability of the optimal collusive discriminatory price schedule *does not depend* on the product differentiation degree. The fact that the substitutability of the products of the two firms is high or low does not have any impact on the likelihood that the tacit agreement will be disrupted by the defection of one member of the cartel. When firms can perfectly price discriminate during the deviation and the punishment phase, collusion can be sustained if and only the market discount factor exceeds 1/2, irrespectively of the position of the firms in the market.

Optimal collusion on uniform price

Suppose now that the firms, instead of colluding on the optimal discriminatory price schedule, collude on the optimal uniform price, $\overline{p}^C$. Chang (1991) and Hackner (1995) show that, under the hypothesis that $w \geq 5/4$, joint profit maximization implies full market coverage. Therefore, profits are maximized by raising the price until the farthest consumer is indifferent between buying and not buying. It follows that when $a \leq 1/4$ the consumer located in the middle of the segment ($x = 1/2$) receives zero utility at the profit maximizing collusive uniform price. Similarly, when $a \geq 1/4$, the consumers at the endpoints of the segment ($x = 0$ and $x = 1$) receive zero utility at the profit maximizing collusive uniform price. Hence, the optimal collusive uniform price is given by:

$$\overline{p}^C = \begin{cases} v - t(1/2 - a)^2 & if \quad a \leq 1/4 \\ v - ta^2 & if \quad a \geq 1/4 \end{cases} \tag{11}$$

At this point it is convenient to handle separately the case of $a \leq 1/4$ and the case of $a \geq 1/4$. The relevant equations will be identified by the appropriate subscript. We start from the case in which the firms are highly differentiated, $a \leq 1/4$. The collusive profits of each firm are the following:

$$\Pi^C_{a\leq 1/4} = \frac{v}{2} - \frac{t}{2}(\frac{1}{2} - a)^2 \tag{12}$$

Suppose that firm *A* deviates. Define $\hat{p}_{2;a\leq 1/4}$ as the price that makes the consumer located in *x* indifferent between buying from firm *A* and from firm *B*, which is setting the collusive price $\overline{p}^C$. Solving $u^A_x(\hat{p}_{2;a\leq 1/4}) = u^B_x(\overline{p}^C)$ with respect to $\hat{p}_{2;a\leq 1/4}$ we get: $\hat{p}_{2;a\leq 1/4} = \overline{p}^C + t(1 - 2x - 2a + 4ax)$. Following the reasoning introduced in the previous subsection, the deviation price is equal to $\hat{p}_{2;a\leq 1/4}$, provided that $\hat{p}_{2;a\leq 1/4}$ is lower than p^* and higher than 0. That is:

$$p^D_{a\leq 1/4} = \max\{0; \min[p^*; \hat{p}_{2;a\leq 1/4}]\} \tag{13}$$

The following lemma describes the deviation price schedule:

<u>Lemma 2</u>. *Suppose* $a \leq 1/4$. *When firms collude on* $\overline{p}^C$ *the deviation price schedule is*:

$$p^D_{a\leq 1/4} = v - t(-4ax - \frac{3}{4} + a^2 + a + 2x) \tag{14}$$

Proof. First, we show that: $\hat{p}_{2;a\leq 1/4} \leq p^*$. The utility of each consumer (except the farthest one) paying $\overline{p}^C$ has to be positive, since $\overline{p}^C$ is obtained by setting the utility of the farthest consumer equal to zero. Therefore, $u^A_x(\overline{p}^C) \geq 0$. Given the indifference condition, it follows that $u^A_x(\hat{p}_{2;a\leq 1/4}) \geq 0$. Recall that the optimal discriminatory price schedule yields zero consumer surplus, that is: $u^A_x(p^*) = 0$. Therefore: $u^A_x(\hat{p}_{2;a\leq 1/4}) \geq u^A_x(p^*)$, which in turn implies $\hat{p}_{2;a\leq 1/4} \leq p^*$. Next, we prove that $\hat{p}_{2;a\leq 1/4} \geq 0$. Using equation (11) in $\hat{p}_{2;a\leq 1/4}$ we obtain the following equation: $\hat{p}_{2;a\leq 1/4} = v - t(-4ax - 3/4 + a^2 + a + 2x)$. After some manipulations, the condition $\hat{p}_{2;a\leq 1/4} \geq 0$ can be rewritten as: $w > -4ax - 3/4 + a^2 + a + 2x$. Since $w \geq 5/4$, the condition is always verified when: $2(1-x) > a(1+a-4x)$. Both the l.h.s. and the r.h.s. of the last inequality are linearly decreasing in *x*. Therefore, it is sufficient to consider the extreme values of *x*: when $x = 0$ we get $2 > a(1+a)$, and when $x = 1$ we get $0 > a(a-3)$. Hence, the l.h.s. is always larger than the r.h.s.. It follows that $\hat{p}_{2;a\leq 1/4} \geq 0$ $\forall x$. Therefore, by equation (13), $p^D_{a\leq 1/4} = \hat{p}_{2;a\leq 1/4} = v - t(-4ax - 3/4 + a^2 + a + 2x)$. ■

Given the deviation price schedule, all consumers are served by the deviating firm, and the deviation profits are:

$$\Pi^{D}_{a\le 1/4}(a) = v - t(\frac{1}{2} - a)^2 \tag{15}$$

By inserting equations (4), (12) and (15) into equation (1) we obtain the critical discount factor:

$$\delta^{*}_{a\le 1/4}(a,w) = \frac{\frac{w}{2} - \frac{1}{8} - \frac{1}{2}a^2 + \frac{1}{2}a}{w - \frac{1}{2} - a^2 + \frac{3}{2}a} \tag{16}$$

Consider now the case of lower product differentiation degree, $a \ge 1/4$. The collusive profits are:

$$\Pi^{C}_{a\ge 1/4}(a) = \frac{v}{2} - \frac{t}{2}a^2 \tag{17}$$

Suppose firm A cheats. As usual, define $\hat{p}_{2;a\ge 1/4}$ as the price which solves $u^{A}_{x}(\hat{p}_{2;a\ge 1/4}) = u^{B}_{x}(\overline{p}^{C})$. Therefore: $\hat{p}_{2;a\ge 1/4} = \overline{p}^{C} + t(1-2x-2a+4ax)$, which is the deviation price schedule provided that it is lower than p^* and higher than 0. Then:

$$p^{D}_{a\ge 1/4} = \max\{0; \min[p^*; \hat{p}_{2;a\ge 1/4}]\} \tag{18}$$

The deviation price schedule is fully characterized by the following lemma.

Lemma 3. *Suppose* $a \ge 1/4$. *When firms collude on* $\overline{p}^{C}$ *the deviation price schedule is*:

$$p^{D}_{a\ge 1/4} = v - t(a^2 - 4ax - 1 + 2x + 2a) \tag{19}$$

Proof. The proof for $\hat{p}_{2;a\geq 1/4} \leq p^*$ is identical to the case described in Lemma 2. We prove now that: $\hat{p}_{2;a\geq 1/4} \geq 0$. By substituting equation (11) into $\hat{p}_{2;a\geq 1/4}$, we obtain: $\hat{p}_{2;a\geq 1/4} = v - t(a^2 - 4ax - 1 + 2x + 2a)$. Rearranging, the condition $\hat{p}_{2;a\geq 1/4} \geq 0$ can be rewritten as: $w > a^2 - 4ax - 1 + 2x + 2a$. Since $w \geq 5/4$, the condition is always verified when: $9/4 - 2x > a(a - 4x + 2)$. Both the l.h.s. and the r.h.s. of the last inequality are linearly decreasing in *x*. Therefore, it is sufficient to consider the extreme values of *x*: when $x = 0$ we get $9/4 > a(a+2)$, and when $x = 1$ we get $1/4 > a(a-2)$. Hence, the l.h.s. is always larger than the r.h.s.. It follows that $\hat{p}_{2;a\geq 1/4} \geq 0$ for any *x* and $1/4 \leq a \leq 1/2$. Therefore, by equation (18), $p^D_{a\geq 1/4} = \hat{p}_{2;a\geq 1/4} = v - t(a^2 - 4ax - 1 + 2x + 2a)$.
■

Therefore, the deviating firm serves the whole market and the deviation profits are:

$$\Pi^D_{a\geq 1/4}(a) = v - ta^2 \tag{20}$$

By inserting equations (4), (17) and (20) into (1), we obtain the critical discount factor:

$$\delta^*_{a\geq 1/4}(a,w) = \frac{\frac{w}{2} - \frac{1}{2}a^2}{w - \frac{1}{4} + \frac{1}{2}a - a^2} \tag{21}$$

Define:

$$\delta^*_2(a,w) = \begin{cases} \delta^*_{a\leq 1/4}(a,w) & if \quad a \leq 1/4 \\ \delta^*_{a\geq 1/4}(a,w) & if \quad a \geq 1/4 \end{cases} \tag{22}$$

We state the following proposition:

Proposition 2: $\delta_2^*(a,w)$

a) is a continuous function;

b) is monotonically decreasing both in a and w;

c) takes values between 1/2 and $g(w)$, where $g(w)\in[1/2\,;2/3]$ and $\partial g(.)/\partial w<0$.

Proof.

a) In order to prove that $\delta_2^*(a,w)$ is a function we need to verify that it takes a unique value for all points of its domain. This is certainly true when $a\in[0,1/4)$ and $a\in(1/4,1/2]$. It remains to verify it when $a=1/4$. Hence, we need to prove that $\delta_{a\le 1/4}^*(a=1/4\,;w)=\delta_{a\ge 1/4}^*(a=1/4\,;w)$. By substituting $a=1/4$ in equations (16) and (21), we get: $\delta_{a\le 1/4}^*(a=1/4\,;w)=\dfrac{w-1/16}{2w-3/8}=\delta_{a\ge 1/4}^*(a=1/4\,;w)$. Moreover, since $\delta_{a\le 1/4}^*(a,w)$ and $\delta_{a\ge 1/4}^*(a,w)$ are continuous, $\delta_{a\le 1/4}^*(a=1/4\,,w)=\delta_{a\ge 1/4}^*(a=1/4\,,w)$ implies that $\delta_2^*(a,w)$ is continuous in $a\in[0,1/2]$ and $w\in[5/4\,,\infty)$.

b) Consider the derivative of δ_2^* with respect to *w*. When $a\le 1/4$ we obtain: $\partial\delta_{a\le 1/4}^*\big/\partial w=(2a-1)\big/[2(1-3a+2a^2-2w)^2]\le 0$, while when $a\ge 1/4$ we obtain: $\partial\delta_{a\ge 1/4}^*\big/\partial w=(4a-2)\big/(1-2a+4a^2-4w)^2\le 0$. Therefore δ_2^* decreases as *w* increases. Consider now the derivative of δ_2^* with respect to *a*. When $a\le 1/4$ we get: $\partial\delta_{a\le 1/4}^*\big/\partial a=-[4w+(1-2a)^2]\big/4(1-3a+2a^2-2w)^2<0$. When $a\ge 1/4$ we obtain: $\partial\delta_{a\ge 1/4}^*\big/\partial a=-4(w-a+a^2)\big/(1-2a+4a^2-4w)^2$. The derivative is negative if and only if $w>a-a^2$, which is always true since the maximum value of the r.h.s. is 1/4.

c) Since $\delta_2^*(a,w)$ is decreasing both in *a* and in *w*, its maximum is in $(a=0;w=5/4)$. It results: $\delta_2^*(a=0;w=5/4)=2/3$. On the contrary, the critical discount factor takes the minimum value when $a=1/2$ and $w\to\infty$. Note that $\delta_2^*(a=1/2\,,w)=1/2$ for any *w*. Conversely, since $\lim_{w\to\infty}\delta_{a\le 1/4}^*=\lim_{w\to\infty}\delta_{a\ge 1/4}^*=1/2$ for any *a*, it follows that $\lim_{w\to\infty}\delta_2^*=1/2$ for any *a*. ■

Proposition 2 shows that there is a *negative* relationship between the product differentiation degree and the sustainability of the collusive agreement, as well as a *negative* relationship between the transportation costs and the sustainability of the collusive agreement. These findings contrast with Chang (1991) model, where a *positive* relationship between the product differentiation degree and the sustainability of the collusive agreement as well as a *positive* relationship between the transportation costs and the sustainability of the collusive agreement are shown to exist. In the following, we try to describe the mechanism behind such reversion. First, note that the sign of the derivative of the critical discount factor depends on the value taken by the following function, which is simply the numerator of the derivative of the critical discount factor (equation (1)) with respect to the variable *i*:

$$\Gamma = \frac{\partial \Pi^D}{\partial i}(\Pi^C - \Pi^N) - \frac{\partial \Pi^C}{\partial i}(\Pi^D - \Pi^N) + \frac{\partial \Pi^N}{\partial i}(\Pi^D - \Pi^C),$$

where $i = t, a$. That is, when $\Gamma > 0$ the derivative of the critical discount factor is positive, and vice-versa.

From equations (4), (15) and (20) of this paper and equations (6), (9) and (10) in Hackner (1995)[7] paper it is immediate to note that: $\Pi_u^N > \Pi_d^N$ and $\Pi_u^D < \Pi_d^D$, where the subscript indicates the uniform price model (Chang, 1991, Hackner, 1995) and the discriminatory price model (our paper) respectively, while the superscript indicates the Nash profits and the deviation profits respectively. The explanation is the following. When a firm can use discriminatory deviation prices, it can better target the prices it uses to steal consumers from the rival, and therefore deviation profits are larger. On the contrary, when *both* firms compete with discriminatory prices, competition is fiercer, and consequently Nash profits are lower.

Consider now $i = t$. By comparing the derivatives of equations (4), (15) and (20) of this chapter with the derivatives of equations (6), (9) and (10) in Hackner (1995) we get:

[7] Hackner (1995) defines the relevant equations for the model with uniform price, while in Chang (1991) they are left implicit. So we refer directly to Hackner's paper. Moreover, in order to simplify the exposition, we refer only to the case where the deviating firm serves the whole market: a sufficient condition for this to occur in the uniform price model is $v/t \geq 13/4$ (see Hackner, 1995, pag. 296).

$\frac{\partial \Pi_u^D}{\partial t} < \frac{\partial \Pi_d^D}{\partial t} < 0$ and $\frac{\partial \Pi_u^N}{\partial t} > \frac{\partial \Pi_d^N}{\partial t} > 0$. The intuition behind the first inequality is the following. When transportation costs increase, each consumer is more "loyal" to the nearer firm. Therefore, it becomes more difficult for the cheating firm to steal consumers from the rival. When the cheating firm uses a uniform price, the deviation price reduces for *all consumers* as a consequence of a larger *t*. Instead, when the deviating firm uses discriminatory prices, a larger *t* allows *increasing* prices on those consumers which are nearer to the cheating firm[8]. This effect partially counterbalances the reduction of the prices applied on the more distant consumers, and therefore the deviation profits are less sensitive (in absolute value) to variations in *t* in the discriminatory price model. The intuition behind the second inequality is the following. The equilibrium Nash price in the uniform price model is given by[9]: $t(1-2a)$, while the equilibrium Nash prices in the discriminatory price model are given by[10]: $t(1-2a-2x+4ax)$. With discriminatory prices, the individual price does not depend only on the transportation costs and on the distance between the firms as in the uniform price model, but also on the location of the consumer, *x*. In particular, the more the consumer is indifferent between the firms, the less the price depends on *t*: at the limit, when the consumer is completely indifferent between the firms ($x = 1/2$), the equilibrium price is 0 for every *t* (i.e. transportation costs do not matter for the equilibrium price on this consumer). In general, the dependency of the equilibrium discriminatory prices on *x* reduces the dependency of equilibrium discriminatory prices on *t*: therefore the Nash profits in the discriminatory price model are less sensitive to *t* with respect to the uniform price model.

Finally, from equations (12) and (17)[11] of this paper, we get: $\frac{\partial \Pi^C}{\partial t} < 0$. In fact, the greater is *t* the smaller is the collusive price needed to serve the furthest consumer.

[8] Consider the derivative of equations (14) and (19) of this paper with respect to *t* and observe that they are positive for low values of *x*.

[9] See, for example, D'Aspremont *et al.*, (1979).

[10] See Proposition 1. Here we consider only consumers located in the first half of the segment. The analysis proceeds in the same way for the other consumers.

[11] Equations (12) and (17) of this paper coincide with equations (4) and (5) in Hackner (1995) paper respectively.

Now, it is possible to identify the impact of the discriminatory price assumption over the Γ-function. *Ceteris paribus*, the fact that the derivative of the deviation profits is smaller (in absolute value) in the discriminatory price model increases Γ with respect to the uniform price model; the fact that the derivative of the Nash profits is smaller in the discriminatory price model decreases Γ with respect to the uniform price model; the fact that the deviation profits are greater in the discriminatory price model increases Γ with respect to the uniform price model; and, finally, the fact that the Nash profits are smaller in the discriminatory price model decreases Γ with respect to the uniform price model[12]. In our framework, the lower sensitivity of the deviation profits and the higher level of the deviation profits in the discriminatory price model outweigh the impact of the lower sensitivity and the lower level of the Nash profits, and change the sign of Γ: the relationship between the transportation costs and the critical discount factor thus reverses with respect to the uniform price model.

Consider now $i = a$, with $a \leq 1/4$. Again, from equations (4) and (15) of this paper and equations (6) and (9) in Hackner (1995) paper it is easy to obtain that: $\frac{\partial \Pi_u^N}{\partial a} < \frac{\partial \Pi_d^N}{\partial a} < 0$ and $\frac{\partial \Pi_u^D}{\partial a} > \frac{\partial \Pi_d^D}{\partial a} > 0$ [13]. Moreover, from equation (12), we get: $\frac{\partial \Pi^C}{\partial a} > 0$ [14]. Therefore, *ceteris paribus*, the fact that the derivative of the deviation profits is smaller in the discriminatory price model decreases Γ with respect to the uniform price model; the fact that the derivative of the Nash profits is smaller (in absolute value) in the discriminatory price model increases Γ with respect to the uniform price model; the fact that the deviation profits are greater in the discriminatory price model decreases Γ with respect to the uniform price model; and, finally, the fact that the Nash profits are smaller in the discriminatory price model increases Γ with respect to the uniform price

---

[12] From equations (12) and (15) of this paper, we get: $\partial\Gamma / \partial\Pi_d^N = -\partial\Pi_d^D / \partial t + \partial\Pi_d^C / \partial t = 1/2\,(1/2 - a)^2 > 0$ for $a \leq 1/4$; from equations (17) and (20), we get: $\partial\Gamma / \partial\Pi_d^N = -\partial\Pi_d^D / \partial t + \partial\Pi_d^C / \partial t = a^2/2 > 0$ for $a \geq 1/4$.

[13] The sign of the derivatives with respect to *a* is the opposite of the sign of the derivatives with respect to *t*. The intuition behind the lower (absolute) value of the derivatives in the discriminatory price model is analogous to the explanation developed for the transportation costs, once one takes into account the reversion of the sign of the derivatives.

[14] Indeed, when firms move from the endpoints of the segment to 1/4 and 3/4 the furthest consumers become nearer.

model[15]. Again, the lower sensitivity of the deviation profits and the higher level of the deviation profits in the discriminatory price model outweigh the impact of the lower sensitivity and the lower level of the Nash profits, and change the sign of Γ: the higher is the product differentiation degree, the higher is the critical discount factor, while the opposite is true in the uniform price model.

Finally, consider $i = a$, with $a \geq 1/4$. Now, from equations (4), (17) and (20) of this paper and equations (6) and (10) in Hackner (1995) paper we obtain the following inequalities: $\frac{\partial \Pi_u^N}{\partial a} < \frac{\partial \Pi_d^N}{\partial a} < 0$, $\frac{\partial \Pi_u^D}{\partial a} > 0$, $\frac{\partial \Pi_d^D}{\partial a} < 0$, and $\frac{\partial \Pi^C}{\partial a} < 0$[16]. Then, *ceteris paribus*, the fact that the derivative of the deviation profits in the discriminatory price model is negative instead of positive decreases Γ with respect to the uniform price model; the fact that the derivative of the Nash profits is smaller (in absolute value) in the discriminatory price model increases Γ with respect to the uniform price model; the fact that the deviation profits are greater in the discriminatory price model decreases Γ with respect to the uniform price model[17]; and, finally, the fact that the Nash profits are smaller in the discriminatory price model decreases Γ with respect to the uniform price model[18]. The reversion of the sign of the derivative of the deviation profits together with the higher level of the deviation profits and the lower level of the Nash profits in the discriminatory price model outweigh the impact of the lower sensitivity of the Nash profits, and change the sign of Γ: the relationship between the product differentiation degree and the critical discount factor is therefore reverted with respect to the uniform price model.

---

[15] Note from equations (12) and (15) that: $\partial\Gamma / \partial\Pi_d^N = -\partial\Pi_d^D / \partial a + \partial\Pi_d^C / \partial a = -t(1/2 - a) < 0$.

[16] The intuition behind $\partial\Pi_u^N / \partial a < \partial\Pi_d^N / \partial a < 0$ is analogous to the intuition developed for *t*, when one takes into account the reversion of the sign of the derivatives. $\partial\Pi^C / \partial a < 0$ is due to the fact that when firms move from 1/4 and 3/4 to the middle of the segment, the furthest consumers (located at the endpoints of the segment) become more distant. With regard to $\partial\Pi_u^D / \partial a > 0$ and $\partial\Pi_d^D / \partial a < 0$, Chang (1991, pag. 464) notices that when $a \geq 1/4$ a lower product differentiation degree has two opposite effects on the deviation profits. First, for a given collusive price, a lower product differentiation degree allows for a higher deviation price, which in turn induces greater deviation profits; second, the collusive price is lower when firms are nearer, and this reduces the deviation profits. In the uniform price model the first effect prevails, and therefore the deviation profits increase with *a*; in the discriminatory price model the second effect dominates, and therefore the deviation profits decrease with *a*.

[17] In fact, from equations (4) and (17), we get: $\partial\Gamma / \partial\Pi_d^D = -\partial\Pi^C / \partial a + \partial\Pi_d^N / \partial a = ta - t/2 < 0$.

[18] Note from equations (17) and (20) that: $\partial\Gamma / \partial\Pi_d^N = -\partial\Pi_d^D / \partial a + \partial\Pi_d^C / \partial a = ta > 0$.

Collusion not to discriminate

In the third collusive scheme the firms do not jointly fix the price schedules. Instead, they agree not to price discriminate. Once firms have established to set a uniform price to all consumers, competition determines which price is effectively applied by the firms. Define with $\breve{p}^C$ the uniform price which results from the competition between the firms, when firms have collusively decided not to price discriminate. It is well known that the equilibrium uniform price and the equilibrium profits are respectively (D'Aspremont *et al.*, 1979):

$$\breve{p}^C = t(1-2a) \tag{23}$$

$$\breve{\Pi}^C = \frac{t}{2}(1-2a) \tag{24}$$

A straightforward implication of equation (23) is that when $a = 1/2$ collusive profits are nil and equal to the punishment profits. We simplify the analysis making the reasonable assumption that in this case firms have no incentive to collude. Therefore, the rest of the analysis is limited to the case of $a < 1/2$.

Suppose that firm *A* deviates. Define $\hat{p}_3$ as the price which solves $u_x^A(\hat{p}_3) = u_x^B(\breve{p}^C)$. Therefore: $\hat{p}_3 = \breve{p}^C + t(1-2x-2a+4ax)$. Then:

$$\breve{p}^D = \max\{0; \min[p^*; \hat{p}_3]\} \tag{25}$$

The deviation price schedule is fully characterized by the following lemma.

<u>Lemma 4</u>. *When firms collude not to discriminate, the deviation price schedule is*:

$$\breve{p}^D = t(2-4a-2x+4ax) \tag{26}$$

Proof. The proof for $\hat{p}_3 \leq p*$ is identical to the case described in Lemma 2. We prove that: $\hat{p}_3 \geq 0$. Substituting equation (23) into $\hat{p}_3$, we obtain: $\hat{p}_3 = t(2-2x-4a+4ax)$. Note that $\hat{p}_3$ is continuous and strictly decreasing in *x*. Therefore, $\hat{p}_3$ is positive for every *x* if and only if it is non-negative when $x=1$. Since $\hat{p}_3(x=1)=0$, $\hat{p}_3$ is strictly positive for every $x<1$. Hence, $\breve{p}^D = \hat{p}_3 = t(2-2x-4a+4ax)$ ■

Therefore, the cheating firm serves the whole market and the deviation profits are:

$$\breve{\Pi}^D = t(1-2a) \tag{27}$$

During the punishment phase the firms compete fiercely. The equilibrium price schedules are defined by equations (2) and (3), while the punishment profits are defined by equation (4). Therefore, the critical discount factor is obtained inserting equations (4), (24) and (27) into equation (1). It follows:

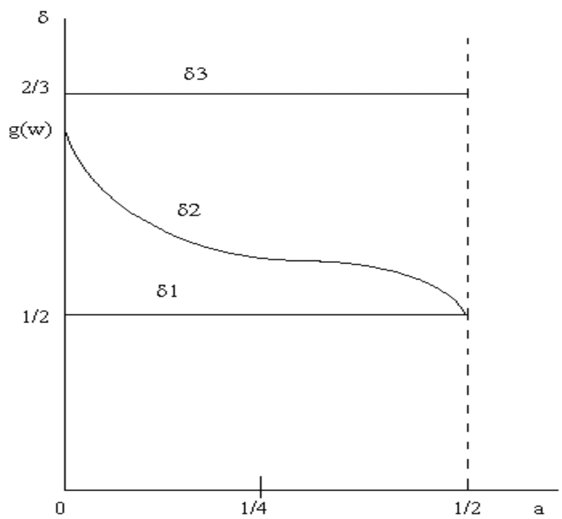
$$\delta_3^* = \frac{2}{3} \tag{28}$$

As for the first collusive scheme, the product differentiation degree does not influence the sustainability of the collusive agreement, since the critical discount factor is equal to 2/3 for every value of a.

Figure 1 summarizes the results. The critical discount factor of the first and the third collusive scheme does not depend on the position of the firms in the market. Instead, the critical discount factor of the second collusive scheme decreases when the product differentiation degree decreases. Moreover, the critical discount factor of the second scheme is always between the critical discount factor of the first scheme (1/2) and the critical discount factor of the third scheme (2/3). This implies that the first collusive agreement is always easier to sustain than the second collusive agreement, which in turn is always easier to sustain than the third collusive agreement. Compare now the collusive profits in the three collusive schemes (equations (6), (12), (17) and (24)).

Obviously, the first collusive scheme yields the largest collusive profits, while the third collusive scheme yields the smallest collusive profits. The collusive profits under the second collusive scheme are in an intermediate position. Therefore, the first collusive scheme dominates the other two collusive schemes: it yields greater profits and it is easier to sustain. However, this does not imply that collusion on a uniform price or collusion not to discriminate will never arise. Even if these schemes are less profitable and more difficult to sustain, they may be less "costly" to implement than the collusion on discriminatory prices, since an agreement regarding a huge number of prices (as the first collusive scheme) may be very time-demanding and difficult to reach. Unfortunately, our model does not allow taking into consideration this aspect.

Figure 1



5. Suboptimal collusion

In the previous section we considered the sustainability of optimal collusion. Now we ask whether suboptimal collusion is sustainable when optimal collusion is not sustainable. In the following we show that suboptimal collusion is never sustainable when optimal collusion is not sustainable[19].

[19] Notice that the answer cannot be known *a priori* if the suboptimal collusive price is lower than the optimal collusive price: both the collusive profits and the deviation profits are lower and the net effect on

Suboptimal collusion on discriminatory prices

Consider the following setting. The two firms collude on the prices to set to the consumers located at points x' and $1-x'$, with $x'<1/2$. Suppose first that the firms collude in the optimal way, that is, they set the collusive prices $p^C(x')=p^*(x')$ and $p^C(1-x')=p^*(1-x')$, respectively on consumer x' and consumer $1-x'$. Of course, $p^*(x')=p^*(1-x')$. The collusive profits each firm obtains from these consumers are:

$$\Pi^C(x';1-x')=p^*(x') \tag{29}$$

Define: $T(x')\equiv t(1-2x'-2a+4ax')$. Note that: $T(x')\geq 0$ if $x\leq 1/2$ and $T(x')\leq 0$ if $x\geq 1/2$ [20]. Moreover, note that: $T(x')=-T(1-x')$.

Consider the punishment profits. From equations (2) and (3) we can write:

$$\Pi^N(x';1-x')=T(x') \tag{30}$$

Suppose that firm *A* deviates. Using Lemma 1 we get:

$$p^D\equiv[p^D_{x'};p^D_{1-x'}]=[p^*(x');p^*(x')-T(x')] \tag{31}$$

It follows that the deviation profits firm *A* obtains from consumers x' and $1-x'$ are:

$$\Pi^D(x';1-x')=2p^*(x')-T(x') \tag{32}$$

the critical discount factor is *a priori* ambiguous. On the contrary, the answer is *a priori* negative if the suboptimal collusive price is higher than the optimal collusive price, since collusive profits are lower and the deviation profits are higher. Therefore, in the remaining part of this section we refer only to suboptimal collusive prices which are lower than the optimal collusive price.

[20] Indeed, $T(x')\equiv t(1-2x'-2a+4ax')=t(1-x'-a)^2-t(x'-a)^2$ measures the advantage (disadvantage) of firm *A* over firm *B* in serving consumer $x'\leq 1/2$ ($\geq 1/2$), because it says how much firm *A* may increase (decrease) its price above (below) the price set by the rival without loosing (serving) consumer x'.

Now, observe that:

$$\Pi^D(x';1-x')-\Pi^N(x';1-x')=2[p^*(x')-T(x')]=2[\Pi^D(x';1-x')-\Pi^C(x';1-x')] \qquad (33)$$

Obviously, if optimal collusion regards not only consumers x' and $1-x'$ but also, say, consumers x'' and $1-x''$, x''' and $1-x'''$, and so on, the previous result does not change: the difference between the deviation profits and the Nash profits is always equal to the double of the difference between the deviation profits and the collusive profits. Define W as the set of consumers served with optimal collusive discriminatory prices. From (33) it follows that:

$$Z=\sum_{x\in W}[\Pi^D(x;1-x)-\Pi^N(x;1-x)]=2\sum_{x\in W}[\Pi^D(x;1-x)-\Pi^C(x;1-x)]=2\Omega \qquad (34)$$

Suppose now that the two firms set suboptimal collusive prices on the consumers located at $\tilde{x}$ and $1-\tilde{x}$, with $\tilde{x}\le 1/2$. Suboptimal collusive prices are defined as the optimal collusive prices minus a strictly positive amount. That is: $p^C(\tilde{x})=p^*(\tilde{x})-k$ and $p^C(1-\tilde{x})=p^*(1-\tilde{x})-k$, respectively for consumer $\tilde{x}$ and consumer $1-\tilde{x}$, with $k>0$. The collusive profits each firm obtains from consumers $\tilde{x}$ and $1-\tilde{x}$ are:

$$\Pi^C(\tilde{x};1-\tilde{x})=p^*(\tilde{x})-k \qquad (35)$$

The punishment profits are:

$$\Pi^N(\tilde{x};1-\tilde{x})=T(\tilde{x}) \qquad (36)$$

Suppose that firm A deviates from the collusive agreement. Define $\hat{p}_S$ as the price which solves: $u_x^A(\hat{p}_s)=u_x^B(p^*-k)$, with $x=\tilde{x},1-\tilde{x}$. It is: $\hat{p}_S=p^*(x)-k+T(x)$, with $x=\tilde{x},1-\tilde{x}$. Then:

$$p_S^D=\max\{0;\min[p^*(x);p^*(x)-k+T(x)]\}\ ,\quad x=\tilde{x},1-\tilde{x} \qquad (37)$$

We look for: $p_S^D \equiv [p_{\tilde{x}}^D; p_{1-\tilde{x}}^D]$, that is, the deviation prices applied by firm *A* on consumer $\tilde{x}$ and consumer $1-\tilde{x}$ respectively. First, note that $p_{\tilde{x}}^D = 0$ is impossible for every *k*. In fact, for it to be possible, it must be: $0 > \min[p^*(\tilde{x}); p^*(\tilde{x}) - k + T(\tilde{x})]$. But $0 > p^*(\tilde{x})$ is clearly impossible, and $0 > p^*(\tilde{x}) - k + T(\tilde{x})$ is impossible as well, since it contradicts the assumption that the collusive profits must be higher than the punishment profits[21]. Second, note that $p_{1-\tilde{x}}^D = p^*(\tilde{x})$ is impossible for every *k*, since it would imply: $p^*(\tilde{x}) = \min[p^*(\tilde{x}); p^*(\tilde{x}) - k - T(\tilde{x})]$.

Assume for the moment $a < 1/2$. Depending on the value of *k*, four cases are possible:

Case 1) $p_S^D = [p^*(\tilde{x});0]$. It occurs if and only if the following conditions hold:

$$\begin{cases} 0 < p^*(\tilde{x}) = \min[p^*(\tilde{x}), p^*(\tilde{x}) - k + T(\tilde{x})] & (38) \\ 0 > p^*(\tilde{x}) - k - T(\tilde{x}) = \min[p^*(\tilde{x}), p^*(\tilde{x}) - k - T(\tilde{x})] & (39) \end{cases}$$

Case 2) $p_S^D = [p^*(\tilde{x}); p^*(\tilde{x}) - k - T(\tilde{x})]$. It occurs if and only if the following conditions hold:

$$\begin{cases} 0 < p^*(\tilde{x}) = \min[p^*(\tilde{x}), p^*(\tilde{x}) - k + T(\tilde{x})] & (40) \\ 0 < p^*(\tilde{x}) - k - T(\tilde{x}) = \min[p^*(\tilde{x}), p^*(\tilde{x}) - k - T(\tilde{x})] & (41) \end{cases}$$

Case 3) $p_S^D = [p^*(\tilde{x}) - k + T(\tilde{x}); p^*(\tilde{x}) - k - T(\tilde{x})]$. It occurs if and only if the following conditions hold:

$$\begin{cases} 0 < p^*(\tilde{x}) - k + T(\tilde{x}) = \min[p^*(\tilde{x}), p^*(\tilde{x}) - k + T(\tilde{x})] & (42) \\ 0 < p^*(\tilde{x}) - k - T(\tilde{x}) = \min[p^*(\tilde{x}), p^*(\tilde{x}) - k - T(\tilde{x})] & (43) \end{cases}$$

[21] Indeed, $\Pi^C > \Pi^N$ implies: $p^*(\tilde{x}) - k > T(\tilde{x})$, which is impossible if $p^*(\tilde{x}) - k + T(\tilde{x}) < 0$.

Case 4) $p_S^D = [p^*(\tilde{x}) - k + T(\tilde{x});0]$. It occurs if and only if the following conditions hold:

$$\begin{cases} 0 < p^*(\tilde{x}) - k + T(\tilde{x}) = \min[p^*(\tilde{x}), p^*(\tilde{x}) - k + T(\tilde{x})] & (44) \\ 0 > p^*(\tilde{x}) - k - T(\tilde{x}) = \min[p^*(\tilde{x}), p^*(\tilde{x}) - k - T(\tilde{x})] & (45) \end{cases}$$

Clearly, the deviation profits firm A obtains from consumers $\tilde{x}$ and $1-\tilde{x}$ depend on the deviation prices. Therefore:

Case 1) $\Pi^D(\tilde{x};1-\tilde{x}) = p^*(\tilde{x})$ (46)

Case 2) $\Pi^D(\tilde{x};1-\tilde{x}) = 2p^*(\tilde{x}) - k - T(\tilde{x})$ (47)

Case 3) $\Pi^D(\tilde{x};1-\tilde{x}) = 2p^*(\tilde{x}) - 2k$ (48)

Case 4) $\Pi^D(\tilde{x};1-\tilde{x}) = p^*(\tilde{x}) - k + T(\tilde{x})$ (49)

We state the following Lemma:

Lemma 5: the following inequalities hold:

Case 1) $\Pi^D(.) - \Pi^N(.) = p^*(\tilde{x}) - T(\tilde{x}) < 2k = 2[\Pi^D(.) - \Pi^C(.)]$ (50)

Case 2) $\Pi^D(.) - \Pi^N(.) = 2p^*(\tilde{x}) - k - 2T(\tilde{x}) < 2(p^*(\tilde{x}) - T(\tilde{x})) = 2[\Pi^D(.) - \Pi^C(.)]$ (51)

Case 3) $\Pi^D(.) - \Pi^N(.) = 2p^*(\tilde{x}) - 2k - T(\tilde{x}) < 2(p^*(\tilde{x}) - k) = 2[\Pi^D(.) - \Pi^C(.)]$ (52)

Case 4) $\Pi^D(.) - \Pi^N(.) = p^*(\tilde{x}) - k < 2T(\tilde{x}) = 2[\Pi^D(.) - \Pi^C(.)]$ (53)

Proof. Inequalities (51) and (52) are immediately verified. Consider inequality (50). Recall that $p_S^D = [p^*(\tilde{x}),0]$ occurs only if: $p^*(\tilde{x}) - k - T(\tilde{x}) < 0$ (condition (39)). For inequality (50) not to hold it must be: $p^*(\tilde{x}) - T(\tilde{x}) - k > k$, but this contradicts condition (39). Therefore, condition (39) always implies inequality (50). Consider inequality (53). Recall that $p_S^D = [p^*(\tilde{x}) - k + T(\tilde{x}),0]$ occurs only if: $p^*(\tilde{x}) - k - T(\tilde{x}) < 0$ (condition (45)). For inequality (53) not to hold it must be:

$p^*(\tilde{x})-k-T(\tilde{x})>T(\tilde{x})$, but this contradicts condition (45). Therefore, condition (45) always implies condition (53). ■

Obviously, if suboptimal collusion regards not only consumers $\tilde{x}$ and $1-\tilde{x}$ but also, say, consumers $\tilde{x}'$ and $1-\tilde{x}'$, $\tilde{x}''$ and $1-\tilde{x}''$, and so on, the previous result does not change: the difference between the deviation profits and the Nash profits is always less than the double of the difference between the deviation profits and the collusive profits. Define $\tilde{W}$ as the set of consumers served with suboptimal collusive discriminatory prices. From lemma 5) it follows:

$$\Delta \equiv \sum_{x\in\tilde{W}}[\Pi^D(x;1-x)-\Pi^N(x;1-x)] < 2\sum_{x\in\tilde{W}}[\Pi^D(x;1-x)-\Pi^C(x;1-x)] \equiv 2\Psi \tag{54}$$

Since it must be $W\cup\tilde{W}=X$, where X is the set of all consumers, and $W\cap\tilde{W}=\emptyset$, putting together equation (34) and inequality (54) it follows:

$$\delta^* = \frac{\sum_{x\in X}\Pi^D(x;1-x)-\sum_{x\in X}\Pi^C(x;1-x)}{\sum_{x\in X}\Pi^D(x;1-x)-\sum_{x\in X}\Pi^N(x;1-x)} =$$

$$= \frac{\sum_{x\in W}\Pi^D(x;1-x)+\sum_{x\in\tilde{W}}\Pi^D(x;1-x)-\sum_{x\in W}\Pi^C(x;1-x)-\sum_{x\in\tilde{W}}\Pi^C(x;1-x)}{\sum_{x\in W}\Pi^D(x;1-x)+\sum_{x\in\tilde{W}}\Pi^D(x;1-x)-\sum_{x\in W}\Pi^N(x;1-x)-\sum_{x\in\tilde{W}}\Pi^N(x;1-x)} = \frac{\Omega+\Psi}{Z+\Delta} \geq \frac{1}{2}$$

Since $Z=2\Omega$ and $\Delta<2\Psi$, the critical discount factor is equal to 1/2 only when Ψ and Δ are equal to 0, that is, when $\tilde{W}=\emptyset$, or, in other words, when all consumers are served with optimal collusive discriminatory prices. Conversely, when Ψ and Δ are different from 0 (that is, when $\tilde{W}\neq\emptyset$), the critical discount factor is strictly greater than 1/2. The straightforward implication is that if optimal collusion is not sustainable, sub-optimal collusion is not sustainable too.

Finally, consider the case of $a=1/2$. It implies $T(\tilde{x})=0$. Consider again conditions (38) – (45). It is immediate to see that conditions (38) and (40) are never verified.

Moreover, conditions (44) and (45) cannot be contemporaneously verified. It follows that cases 1), 2) and 4) never occur. Conversely, conditions (42) and (43) are always verified. Therefore, case 3) always occurs, and the deviation price schedule is given by: $p_S^D = [p^*(\tilde{x}) - k; p^*(\tilde{x}) - k]$. The deviation profits are: $\Pi^D = 2p^*(\tilde{x}) - 2k$, while the punishment profits are: $\Pi^N = 0$. It follows that:

$$\Pi^D(.) - \Pi^N(.) = 2p^*(\tilde{x}) - 2k = 2[\Pi^D(.) - \Pi^C(.)] \tag{55}$$

When $a = 1/2$ and firms collude in a suboptimal way, the difference between the deviation profits and the Nash profits is equal to the double of the difference between the deviation profits and the collusive profits. It follows that the critical discount factor is equal to 1/2.

Note that these results are consistent with Proposition 2. Optimal collusion on a uniform price can be seen as a form of suboptimal collusion with respect to optimal collusion on discriminatory prices: Proposition 2 says that the critical discount factor is strictly larger than 1/2 when firms are differentiated and it is equal to 1/2 when firms are undifferentiated. In this section we have shown that *any possible suboptimal collusive agreement* induces a critical discount factor larger than 1/2 if firms are differentiated and equal to 1/2 if firms are undifferentiated.

Suboptimal collusion on uniform price

We consider now the second collusive scheme: firms collude on a uniform price to be applied to all consumers. What happens to the critical discount factor if firms collude on a suboptimal uniform collusive price?

Proposition 3: *the critical discount factor is a decreasing function of the collusive price.*

Proposition 3 implies that when optimal collusion is not sustainable, suboptimal collusion cannot be sustainable too, because it increases the critical discount factor. In what follows we prove Proposition 3.

Let $\tilde{p}^C \leq \overline{p}^C$ denote the collusive uniform price[22]. When $\tilde{p}^C = \overline{p}^C$ we are clearly in the optimal collusion case we described in section 4. Therefore, we concentrate on $\tilde{p}^C < \overline{p}^C$. First, since when $\tilde{p}^C = \overline{p}^C$ all consumers are served, the same must be true when firms sub-optimally collude, that is when $\tilde{p}^C < \overline{p}^C$. The sub-optimal collusive profits of each firm are therefore the following:

$$\tilde{\Pi}^C = \frac{\tilde{p}^C}{2} \tag{56}$$

Now consider the deviation price. Suppose that firm *A* cheats. Define $\hat{p}_U$ as the price which solves: $u_x^A(\hat{p}_U) = u_x^B(\tilde{p}^C)$. We get: $\hat{p}_U = \tilde{p}^C + t(1-2x-2a+4ax)$. The deviation price is therefore:

$$\tilde{p}_U^D = \max\{0; \min[p^*; \tilde{p}^C + t(1-2x-2a+4ax)]\} \tag{57}$$

Before proceeding, note that the following equation is always true: $\tilde{p}^C + t(1-2x-2a+4ax) = \min[p^*; \tilde{p}^C + t(1-2x-2a+4ax)]$. The intuition is simple. Lemma 2 and Lemma 3 show that: $\overline{p}^C + t(1-2x-2a+4ax) < p^*$. Since $\tilde{p}^C < \overline{p}^C$, it must be: $\tilde{p}^C + t(1-2x-2a+4ax) < p^*$. Therefore, the deviation price is simply:

$$\tilde{p}_U^D = \max\{0; \tilde{p}^C + t(1-2x-2a+4ax)\} \tag{58}$$

Intuitively, the smaller is the suboptimal collusive uniform price the more difficult for the cheating firm is to steal a consumer without setting a price lower than the marginal cost. This allows us to derive a condition for the entire market to be served by the cheating firm. First, note that the second term in (58) is decreasing in *x*. Therefore, if

---

[22] Note that we do not consider $\tilde{p}^C > \overline{p}^C$, because in this case the critical discount factor is unambiguously higher than under optimal collusion (see also footnote 18).

the consumer located in x is served by the deviating firm, all consumers located at the left of x must be served as well. The consumer located in x is served by the deviating firm when: $\tilde{p}^C + t(1-2x-2a+4ax) \geq 0$, from which it follows:

$$x \leq \frac{\tilde{p}^C}{2t(1-2a)} + \frac{1}{2} \equiv \zeta(\tilde{p}^C) \tag{59}$$

On the contrary, if $x > \zeta(\tilde{p}^C)$ the consumer located in x cannot be stolen by the deviating firm. It follows that if $\zeta(\tilde{p}^C)$ is higher than 1, all consumers are served by firm A. By solving $\zeta(\tilde{p}^C) \geq 1$ with respect to the price we obtain the necessary and sufficient condition for the whole market to be served by the deviating firm. This condition reduces to:

$$\tilde{p}^C \geq t(1-2a) \tag{60}$$

Hence, if the suboptimal collusive uniform price is high enough (i.e. if condition (60) is satisfied) the entire market is served by the cheating market, otherwise a subset of consumers continues to be served by firm B. Incidentally, note that condition (60) is always satisfied when $a = 1/2$.

Suppose first that inequality (60) is satisfied. The deviation profits are the following (the subscript w indicates that the whole market is served by the deviating firm):

$$\tilde{\Pi}_w^D = \int_0^1 [\tilde{p}^C + t(1-2x-2a+4ax]dx = \tilde{p}^C \tag{61}$$

By substituting equations (4), (56) and (61) into equation (1) we get the critical discount factor:

$$\tilde{\delta}_w^*(\tilde{p}^C) = \frac{2\tilde{p}^C}{4\tilde{p}^C - t(1-2a)} \tag{62}$$

The derivative of the critical discount factor with respect to the collusive price is the following:

$$\frac{\partial\widetilde{\delta}_w^*(\widetilde{p}^C)}{\partial\widetilde{p}^C} = -\frac{2t(1-2a)}{[4\widetilde{p}^C - t(1-2a)]^2} \leq 0 \tag{63}$$

Therefore, colluding on a suboptimal collusive uniform price increases the critical discount factor (or leaves it unchanged when firms are not differentiated), and this makes the collusive agreement less (or equally) sustainable.

Suppose now that condition (60) does not hold. The deviating firm does not serve all consumers, but only the consumers located at the left of x^*, where $x^* = \zeta(\widetilde{p}^C) < 1$. Hence, the deviation profits are the following (the subscript f indicates that only a fraction of the market is served by the deviating firm):

$$\widetilde{\Pi}_f^D = \int_0^{\zeta(\widetilde{p}^C)} [\widetilde{p}^C + t(1-2x-2a+4ax)]dx = \frac{1/2-a}{2t(1-2a)^2}\widetilde{p}^{C2} - \frac{t}{2}(a-\frac{1}{2}) + \frac{\widetilde{p}^C}{2} \tag{64}$$

By substituting equations (4), (56) and (64) into equation (1) we obtain the critical discount factor:

$$\widetilde{\delta}_f^*(\widetilde{p}^C) = \frac{K\widetilde{p}^{C^2} + Z}{K\widetilde{p}^{C^2} + \widetilde{p}^C} \tag{65}$$

with $K \equiv \frac{1/2-a}{t(1-2a)^2}$ and $Z \equiv t(\frac{1}{2}-a)$. After some manipulations, the derivative of the critical discount factor can be written as follows:

$$\frac{\partial\widetilde{\delta}_f^*(\widetilde{p}^C)}{\partial\widetilde{p}^C} = \frac{K\widetilde{p}^C(\widetilde{p}^C - 2Z) - Z}{(K\widetilde{p}^{C^2} + \widetilde{p}^C)^2} \tag{66}$$

Since $K > 0$ and $Z > 0$, a sufficient condition for the derivative to be negative is $\tilde{p}^C - 2Z < 0$. Note that this condition coincides with: $\tilde{p}^C < t(1-2a)$, which is always satisfied when the deviating firm cannot serve the whole market (see condition (60)). Therefore, the derivative of the critical discount factor with respect to price is always negative. This means that lowering the collusive price below the optimal uniform collusive price makes the collusion less sustainable, even in the case in which the deviating firm cannot serve the entire market. This completes the proof of Proposition 3.

6. Conclusion

In this paper we have analyzed the relationship between the product differentiation degree and the sustainability of three different collusive schemes. The main innovation of our analysis is represented by the possibility for firms to perfectly price discriminate. We obtain the following results. The critical discount factor for the first collusive scheme (optimal collusion on discriminatory prices) is equal to 1/2 for any product differentiation degree. The second collusive scheme (optimal collusion on uniform price) is more difficult to sustain than the first, since the critical discount factor is between 1/2 and 2/3. Moreover, the greater is the product differentiation degree and the greater are the transportation costs, the greater is the critical discount factor: these findings contrast with previous results obtained under the hypothesis of uniform price (Chang, 1991; Chang, 1992 and Hackner, 1995). Finally, the sustainability of the third collusive scheme (collusion not to discriminate) does not depend on the product differentiation degree, and it is always equal to 2/3. In the last section we extend the model to consider the possibility for the firms to sub-optimally collude. Both when firms collude on a suboptimal discriminatory price schedule and when they collude on a suboptimal uniform price, the critical discount factor is always greater or equal to the critical discount factor obtained under optimal collusion.

Appendix

In this appendix we study the sustainability of the second and the third collusive scheme when imperfect direct price discrimination *a la* Liu and Serfes (2004, 2007) is assumed. Following Liu and Serfes (2004, 2007), we suppose that there is an information technology which allows firms to partition the consumers into different groups. We assume that the technology partitions the linear market into *n* sub-segments indexed by *m*, with $m = 1,...,n$. Each sub-segment is of equal length, $1/n$. It follows that sub-segment *m* can be expressed as the interval $\left[\frac{m-1}{n};\frac{m}{n}\right]$. A firm can price discriminate between consumers belonging to different sub-segments, but not between the consumers belonging to the same sub-segment. The cost of using the information technology is zero. Define with p_m^J the price set by firm *J*=*A*,*B* on consumers belonging to sub-segment *m*. Clearly, when firm *J* cannot price discriminate, it must be $p_m^J = p_{m'}^J$, $\forall m, m'$. Finally, assume that $n = 2^k$, with $k = 1,2,3,4...$ Therefore, *n* measures the precision of consumer information: an higher *n* means an higher information precision.

The Nash profits are the following[23]:

$$\Pi^N = \frac{t(1-2a)(9n^2 - 18n + 40)}{36n^2} \tag{67}$$

The second collusive scheme. Suppose that firm *A* deviates. Assume that *v* is sufficiently high, so that it is always optimal for the deviating firm to serve the whole market. Therefore, the deviation price schedule is defined in such a way to make the consumers located at the endpoint of each sub-segment indifferent between buying from the deviating firm and the colluding firm[24]. Such consumers are located at m/n, with

[23] The Nash prices can be easily calculated following the proof of Proposition 1 in Liu and Serfes (2004): the only difference is that we allow firms being located at *a* at 1 – *a* instead of 0 and 1. Nash profits follow directly from the Nash prices. All the calculations are available from the author upon request.
[24] If the consumer located at the endpoint of a sub-segment receives the same utility from firm *A* and from firm *B*, it follows that all consumers in the same sub-segment at the left of such consumer receives a (strictly) higher utility from firm *A* than from firm *B*, because the transportation costs are lower.

$m=1,...,n$, and the indifference condition is: $v-p^D-t(\frac{m}{n}-a)^2=v-\overline{p}^C-t(\frac{m}{n}-1+a)^2$,

from which it follows:

$$p^D=\begin{cases} v-t(\frac{1}{2}-a)^2+t(\frac{m}{n}-1+a)^2-t(\frac{m}{n}-a)^2 & \forall a\le\frac{1}{4} \\ v-ta^2+t(\frac{m}{n}-1+a)^2-t(\frac{m}{n}-a)^2 & \forall a\ge\frac{1}{4} \end{cases}$$

The deviation profits are:

$$\Pi^D=\sum_{m=1}^{n}\frac{p^D}{n}=\begin{cases} \frac{4vn-t(2a-1)[n(2a-1)-4]}{4n} & \forall a\le\frac{1}{4} \\ \frac{vn-t(1-2a+na^2)}{n} & \forall a\ge\frac{1}{4} \end{cases} \tag{68}$$

Inserting equations (12), (17), (67) and (68) into equation 1, we get the critical discount factor:

$$\delta^*=\begin{cases} \frac{9n[(2a-1)(n(2a-1)-8)-4wn]}{4[(2a-1)(-20-9n+9n^2(a-1))-18wn^2]} & \forall a\le\frac{1}{4} \\ \frac{18n(2-4a+na^2-wn)}{40+18+9n^2+36a^2n^2-2a(40+18n+9n^2)-36wn^2} & \forall a\ge\frac{1}{4} \end{cases}$$

Differentiating δ^* with respect to *a* and *w* we get respectively:

$$\frac{\partial\delta^*}{\partial a}=\begin{cases} -\frac{9n^2(40-54n+9n^2)(1-4a+4a^2+4w)}{4[20+9n+18a^2n^2-a(40+18n+27n^2)+n^2(9-18w)]^2} & \forall a\le\frac{1}{4} \\ -\frac{36n^2(40-54n+9n^2)(w-a+a^2)}{[40+18+36a^2n^2-2a(40+18n+9n^2)+n^2(9-36w)]^2} & \forall a\ge\frac{1}{4} \end{cases}$$

$$\frac{\partial \delta^*}{\partial w} = \begin{cases} -\dfrac{9n^2(1-2a)(40-54n+9n^2)}{2[20+9n+18a^2n^2-a(40+18n+27n^2)+n^2(9-18w)]^2} & \forall a \le \dfrac{1}{4} \\ -\dfrac{18n^2(1-2a)(40-54n+9n^2)}{[40+18+36a^2n^2-2a(40+18n+9n^2)+n^2(9-36w)]^2} & \forall a \ge \dfrac{1}{4} \end{cases}$$

It is easy to see that for $n \ge 8$ the critical discount factor is decreasing both in *a* and *w*, while for $n = 2$ and $n = 4$ it in increasing both in *a* and *w*. Therefore, for a sufficiently high information quality, the more the firms are differentiated and the higher are the transportation costs, the less collusion is sustainable.

The third collusive scheme. As before, suppose that firm *A* deviates and assume that *v* is sufficiently high, so that it is always optimal for the cheating firm to serve the whole market. Therefore, the deviation price schedule is obtained by solving the following indifference condition: $v - p^D - t(\frac{m}{n} - a)^2 = v - \breve{p}^C - t(\frac{m}{n} - 1 + a)^2$. We get:

$$p^D = t(1-2a) + t(\frac{m}{n} - 1 + a)^2 - t(\frac{m}{n} - a)^2$$

The deviation profits are:

$$\Pi^D = \sum_{m=1}^{n} \frac{p^D}{n} = \frac{t(1-2a)(n-1)}{n} \tag{69}$$

Inserting equations (24), (67) and (69) we obtain the critical discount factor:

$$\delta^3{*} = \frac{18n^2 - 36n}{27n^2 - 18n - 40}$$

As for perfect price discrimination, the sustainability of collusion does depend neither on the product differentiation degree nor on transportation costs.

References

Chang, M.H. (1991), "The Effects of Product Differentiation on Collusive Pricing", *International Journal of Industrial Organization*, 3, 453-470.

Chang, M.H. (1992), "Intertemporal Product Choice and its Effects on Collusive Firm Behavior", *International Economic Review*, 4, 773-793.

D'Aspremont, C., Gabszewicz, J.J. and J.F. Thisse (1979): "On Hotelling's Stability in Competition", *Econometrica*, vol. 47, 1145 – 1150, 1979.

Friedman, J.W. (1971), "A Non-Cooperative Equilibrium for Supergames", *Review of Economic Studies*, 38, 1-12.

Hackner, J. (1995), "Endogenous Product Design in an Infinitely Repeated Game", *International Journal of Industrial Organization*, 13, 277-299.

Hamilton, J.H., Thisse, J.F. and A. Weskamp (1989): "Spatial Discrimination: Bertrand vs Cournot in a Model of Location Choice", *Regional Science and Urban Economics*, 19, 87-102.

Hamilton, J.H. and J.F. Thisse (1992): "Duopoly with Spatial and Quantity-Dependent Price Discrimination", *Regional Science and Urban Economics*, 22, 175-185.

Hotelling, H. (1929), "Stability in Competition", *Economic Journal*, 39, 41-57.

Hurter, A.P. and P.J. Lederer (1985) "Spatial Duopoly with Discriminatory Pricing", *Regional Science and Urban Economics* 15, 541-533.

Lederer, P.J. and A.P. Hurter (1986), "Competition of Firms: Discriminatory Pricing and Locations", *Econometrica*, 54, 623 – 640.

Liu, Q. and K. Serfes (2004), "Quality of Information and Oligopolistic Price Discrimination", *Journal of Economics and Management Strategy*, 13, 671-702.

Liu, Q. and K. Serfes (2007), "Market Segmentation and Collusive Behaviour", *International Journal of Industrial Organization*, 25, 353-378.

Thisse, J. F. and X. Vives (1988), "On the Strategic Choice of Spatial Price Policy", *American Economic Review*, 78, 122-137.

Elenco Quaderni già pubblicati

1. L. Giuriato, *Problemi di sostenibilità di programmi di riforma strutturale,* settembre 1993.
2. L. Giuriato, *Mutamenti di regime e riforme: stabilità politica e comportamenti accomodanti,* settembre 1993.
3. U. Galmarini, *Income Tax Enforcement Policy with Risk Averse Agents*, novembre 1993.
4. P. Giarda, *Le competenze regionali nelle recenti proposte di riforma costituzionale*, gennaio 1994.
5. L. Giuriato, *Therapy by Consensus in Systemic Transformations: an Evolutionary Perspective*, maggio 1994.
6. M. Bordignon, *Federalismo, perequazione e competizione fiscale. Spunti di riflessione in merito alle ipotesi di riforma della finanza regionale in Italia,* aprile 1995.
7. M. F. Ambrosanio, *Contenimento del disavanzo pubblico e controllo delle retribuzioni nel pubblico impiego*, maggio 1995.
8. M. Bordignon, *On Measuring Inefficiency in Economies with Public Goods: an Overall Measure of the Deadweight Loss of the Public Sector*, luglio 1995.
9. G. Colangelo, U. Galmarini, *On the Pareto Ranking of Commodity Taxes in Oligopoly*, novembre 1995.
10. U. Galmarini, *Coefficienti presuntivi di reddito e politiche di accertamento fiscale*, dicembre 1995.
11. U. Galmarini, *On the Size of the Regressive Bias in Tax Enforcement*, febbraio 1996.
12. G. Mastromatteo, *Innovazione di Prodotto e Dimensione del Settore Pubblico nel Modello di Baumol*, giugno 1996.
13. G. Turati, *La tassazione delle attività finanziarie in Italia: verifiche empiriche in tema di efficienza e di equità,* settembre 1996.
14. G. Mastromatteo, *Economia monetaria post-keynesiana e rigidità dei tassi bancari,* settembre 1996.
15. L. Rizzo, *Equalization of Public Training Expenditure in a Cross-Border Labour Market*, maggio 1997.
16. C. Bisogno, *Il mercato del credito e la propensione al risparmio delle famiglie: aggiornamento di un lavoro di Jappelli e Pagano*, maggio 1997.
17. F.G. Etro, *Evasione delle imposte indirette in oligopolio. Incidenza e ottima tassazione*, luglio 1997.
18. L. Colombo, *Problemi di adozione tecnologica in un'industria monopolistica*, ottobre 1997.
19. L. Rizzo, *Local Provision of Training in a Common Labour Market*, marzo 1998.
20. M.C. Chiuri, *A Model for the Household Labour Supply: An Empirical Test On A Sample of Italian Household with Pre-School Children*, maggio 1998.
21. U. Galmarini, *Tax Avoidance and Progressivity of the Income Tax in an Occupational Choice Model*, luglio 1998.
22. R. Hamaui, M. Ratti, *The National Central Banks' Role under EMU. The Case of the Bank of Italy*, novembre 1998.

23. A. Boitani, M. Damiani, *Heterogeneous Agents, Indexation and the Non Neutrality of Money*, marzo 1999.
24. A. Baglioni, *Liquidity Risk and Market Power in Banking*, luglio 1999.
25. M. Flavia Ambrosanio, *Armonizzazione e concorrenza fiscale: la politica della Comunità Europea*, luglio 1999.
26. A. Balestrino, U. Galmarini, *Public Expenditure and Tax Avoidance*, ottobre 1999.
27. L. Colombo, G. Weinrich, *The Phillips Curve as a Long-Run Phenomenon in a Macroeconomic Model with Complex Dynamics*, aprile 2000.
28. G.P. Barbetta, G. Turati, *L'analisi dell'efficienza tecnica nel settore della sanità. Un'applicazione al caso della Lombardia*, maggio 2000.
29. L. Colombo, *Struttura finanziaria delle imprese, rinegoziazione del debito Vs. Liquidazione. Una rassegna della letteratura*, maggio 2000.
30. M. Bordignon, *Problems of Soft Budget Constraints in Intergovernmental Relationships: the Case of Italy*, giugno 2000.
31. A. Boitani, M. Damiani, *Strategic complementarity, near-rationality and coordination*, giugno 2000.
32. P. Balduzzi, *Sistemi pensionistici a ripartizione e a capitalizzazione: il caso cileno e le implicazioni per l'Italia*, luglio 2000.
33. A. Baglioni, *Multiple Banking Relationships: competition among "inside" banks*, ottobre 2000.
34. A. Baglioni, R. Hamaui, *The Choice among Alternative Payment Systems: The European Experience*, ottobre 2000.
35. M.F. Ambrosanio, M. Bordignon, *La concorrenza fiscale in Europa: evidenze, dibattito, politiche*, novembre 2000.
36. L. Rizzo, *Equalization and Fiscal Competition: Theory and Evidence*, maggio 2001.
37. L. Rizzo, *Le Inefficienze del Decentramento Fiscale*, maggio 2001.
38. L. Colombo, *On the Role of Spillover Effects in Technology Adoption Problems*, maggio 2001.
39. L. Colombo, G. Coltro, *La misurazione della produttività: evidenza empirica e problemi metodologici*, maggio 2001.
40. L. Cappellari, G. Turati, *Volunteer Labour Supply: The Role of Workers' Motivations*, luglio 2001.
41. G.P. Barbetta, G. Turati, *Efficiency of junior high schools and the role of proprietary structure*, ottobre 2001.
42. A. Boitani, C. Cambini, *Regolazione incentivante per i servizi di trasporto locale*, novembre 2001.
43. P. Giarda, *Fiscal federalism in the Italian Constitution: the aftermath of the October 7th referendum*, novembre 2001.
44. M. Bordignon, F. Cerniglia, F. Revelli, *In Search for Yardstick Competition: Property Tax Rates and Electoral Behavior in Italian Cities*, marzo 2002.
45. F. Etro, *International Policy Coordination with Economic Unions*, marzo 2002.
46. Z. Rotondi, G. Vaciago, *A Puzzle Solved: the Euro is the D.Mark*, settembre 2002.
47. A. Baglioni, *Bank Capital Regulation and Monetary Policy Transmission: an heterogeneous agents approach*, ottobre 2002.
48. A. Baglioni, *The New Basle Accord: Which Implications for Monetary Policy Transmission?*, ottobre 2002.
49. F. Etro, P. Giarda, *Redistribution, Decentralization and Constitutional Rules*, ottobre 2002.
50. L. Colombo, G. Turati, *La Dimensione Territoriale nei Processi di Concentrazione dell'Industria Bancaria Italiana*, novembre 2002.

51. Z. Rotondi, G. Vaciago, *The Reputation of a newborn Central Bank*, marzo 2003.
52. M. Bordignon, L. Colombo, U. Galmarini, *Fiscal Federalism and Endogenous Lobbies' Formation*, ottobre 2003.
53. Z. Rotondi, G. Vaciago, *The Reaction of central banks to Stock Markets*, novembre 2003.
54. A. Boitani, C. Cambini, *Le gare per i servizi di trasporto locale in Europa e in Italia: molto rumore per nulla?*, febbraio 2004.
55. V. Oppedisano, *I buoni scuola: un'analisi teorica e un esperimento empirico sulla realtà lombarda*, aprile 2004.
56. M. F. Ambrosanio, *Il ruolo degli enti locali per lo sviluppo sostenibile: prime valutazioni*, luglio 2004.
57. M. F. Ambrosanio, M. S. Caroppo, *The Response of Tax Havens to Initiatives Against Harmful Tax Competition: Formal Statements and Concrete Policies*, ottobre 2004.
58. A. Monticini, G. Vaciago, *Are Europe's Interest Rates led by FED Announcements?*, dicembre 2004.
59. A. Prandini, P. Ranci, *The Privatisation Process*, dicembre 2004.
60. G. Mastromatteo, L. Ventura, *Fundamentals, beliefs, and the origin of money: a search theoretic perspective*, dicembre 2004.
61. A. Baglioni, L. Colombo, *Managers' Compensation and Misreporting*, dicembre 2004.
62. P. Giarda, *Decentralization and intergovernmental fiscal relations in Italy: a review of past and recent trends*, gennaio 2005.
63. A. Baglioni, A. Monticini, *The Intraday price of money: evidence from the e-MID market*, luglio 2005.
64. A. Terzi, *International Financial Instability in a World of Currencies Hierarchy*, ottobre 2005.
65. M. F. Ambrosanio, A. Fontana, *Ricognizione delle Fonti Informative sulla Finanza Pubblica Italiana*, gennaio 2006.
66. L. Colombo, M. Grillo, *Collusion when the Number of Firms is Large*, marzo 2006.
67. A. Terzi, G. Verga, *Stock-bond correlation and the bond quality ratio: Removing the discount factor to generate a "deflated" stock index*, luglio 2006.
68. M. Grillo, *The Theory and Practice of Antitrust. A perspective in the history of economic ideas*, settembre 2006.
69. A. Baglioni, *Entry into a network industry: consumers' expectations and firms' pricing policies*, novembre 2006.
70. Z. Rotondi, G. Vaciago, *Lessons from the ECB experience: Frankfurt still matters!*, marzo 2007.
71. G. Vaciago, *Gli immobili pubblici…..ovvero, purché restino immobili*, marzo 2007.
72. F. Mattesini, L. Rossi, *Productivity shocks and Optimal Monetary Policy in a Unionized Labor Market Economy*, marzo 2007.
73. L. Colombo, G. Femminis, *The Social Value of Public Information with Costly Information Acquisition*, marzo 2007.
74. L. Colombo, H. Dawid, K. Kabus, *When do Thick Venture Capital Markets Foster Innovation? An Evolutionary Analysis*, marzo 2007.
75. A. Baglioni, *Corporate Governance as a Commitment and Signalling Device*, novembre 2007.
76. L. Colombo, G. Turati, *The Role of the Local Business Environment in Banking Consolidation*, febbraio 2008.

77. F. Mattesini, L. Rossi, *Optimal Monetary Policy in Economies with Dual Labor Markets*, febbraio 2008.

78. M. Abbritti, A. Boitani, M. Damiani, *Labour market imperfections, "divine coincidence" and the volatility of employment and inflation*, marzo 2008.

79. S. Colombo, *Discriminatory prices, endogenous locations and the Prisoner Dilemma problem*, aprile 2008.

80. L. Colombo, H. Dawid, *Complementary Assets, Start-Ups and Incentives to Innovate*, aprile 2008.

81. A. Baglioni, *Shareholders' Agreements and Voting Power, Evidence from Italian Listed Firms*, maggio 2008.

82. G. Ascari, L. Rossi, *Long-run Phillips Curve and Disinflation Dynamics: Calvo vs. Rotemberg Price Setting*, settembre 2008.

83. A. Baglioni, A. Monticini, *The intraday interest rate under a liquidity crisis: the case of August 2007,* ottobre 2008.

84. M. F. Ambrosanio, M. Bordignon, F. Cerniglia, *Constitutional reforms, fiscal decentralization and regional fiscal flows in Italy*, dicembre 2008.

85. S. Colombo, *Product differentiation, price discrimination and collusion*, marzo 2009.