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Taxation and Predatory Prices in a Spatial Model

Stefano Colombo[♦]

Abstract

Using a spatial model with two separated markets, we study how taxation alters the incentive to prey of an incumbent firm facing a potential entrance by another firm. We show that for intermediate levels of the transportation costs, the higher are taxes the lower are the expected gains from the predatory strategy. We also show that under some conditions setting a positive level of taxes may induce a duopolistic equilibrium instead of a monopolistic one, and this ultimately increases welfare.

JEL codes: D43; L11
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1. Introduction

It is well known that firms may adopt predatory strategies in order to maintain or to acquire monopolistic positions. A typical predatory strategy consists in setting below-cost prices in order to induce the exit of the rival or to convince it not to enter.[1] Predation is characterized by the fact the predator should be prompt to suffer some losses in the current period in change of higher profits in the future (Motta, 2004). Literature has deeply investigated the rationale of predatory strategies, and nowadays there is a large consensus on the rationale of such strategies.

This paper adopts a different prospective in analysing predation. Instead of interrogating about the rationale of predation, we look at predation taking for granted its rationality. In particular, we investigate the relationship between taxation and predation in a spatial model originally developed by Hwang and Mai (1990).[2] Two spatially separated markets are supposed to exist. Transportation costs have to be paid in order to to move one unit of output one unit of distance. In such framework, we suppose that there is an incumbent firm and another firm, which is a potential entrant. In case of entrance, the incumbent may prey on the entrant, driving its profits to zero. The entrant, anticipating this fact, may decide to stay out. We investigate the effects of taxation on the incentives of the incumbent to act in a predatory way.

This approach seems interesting for at least two reasons:

- Predation is a relevant issue for welfare. As long as the adoption of predatory strategies implies lower welfare in equilibrium, a natural question is whether public policy instruments like taxes may modify the incentives to prey, and henceforth the likelihood of the emerging of predation in equilibrium.
- Space is a relevant dimension is many economic situations, including predation, Indeed, the existence of spatially separated markets allows discriminatory prices which make predation less costly for the predator, as the below-cost prices may not involve all markets, but only some selected ones. At the same time, the existence of

[1] Predatory prices are typically defined as prices which are below the marginal costs or the average variable costs (Areeda and Turner, 1975). In this article we adopt this definition. More sophisticated definitions of predatory prices involve average total costs (Joskow and Klevoric, 1979), average avoidable costs (Baumol, 1996) and average incremental costs (Bolton *et al*., 2000).

[2] For applications of the model of Hwang and Mai (1990) see, among the others, Gross and Holahan (2003) and Liang et al. (2006). Also, taxation has received some attention in spatial economics. See for example Heywood and Pal (1996) and Kilkenny and Melkonyan (2002) and the references therein.

separated markets allows the potential entrant to enter in a different market from the incumbent, which in turn increases its possibility to survive to predation. Therefore, the possibility that predation occurs in equilibrium is likely to depend on distance and transportation costs.

By interacting these two factors (taxation and space) with the incentive to prey of the incumbent, we obtain some interesting results. First, the possibility that taxation may induce a non-predatory equilibrium depends on the fact that transportation costs between the two markets are positive (since with zero transportation costs the two markets collapse in one, positive transportation costs amount to say that the "spatial" dimension is relevant for the economy), but not too large (in this case, two local monopolies always arise in equilibrium). Second, we show that when transportation costs are intermediate, a positive level of taxation may be welfare maximizing. This is due to the fact that taxes decrease the gains from predation. Therefore, setting a positive level of taxes may induce a competitive scenario where the potential entrant decides to enter and the incumbent accommodates, while zero taxes may induce a predatory scenario where the potential entrant stays out because the threat of predation is credible. If this occurs, welfare may increase under a positive taxation.

The paper proceeds as follows. In Section 2 we describe the model. In Section 3 we describe the equilibrium. In section 4 we calculate the welfare-maximizing tax rate and we characterize the main result. Section 5 concludes.

2. The model

Following Hwang and Mai (1990), we assume a linear segment of length one. There are two distinct markets, market 1 and market 2, located at the endpoints of the segment. There are two firms, firm *A* (the incumbent), and firm *B* (the entrant). Firm *A* is located in market 1, while firm *B*, if enters, locates in market 2.[3] Suppose that transporting a unit of the good entails a linear non-negative transportation cost equal to t. Within the same market, transportation costs are negligible (zero). Costs of production

[3] We keep exogenous locations in order to highlight the impact of taxation over predatory prices maintaining the analysis as simple as possible. Moreover, as Liang et al. (2006) has shown, maximal differentiation is the unique locational equilibrium in the duopolistic Hwang and Mai (1990) model. See also Gross and Holahan (2003) for a similar assumption.

are normalized to zero for both firms. An ad valorem tax rate, $k \in [0,1]$, is supposed to exist.[4] Firms can set different prices at different markets, and arbitrage between consumers is excluded. The demand curves in markets 1 and 2 are assumed to be linear and symmetric, and take the following form:

$$q_1 = v - p_1 \tag{1}$$

$$q_2 = v - p_2 \tag{2}$$

where q_i and p_i are the quantity demanded and the delivered price in market $i = 1, 2$ respectively, and v is a strictly positive parameter. At each market, the firm which sets the lowest price gets the whole demand. To avoid ε-equilibria, we assume that when the two firms set the same price, the whole demand is served by the nearest firm. Therefore, the profits of firm *A* at market 1 and market 2 are given respectively by:

$$\Pi_1^A = p_1^A (1-k)(v - p_1^A) \tag{3}$$

$$\Pi_2^A = [p_2^A (1-k) - t](v - p_2^A) \tag{4}$$

provided that $p_1^A \le p_1^B$ and $p_2^A < p_2^B$. Otherwise, they are zero. Similarly, the profits of firm *B* in the two markets are defined by:

$$\Pi_1^B = [p_1^B (1-k) - t](v - p_1^B) \tag{5}$$

$$\Pi_2^B = p_2^B (1-k)(v - p_2^B) \tag{6}$$

provided that $p_1^A > p_1^B$ and $p_2^A \ge p_2^B$. Otherwise, they are zero. The sequence of the moves is the following. At period 0, firm *B* decides whether to enter or to stay out. Entrance costs are zero. If firm *B* has entered, firms compete for two periods, period 1 and period 2. At the end of period 1, firm *B* decides whether to stay or leave. If firm *B*'s profits at the end of period 1 are non-positive it leaves, while firm *A* has no binding

[4] We are implicitly assuming that the two markets belong to the same jurisdiction. Therefore, phenomena of tax competition between competing jurisdictions are excluded. Moreover, the results we obtain are qualitatively the same when unit taxation is supposed instead of ad valorem taxation.

financial constraint.[5] In period 2, if firm *B* has stopped to be active, firm *A* acts as a monopolist, while if firm *B* is still active, firms compete. There is no discount between periods. The sub-game Nash perfect equilibrium concept is used in solving the game.

3. Solution of the model

Period 2

Consider the last period (period 2). We distinguish between three cases: both firms are active and compete for both markets ("duopoly" case); both firms are active and act as local monopolists ("local monopolies" case); only the incumbent is still active ("predation" case).[6]

We proceed analysing each case.

Duopoly

Suppose that in period 2 firm *B* is still active. Therefore, predation by firm *A* is not relevant (firm *A* has no incentive to prey, since there are no periods left to take advantage from predation). Let us define:

$$\bar{k} \equiv 1 - 2t/v$$

Suppose for the moment that $k \leq \bar{k}$. Consider market 1. Following a standard Bertrand argument, the two firms cut the prices until the farthest firm cannot decrease the price further because this would entail a negative mark-up. At this point, the nearest firm sets

[5] Things do not change if firm *B* has a limited access to credit, provided that firm *A* has better access to credit than the rival. This asymmetry in the disposal of credit may be due to the fact that banks have a better knowledge of firm *A* than firm *B* because the former has entered the market first, and therefore the banks are more prompt to give credit to firm *A* than to firm *B*. For more about the credit theory as a rationale for predatory strategies by an incumbent firm, see Motta (2004).

[6] Note that the "predation" case is a monopoly (only one firm operates). However, we use the term "predation" to emphasize that such situation is generated by the predatory strategy adopted by the incumbent in period 1 (see later). On the contrary, the "local monopolies" case consists in two monopolists, one at each market, and it is generated (as it will become clear later) by excessive transportation costs.

the same price and serves the whole market.[7] The furthest firm is *B*, which cannot decrease the price below $t/(1-k)$. Therefore, firm *A* sets $t/(1-k)$ and serves the whole demand at market 1. Therefore the equilibrium price is market 1 is:

$$p_1^{A,D} = t/(1-k) \tag{7}$$

where the superscript *D* refers to the fact that we are considering the "duopoly" case. A similar reasoning shows that firm *B* serves the whole market 2 setting the following price:

$$p_2^{B,D} = t/(1-k) \tag{8}$$

Plugging (7) into (3), and (8) into (6), we get the equilibrium profits in period 2 for both firms when there is competition. Therefore:

$$\Pi^{A,D} = \Pi^{B,D} = t(v - \frac{t}{1-k}) \tag{9}$$

Local monopolies

Suppose now that $k \geq \bar{k}$. Consider market 1. Let us suppose that firm *A* is a monopolist in market 1. It maximizes (3) with respect to p_1^A. This yields:

$$p_1^{A,M} = v/2 \tag{10}$$

where the superscript *M* indicates that we are considering the "local monopolies" case. It is immediate to see that when $k \geq \bar{k}$, the duopoly price, $p_1^{D,M}$, is higher than the local monopoly price, $p_1^{A,M}$. Therefore, when $k \geq \bar{k}$, firm *A* sets the local monopolistic price even if firm *B* is active, because firm *B* is too distant, given the level of the

[7] See Lederer and Hurter (1986) for a technical and formal proof. If the assumption that at equal prices the nearest firm serves the whole demand is removed, the nearest firm slightly undercuts the furthest firm at its lowest possible price and serves the whole demand.

transportation costs, to compete with firm A for market 1. In other words, firm A acts as a local monopolist in market 1. The profits of firm A are:

$$\Pi^{A,M} = v^2(1-k)/4 \tag{11}$$

By symmetry, when $k \geq \bar{k}$, firm A cannot compete with firm B for market 2. Firm B is a local monopolist in market 2, and maximizes (6) with respect to p_2^B. This yields the following price and profits:

$$p_2^{B,M} = v/2 \tag{12}$$

$$\Pi^{B,M} = v^2(1-k)/4 \tag{13}$$

Predation

Suppose now that firm B has left at the end of period 1, due to predation by firm A. Suppose also that $k \leq \bar{k}$. Firm A is a monopolist in period 2 and maximize (3) and (4) with respect to p_1^A and p_2^A respectively. This yields the following equilibrium prices:

$$p_1^{A,P} = v/2 \tag{14}$$

$$p_2^{A,P} = v/2 + t/2(1-k) \tag{15}$$

where the superscript P indicates that we are considering the case of predation. Overall predatory profits of firm A are:

$$\Pi^{A,P} = \Pi_1^{A,P}(p_1^{A,P}) + \Pi_2^{A,P}(p_2^{A,P}) = \frac{2v^2(1-k)^2 - vt(1-k) + t^2}{4(1-k)} \tag{16}$$

Suppose now that $k \geq \bar{k}$. In this case, firm A, even if it is the only active firm, cannot profitably serve market 2. Therefore, overall predatory profits of firm A in period 2 coincide with the local monopolistic profits, $\Pi^{A,M}$.

Future gains from predation. It is well known that predation entails short term costs in order to obtain higher profits in the future. In this simple model, when $k \leq \bar{k}$, the future gains from predation are the difference between the predatory profits and the duopolistic profits. That is:

$$G = \Pi^{A,P} - \Pi^{A,D} = \frac{2v^2(1-k)^2 - 6vt(1-k) + 5t^2}{4(1-k)} \tag{17}$$

Instead, when $k \geq \bar{k}$, firm *A* serves only market 1, both when firm *B* is active and when it is absent, and obtains always $\Pi^{A,M}$. Therefore, the gains from predation are nil.

<u>*Period 1*</u>

Consider firm *A* in period 1. Firm *A* has two possibilities. On one hand, it can act in an aggressive way, in order to induce firm *B* to leave at the end of the period. On the other hand, it can accommodate firm *B*'s entrance.

Predation

Suppose that firm *A* acts in a predatory way. Firm *A* knows that if firm *B* obtains zero profits it leaves. Therefore, firm *A* sets prices in such a way to leave firm *B* with zero demand. The important aspect here is that firm *A* does not care to price below its own marginal costs. Indeed, pricing below costs is precisely the nature of predatory pricing: while this is not a profit-maximizing behaviour in the short term, it may maximize the profits when the two periods are considered together. Consider market 1. Even under duopoly, firm *B* obtains zero profits. So market 1 is not relevant for the analysis of predation. Instead, firm *B* obtains positive profits at market 2 when firm *A* sets the lowest price which guarantees it a non-negative mark-up (see eq. (8)). Therefore, if firm *A* wants to force firm *B* to leave, it has to set such a low price in market 2 that firm *B* obtains non-positive profits for any equal or lower price. It is immediate to see that such predatory price in market 2 is given by: $\hat{p}_2^{A,P} = 0$. At this price, the demand, which is equal to *v*, is entirely served by firm *A* with a negative mark-up equal to: $-t$. Since the equilibrium profits in market 1 are the same as under the duopoly case, the overall predatory profits of firm *A* in period 1 are given by:

$$\hat{\Pi}^{A,P} = \Pi^{A,D} - tv \tag{18}$$

when $k \le \bar{k}$, and are given by:

$$\hat{\Pi}^{A,P} = \Pi^{A,M} - tv \tag{19}$$

when $k \ge \bar{k}$.

Accommodation

Suppose that firm *A* does not prey on firm *B*. This case coincides with the duopoly case analysed in period 2. Therefore, in case of accommodation, the profits of firm *A* in period 1 are given by $\Pi^{A,D}$ when $k \le \bar{k}$, and by $\Pi^{A,M}$ when $k \ge \bar{k}$.

Now, we can calculate the losses from predation, which amount to the reduction of current profits of firm *A* induced by the adoption of a below-cost price in market 2. When $k \le \bar{k}$ (resp. $k \ge \bar{k}$), the firm's losses arising from the predatory strategy are simply the difference between the duopoly (resp. local monopoly) profits and the predatory profits in period 1. Therefore, denoting by *L* the losses from predation, we get:

$$L = tv \tag{20}$$

Now we can address the following question: when does firm *A* price aggressively? Firm *A* prices aggressively when the future gains from predation outweigh the current losses from predation. As Spector (2005, p.193-194) argues: "*a major ingredient of any economic assessment* [of predation] *is a comparison of the short term losses induced by* *temporary low prices and the long-term gains derived from enhanced market power after the rival's eviction*". Consider the case $k \leq \bar{k}$. Let us define:

$$\Gamma \equiv G - L = \frac{2v^2(1-k)^2 - 10vt(1-k) + 5t^2}{4(1-k)} \tag{21}$$

Predation shall occur in equilibrium as long as the function Γ has a positive value, which means that the expected gains from the predatory strategy outweigh the current costs associated with setting a price lower than the marginal costs (predatory price) in the most distant market. On the contrary, when $k \geq \bar{k}$, the gains from predation are zero. Since the losses of predation are positive, it follows that predation is never optimal. This is rather obvious. Predation in period 1 is rational as long as it allows firm *A* to serve market 2 in period 2, which otherwise would be served by firm *B*. However, when $k \geq \bar{k}$, firm *A* cannot profitably enter in market 2 even if firm *B* is not active. Therefore, predation in period 1 does not allow additional future profits.

<u>*Period 0*</u>

In period 0, firm *B* decides whether to enter or not. Let us consider first the case where $k \leq \bar{k}$. Clearly, if Γ is positive, firm *B* anticipates that in case of entry, firm *A* will prey on it. Therefore, it decides to stay out. At the opposite, if Γ is negative, firm *B* anticipates that in case of entrance, firm *A* will not prey on it. Then, it enters. Next, we consider the case where $k \geq \bar{k}$. Firm *B* anticipates that predation shall not occur. Moreover, if enters, firm *B* is a local monopolist in market 2 in both periods. Therefore, firm *B* enters.

<u>**4. Optimal taxation**</u>

In this section, we investigate the tax rate which maximizes welfare. First, we show that, depending on the tax rate, different equilibria may arise. Let us consider function Γ. Solving $\Gamma = 0$ with respect to *k*, we get the following roots:

$$k_1 = 1 - \frac{(5+\sqrt{15})t}{2v} \tag{22}$$

$$k_2 = 1 - \frac{(5-\sqrt{15})t}{2v} \tag{23}$$

where $k_1 < k_2$. When $k \in [k_1, k_2]$, Γ is non-positive; when $k \le k_1$ and $k \ge k_2$, Γ is non-negative. Moreover, note that $k_2 > \bar{k}$. Therefore, we can state the following result:

Result 1. When $k \le k_1$, firm *B* does not enter and firm *A* serves both markets ("predation" case); when $k_1 \le k \le \bar{k}$, firm *B* enters and firms compete for both markets ("duopoly" case); when $k \ge \bar{k}$, firm *B* enters and both firms are monopolists in their own market ("local monopolies" case).

The intuition behind Result 1 is the following. Consider the case $k \le \bar{k}$. We know from Section 3 that the losses from predation do not depend on *k*. Therefore, the impact of the tax rate over the possibility that predation is a profitable option for the incumbent passes through the relationship between *k* and the gains from predation, which in turn are given by the difference between the predatory profits and the duopolistic profits. In order to disentangle the forces operating on the relationship between *k* and the gains from predation, let us consider first the duopolistic profits. Note that the equilibrium duopolistic mark-up of firm *A* does not depend on *k*: in fact, higher taxes are totally transferred into higher prices, so the net price is invariant with *k*. However, higher prices induce lower quantities, which in turn determine lower profits given that the mark-up is constant. As a consequence, the higher is the tax rate, the lower are the duopoly profits. Note that this effect is driven by the impact of *k* on the equilibrium duopoly price (see eq. (7)). Therefore, it positively depends on *t*, but not on *v*. Let us call this effect as the "*price effect*": the higher is *t*, the stronger is the *price effect*. Consider now the predatory profits. Here, the level of the taxes affects the equilibrium profits in two ways. In market 1, where firm *A* does not pay transportation costs, the ad valorem tax works as a tax on profits. Since this effect does not pass through prices (which are invariant with *k*) but directly through profits, we call it the "*profits effect*".

Note that equilibrium predatory profits in market 1 depend positively on v, but not on t (in fact, there are no transportation costs for firm A when it serves market 1). Therefore, the higher is v, the stronger is the *profits effects*. In market 2, where firm A pays transportation costs, the equilibrium predatory mark-up is affected by the tax rate. We call this effect as the "*mark-up effect*". The higher are the transportation costs, t, with respect to the size market parameter, v, the less the equilibrium mark-up depends on the tax rate: that is, the *mark-up effect* depends negatively (positively) on t (v). The *profits effect* and the *mark-up effect* work in the same direction: the higher is the tax rate, the lower are the predatory profits both in market 1 and in market 2. However, whether the predatory profits decrease more or less rapidly than the duopolistic profits when k goes up depends on the ratio t/v. When t/v is low, the *profits effect* and the *mark-up effect* are strong, while the *price effect* is weak. Therefore, the predatory profits decrease more rapidly than the duopoly profits. As a consequence, the gains from predation decrease, and when k goes above k_1, the gains from predation become lower than the losses from predation, thus making predation unprofitable for firm A. After a given threshold of the ratio t/v,[8] the *price effect* dominates the *profits effect* and the *mark-up effect*, the gains from predation start to increase with k, and when k goes above k_2, the gains from predation become higher than the losses from predation. However, as we noted above, the root k_2 is no relevant for the analysis, because for high levels of k the two markets are completely separated. In other words, when $k \geq \bar{k}$ there are always two local monopolies and predation is never profitable for firm A.

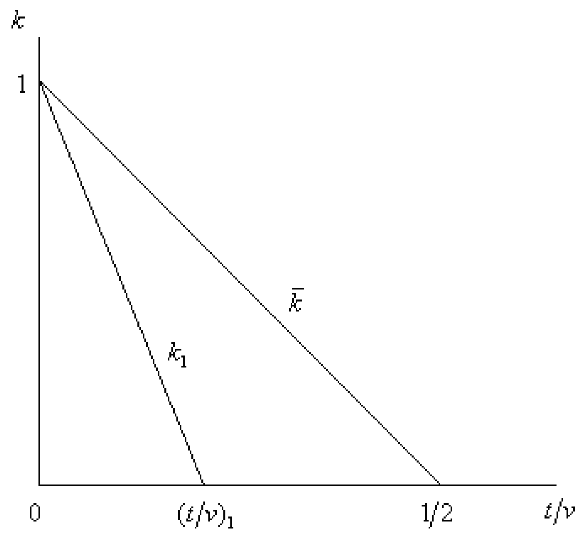
A direct corollary of Result 1 is the following:

Corollary: if the two markets coincide (i.e. $t = 0$), there is always predation; if the two markets are sufficiently far apart (i.e. $t > v/2$), there is never predation and two local monopolies always arise.

[8] Solving $\partial G/\partial k = 0$ with respect to k, it is immediate to verify that the derivative becomes positive at: $k > 1 - \sqrt{5}t/\sqrt{2}v$.

The corollary of Result 1 highlights the importance of the spatial dimension for the emerging of different scenarios depending on *k*. The role of the spatial dimension can be noticed by looking at Figure 1, where k_1 and $\bar{k}$ are depicted as functions of t/v and $(t/v)_1 = 2/(5+\sqrt{15})$.

Figure 1



First, note that the distance between $\bar{k}$ and k_1 increases with t/v. At the lower limit, when t/v goes to zero, $k_1 = \bar{k} = 1$, and all admissible values of *k* must be weakly lower than k_1. At the opposite, when transportation costs are sufficiently high, $t/v \geq 1/2$, all admissible values of *k* are higher than $\bar{k}$. This implies that when the space is not relevant (t/v is zero) or when markets are too distant (t/v is larger than $1/2$), taxation cannot be used to induce different competitive scenarios. When there is no space, predation occurs in case of entrance whatever is the level of taxes; if there is too distance between the markets, there are two separate monopolists whatever is the level of taxes. The intuition of the corollary of Result 1 is straightforward. When the two market coincide ($t = 0$), since firms are undifferentiated, the equilibrium price is zero, *whatever is the tax rate*. Therefore, duopolistic profits are zero as well, thus making predation the unique equilibrium. On the contrary, when there is "too" distance between the two firms ($t > v/2$), each firm is protected in its own market from the competition of the rival, *whatever is the tax rate*. As a consequence, predation is never profitable for the incumbent, and two local monopolies arise.

From Result 1 and its corollary it follows that when the two markets are separated (but not too much, i.e. $0 < t < v/2$), the incentives of predation (and therefore the level of competition in the following stages of the game) depend on the tax rate.

To better clarify how taxation can induce different competitive scenarios, consider Figure 2, 3 and 4, where different values of t/v are supposed. First, consider Figure 2. When $t/v \leq (t/v)_1$ (for example, $(t/v)°$ in the figure), there are three different possible scenarios depending on *k*. If $k \in [0, k_1((t/v)°)]$, predation occurs in case of entry. Then, firm *B* does not enter and firm *A* is the unique active firm in both markets and in both periods. If $k \in [k_1((t/v)°), \bar{k}((t/v)°)]$, predation does not occur in case of entrance. Then, firm *B* enters and firms compete in both markets and in both periods. Finally, if $k \in [\bar{k}((t/v)°), 1]$, predation does not occur in case of entrance. Then, firm *B* enters, but firms are monopolists in their own market in both periods.

Next, consider Figure 3. When $(t/v)_1 \leq t/v \leq 1/2$ (for example, $(t/v)^\wedge$ in the figure), there are two different possible scenarios depending on *k*. If $k \in [0, \bar{k}((t/v)^\wedge)]$, predation does not occur in case of entrance. Then, firm *B* enters and firms compete in both markets in both periods. If $k \in [\bar{k}((t/v)^\wedge), 1]$, predation does not occur in case of entrance. Then, firm *B* enters, but firms are monopolists in their own market in both periods.

Finally, consider Figure 4. When $t/v \geq 1/2$ (for example, $(t/v)^*$ in the figure), whatever is the level of the tax rate, predation does not occur in case of entrance. Then, firm *B* enters and firms are monopolists in their own market in both periods.

Figure 2

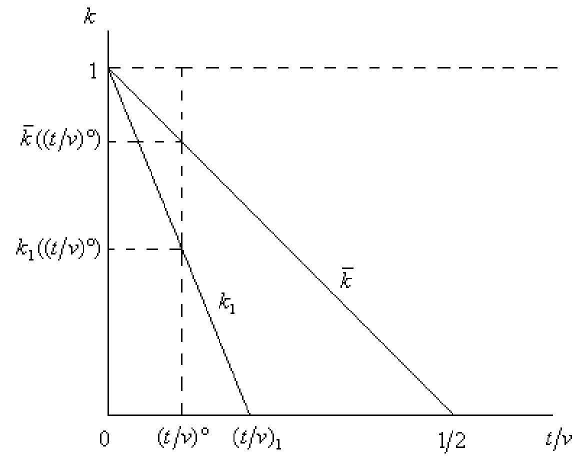


Figure 3

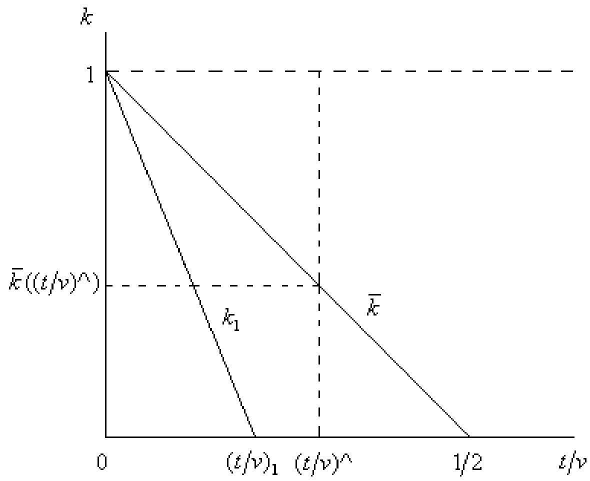
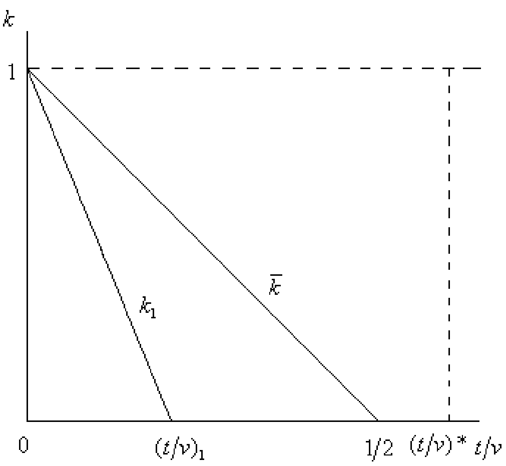


Figure 4



Next, taking into account the fact that depending on the level of k and t/v different scenarios may arise, we calculate the welfare-maximizing level of taxation. Welfare is given by the sum of firms' profits, consumer surplus and tax revenues. Since the equilibrium welfare in period 1 is the same as the equilibrium welfare in period 2,[9] we consider welfare in one period only.

First, we consider the case of predation, which occurs when $k \le k_1$. Firm *B*, anticipating predation, does not enter, and firm *A* sets the predatory prices in both periods. Therefore, using the equilibrium predatory price in the two markets, welfare in the predation case is:

$$W^P = \Pi_1^{A,P} + \Pi_2^{A,P} + \frac{(v - p_1^{A,P})q(p_1^{A,P})}{2} + \frac{(v - p_2^{A,P})q(p_2^{A,P})}{2} + k[p_1^{A,P}q(p_1^{A,P}) + p_2^{A,P}q(p_2^{A,P})] =$$

$$= \frac{6v^2(1-k)^2 - 2vt(3 - 5k + 2k^2) + t^2(3 - 4k)}{4(1-k)^2} \tag{24}$$

where the second and the third term indicate the equilibrium consumer surplus in market 1 and in the market 2 respectively, while the last term indicates the total equilibrium tax revenues.

Next, we consider the case of competition between firms (duopoly case), which occurs when: $k_1 \le k \le \bar{k}$. Firm *B*, anticipating that predation will not occur in case of entrance, enters, and both firms compete in both markets and in both periods. Therefore, using the equilibrium duopoly prices in the two markets, welfare in the duopoly case is:

$$W^D = 2\Pi^{A,D} + 2 \cdot \frac{(v - p_1^{A,D})q(p_1^{A,D})}{2} + 2kp_1^{A,D}q(p_1^{A,D}) = \frac{2[v^2(1-k)^2 - t^2]}{(1-k)^2} \tag{25}$$

where the second term indicates the overall equilibrium consumer surplus, while the last term indicates the total equilibrium tax revenues (recall that in the duopoly case the two firms set the same equilibrium price, so market 2 replicates market 1).

[9] In fact, in equilibrium firm *A* never sets below-cost prices. When predation is a credible threat, firm *B* stays out and the equilibrium prices of firm *A* are $p_1^{A,P}$ and $p_2^{A,P}$ in both periods.

Finally, we consider the case where firms act as monopolists in their own market ("local monopolies" case), which occurs when $k \geq \bar{k}$. In this case, using the equilibrium monopoly prices in the two markets, welfare is:

$$W^{M} = 2\Pi^{A,M} + 2 \cdot \frac{(v - p_1^{A,M})q(p_1^{A,M})}{2} + 2p_1^{A,M}q(p_1^{A,M}) = \frac{3v^2}{2} \tag{26}$$

where the second term indicates the overall equilibrium consumer surplus, while the last term indicates the total equilibrium tax revenues (again, in the "local monopolies" case, the prices in the two markets are identical in equilibrium, so market 2 replicates market 1). Note that the level of welfare under local monopolies does not depend on the tax rate. This is due to the fact that the equilibrium price in both markets does not depend on *k*. Therefore, the ad valorem tax acts as a tax on profits, and it is simply a transfer from firms to Treasury, leaving unchanged the level of welfare. However, recall that a positive level of taxes ($k \geq \bar{k}$) is necessary to induce the monopolistic equilibrium, even if it does not affect the equilibrium welfare within the "local monopolies" case.

In the following, for each case, we calculate the optimal level of taxes. Clearly, within the predatory and the duopoly case, welfare must decrease with taxes. In fact: $\frac{\partial W^P}{\partial k} = -\frac{t[v - vk - t(1-2k)]}{2(1-k)^3} < 0$ and $\frac{\partial W^D}{\partial k} = -\frac{4t^2}{(1-k)^3} < 0$. Therefore, in the predatory case ($k \leq k_1$), the optimal tax rate is: $k = 0$. In the duopoly case ($k_1 \leq k \leq \bar{k}$), the optimal tax rate is: $k = \max[0, k_1]$. Finally, in the monopoly case ($k \geq \bar{k}$), any tax rate such that $k \geq \bar{k}$ is optimal.

Next, we calculate welfare at the optimal tax rate. First, suppose $t/v \leq (t/v)_1$. If $k = 0$, there is predation; if $k = k_1$, there is duopoly; if $k \geq \bar{k}$, there are local monopolies. Substituting $k = 0$ into W^P, we get:

$$W^{P}(k = 0) = \frac{3(2v^2 - 2vt + t^2)}{4} \tag{27}$$

Note that: $W^P(k=0) - W^M = -\frac{3t(2v-t)}{4} < 0$. Therefore, when the transportation costs are sufficiently low, a situation where markets are served monopolistically by different firms is welfare superior to a situation where both markets are served monopolistically by the same firm. The intuition is straightforward. When firm *A* serves market 2 (because firm *B* has not entered), it sets a monopolistic price that is higher than the price firm *B* would set if it was the monopolist in market 2, the reason being that firm *A* suffers transportation costs when serves market 2, while firm *B* does not sustain transportation costs. A higher price is associated with a lower equilibrium quantity. As a consequence, welfare is lower in the predation case than in the case of local monopolies. Now, we compare the welfare in the "local monopolies" case with the welfare in the duopoly case when the taxation is optimal. Substituting $k = k_1$ into W^D, we get:

$$W^D(k = k_1) = \frac{4v^2(18 + 5\sqrt{15})}{(5 + \sqrt{15})^2} \tag{28}$$

Comparing W^D with W^M, we observe: $\Delta = W^M - W^D(k = k_1) = -\frac{v^2(12 + 5\sqrt{15})}{(5 + \sqrt{15})^2} < 0$.

Therefore, the duopoly case is welfare superior to the case of local monopolies. The intuition is that under duopoly, competition lowers the equilibrium prices and increases the quantity sold in equilibrium with respect to monopoly. *Even if the duopolistic equilibrium is obtained with a positive level of taxes* (which are welfare detrimental), the positive effect on welfare due to the higher quantity sold in equilibrium outweighs the negative effect due to positive taxes. Therefore, welfare is higher under competition than under local monopolies, even if k is positive. It follows that the welfare-maximizing tax rate is $k^* = k_1 > 0$.

Consider now the case $(t/v)_1 \le t/v \le 1/2$. The government may induce duopoly by setting $k = 0$ and may induce monopoly by setting $k \ge \bar{k}$. Substituting $k = 0$ into W^D, we get:

$$W^D = 2(v^2 - t^2) \quad (29)$$

Comparing W^D with W^M, we get: $\Delta = W^M - W^D(k=0) = -\frac{v^2}{2} + 2t^2 < 0$. Therefore, welfare is higher under duopoly than under local monopolies. This is not surprising. In fact, the duopoly equilibrium here can be induced with zero taxes. This is welfare increasing, because allows lower equilibrium prices and higher equilibrium quantities. Since monopoly welfare does not depend on taxes, the duopoly equilibrium is necessarily welfare superior than the "local monopolies" equilibrium. It follows that the welfare-maximizing tax rate is $k^* = 0$.

Finally, consider the case where $t/v \geq 1/2$. The unique equilibrium is constituted by two local monopolies. Moreover, as we already noted, the welfare under local monopolies does not depend on *k*, so the level of taxes is irrelevant for equilibrium welfare.
We sum up the results in the following proposition:

Proposition 1: Provided that $t > 0$, we get:
1) if $t/v \leq (t/v)_1$, the welfare-maximizing tax rate is: $k^* = k_1 > 0$, where k_1 decreases with *t* and increases with *v*. In equilibrium, we observe entrance of firm *B* and duopolistic competition.
2) if $(t/v)_1 \leq t/v \leq 1/2$, the welfare-maximizing tax rate is: $k^* = 0$. In equilibrium, we observe entrance of firm *B* and duopolistic competition.
2) if $t/v \geq 1/2$, welfare does not depend on the tax rate. In equilibrium, we observe entrance of firm *B* and a local monopoly at each market.

Proposition 1 shows that choosing a positive level of taxation may be welfare-improving as long as it provides a competitive equilibrium instead of a predatory equilibrium. Therefore, a welfare-maximizing government may face a trade-off when it has to choose the appropriate level of taxation in the presence of spatial competition and the threat of predation by an incumbent. Ceteris paribus, taxes are welfare detrimental, and therefore they should be set at zero. However, zero taxes may induce a bad equilibrium, where the entrant is forced to stay out by the threat of a predatory behaviour by the incumbent in the case of entrance. Therefore, the government may found it optimal to set a positive tax rate in order to provide the right incentives for the incumbent firm to accommodate the entrance of the rival. In fact, the higher are taxes the lower are the gains deriving from predation (see the discussion of Result 1). The spatial dimension of the framework plays a key role in determining the possibility for the government to induce different competitive equilibria setting the appropriate tax level. In fact, when there is no space, predation always occurs, while when the two markets are too distant, two local monopolies always arise. Instead, for intermediate levels of the transportations costs, two situations emerge: when transportation costs are quite low, a positive level of taxation is necessary to induce entrance and competition between firms, while when transportation costs are quite high, a zero tax rate is sufficient to induce entrance and competition between firms.

5. Conclusions

Predatory prices (that is, prices which are lower than the marginal or the average costs) are a relevant issue in economics. Predation is characterized by the fact that the predator is ready to suffer current losses in change of future gains. Notwithstanding the importance of this issue, the literature on predation has focused almost exclusively on the rationale of predatory strategies (see, for example, the comprehensive analysis by Bolton et al., 2000), leaving unexplored many other questions concerning predation.

In this paper, the rationality of predation has been taken for granted. On the contrary, we investigated the relationship between taxation and predation in a spatial competition model with two separate markets (Hwang and Mai, 1990), where an incumbent, located in one market, may use a predation strategy to deter entrance by another firm into the other market in order to maintain a monopolistic position.

We obtain some striking results. Depending on the transportation costs, the impact of the tax rate over the incentive to prey may vary. In particular, the range of tax rates which affect the incentive to prey expands when the transportation costs increase (up to an upper bound). At the inferior limit, when transportation costs are zero, the incentive to prey does not depend on the tax rate. Since zero transportation costs amount to say that the two markets coincide, we first concluded that when there is no spatial dimension in the economy taxes are irrelevant for the incentive to prey. The symmetric result is that *when there is a spatial dimension*, taxes may affect the likelihood that an incumbent firm acts in a predatory way against an incoming firm. In particular, we showed that for positive and moderate transportation costs, the welfare is maximized with a *positive* level of taxes. This is due to the fact that when a positive tax rate is imposed on firms, the incumbent firm accommodates entrance, thus determining a duopolistic equilibrium instead of a monopolistic one. Finally, when transportation costs are too high, two local monopolies always arise, independently on the tax rate.

Although our analysis may be too simple to derive the conclusion that taxes make predatory prices less (or equally) likely to arise, it has at least showed in a stylized spatial model the existence of a non obvious linkage between taxation and the incentive to prey by an incumbent. Of course, further research is needed to check the robustness of such link in different contexts.

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