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Cheap Talk with Correlated Signals

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Abstract

We consider a game of information transmission, with one informed decision maker gathering information from one or more informed senders. Private information is (conditionally) correlated across players, and communication is cheap talk. For the one sender case, we show that correlation unambiguously tightens the existence conditions for a truth-telling equilibrium. We then generalize the model to an arbitrary number of senders, and we find that, in this case, the effect of correlation on the incentives to report information truthfully is non monotone, and correlation may *discipline* senders' equilibrium behavior, making it easier to sustain truth-telling.

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1 Introduction

The importance of strategic information transmission in economic contexts has long been recognized. Following the seminal work of Crawford and Sobel (1982), a large body of literature has studied the conditions under which information is transmitted truthfully in equilibrium and the impact of different informational and strategic conditions on such equilibria. Few papers (e.g. Ottaviani and Sørensen (2006) in a model with reputation) have dealt with the *quality* of information and none has investigated the impact of quality on truth-telling behavior when information is transmitted between more than two players.

The quality of information is usually measured by its accuracy. However, when different pieces of information come from multiple sources, correlation also affects their informative content in an intuitive way. In the limit case of perfect correlation, observing one single piece of information is equivalent to observing any number of pieces, and communication becomes worthless. Correlation is, indeed, a common feature of many problems where the strategic transmission of information is a relevant issue: a legislative commission audits several experts on a given matter and the experts have partially coincident information sources; a perspective voter wants to form an opinion and joins an online political discussion group whose members lean towards the same party and form their opinions sourcing from similar media; witnesses in a cross-examination may have information that overlaps to different extents; etc. In general, correlation of private information may be caused either by external factors (e.g. a small number of information sources is available) or by preferences (e.g. people with similar views and preferences may source from similar sources and have similar acquaintances).

While it is clear that correlation weakens the welfare gains from information transmission, the way in which it affects the incentives to strategically disclose truthful information is not *a priori* clear. In this paper we address this problem in the context of a cheap talk game where a partially informed receiver (she) gathers information from one or more partially informed senders before taking action. To model strategic communication we follow the literature and adopt the binary signals framework introduced by Morgan and Stocken (2008) in the context of information aggregation. In their paper, the signals of the sender and the receiver are conditionally independent. To incorporate correlation into their binary signals framework, we rely on the work of Bahadur

(1961) that first discusses correlation among binary signals. Correlation is measured by a single parameter, denoting the probability that both players observe signals from the same source, while with complementary probability they access independent sources.

For the model with one sender, we show that correlation unambiguously tightens the existence conditions for a truth-telling equilibrium: credibly revealing information becomes more difficult as correlation increases. The intuition is straightforward: by decreasing the informational content of each signal, correlation weakens the impact of information transmission on the receiver's decision and minimizes the risk of an excessive reaction (overshooting), enabling the sender to move the receiver's action in a profitable way.

We then generalize the information structure to the case of more than two players, by introducing a multinomial distribution over signals with a single correlation parameter. Again, the receiver possesses a prior belief on the likelihood that signals come from either a perfectly correlated or from an independent binomial process. We find that the effect of correlation on the incentives to reveal information is non monotone, and that there exists a critical level of correlation such that truth-telling is always an equilibrium. Moreover, this critical level is decreasing in the number of players. This characterization is driven by two conflicting forces that shape each sender's incentives to misreport his observed signal. The first is the direct effect that one additional piece of information has on the receiver's action, also at work in the case of two players. The second is an indirect effect we refer to as *discipline*: any report not in line with the rest of the senders causes the receiver to believe that information sources are independent. As a consequence, the perceived informational content of the rest of the signals grows larger, decreasing or even reverting the sender's incentives to misreport the observed signal and favoring the emergence of truth-telling. For an interval of values of the correlation parameter, the indirect effect is strong enough to offset the direct one and deter any sender from telling a lie, thereby acting as a disciplining force supporting truth-telling.

The paper is organized as follows: Section 2 lays down the model with two players, introduces our information generating structure and characterizes the truth-telling equilibrium. Section 3 generalizes the model to more than two players, presents the non monotonicity result and discusses the discipline effect. Section 5 concludes.

2 Two Players

Players. There are two players, a Sender i (he) and a Receiver j (she). The receiver takes an action $y \in \mathbb{R}$ which affects both players' utilities. These also depend on the state of the world θ, which is unknown to both players. Before the receiver takes action, each player observes a binary signal about the state θ. Let the signal of the sender be s_i and the signal of the receiver be s_j with $s_i, s_j \in \{0, 1\}$. After observing s_i, the sender sends a message $t \in \{0, 1\}$ to the receiver. After hearing t and observing s_j, the receiver takes action y. We consider quadratic loss utility functions: the utility of the receiver is $U^j(y, \theta, b_j) = -(y - \theta - b_j)^2$ and the utility of the sender is $U^i(y, \theta, b_i) = -(y - \theta - b_i)^2$, with b_i and b_j representing individual preferences.

Information structure. The state of the world is known to be uniformly distributed, $\theta \sim U(0, 1)$. To generate a simple one parameter correlation structure we make the following assumptions:

A1 There are (at least) 2 sources of information, each generating an informative and independent signal according to the binomial distribution $\Pr(s = 1|\theta) = \theta$.

A2 Players either collect information from independent sources with probability $1 - k$ or, with probability k, they collect information from the same source.

In words: before the game is played, each player independently picks one information source among those available and observes the corresponding signal. While each player is not aware of the source picked by the other, it is known that the probability that each player picks the same source is $k \in [0, 1)$. Since with some probability players collect information from the same source, their signals are correlated.

The information acquisition process just described leads to the joint distribution of players' signals conditional on the state $\Pr(s_i, s_j|\theta)$, which was first discussed by Bahadur (1961) and is summarized in Table 1.

Since θ is uniformly distributed, the marginals are $\Pr(s_i = 1) = \Pr(s_j = 1) = \int_0^1 \theta \, d\theta = 1/2$ and, similarly, $\Pr(s_i = 0) = \Pr(s_j = 0) = \int_0^1 (1 - \theta) \, d\theta = 1/2 = 1 - P(s_i = 1)$. Notice also that, because of the uniform assumption on the state θ, its density is $f(\theta) = 1$ which implies that $\Pr(s_i, s_j|\theta) = \frac{f(s_i, s_j, \theta)}{f(\theta)} = f(s_i, s_j, \theta)$ where $f(s_i, s_j, \theta)$ is the joint density function of signals and state. Thus, the joint distribution of players' signals conditional on the state equals the unconditional joint density on signals and state.

Table 1: Signals' joint distribution conditional on θ

	$s_j = 0$	$s_j = 1$
$s_i = 0$	$(1-\theta)\,k + (1-\theta)^2\,(1-k)$	$\theta\,(1-\theta)\,(1-k)$
$s_i = 1$	$\theta\,(1-\theta)\,(1-k)$	$\theta k + \theta^2\,(1-k)$

Conditionals can be easily derived from Table 1: the probability of the receiver observing signal s_j given the sender observed signal s_i and the state θ is:

$$\Pr\left(s_j|s_i,\theta\right) = \frac{\Pr(s_j,s_i|\theta)}{\Pr(s_j,s_i|\theta)+\Pr(1-s_j,s_i|\theta)}.$$

To understand the conditionals, take the state θ and let the sender have observed $s_i = 0$. Then, on the one hand, the receiver may observe $s_j = 0$ from the same source — which happens with probability k — or from an independent source — which happens with probability $1-k$ —, in which case the probability she observes $s_j = 0$ is $1-\theta$. Hence, the receiver observes $s_j = 0$ with probability $\Pr\left(s_j = 0|s_i = 0,\theta\right) = k + (1-k)\,(1-\theta)$. On the other hand, the receiver can only observe $s_j = 1$ from a source different from that of the sender — which happens with probability $1-k$ — in which case the probability she observes $s_j = 1$ is θ, from which it follows that the receiver observes $s_j = 1$ with probability $\Pr\left(s_j = 1|s_i = 0,\theta\right) = (1-k)\,\theta$. Similar reasoning apply for the case of $s_i = 1$. By symmetry of the information structure, the same conditionals hold for the sender.

Notice that k denotes the Pearson's correlation coefficient between the random binomial variables s_i and s_j, i.e.

$$k = \frac{Cov\left(s_i,s_j\right)}{\sigma_{s_i}\sigma_{s_j}}.$$

Finally, it is important to notice that the information structure we have introduced allows for non-negative correlation only, i.e. it must be $k \in [0,1)$. Indeed the NE and SW quadrants of Table 1 are non-negative for all θ only if $k \geq 0$, while the NW and SE quadrants are non-negative for all θ only if $k \leq 1$.

Equilibrium. Since the game is sequential and involves asymmetric information, the equilibrium concept is weak Perfect Bayesian Nash Equilibrium (see, e.g., Mas-Colell, Whinston, and Green (1995)). Such equilibrium is defined by the strategies $t(s_i)$ of the sender and $y(t,s_j)$ of the receiver such that:

- $t(s_i)$ maximizes the expected utility of the sender, i.e.

$$t\left(s_i\right) = \arg\max_{t\in\{0,1\}} \sum_{s_j=\{0,1\}} \int_0^1 -\left(y\left(t,s_j\right)-\theta-b_i\right)^2 f\left(s_j,\theta|s_i\right)\, d\theta$$

- $y(t,s_j)$ maximizes the expected utility of the receiver, i.e.

$$y\left(t,s_j\right) = \arg\max_{y\in\mathbb{R}} \int_0^1 -\left(y-\theta-b_j\right)^2 f\left(\theta|t,s_j\right)\, d\theta.$$

2.1 Truth-telling Equilibrium

As it is common in cheap talk games, multiple equilibria exist and, in particular, a babbling equilibrium in which no information is transmitted is always an equilibrium. We focus, however, on truth-telling equilibrium $t\left(s_i\right) = s_i$ because such equilibrium Pareto dominates all other equilibria.[1] In a truth-telling equilibrium the receiver correctly learns the signal s_i from the sender. Letting $y_{s_i,s_j} \equiv y(s_i,s_j)$ to simplify notation, the utility maximizing action of the receiver after observing her own signal s_j and being truthfully informed about signal s_i from the sender, is:

$$\begin{aligned} y_{s_i,s_j} &= \arg\max_{y\in\mathbb{R}} \int_0^1 -(y-\theta-b_j)^2 f(\theta|s_i,s_j)\, d\theta \\ &= b_j + \int_0^1 \theta f(\theta|s_i,s_j)\, d\theta \\ &= b_j + \mathrm{E}[\theta|s_i,s_j] \end{aligned} \tag{1}$$

where

$$f(\theta|s_i,s_j) = \frac{\Pr(s_i,s_j|\theta)}{\int_0^1 \Pr(s_i,s_j|\theta)d\theta}.$$

Simple algebra (see appendix) yields:

$$y_{0,0} = b_j + \frac{1+k}{2(2+k)}, \qquad y_{0,1} = b_j + \frac{1}{2} = y_{1,0}, \qquad y_{1,1} = b_j + \frac{3+k}{2(2+k)}. \tag{2}$$

Notice that only the actions based on identical signals depend on k. In fact, when the receiver receives from the sender the same signal she has observed, she believes that with probability k she has acquired information from the same source as the sender. To the contrary, when the receiver hears a different signal from the sender, she infers

[1] See, e.g., Galeotti, Ghiglino, and Squintani (2013).

that their information certainly comes from independent sources. Notice also that, as expected, $y_{0,0} < y_{0,1} < y_{1,1}$ for all $k \in (0,1)$.

We now study the sender's incentives to report truthfully the observed signal. the sender reports signal s_i instead of the false signal $1 - s_i$ if

$$\sum_{s_j \in \{0,1\}} \int_0^1 -\left(y_{s_i,s_j} - \theta - b_i\right)^2 f\left(s_j, \theta | s_i\right) \, d\theta \geq \tag{3}$$

$$\geq \sum_{s_j \in \{0,1\}} \int_0^1 -\left(y_{1-s_i,s_j} - \theta - b_i\right)^2 f\left(s_j, \theta | s_i\right) \, d\theta,$$

which, substituting $f\left(s_j, \theta | s_i\right) = f\left(\theta | s_i, s_j\right) \Pr\left(s_j | s_i\right)$ by Bayes rule and integrating, simplifies to

$$\sum_{s_j \in \{0,1\}} -\left(y_{s_i,s_j} - \mathrm{E}_f\left[\theta | s_i, s_j\right] - b_i\right)^2 \Pr\left(s_j | s_i\right) \geq$$

$$\geq \sum_{s_j \in \{0,1\}} -\left(y_{1-s_i,s_j} - \mathrm{E}_f\left[\theta | s_i, s_j\right] - b_i\right)^2 \Pr\left(s_j | s_i\right).$$

Since $y_{s_i,s_j} = b_j + \mathrm{E}_f\left[\theta | s_i, s_j\right]$ from (1), condition (3) further simplifies to:

$$\sum_{s_j \in \{0,1\}} \Pr\left(s_j | s_i\right) \frac{\left(y_{1-s_i,s_j} - y_{s_i,s_j}\right)^2}{2} \geq \left(b_i - b_j\right) \sum_{s_j \in \{0,1\}} \Pr\left(s_j | s_i\right) \left(y_{1-s_i,s_j} - y_{s_i,s_j}\right). \tag{4}$$

We now use $f\left(s_i, s_j, \theta\right) = \Pr\left(s_i, s_j | \theta\right)$ and $\Pr\left(s_i\right) = \frac{1}{2}$ and rewrite $\Pr\left(s_j | s_i\right)$ as:

$$\Pr\left(s_j | s_i\right) = \int_0^1 f\left(s_j, \theta | s_i\right) \, d\theta = \int_0^1 \frac{f\left(s_i, s_j, \theta\right)}{\Pr\left(s_i\right)} \, d\theta = 2 \int_0^1 \Pr\left(s_i, s_j | \theta\right) \, d\theta.$$

We then substitute y_{s_i,s_j} from (2) and $\Pr\left(s_j | s_i\right)$ using Table 1 in the truth telling condition (4). Whenever $s_i = 0$ truth-telling requires

$$b_i - b_j \leq \frac{1}{8 + 4k},$$

while, when $s_i = 1$, it requires

$$b_i - b_j \geq -\frac{1}{8 + 4k}.$$

The following proposition characterizes the truth-telling equilibrium.

Proposition 1. *Under* ***A1*** *and* ***A2*** *a truth-telling equilibrium exists if and only if*

$$d_{ij} \leq \frac{1}{8+4k} \tag{5}$$

where $d_{ij} = |b_i - b_j|$ *and* $k \in [0,1)$. *It follows that the maximal distance in preferences consistent with truthful information revelation decreases as the correlation between information sources increases.*

To understand Proposition 1 it is useful to refer to the so called *overshooting* effect — see Morgan and Stocken (2008) p.871 — which pins down threshold (5). The sender's incentive to lie comes from his desire to drag the receiver's action closer to his bliss point. However, when the displacement in the receiver's action caused by a lie is large compared to the distance in preferences, her action may end up being even further away from the sender's bliss point than in case of truth-telling. This concern prevents senders who are close in preferences to the receiver from lying.

Once the overshooting mechanism is clear, the intuition behind Proposition 1 is straightforward. The strength of the overshooting effect depends on how informative the signal is for the receiver. When k is large, the informative content of a message is low. Accordingly, its impact on the action will be lower on average, reducing the risk of overshooting. This, in turn, increases the incentives to tell a lie and makes truth-telling an optimal strategy only for senders with close preferences.[2]

3 More than Two Players

We now generalize the model to n players: a receiver j (she) and $n-1$ senders $i \neq j$. Players observe each a binary signal, $s_i, s_j \in \{0,1\}$, and their utility depends on the action $y \in \mathbb{R}$ taken by the receiver, on the state of the world θ and on the individual preference parameter. The game goes as follows: first players observe signals, then each sender independently reports a message $t_i \in \{0,1\}$ to the receiver, who then takes

[2] Finally, notice that when $k = 0$, we have $d_{ij} \leq \frac{1}{8}$ corresponding to Corollary 1 in Galeotti, Ghiglino, and Squintani (2013). Moreover, the general principle that more informative signals sustain truth-telling equilibrium for larger distance in preferences is behind other results in the literature such as Morgan and Stocken (2008) and, more recently, Hagenbach and Koessler (2010) and Galeotti, Ghiglino, and Squintani (2013) — all featuring conditionally independent signals.

action y. Utilities are loss quadratic: $U^j(y,\theta,b_j) = -(y-\theta-b_j)^2$ for the receiver and $U^i(y,\theta,b_i) = -(y-\theta-b_i)^2$ for sender i.

The information structure is such that the world is again known to be uniformly distributed, $\theta \sim U(0,1)$. As to the information sources and the signals acquisition process we retain assumption **A2** and we change assumption **A1** to:

A3 There are (at least) n sources of information, each generating an informative and independent signal according to the binomial distribution $\Pr(s=1|\theta)=\theta$.

Assumptions **A1** and **A3** allow us to generate a multinomial distribution of correlated signals with a single correlation parameter. We now illustrate its basic properties.

The joint probability distribution of signals conditional on the state is now

$$\begin{cases} \Pr(\mathbf{1}_n|\theta) = \theta^n(1-k)+\theta k \\ \Pr(\mathbf{0}_n|\theta) = (1-\theta)^n(1-k)+(1-\theta)k \\ \Pr(\mathbf{1}_l,\mathbf{0}_{n-l}|\theta) = \theta^l(1-\theta)^{n-l}(1-k) \quad \text{if} \quad 0<l<n \end{cases}$$

where $\mathbf{1}_x$ (resp. $\mathbf{0}_x$) is a vector of ones (resp. zeros) of dimension x and $\mathbf{1}_x,\mathbf{0}_y$ is a vector of $x+y$ signals containing x ones and y zeros in any ordering. Notice that, by the information acquisition process we have introduced, a mixed sequence of signals can only realize when signals come from independent sources, which happens with probability $1-k$. To the contrary, a full string of zeros or ones may be the result of a series of independent draws as well as the outcome of players observing the same source. This is why, in the last raw, the k-weighted term does not appear, because the probability of a mixed sequence of signals coming from the same source is zero.

Given the integers a, b, c and d with $a+b+c+d \leq n$, define the conditional probability $\Pr(\mathbf{1}_a,\mathbf{0}_b|\mathbf{1}_c,\mathbf{0}_d,\theta)$ as the probability that a string of a one-signals and b zero-signals is observed given a string of c one-signals and d zero-signals has already been observed and the state is θ. Such probability is

$$\Pr(\mathbf{1}_a,\mathbf{0}_b|\mathbf{1}_c,\mathbf{0}_d,\theta) = \frac{\Pr(\mathbf{1}_{a+c},\mathbf{0}_{b+d},\theta)}{\Pr(\mathbf{1}_c,\mathbf{0}_d,\theta)} = \begin{cases} \theta^a(1-\theta)^b & \text{if} \quad c\neq 0 \wedge d\neq 0 \\ \frac{\Pr(\mathbf{1}_{a+c},\mathbf{0}_b|\theta)}{\Pr(\mathbf{1}_c|\theta)} & \text{if} \quad c\neq 0 \wedge d=0 \\ \frac{\Pr(\mathbf{1}_a,\mathbf{0}_{b+d}|\theta)}{\Pr(\mathbf{0}_d|\theta)} & \text{if} \quad c=0 \wedge d\neq 0 \end{cases} \tag{6}$$

The first line of (6) has a straight interpretation: once (at least) two different signals have been observed ($c \neq 0$ and $d \neq 0$) it is immediately inferred that the information sources are independent, hence the simple binomial expression. To the contrary, when either $c = 0$ or $d = 0$, the conditional probability depends on the marginals as reported in lines two and three of (6). The following Lemma provides an expression for the marginal probability of any subset of signals given the state θ. It also shows that the probability structure of a generic subset of signals is "regular" in that it follows the behavior of a standard binomial probability distribution.

Lemma. *Assume **A1** and **A3** and consider a $(n-l)$-tuple of signals, $0 < l < n$. Its joint probability distribution is*

$$\begin{cases} \Pr\left(\mathbf{1}_{n-l}|\theta\right) = \theta^{n-l}\left(1-k\right) + \theta k \\ \Pr\left(\mathbf{0}_{n-l}|\theta\right) = \left(1-\theta\right)^{n-l}\left(1-k\right) + \left(1-\theta\right)k \\ \Pr\left(\mathbf{1}_j, \mathbf{0}_{n-l-j}|\theta\right) = \theta^j(1-\theta)^{n-l-j}\left(1-k\right) \quad \textit{if} \quad 0 < l < n \end{cases}$$

where $\Pr\left(\mathbf{1}_j, \mathbf{0}_{n-l-j}|\theta\right)$ is the probability of an unsorted sequence of signals containing j ones, $n-l-j$ zeros and l other one or zero signals.

A natural question arises: what does the parameter k represent in the model with n players? It turns out that, as in the two players case, k denotes the Pearson's correlation coefficient of the joint probability distribution of any two signals. To see this, apply the Lemma to any couple of signals: their probability distribution is exactly as in Table 1. Thus, the interpretation of k is unaltered in the model with n-players.

3.1 Truth-telling Equilibrium

A Perfect Bayesian Nash Equilibrium (PBNE) of this game is defined in the same way as for the case of two players except that the senders and the receiver now take into account the messages of other senders when maximizing their utility functions.

Let $\mathbf{t}$ denote the vector of all messages sent by senders, and $\mathbf{t}_{-i}$ denote the same vector without the *i-th* component. A PBNE of this game is defined by a strategy $t_i(s_i)$ for each sender i, and a strategy $y(\mathbf{t}, s_j)$ for the receiver, such that:

- $t_i(s_i)$ maximizes the expected utility for all i:

$$t_i(s_i) = \max_{t_i \in \{0,1\}} \sum_{(\mathbf{t}_{-i}, s_j) \in \{0,1\}^{n-1}} \int_0^1 -(y(\mathbf{t}, s_j) - \theta - b)^2 f(\mathbf{t}_{-i}, s_j, \theta | s_i) \, d\theta.$$

- $y(\mathbf{t}, s_j)$ maximizes the expected utility of the receiver, i.e.

$$y(\mathbf{t}, s_j) = \max_{y \in \mathbb{R}} \int_0^1 -(y - \theta)^2 f(\theta | \mathbf{t}, s_j) \, d\theta$$

Letting $\mathbf{s}$ be the vector of signals, we focus on truth-telling equilibrium, where $(\mathbf{t}, s_j) = \mathbf{s}$. Let also $y(\mathbf{s}) \equiv y_{\mathbf{s}}$ be the utility maximizing action of the receiver after observing the vector of signals $\mathbf{s}_{-j}$, defined as follows

$$y_{\mathbf{s}} = \max_{y \in \mathbb{R}} \int_0^1 -(y - \theta)^2 f(\theta | \mathbf{s}) \, d\theta$$
$$\Rightarrow y_{\mathbf{s}} = \mathrm{E}[\theta | \mathbf{s}] = \int_0^1 \theta f(\theta | \mathbf{s}) \, d\theta$$

where $f(\theta | \mathbf{s}) = \frac{f(\mathbf{s}|\theta)}{\int_0^1 f(\mathbf{s}|\theta) \, d\theta}$. It can be shown that the optimal action takes the following values depending on $\mathbf{s}$:

$$y_{\mathbf{0}_n} = \frac{6 + k(n-1)(n+4)}{3(n+2)(2-k+kn)}, \quad y_{\mathbf{1}_l, \mathbf{0}_{n-l}} = \frac{1+l}{n+2}, \quad y_{\mathbf{1}_n} = \frac{2(n+1)(3-k+kn)}{3(n+2)(2-k+kn)}. \tag{7}$$

Note that, while $y_{\mathbf{1}_l, \mathbf{0}_{n-l}}$ does not depend on k as expected, $y_{\mathbf{1}_n}$ decreases and $y_{\mathbf{0}_n}$ increases when k grows larger. In other words, as the correlation increases — reducing each signal's informative content — the distance between the optimal actions following two strings of one and zero signals shrinks.

In a truth-telling equilibrium, the incentive compatibility constraint for player i is the following:

$$\int_0^1 - \sum_{\mathbf{s}_{-i} \in \{0,1\}^{n-1}} (y_{s_i, \mathbf{s}_{-i}} - \theta - b_i)^2 f(\mathbf{s}_{-i}, \theta | s_i) \, d\theta$$
$$\geq \int_0^1 - \sum_{\mathbf{s}_{-i} \in \{0,1\}^{n-1}} (y_{1-s_i, \mathbf{s}_{-i}} - \theta - b_i)^2 f(\mathbf{s}_{-i}, \theta | s_i) \, d\theta.$$

Expanding squares and rearranging terms as we did in equation (4), the above constraint simplifies to:

$$\sum_{\mathbf{s}_{-i} \in \{0,1\}^{n-1}} \Pr(\mathbf{s}_{-i} | s_i) \frac{\Delta^2(\mathbf{s}_{-i} | s_i)}{2} \geq (b_i - b_j) \sum_{\mathbf{s}_{-i} \in \{0,1\}^{n-1}} \Pr(\mathbf{s}_{-i} | s_i) \Delta(\mathbf{s}_{-i} | s_i)$$

where $\Delta(\mathbf{s}_{-i}|s_i) \equiv y_{1-s_i,\mathbf{s}_{-i}} - y_{s_i,\mathbf{s}_{-i}}$ measures the displacement of the optimal receiver's action following a lie.

The following proposition characterizes the truth-telling equilibrium for the case with n players.

Proposition 2. *Assume* ***A1*** *and* ***A3*** *and let* $k \in [0,1)$.

1. *If k is such that $\sum_{\mathbf{s}_{-i}\in\{0,1\}^{n-1}} \Pr(\mathbf{s}_{-i}|s_i)\Delta(\mathbf{s}_{-i}|s_i) \neq 0$, then a truth-telling equilibrium exists if and only if $d_{ij} \leq d^*_{i,j}(n,k)$, where:*

$$d^*_{i,j}(n,k) \equiv \frac{1}{2}\left|\frac{\sum_{\mathbf{s}_{-i}\in\{0,1\}^{n-1}} \Pr(\mathbf{s}_{-i}|s_i)\Delta^2(\mathbf{s}_{-i}|s_i)}{\sum_{\mathbf{s}_{-i}\in\{0,1\}^{n-1}} \Pr(\mathbf{s}_{-i}|s_i)\Delta(\mathbf{s}_{-i}|s_i)}\right|.$$

2. *If k is such that $\sum_{\mathbf{s}_{-i}\in\{0,1\}^{n-1}} \Pr(\mathbf{s}_{-i}|s_i)\Delta(\mathbf{s}_{-i}|s_i) = 0$, then a truth-telling equilibrium exists for any distance in preferences $d_{i,j}$.*

Proposition 2 extends and qualifies the result of Proposition 1 to the case of more than two players. The presence of more than one seller brings new and qualitatively different features in the truth-telling conditions. There exists a specific value of k such that truth-telling is consistent with arbitrarily distant preferences. We will discuss this in full detail in the next section, where we also analyze the role of n and its interplay with k.

3.2 The Disciplining Effect of Correlation

Figure 1 presents a plot of the thresholds that define the maximum distance in preferences allowing for truth-telling, for various levels of n. As we can see, for more than three players the maximal distance d^*_{ij} is a non monotonic function of the correlation index k, decreasing for low values of k, and then increasing up to a critical value $\bar{k}$ at which any distance is consistent with truth-telling (point 2 of proposition 2). Moreover, the critical correlation level $\bar{k}$ is lower the larger the number of senders. These graphical insights are proved in the next proposition for any arbitrary number of sellers.

Proposition 3. *For $n \leq 4$, d^*_{ij} is bounded for all $k \in [0,1)$. For $n \geq 5$, d^*_{ij} is unbounded if and only if $k = \bar{k}$, where*

$$\bar{k} = \frac{1}{2(1+n)}\left(1 + \sqrt{1 + 24\frac{n(1+n)}{(n-2)(n-1)}}\right)$$

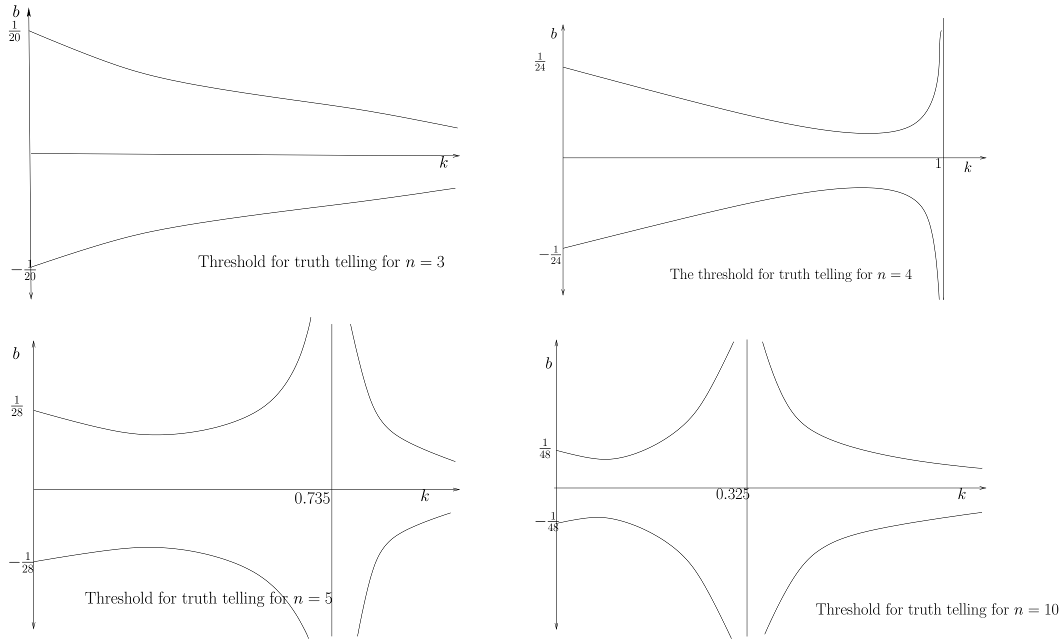

Figure 1: Threshold bands $d_{i,j} \leq \left|d^*_{i,j}\right|$ for $n = 3, 4, 5, 10$.

is a decreasing function of n *such that* $\lim_{n\to\infty} \bar{k} = 0$. *Finally,* d^*_{ij} *is strictly decreasing in* k *at* $k = 0$.

To understand the forces behind the non monotonicity of d^*_{ij}, we need to consider the details of the random process that governs the observation of signals and the reaction of the receiver to the available information.

Consider sender i pondering whether to report truthfully his signal, taking as given all the other signals learnt by the receiver in a truth-telling equilibrium. When anticipating the expected equilibrium action, sender i finds himself in one of two situations about the signals $\mathbf{s}_{-i}$ reported by the other senders: either $\mathbf{s}_{-i}$ contains at least two different signals, in which case the receiver infers that signals come from an independent distribution, or $\mathbf{s}_{-i}$ contains $n-1$ identical signals, in which case the receiver's belief (after learning all senders' reports) crucially depend on sender i's choice. In particular, if sender i's report is in line with that of the other senders, the receiver will believe that signals are independent with probability $1-k$ and perfectly correlated with probability k; if sender i's report contradicts the other senders', the receiver will believe that all signals come from an independent process.

Sender i's incentives to tell the truth depend on the extent and direction of the displacement in the receiver's action caused by i's misreport. This displacement is in turn affected by the update that the receiver makes on the signal generating process. To fix ideas, consider the case in which sender i observes $s_i = 0$ and $b_i > b_j$.[3] Suppose all other senders' have reported $t = 0$, and that also sender i has observed $s_i = 0$. A misreport by sender i has two effects. First, it provides one additional bit of information to the receiver - a direct effect which tends to displace the action to the right. Second, it modifies the (perceived) informational content of the 0 signals of the other senders through the updating process described above - an indirect effect. As a result, the receiver's action may move in either direction, depending on the relative strength of each effect. The larger k, the stronger the indirect effect (due to a stronger revision of the sender's beliefs), and the weaker the direct effect, due to the lower informative content of the extra signal (as explained in Proposition 1). This trade off is novel, and directly stems from the presence of correlation in the signals' generating process.

The interplay of direct and indirect effects, together with the fact that the probability of identical reports from the other senders increases with k, accounts for the U-shaped pattern of the threshold d^*_{ij} in Figure 1. The threshold for truth-telling is in fact defined in (1) as (half) the ratio of two terms: the expected displacement in case of lying (the denominator) and the expected squared displacement (the numerator). As k increases, the denominator decreases as the indirect (negative) effect counteracts the direct (positive) one, and tends to zero as k approaches $\bar{k}$ from the left. The numerator is, instead, bounded away from zero being a sum of squares, in which some elements (those relative to the case in which other senders have reported contradictory signals) are constant in k and strictly positive. As a result, as we approach $\bar{k}$ from the left, the value of the threshold increases, and tends asymptotically to infinite in the limit.

To sum up, adding senders has qualitative implications for our understanding of information transmission. While in a model with two informed players correlation between information sources always weakens the incentives to truthfully reveal private information (and this is due to the reduced informational content of messages as correlation increases), in a model with three or more players, correlation has additional and non monotonic effects on truthful information transmission. Non homogeneous strings of reports from the senders are interpreted (in equilibrium) as evidence of independent

[3] A symmetric analysis applies when $s_i = 1$.

sources. When reporting a signal not in line with the other reports, each sender takes this into account and anticipates the possibly averse effect on the receiver's action, facing, as a result, weaker incentives to misreport. In this sense, correlation *disciplines* the sender's reporting behavior by increasing the relative profitability of consistent reports. Due to the mechanics discussed above, this discipline becomes stronger as k nears $\bar{k}$.

4 Choosing Who to Audit

In this final section we wish to discuss some implications of our analysis for how an informed receiver would select a sender to audit from a given population. The distinctive feature of this section is the assumption that the amount of (conditional) correlation between the sender's and the receiver's signals is negatively related to their distance in preferences. We use this assumption as a reduced form of a model where players with closer preferences are more likely to refer to the same source. This assumption seems to apply well to political discussion, were people with similar ideology generally read the same newspapers, refer to the same media and talk to a common pool of friends to acquire information.

Formally, we assume that the distribution of preferences is associated with the distribution of signal correlations according to the linear function $b(k) = B - ak$, where both $B, a > 0$. The slope a measures how much additional distance in preferences is needed to generate sensitive given decrease in correlation. We interpret this parameter as an index of *polarization* of society: a large a indicates that very distant preferences do not imply a very different correlation of information. This would be the case if, for instance, all agents acquired information from similar sources, regardless of their preferences — a case of weak polarization. The position of the line $b(k)$, given by the intercept B, measures the general level of correlation of information: a parallel shift to the right captures an increase of correlation for each possible distance in preferences. We interpret B as an index of correlation in the information disclosed by primary sources (e.g., newspapers) to the senders.

We address the simple problem of a receiver that chooses a sender to audit within the set of feasible alternatives, given the exogenous constraint $b(k) = B - ak$. One important issue that we can address in this framework is the distance in preference (and therefore the level of correlation) that characterizes information transmission. The inverse of this distance can be thought of as an index of "homophily" in information transmission, being the outcome of the voluntary choice of who to audit.

In addressing this problem, we restrict to choices that allow for a truth telling equilibrium, that is, for distance in preferences that satisfy the condition in Proposition 1. Among such equilibria, those with lowest correlation level k are preferred by the receiver, as they bear higher informational content. We conclude that for $B = B_1, B_2, B_3$ in Figure 2, the receiver would chose to obtain information from a sender whose preferences lie at the intercept B with the y-axis, with $k = 0$.

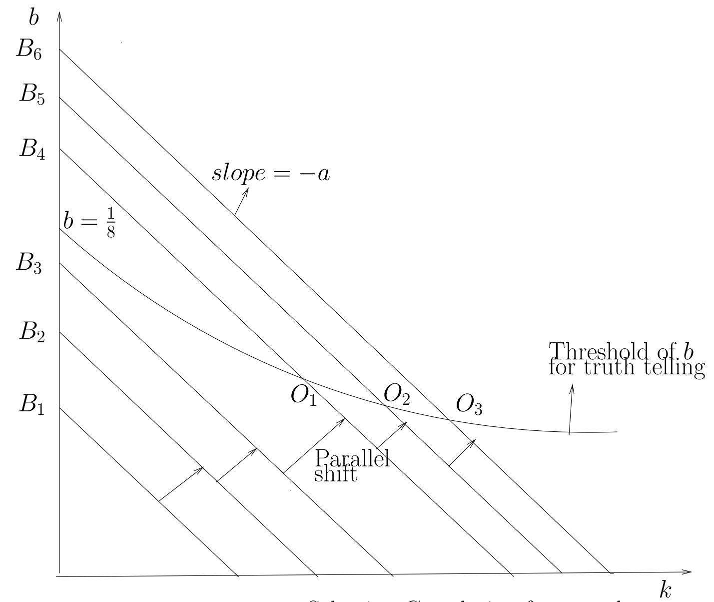

Figure 2: The effect of change in B fixing a with $B, a > 0$

In contrast, for $B = B_4$, the lowest available correlation among senders is the level corresponding to the point O_1, at the intercept of the linear constraint and the truth-telling equilibrium threshold.

Note how the effect on homophily of an increase in the general correlation parameter B crucially depends on the current level of correlation. At low levels of B, we observe that the optimal choice of the receiver displays lower and lower degrees of homophily as correlation increases, while the transmitted information is totally uncorrelated (points B_1, B_2, B_3). However, at higher levels of B, the equilibrium is found on the intercept of the two *loci*, and further increases in B shift the equilibrium to the right along the truth-telling threshold, progressively increasing the degree of homophily in information transmission. At the same time, also the degree of homophily in information transmission tends to increase. These insights are summarized in the following proposition.

Proposition 4. *An increase in the correlation of information sources (B) has the effect of decreasing homophily (with no effect on the receiver's welfare) when B is small, and to increase homophily (with a decrease in the receiver's welfare) when B is large.*

5 Conclusion

We have studied the role of conditional correlation of private information for truth-telling in a cheap talk game. We have found that in a model with only one sender, correlation of the sender's and the receiver's signals shrinks the interval of biases that support truth telling as an equilibrium. With more than one sender, correlation has non monotonic effects on truth-telling, and around a specific value of the correlation parameter truth-telling is an equilibrium regardless of the distance in preferences. A wide range of applications of the cheap talk game have correlation as a natural ingredient, including auditing of experts, political discussion, cross questioning in criminal investigations, etc. In such context, it would be possible to explicitly model the origins of correlation as a result of either scarcity of information sources or of the directed information acquisition by senders. We leave this and other developments for future research.

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Appendix

Proof that k is the Pearson correlation coefficient for the distribution of Table 1. Notice first that:

$$\mathrm{E}\left(s_i\right) = 0 \cdot \Pr\left(0|\theta\right) + 1 \cdot \Pr\left(1|\theta\right) = \theta$$

and

$$\begin{aligned}
\sigma_{s_i} &= \sqrt{\mathrm{E}\left[\left(s_i - \mathrm{E}\left(s_i\right)\right)^2\right]} \\
&= \sqrt{\left(0 - \mathrm{E}(s_i)\right)^2 \Pr\left(0|\theta\right) + \left(1 - \mathrm{E}(s_i)\right)^2 \Pr\left(1|\theta\right)} \\
&= \sqrt{\theta\left(1-\theta\right)}
\end{aligned}$$

implying $\mathrm{E}(s_i) = \mathrm{E}(s_j)$ and $\sigma_{s_i} = \sigma_{s_j}$. The covariance between signals is:

$$\begin{aligned}
Cov\left(s_i, s_j\right) &= \mathrm{E}\left[\left(s_i - \mathrm{E}\left(s_i\right)\right)\left(s_j - \mathrm{E}\left(s_j\right)\right)\right] \\
&= \mathrm{E}\left[\left(s_i - \theta\right)\left(s_j - \theta\right)\right] \\
&= \sum_{(s_i, s_j) \in \{0,1\}^2} \Pr\left(s_i, s_j|\theta\right)\left(s_i - \mathrm{E}\left(s_i\right)\right)\left(s_j - \mathrm{E}\left(s_j\right)\right) \\
&= \theta\left(1-\theta\right)k.
\end{aligned}$$

It follows that

$$\frac{Cov\left(s_i, s_j\right)}{\sigma_{s_i}\sigma_{s_j}} = k.$$

Derivation of $f\left(\theta|s_i, s_j\right)$ and y_{s_i,s_j}. Applying Bayes rule and noticing that $f\left(\theta\right) = 1$ because $\theta \sim U\left(0,1\right)$, the general expression for the conditional distribution of θ given a pair of signals $\left(s_i, s_j\right)$ is:

$$\begin{aligned}
f\left(\theta|s_i, s_j\right) &= \frac{\Pr\left(s_i, s_j|\theta\right) f\left(\theta\right)}{\int_0^1 \Pr\left(s_i, s_j|\theta\right) f\left(\theta\right) d\theta}, \\
&= \frac{\Pr\left(s_i, s_j|\theta\right)}{\int_0^1 \Pr\left(s_i, s_j|\theta\right) d\theta}.
\end{aligned}$$

Using Table 1 we calculate the conditional density functions given all possible pairs of signals:

$$\begin{aligned}
f\left(\theta|s_i = 0, s_j = 0\right) &= \frac{6}{2+k}\left[\left(1-\theta\right)^2 + \theta\left(1-\theta\right)k\right], \\
f\left(\theta|s_i = 0, s_j = 1\right) &= 6\theta(1-\theta) = f\left(\theta|s_i = 1, s_j = 0\right), \\
f\left(\theta|s_i = 1, s_j = 1\right) &= \frac{6}{2+k}\left[\theta^2 + \theta\left(1-\theta\right)k\right].
\end{aligned} \tag{8}$$

The optimal action of the receiver when she observes the pair of signals (s_i, s_j) is:

$$\begin{aligned} y_{s_i,s_j} &= b_j + \mathrm{E}\left[\theta|s_i, s_j\right], \\ &= b_j + \int_0^1 \theta f\left(\theta|s_i, s_j\right) d\theta, \end{aligned}$$

which, using (8), yields:

$$y_{0,0} = b_j + \frac{1+k}{2(2+k)}, \qquad y_{0,1} = b_j + \frac{1}{2} = y_{1,0}, \qquad y_{1,1} = b_j + \frac{3+k}{2(2+k)}.$$

Proof of Lemma (3). Using the notation introduced in Section 3 and letting $\mathbf{s}_l$ be a generic l-dimensional vector of zero or one signals, we can write:

$$\begin{aligned} \Pr\left(\mathbf{0}_{n-l}|\theta\right) &= \sum_{\mathbf{s}_l \in \{0,1\}^l} \Pr\left(\mathbf{0}_{n-l}, \mathbf{s}_l|\theta\right), \\ &= \Pr\left(\mathbf{0}_n\right) + \sum_{j=0}^{l-1} \Pr\left(\mathbf{0}_{n-l}, \left(\mathbf{0}_j, \mathbf{1}_{l-j}\right)\right), \\ &= \underbrace{\left(1-\theta\right)k + \left(1-\theta\right)^n\left(1-k\right)}_{\Pr\left(\mathbf{0}_n\right)} + \\ &\qquad + \underbrace{\left(1-\theta\right)^{n-l}\left(1-k\right)\sum_{j=0}^{l-1}\frac{l!}{(l-j)!j!}\left(1-\theta\right)^j\theta^{l-j}}_{\sum_{j=0}^{l-1}\Pr\left(\mathbf{0}_{n-l},\left(\mathbf{0}_j,\mathbf{1}_{l-j}\right)\right)}, \\ &= \left(1-\theta\right)k + \left(1-k\right)\left(1-\theta\right)^{n-l}. \end{aligned}$$

In a similar fashion it is easy to show that $\Pr\left(\mathbf{1}_{n-l}|\theta\right) = \theta k + \left(1-k\right)\theta^{n-l}$. As to the conditional probability of a string of signals containing both ones and zeros, let $\left(\mathbf{0}_{n-l-q}, \mathbf{1}_q, \mathbf{s}_{n-l}\right)$ be an unsorted vector containing $n-l-q$ zeros and q ones where $n > l+q > q > 0$. Then,

$$\begin{aligned} \Pr(\mathbf{0}_{n-l-q}, \mathbf{1}_q|\theta) &= \sum_{\mathbf{s}_l \in \{0,1\}^l} \Pr\left(\mathbf{0}_{n-l-q}, \mathbf{1}_q, \mathbf{s}_l|\theta\right), \\ &= \left(1-\theta\right)^{n-l-q}\theta^q\left(1-k\right)\sum_{j=0}^{l}\frac{l!}{(l-j)!j!}\left(1-\theta\right)^j\theta^{l-j}, \\ &= \left(1-k\right)\left(1-\theta\right)^{n-l-q}\theta^q. \end{aligned}$$

Proof of Proposition (2). We proceed as we did in the text for the model with two players, the only difference being that expectations are taken over a vector of $n-1$

signals rather than on a single one. Thus, given $\mathbf{s}_{-i}$ in a truth-telling equilibrium, sender i reports his signal truthfully if:

$$\sum_{\mathbf{s}_{-i}\in\{0,1\}^{n-1}} \Pr\left(\mathbf{s}_{-i}|s_i\right)\Delta^2\left(\mathbf{s}_{-i}|s_i\right) \geq 2\left(b_i - b_j\right) \sum_{\mathbf{s}_{-i}\in\{0,1\}^{n-1}} \Pr\left(\mathbf{s}_{-i}|s_i\right)\Delta\left(\mathbf{s}_{-i}|s_i\right) \tag{9}$$

where $\Delta(\mathbf{s}_{-i}|s_i) = y_{1-s_i,\mathbf{s}_{-i}} - y_{s_i,\mathbf{s}_{-i}}$ (corresponding to relation (4) in the two players model).

Let us first focus on the Δ-terms and notice that:

$$\Delta\left(\mathbf{s}_{-i}|s_i\right) = -\Delta\left(\mathbf{s}_{-i}|1-s_i\right). \tag{10}$$

Using (7) and (10), the Δ-terms of (9) are

$$-\Delta\left(\mathbf{0}_{n-1}|0\right) = \frac{-6+k\left(2-3n+n^2\right)}{3\left(2+k\left(n-1\right)\right)\left(2+n\right)} = \Delta\left(\mathbf{1}_{n-1}|1\right), \tag{11}$$

$$-\Delta\left(\mathbf{0}_{n-1-l},\mathbf{1}_l|0\right) = -\frac{1}{2+n} = \Delta\left(\mathbf{0}_{n-1-l},\mathbf{1}_l|1\right) \qquad \left(l\neq 0, n-1\right), \tag{12}$$

which we shall write as:

$$\Delta_1\left(s_i\right) = \Delta\left(\mathbf{0}_{n-1}|s_i\right) = \Delta\left(\mathbf{1}_{n-1}|s_i\right),$$
$$\Delta_2\left(s_i\right) = \Delta\left(\mathbf{0}_{n-1-l},\mathbf{1}_l|s_i\right) \qquad \left(l\neq 0, n-1\right).$$

Turning to the probability terms in (9), we shall notice that, given $\Pr(s_i) = \frac{1}{2}$, the conditionals can be rewritten as

$$\Pr\left(\mathbf{s}_{-i}|s_i\right) = \frac{\Pr\left(\mathbf{s}_{-i},s_i\right)}{\Pr\left(s_i\right)} = 2\Pr\left(\mathbf{s}_{-i},s_i\right), \tag{13}$$

and unconditional probabilities are:

$$\Pr\left(\mathbf{0}_n\right) = \frac{2+k\left(n-1\right)}{2\left(n+1\right)} = \Pr\left(\mathbf{1}_n\right), \tag{14}$$

$$\Pr\left(\mathbf{0}_{n-l},\mathbf{1}_l\right) = \frac{\left(1-k\right)\left(n-l\right)!\,l!}{\left(n+1\right)!} = \Pr\left(\mathbf{0}_l,\mathbf{1}_{n-l}\right) \qquad \left(l\neq 0, n-1\right). \tag{15}$$

Notice further that

$$\begin{aligned}\sum_{l=1}^{l=n-2}\Pr\left(\mathbf{0}_{n-l-1},\mathbf{1}_l|s_i\right) &= 1-\Pr\left(\mathbf{0}_{n-1}|s_i\right)-\Pr\left(\mathbf{1}_{n-1}|s_i\right),\\ &= 1-2\Pr\left(\mathbf{0}_{n-1},s_i\right)-2\Pr\left(\mathbf{1}_{n-1},s_i\right),\end{aligned}$$

where the latter follows from (13). Now define

$$P^* \equiv \Pr\left(\mathbf{0}_{n-1}, s_i\right) + \Pr\left(\mathbf{1}_{n-1}, s_i\right),$$

and notice that it can be easily shown that P^* is equal across $s_i = 0, 1$. Summing up, we can rewrite the RHS and LHS sums of equation (9) respectively as:

$$\sum_{\mathbf{s}_{-i}\in\{0,1\}^{n-1}} \Delta\left(\mathbf{s}_{-i}|s_i\right)\Pr\left(\mathbf{s}_{-i}|s_i\right) = 2P^*\Delta_1\left(s_i\right) + \left(1-2P^*\right)\Delta_2\left(s_i\right), \tag{16}$$

$$\sum_{\mathbf{s}_{-i}\in\{0,1\}^{n-1}} \Delta^2\left(\mathbf{s}_{-i}|s_i\right)\Pr\left(\mathbf{s}_{-i}|s_i\right) = 2P^*\Delta_1^2\left(s_i\right) + \left(1-2P^*\right)\Delta_2^2\left(s_i\right). \tag{17}$$

While (17) is clearly positive, (16) may be either positive, negative or equal to zero for $s_i \in \{0, 1\}$. From (9) define now:

$$T_{i,j}\left(n,k\right) \equiv \frac{1}{2}\frac{\sum_{\mathbf{s}_{-i}\in\{0,1\}^{n-1}}\Pr(\mathbf{s}_{-i}|s_i)\Delta^2(\mathbf{s}_{-i}|s_i)}{\sum_{\mathbf{s}_{-i}\in\{0,1\}^{n-1}}\Pr(\mathbf{s}_{-i}|s_i)\Delta(\mathbf{s}_{-i}|s_i)}, \tag{18}$$

(notice that $d^*_{i,j}\left(n,k\right) = \left|T_{i,j}\left(n,k\right)\right|$).

Suppose the sender observes $s_i = 0$: if $T_{i,j}\left(n,k\right) \geq 0$ truth-telling requires $b_i - b_j \leq T_{i,j}\left(n,k\right)$; if $T_{i,j}\left(n,k\right) < 0$ then it must be $b_i - b_j > T_{i,j}\left(n,k\right)$. Suppose instead the sender observes $s_i = 1$: then if $T_{i,j}\left(n,k\right) \geq 0$ truth-telling requires $b_i - b_j \geq -T_{i,j}\left(n,k\right)$; if $T_{i,j}\left(n,k\right) < 0$ then it must be $b_i - b_j < -T_{i,j}\left(n,k\right)$. Recalling $d_{ij} = |b_i - b_j|$, it is easy to see that the observations just made, taken altogether, imply point a of the Proposition. As to point b, it is clear that, whenever the denominator of $T_{i,j}\left(n,k\right)$ is equal to zero, condition (9) is always satisfied, no matter how large d_{ij} is.

Proof of Proposition (3). Using (11), (12), (14) and (15) it is just a matter of algebra to show that

$$T_{i,j}\left(n,k\right) = \frac{36n+k(n-2)(n-1)(12+k(-17+k(n-5)(n-1)(n+1)+n(2n-9)))}{12(2+k(n-1))(2+n)(6n-k(n-2)(n-1)(k(n+1)-1))}.$$

Further algebra shows that the denominator equal to zero when $k = \bar{k}$ with

$$\bar{k} \equiv \frac{1}{2\left(1+n\right)}\left(1+\sqrt{1+24\frac{n(1+n)}{(n-2)(n-1)}}\right).$$

It is easy to show that the term $\frac{n(1+n)}{(n-2)(n-1)}$ inside the square root is decreasing in n so that $\bar{k}$ is clearly decreasing in n as well. The main claim of the Proposition follows from the observation that $\bar{k} < 1$ for $n \geq 5$, while $\bar{k} \geq 1$ for $n \leq 4$. It is also easy to check that $\lim_{n \to \infty} \bar{k} = 0$.

Finally, noticing that $\bar{k} > 0$ for any finite n and that it holds $d_{ij}^* (n, k) = T_{i,j} (n, k)$ for $k < \bar{k}$, tedious algebra yields:

$$\frac{\partial d_{ij}^* (n, k)}{\partial k} = \left(n^2 - 1\right) \cdot$$
$$\cdot \frac{k(n-2)\left(k^3(n-2)(n-1)^2(n(n-7)-2)+8k^2(n-1)^2(2(n-4)n-1)+4k(n-1)(13(n-3)n-1)+48(n-4)n\right)-144n}{12(n+2)(k(n-1)+2)^2(k(n-2)(n-1)(k(1+n)-1)-6n)^2},$$

and

$$\lim_{k \to 0} \frac{\partial d_{ij}^* (n, k)}{\partial k} = -\frac{n^2 - 1}{12n \, (n + 2)} < 0,$$

which proves the last claim of the Proposition.

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