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DISCOVERING SIFIs IN INTERBANK COMMUNITIES.

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Abstract

This paper proposes a new methodology based on non-negative matrix factorization to detect communities and to identify Systemically Important Financial Institutions in the interbank network as well as within communities.

The method is specifically designed for directed weighted networks and it is able to take into account exposures on both sides of banks' balance sheets, distinguishing between Systemically Important Borrowers and Lenders.

Using interbank transactions data from the e-Mid platform, we show that the systemic importance associated with Italian banks decreased during the 2007-2009 financial crisis while the opposite happened for foreign institutions.

We also show that, as the transactions volume grew, the number of communities rose as well. The contrary happened during the crisis phase.

Moreover results indicate that, during financial crisis, banks strongly operate into non overlapping communities with few institutions playing the role of SIFIs. On the contrary during *business as usual* times banks act in several and overlapping modules.

JEL: D8, L14, C02

Keywords: Financial networks, community detection, systemic risk

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1 Introduction

The 2007-2009 financial crisis has stressed the need of looking at the banking system as a network of economic institutions whose financial linkages play a fundamental role. In this respect, macroprudential policies aim at measuring and monitoring the risks arising from Systemically Important Financial Institutions (SIFIs). Determining the importance of individual financial institutions within an interconnected network is key to design policies that try to prevent and mitigate financial contagion. The Basel III framework [1] proposed an indicator-based measure to identify SIFIs. This indicator is composed of five categories that should reflect the systemic importance of individual intermediaries.¹ One of the categories included in the measure, *interconnectedness*, aims at capturing the impact that an institution's bilateral exposures have on the other institutions within the banking system. Interconnectedness is thus related to the detection of the most important player in a network. Not surprisingly, the research in network theory has dedicated a vast amount of effort to deal with this topic (see [2], [3], [4], [5], [6] among others).

Moreover, several studies analyzed the empirical characteristics of interbank networks in different jurisdictions finding the existence of a community structure in such markets (see [7], [8], [9], [10], [11], [12] and [13] among others). This property indicates the presence of sets of banks usually defined as very dense subgraphs, with few connections between them, as a result of preferential lending relationships on the micro-level (see [10], [14] among others).

In general, centrality measures rank vertices according to their importance without paying attention to whether the network is characterized by a community structure. However the identification of the modularity structure is important in financial networks to detect the most plausible areas of contagion of institutions' possible defaults. Indeed, a bank's default will not affect homogeneously all the other components of the system but, in primis, players belonging to the same community of the defaulted intermediary.

On the other hand, not all banks are equal in a community, and some institutions might be special in the sense that they are linked with almost all others. Thus, developing a ranking measure able to capture the risk that individual institutions place in the communities they operate, is crucial to enhance the understanding of the interbank market and the lending relationships between banks.

Despite the fact that centrality and community detection have been widely studied as independent phenomena from one another, to the best of our knowledge no unifying view of the two problems exists for directed networks, as the bilateral interbank exposures network.

In this article, we try to fill this gap. We propose a new methodology to identify systemically important nodes and, simultaneously, the community structure of the net-

¹The five categories are: size, cross-jurisdictional activity, interconnectedness, substitutability/financial institution infrastructure and complexity. Each category has the same weight (20%) in the overall measure which is rescaled such that an overall score is given in basis points. See [1] for further details.

work as well as the systemic importance of each node within communities. In particular our framework is suitable to investigate relevant economic topics such as the systemic importance of each institution as a borrower and/or as a lender in the whole network or in the community it belongs to and the identification of areas of contagion after banks default.

The method is based on non-negative matrix factorization (NMF). The NMF has been widely studied in the data mining and machine learning areas since the initial work of Lee et al. [15]. It has been applied to a number of different areas such as pattern recognition [16], multimedia data analysis [17] and text mining [18]. Extensions of NMF have also been developed to accommodate various cost functions as needed in different data analysis problems, such as classification [19] and clustering [20]. Only recently NMF has been adapted to community detection. Zarei et al. [21] proposed a NMF-based algorithm for identifying fuzzy communities. Psorakis et al. [22] presented a community detection approach that employ a Bayesian NMF model to extract soft modules from networks. However, all of these NMF based methods only focus on the detection of communities, but none of them take into account the identification of central nodes. Recently, in [23], the authors proposed a novel model to identify overlapping communities and central nodes in undirected network. Here we extend their methodology to directed graphs.

Specifically the method suggested in the present paper is designed for directed networks and therefore able to take into account exposures on both sides of banks' balance sheets². Since links represent flows of funds between lenders and borrowers, it seems appropriate to distinguish between Systemically Important Borrowers (SIBs) and Systemically Important Lenders (SILs). SIBs are entities more vulnerable to liquidity shocks, i.e. shocks that affect the liabilities of financial intermediaries because of a huge withdrawal of deposits, or a refusal to roll over on the counterpart. SILs are institutions more prone to devaluations shocks, i.e. shocks that hit the assets side of the balance sheets because of the default of some borrowers or for a fall of the market value of financial assets in the lenders' portfolio.

In a nutshell, we consider the weighted adjacency matrix representing the interbank exposures network as a mean of representing banks which are connected (or adjacent) to other banks. Using the NMF, such a matrix is approximated as the outer product of two lower dimensional matrices called borrowing and lending matrix respectively. Each element of these matrices represents the borrowing and lending systemic importance of each bank in each community. In order to find this matrices we develop an algorithm that exploits the connectivity information of the network highlighting the reinforcement relationships among nodes (see also [24]) meaning that systemically important borrowers are pointed to by many systemically important lenders and systemically important lenders point to many systemically important borrowers. This reinforcement relation-

²While a bank can be a systemically important borrower and lender at the same time, this is not always the case. A bank can borrow relatively small amounts in the interbank market and yet be systemically important as a result of lending activities. The risks for such a bank lie on the asset side of its balance sheet, and will be transmitted to the rest of the system. On the contrary, banks borrowing large volumes face and distribute risks to the system through the liability side of the balance sheet.

ship suggests that nodes that make themselves systemically important borrowers and lenders each other can be placed together in the same community (see Methods for a formal definition).

Even if much of the focus within community detection methods has been put on identifying disjoint communities, it is well understood that nodes in a network are naturally characterized by multiple community memberships ([25], [26]). Also in financial networks, it is very common for institutions to participate in more than one community, i.e. communities may often overlap. Our method takes into account this feature. Specifically, communities are retrieved independently from each other and vertices can belong to more than one community.

We test our method on the bilateral interbank exposures of the e-MID platform in order to evaluate the systemic importance of each bank within this market and within the existing communities. Data are taken from the Italian electronic broker market MID (Market for Interbank Deposits) run by e-MID S.p.A. Società Interbancaria per l'Automazione (SIA), Milan. The Italian electronic broker Market for Interbank Deposit (e-MID) covers the entire overnight deposit market in Italy. The information about the parties involved in a transaction allows us to perform an accurate analysis of the connectivity among banks and its change over time.

The results reveal that the risk associated with Italian banks decreased during the recent financial crisis while the opposite happened for foreign institutions. The borrowing and lending scores, calculated without assessing the presence of a community structure inside the network, although being informative about some market dynamics, fail to recover the market euphoria and the subsequent crash of the recent past. In fact as the transaction volume increased, the number of communities into the market rose as well. On the contrary, during the burst phase, when most of the banks interrupted transactions, also the number of communities decreased.

We also investigate whether the increase in the number of communities is associated with a stronger partition of financial institutions within each community or whether banks increase operations across different communities while the number of communities rises. Results indicate a different behavior affecting financial institutions in normal time or in periods of distress. Together with the growth of the number of communities, the e-Mid interbank market was affected by a strong split of banks within each community during the crisis period with few banks operating as SIBs or SILs within each community.

2 Results

In this section we present the application of our method on the e-Mid dataset. We consider a set of 354 banks, each of them represented by the amount of their exposures vis-a-vis the rest of the reporting banks, measured on monthly basis from the beginning of 1999 to the end of 2012.

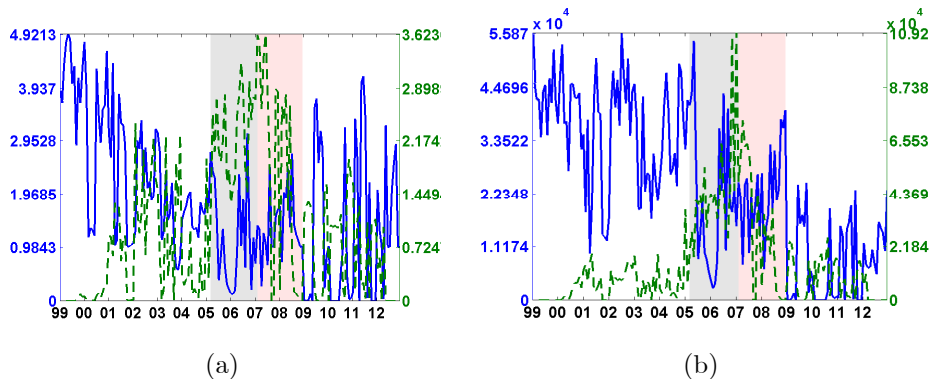


Figure 1: Time evolution of the borrowing score (a) and of the lending score (b) for rank-1 NMF from 1999 to 2012 at a monthly frequency. The solid blue line refers to Italian banks while the dashed green line is associated to foreign banks. The gray shaded area emphasizes the pre-crisis period (2005-Q1:2007-Q1) while the red area indicates the crisis period (2007-Q1:2008-Q4).

Let us consider first the Borrowing and Lending scores obtained disregarding the presence of a network structure, namely setting the number of communities equal to one. Notice that this procedure leads to the same results obtained by the HITS algorithm [24]. Supplementary Information presents a formal definition of the HITS and its relationship with the NMF. Being a feedback centrality measure, the role of a bank in the system is calculated on the basis of its neighbors behavior, and the neighbors centrality scores, in turn, will be calculated taking into account the neighbors of neighbors business etc. Thus, two banks will be ranked differently as SIL even if they lend the same amount of funds, depending on the behavior of their borrowers. The algorithm will rank higher the bank that lends to the most Systemically Important Borrower. The same happens for the SIB with respect to the lender: two banks that borrow the same amount of funds will be ranked differently depending from the lender they borrow from.

Fig. 1 presents the temporal evolution of the borrowing (a) and of the lending (b) scores³. For each measure we aggregate the scores associated to Italian (solid blue line) and to foreign banks (dashed green line). The course of the two scores displays similar pattern, indicating that the systemic importance of Italian banks decreased during the recent financial crisis while the opposite happened for foreign institutions. Moreover while Italian banks' borrowing scores approximately turn back to the pre-crisis level after 2009, the lending score settles down to lower values. The scores of foreign banks peak at the beginning of the crisis: the borrowing score starts rising from 2005 and it keeps increasing until 2007 whereas the lending score have a steep buildup from 2005, collapsing after 2007.

³Although the order of magnitudes of lending score is 4 times the one of the borrowing score the ranking position of the nodes in the two index are not affected by this problem because NMF is scale invariant. One can multiply \mathbf{B} by some constant c and \mathbf{L} by $1/c$ to obtain different \mathbf{B}, \mathbf{L} estimates without changing their product.

These dynamics underline different economic trends. During the years 1999-2005 foreign financial institutions joined the e-Mid interbank market, borrowing mostly from Italian banks. During the pre-crisis period (gray background) this trend grew up, but the most systemically important lenders turned out to be other foreign financial institutions. The dynamics reverted when the crisis unfolded (red background): foreign banks suddenly stopped to lend to other institutions, and smoothly decreased their borrowing operations. Italian banks, on the contrary, increased their lending activities.

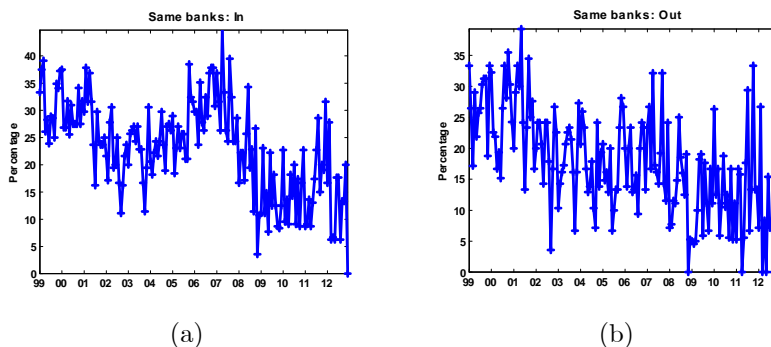


Figure 2: NMF method vs weighted degree measures. Panel (a) shows the percentage of banks identified as SIBs (SIFIs for the borrowing component) by both methods. Panel (b) shows the percentage of banks identified as SILs (SIFIs for the lending component) by both methods.

We compare the ranking obtained using the NMF method with the ones obtained with the methodology proposed by Basel III (in- and out-weighted degrees)⁴. Specifically, we ask the following question: "how many of the SIFIs that we identify with our methodology are also picked out by the approach employed by the Basel Committee?" Basel III applies a bucketing approach with a certain cutoff point and labels as Systemically Important banks those that lie above the threshold. Accordingly, 28 banks were classified as Globally-Systemically Important in November 2012 (see [1]). We adopt a similar bucketing approach and label banks as systemically important if their ranking falls within the upper 20-th percentile of importance⁵.

⁴In the case of directed graphs, a distinction needs to be made between the weighted in-degree and the weighted out-degree, which measure the total amount of borrowing (ingoing) and lending (outgoing) respectively. Formally

$$k_{i,in}^w = \sum_{j=1}^n \mathbf{W}_{i,j} \quad k_{i,out}^w = \sum_{i=1}^n \mathbf{W}_{i,j}$$

⁵We have experimented with several cutoff values and the results remain qualitatively unchanged.

The Basel Committee admittedly relies on more indicators than the bilateral exposures to identify SIFIs, as we do here. Our dataset however, includes only the anonymized identities of both lenders and borrowers, which prevents us from compiling additional information like balance sheets, market capitalization, etc., for the banks that use the e-MID platform.

The results are shown in Fig. 2 (a)-(b). The figure displays the percentage of banks that were labeled as SIFIs within our method and within the Basel III technique simultaneously. The difference between the two methods is considerably large. In the best of the cases, there is an approximate 40%-50% coincidence among the banks identified by the borrowing and lending scores and the Basel III method. The percentage of coincidence then reaches zero toward the end of the sample.

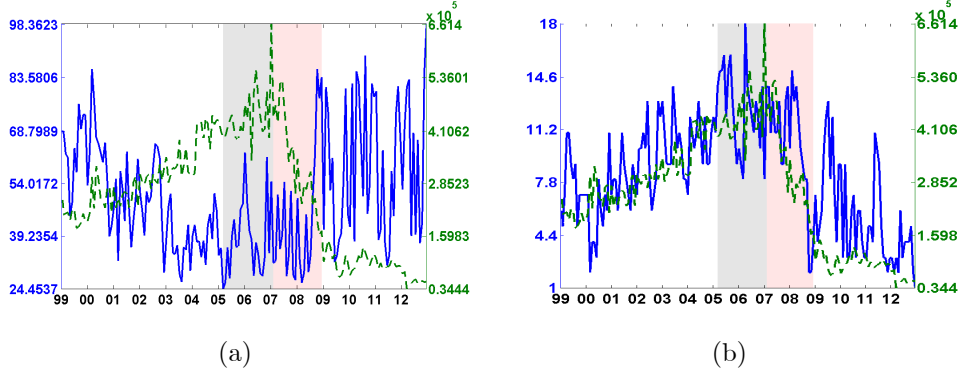


Figure 3: Time evolution of the model fit (blue line) versus the traded volume (green dashed line) is shown in the panel (a). Panel (b) displays the number of communities (blue line) versus the traded volume (dashed green line). The y-right axis displays the traded volume in million of dollars while on the y-left axis we report the model fitting (left panel), and the number of communities (right panel).

Despite the centrality measures help the understanding of the relative position (the systemic importance) of each bank during different time periods, the model explanatory power⁶ widely oscillates from 24 to 95%, and it also displays a negative correlation with the traded volume during the whole sample, as reported by Fig. 3 (a). The rank-1 NMF decomposition is well suited to describe the borrowing and lending relationships only at the beginning of the time sample, from 1999 to 2002, or after 2008, when the transaction volume lowered. However, it leaves out substantial topological informations while computing the systemic importance of financial institutions during the market euphoria and the subsequent crash, namely from 2003 to 2008.

This opens the issue of whether a better micro investigation, at a community level, can enhance the understanding of the systemic importance associated to each financial institution. Thus we investigate the clusterization trend that affected the interbank market during the last decade along with the centrality scores of institutions within

⁶In order to evaluate what is the percentage of data variability that the two indices are able to take into account we use the percentage fitting:

$$Fit(\%) = \left[1 - \frac{\|\hat{\mathbf{W}} - \mathbf{W}\|_2^2}{\|\mathbf{W} - E(\mathbf{W})\|_2^2} \right] \times 100$$

each community. To do so, we adopt an heuristic approach fixing⁷ at 90% the data variability we want to replicate, and looking for the number of communities that can jointly meet this goodness of fit.

Fig. 3 (b) emphasizes the evolution of the number of communities (solid blue line) and the traded volume (dashed green line) over time. The positive correlation between the two quantities clearly appears until the end of the crisis. The growing traded volume is positively related with the increasing number of communities of the interbank market before the 2007 collapse. When most of the banks interrupted transactions also number of communities decreased as well. It is worth mentioning that after 2009 the modules widely oscillated even if the overall traded volume remains low.

The link between the number of communities and the traded volume is helpful in understanding the interbank market dynamics. Banks are repeatedly engaged in transactions with other banks within the same community, while transactions between banks of different communities are much lower. Several factors can explain why banks form modules in the interbank network. It is well known that information asymmetries, moral hazard, adverse selection and market frictions influence the behavior of banks in the interbank network. Moreover, differently from [27], [28], [9], [12], we provide evidence that the e-MID interbank network, although being characterized by communities, does not display a persistent structure over time.

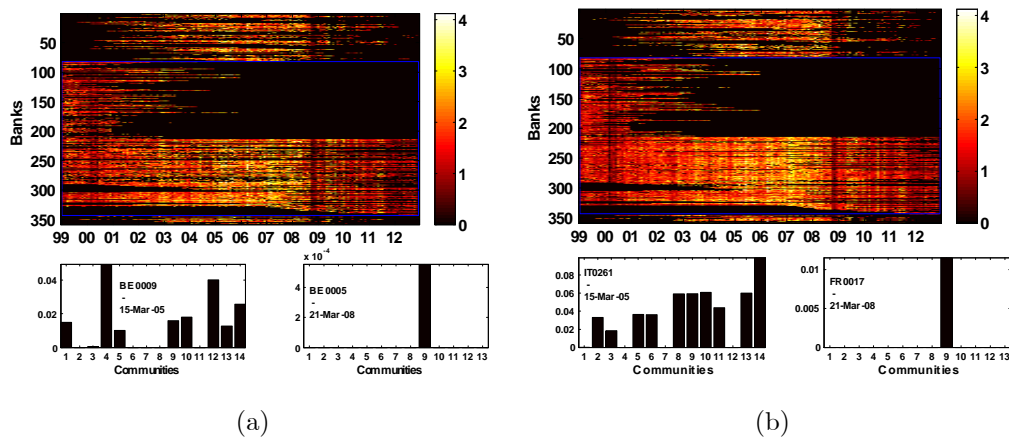


Figure 4: Time evolution of the coefficient of variation for the borrowing score (top panel (a)) and for the lending score (top panel (b)). We distinguish between Italian and foreign banks behavior encapsulating Italian banks into a blue rectangle. The y-left axis shows the number of anonymized banks operating while the x-axis denotes years. The color bars emphasize the coefficient of variation value. The bottom panels show an example of borrowing (a) and lending

⁷We also compute the number of communities able to explain the 95% and the 99% of the data variability. While in the first case the number of communities ranges from 5 to 25, in the second case the number of communities reaches 47. We choose the more conservative approach because using a high number of components faces the risk of overfitting noises.

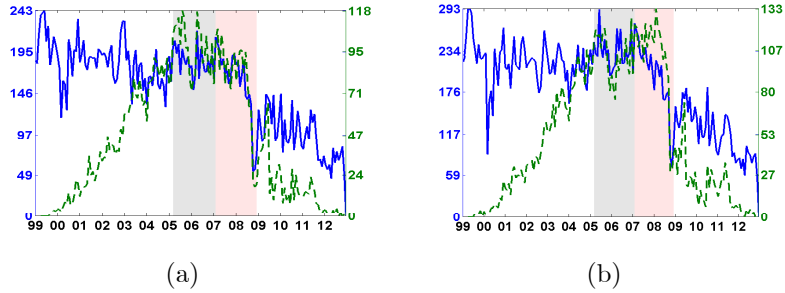
- (b) scores associated to a particular bank: this score can be dispersed across communities or fully concentrated into a particular module.

Additionally, since this technique admits an overlapping or soft-partitioning solution, i.e. communities are allowed to share members, it seems natural to investigate the soft-membership distributions of the two scores across time, which quantify *how strongly* each individual participates in each group as a borrower or as a lender. In other words we can explore the degree of fuzziness in the network by collecting, for each time and for each bank, the coefficient of variation of the borrowing and lending scores across communities. The coefficient is defined as the ratio of the standard deviation of the scores to the mean.

$$V_{i,t}^B = \frac{\left(\frac{1}{K_t} \sum_{k=1}^{K_t} \left(B_{i,k}^t - \bar{B}_i^t \right)^2 \right)^{\frac{1}{2}}}{\bar{B}_i^t}$$

where K_t is the number of communities at time t , $B_{i,k}^t$ is the borrowing scores of the i -th nodes in the k -th community at t and \bar{B}_i^t is the average borrowing score for the i -th nodes across communities in which it participates at t , $\bar{B}_i^t = \frac{1}{K_t} \sum_{k=1}^{K_t} B_{i,k}^t$. The same index can be applied to the lending scores⁸. The coefficient of variation is helpful in comparing the degree of variation of borrowing and lending scores when their means are considerably different from each other.

A financial institution that presents a low coefficient of variation value, having a membership distribution that is closer to uniform, belongs to different communities. On the contrary, if the borrowing or lending score values are associated with a high coefficient of variation, the involved bank, having an unimodal membership distribution, belongs only to the corresponding community.



⁸The index can be written as:

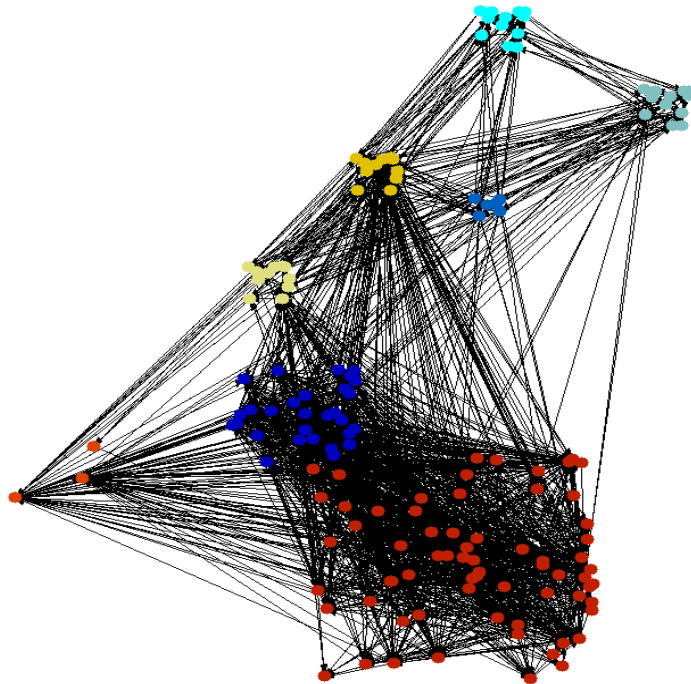
$$V_{i,t}^L = \frac{\left(\frac{1}{K_t} \sum_{k=1}^{K_t} \left(L_{i,k}^t - \bar{L}_i^t \right)^2 \right)^{\frac{1}{2}}}{\bar{L}_i^t}$$

Figure 5: Time evolution of the coefficient of variation for the borrowing (a) and for the lending (b) scores. We make a distinction between the behavior of Italian (solid blue line) and foreign (dashed green line) borrowers and lenders respectively.

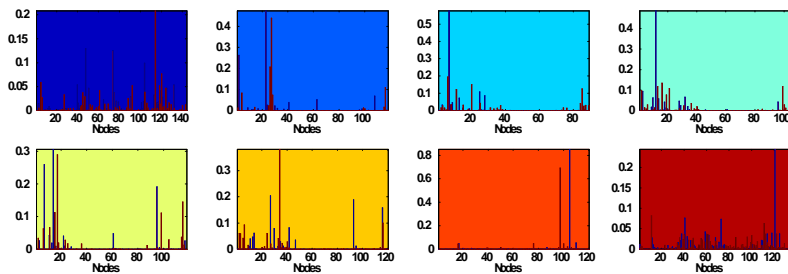
Fig. 4 (upper panels) displays the coefficient of variation of the borrowing (a) and lending (b) scores across communities. The coefficient displays approximately the same behavior for both indices, signaling an increase in the variability during the pre-crisis and crisis years. Therefore institutions, during the pre-crisis and crisis time, increased operations inside each community, without (or with small) overlapping. On the contrary, in business as usual periods, not only the number of communities is lower than during crisis time (see Fig. 3), but banks participate in different clusters as a borrower or lender. The figures also indicate that half of the Italian banks were active for the whole sample size while other stopped to exchange fund during the first years (black area). On the contrary foreign banks were particularly active during the years of the recent financial crisis. In other words, the results indicate a different behavior of financial institutions in normal time or in periods of distress. Together with the growth of the number of communities, the e-Mid interbank market was affected by a strong split of banks within each community during the recent financial crisis. As an example the small bottom plots of Fig. 4 show the soft-membership distribution across communities of the lending (a) and borrowing (b) scores for specific banks at different time periods.

In Fig. 5, we collect the coefficient of variation of the Italian and foreign financial institutions for the borrowing (a) and lending scores (b). Italian banks display a high coefficient of variation until the end of the financial crisis. Foreign banks behave differently. They show an increasing coefficient of variation that peaks during the pre-crisis time for the borrowing score and during crisis for the lending score. Thus they switch from a soft partition behavior during business as usual time to a hard partition scheme during the financial crisis.

Finally, in order to give a simple overview of the results obtained by the application of NMF to the e-Mid dataset, we show the network community structure and the relative scores for banks in September 2008 when Lehman Brothers bankruptcy occurred .



(a)



(b)

Figure 6: The network community structure of the e-Mid interbank market during September 2008 (a) together with the Borrowing (blue bars) and Lending (red bars) scores for each community (b). The communities are emphasized with different background colors. On the x-axis we display the number of nodes, on the y-axis the strenght of the scores. While we use a hard partition scheme in order to visualize the network, the scores are calculated using a soft partition scheme.

The interbank network displays eight communities, emphasized in different colors. Fig. 6 (a) gives a network representations of the relationships among banks, using a hard

partition scheme, where nodes are assigned to the community that most contribute to their scores. On Fig. 6 (b) we show the borrowing (blue bar) and lending (red bar) scores for each community. The relative community is emphasized using the correspondent background color. Differently from the network visualization, the nodes are associated with each community via a soft partition scheme, therefore, a single banks can belong to a multiplicity of communities. The scores indicate that, except for the "red" and "blue" communities, few banks operate as SIBs or SILs within each community.

2.1 Evaluating the algorithm

In this section we compare our algorithm with several well-known community detection methods. Since our method produces both soft and hard partition schemes, we compare the goodness of the communities obtained by both solutions against methods that produce crisp assignment (non-fuzzy) or fuzzy assignment. With crisp assignment, the relationship between a node and a cluster is binary. That is, a node i either belongs to cluster c or does not. With fuzzy assignment, each node is associated with communities in proportion to a belonging factor. Thus we compare our hard partition solution against methods that produce crisp assignment and the soft partition solution against methods that produce fuzzy assignment.

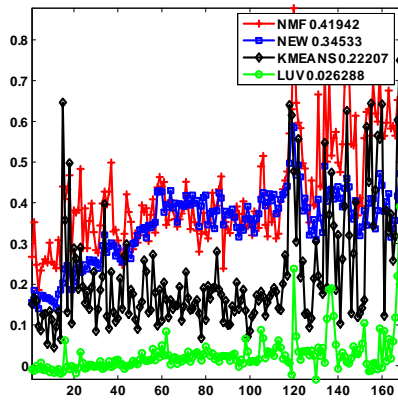
In particular we consider the modularity maximization method [29],[30], the Louvain method [31] and the K-means algorithm [32] for crisp assignment; the C-means algorithm [33],[34] and the Clique Percolation Method [35] for fuzzy assignment.

Since our algorithm is applied to networks for which the communities are not known in advance, we need a measure to quantify the goodness of the communities detected by each technique. In other words, we would like to know which of the divisions produced by the different algorithms are the best ones for the given network. To answer this question, we define two modularity measures that show the quality of a particular division of a network. These two measures are the crisp and the fuzzy modularity for directed weighted network which are defined as:

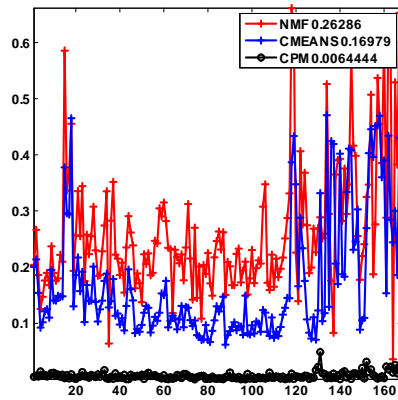
$$Q^C = \frac{1}{m} \sum_{i,j} \left[w_{ij} - \frac{s_i^{in} s_j^{out}}{m} \right] \delta(c_i, c_j)$$

$$Q^F = \frac{1}{m} \sum_c \sum_{i,j} \left[w_{ij} - \frac{s_i^{in} s_j^{out}}{m} \right] a_{ic} a_{jc}$$

respectively. Where s_i^{in} and s_i^{out} are the in- and out-strength respectively, $m = \sum_i s_i^{in} = \sum_j s_j^{out}$. The difference between the two measures relies on the last term: $\delta(c_i, c_j)$ is the Kronecker delta symbol, and c_i (c_j) is the label of the community to which node i (j) is assigned; a_{ic} (a_{jc}) is the degree of membership of node i (j) in the community c .



(a)



(b)

Figure 7: The modularity measure of different algorithms for hard partition solutions (a) and for soft partition solutions (b). Near the names of each algorithm we report the average of the modularity values over the entire data sample. In the legends, NMF refers to our algorithm, Von Newman (NEW), K-means (KMEANS) and Louvain method (LUV) for hard partition solution. The soft partition solution of our method (NMF) is also compared with the C-means algorithm (CMEANS) and with the Clique Percolation Method (CPM).

Fig. 7 reports the results for the different methods. We calculate the modularity metrics for each period in the data sample and in the legend, near the name of each algorithm, we report the average modularity value. Fig. 7 (a) encompasses the results about the hard partition solutions, while in the Fig. 7 (b) we show the modularities for the soft partition solutions. In both cases, on average, our method outperforms the other algorithms even if in some period the other techniques provide a higher modularity. Moreover comparing the soft and the hard partition solutions of our method, one can notice that in the middle of the data sample, when the number of community increases, with banks operating in different communities with low overlapping, the modularity of the hard partition solution becomes higher than the one obtained with a soft partition solution.

3 Discussion

Methods able to effectively aid financial authorities to identify Systemically Important Financial Institutions (SIFIs) are particularly valuable to enhance policy-making (e.g. prudential regulation, oversight and supervision) and decision-making (e.g. resolving, restructuring or providing emergency liquidity) capabilities. In this article, we proposed a new methodology to identify central nodes and, simultaneously, to detect the

community structure in directed graphs. SIFIs inside communities are identified according to two indicator-based measures that we name borrowing score and lending score. In so doing we are able to distinguish between risks arising from exposures on the asset and on the liability side of banks' balance sheet. In other words we discriminate banks as Systemically Important Borrowers or Systemically Important Lenders as suggested by the reform proposed by Basel III. Moreover, we can intend the communities founded within this technique as the most plausible areas of contagion of a banks possible default.

Our work reveals that the risk associated with Italian banks decreased during the recent financial crisis while the opposite happened for foreign institutions. Since interbank market displays the existence of a community structure, our method outperforms, in term of goodness of fit, the centrality scores calculated along with a rank-1-factorization. Indeed, the borrowing and lending scores, calculated without assessing the presence of a community structure, although being informative about some market dynamics, fail to recover the market euphoria and the subsequent crash of the recent past. In fact, as the transaction volume increased, the number of communities into the market rose as well. On the contrary, during the burst phase, when most of the banks interrupted transactions, also the number of communities decreased on average.

We also investigated whether the increase in the number of communities is associated with a stronger partition of the financial institutions within each community or whether banks operate across different communities. Results indicate a different behavior affecting financial institutions in normal time or in periods of distress. Together with the growth in the number of communities, the e-Mid interbank market was affected by a strong split of banks within each community during the recent financial crisis with few banks operating as SIBs or SILs.

4 Methods

Let $G = (V, E)$ be a directed and weighted graph representing the financial transactions taking place in the interbank market, where V is the n -dimensional set of banks and E the m -dimensional set of financial transactions. Graphically, banks are represented by nodes and the transactions by edges. Let $\mathbf{W}_{i,j}$ be the amount that bank j lends to bank i in a certain period. The collection of all the interbank transactions between financial institutions during the same period leads to the matrix of exposures $\mathbf{W}_{n \times n}$, where $\mathbf{W}_{i,j} > 0$ if a transaction between i and j takes place while $\mathbf{W}_{i,j} = 0$ otherwise. We call this matrix the *weighted-adjacency transaction* matrix. Let $K \in \mathbb{N}$ be the maximum number of communities in the network at a certain time. In empirical works though, K needs to be fixed on the basis of the desired level of detail: a low number of components only yields the strongest structures, whereas using a high number of components faces the risk of overfitting noise. In the extreme case of $K = 1$ borrowing and lending scores are computed for the whole network structure, but without assessing the presence of a community structure inside the network. In what follows we assume K is known a priori (while we will relax this assumption in Section 4).

The NMF method consists in factorizing the exposures matrix \mathbf{W} into two matrices, \mathbf{B} and \mathbf{L} , such that both matrices have no negative elements, i.e. $\mathbf{B} \in \mathbb{R}_+^{n \times K}$ and $\mathbf{L} \in \mathbb{R}_+^{K \times n}$. The element \mathbf{B}_{ik} corresponds to the borrowing systemic importance of bank i within community k . By analogy, the element \mathbf{L}_{ki} is the systemic importance of bank i within the community k in terms of its lending activity. It is straightforward to interpret $\mathbf{B}_{ik}\mathbf{L}_{kj}$ as the contribution, in terms of model fitting, of the k -th community to the edge \mathbf{W}_{ij} . In other words, the interaction $\hat{\mathbf{W}}_{ij} = \sum_{k=1}^K \hat{\mathbf{W}}_{ij}^k = \sum_{k=1}^K \mathbf{B}_{ik}\mathbf{L}_{kj}$ between nodes i and j is the result of the sum of their participation in the same communities. Therefore, $\hat{\mathbf{W}}$ is a summation of K rank-1 matrices and each $\hat{\mathbf{W}}_{ij}^k$ denotes the number of pairwise interactions in the context of community k (see [22], [36]). Thus $\hat{\mathbf{W}}$ is an approximation of the original matrix \mathbf{W} .

We call the sum over each column of the matrix \mathbf{B} and over each row of \mathbf{L} as $\mathbf{s}^B = \sum_k \mathbf{B}_{ik}$ and $\mathbf{s}^L = \sum_k \mathbf{L}_{kj}$ respectively. If each column of $\mathbf{B}_{:,k}$ and each row of $\mathbf{L}_{k,:}$ is normalized to one, dividing it by $\mathbf{s}_{1,k}^B$ and $\mathbf{s}_{k,1}^L$, the elements \mathbf{B}_{ik} and \mathbf{L}_{ki} can be seen as the proportion of borrowing and lending systemic importance of bank i into community k since now $\sum_k \mathbf{B}_{ik} = 1$ and $\sum_k \mathbf{L}_{ki} = 1$.

Since we are dealing with overlapping communities, a soft partition⁹ scheme is proposed by assigning to each node the percentage of its strength centrality that belong to that community. In other words we calculate for each node its strength in each $\hat{\mathbf{W}}^k$ and we stack this measure in a matrix $\mathbf{D}^{n \times K}$ where each element \mathbf{D}_{ik} represents the weighted degree centrality of node i in community k . Normalizing each row of \mathbf{D} by $\mathbf{d} = \sum_k \mathbf{D}_{ik}$ we obtain the soft partition solution. Such an edge decomposition can then be used also to assign nodes to communities according to a hard partition scheme, assigning each bank to the community in which it has the highest impact in terms of strength.

In order to compute \mathbf{B} and \mathbf{L} , we consider the following minimization problem

$$\min_{\mathbf{B} \in \mathbb{R}_+^{n \times K}, \mathbf{L} \in \mathbb{R}_+^{K \times n}} \frac{1}{2} \|\mathbf{W} - \mathbf{BL}\|_F^2 \quad (1)$$

where $\|\bullet\|_F^2$ is the Frobenius norm. The optimization problem results in

$$\mathbf{B} \geq \mathbf{0}, \quad (2)$$

$$\mathbf{L} \geq \mathbf{0}$$

$$\nabla_{\mathbf{B}} = \mathbf{BLL}^T - \mathbf{WL}^T \geq \mathbf{0}, \quad (3)$$

$$\nabla_{\mathbf{L}} = \mathbf{B}^T \mathbf{BL} - \mathbf{B}^T \mathbf{W} \geq \mathbf{0}$$

$$\mathbf{B} \otimes (\mathbf{BLL}^T - \mathbf{WL}^T) = \mathbf{0}, \quad (4)$$

$$\mathbf{L} \otimes (\mathbf{B}^T \mathbf{BL} - \mathbf{B}^T \mathbf{W}) = \mathbf{0}$$

⁹We distinguish between soft and hard membership distributions. In the first case nodes can belong to more than one community. In the hard partition scheme, overlapping communities are not allowed.

where \otimes is the Hadamard product and equations (3)-(4) are the Karush-Kuhn-Tucker conditions.

We can solve this problem using the gradient descent method ([37]) by choosing a set of initial values for \mathbf{B} and \mathbf{L} .

$$\mathbf{B}_{ik} \leftarrow \mathbf{B}_{ik} \frac{(\mathbf{W}\mathbf{L}^T)_{ik}}{(\mathbf{B}\mathbf{L}\mathbf{L}^T)_{ik}} \quad (5)$$

$$\mathbf{L}_{jk} \leftarrow \mathbf{L}_{jk} \frac{(\mathbf{B}^T\mathbf{W})_{jk}}{(\mathbf{B}\mathbf{B}^T\mathbf{L})_{jk}} \quad (6)$$

The expressions in (5) and (6) represent the borrowing and the lending score of banks i and j in community k respectively. For example, the borrowing score \mathbf{B}_{ik} , which measure the capability of bank i to borrow from banks belonging to community k , is obtained by multiplying the i -th row of matrix \mathbf{W} (which collect flows borrowed by bank i) with the k -th column of matrix \mathbf{L} (which collect the lending score of each bank in community k). A similar argument applies to \mathbf{L}_{jk} .

Once we have matrices \mathbf{L} and \mathbf{B} , we can calculate the weighted-adjacency transaction matrix approximation $\hat{\mathbf{W}}^k$ belonging to each community and then we can assign nodes to communities depending on the normalized degree that each bank has in each community.

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References

- [1] Basel Committee on Banking Supervision. Global Systemically Important Banks: Updated Assessment Methodology and the Higher Loss Absorbency Requirement. Technical Report, Bank for International Settlements, (2013).
- [2] Arregui, M. N. et.al. Addressing Interconnectedness: Concepts and Prudential Tools. *International Monetary Fund* 13, 199 (2013).
- [3] Battiston, S., Di Iasio, G., Infante, L., & Pierobon, F. Capital and contagion in financial networks. *IFC Bullettins chapters* 39 (2015).
- [4] Battiston, S. & Caldarelli, G. Systemic Risk in Financial Networks. *J Financ Manag Mark Inst* 1, 129-154 (2013).
- [5] Battiston, S., Delli Gatti, D., Gallegati, M., Greenwald, B. & Stiglitz, J. E. Liaisons dangereuses: Increasing connectivity, risk sharing, and systemic risk. *J Econ Dyn Control* 36, 1121-1141 (2012).
- [6] Battiston, S., Puliga, M., Kaushik, R., Tasca, P., & Caldarelli, G. Debtrank: Too central to fail? Financial networks, the fed and systemic risk. *Sci Rep*, 2 (2012).
- [7] Soramäki, K., Bech, M. L., Arnold, J., Glass, R. J., & Beyeler, W. E. The topology of interbank payment flows. *Physica A* 379, 317-333 (2007).
- [8] Iori, G., De Masi, G., Precup, O. V., Gabbi, G., & Caldarelli, G. A network analysis of the Italian overnight money market. *J Econ Dyn Control* 32, 259-278 (2008).
- [9] Iori, G., Reno, R., De Masi, G., & Caldarelli, G. Trading strategies in the Italian interbank market. *Physica A* 376, 467-479 (2007).
- [10] Cocco, J. F., Gomes, F. J., & Martins, N. C. Lending relationships in the interbank market. *J Financ Intermed* 18, 24-48 (2009).
- [11] Craig, B., & Von Peter, G. Interbank tiering and money center banks. *J Financ Intermed* 23, 322-347 (2014).
- [12] Fricke, D. Trading strategies in the overnight money market: Correlations and clustering on the e-MID trading platform. *Physica A* 391, 6528-6542 (2012).
- [13] Fricke, D., & Lux, T. On the distribution of links in the interbank network: Evidence from the e-mid overnight money market. *Empir Econ* 49, 1463-1495 (2015).
- [14] Fricke, D., & Lux, T. Core-periphery structure in the overnight money market: evidence from the e-mid trading platform. *Comput Econ* 45, 359-395 (2012).
- [15] Lee, D. D., & Seung, H. S. Learning the parts of objects by non-negative matrix factorization. *Nature* 401, 788-791 (1999).

- [16] Li, S. Z., Hou, X., Zhang, H., & Cheng, Q. Learning spatially localized, parts-based representation. In *Computer Vision and Pattern Recognition. Proceedings of the 2001 IEEE Computer Society Conference 1*, 207-212.
- [17] Cooper, M., & Foote, J. Summarizing video using non-negative similarity matrix factorization. In *Multimedia Signal Processing, 2002 IEEE Workshop 25-28*.
- [18] Pauca, V. P., Shahnaz, F., Berry, M. W., & Plemmons, R. J. Text Mining Using Non-Negative Matrix Factorizations. In *SDM* 452-456 (2004).
- [19] Sha, F., Saul, L. K., & Lee, D. D. Multiplicative updates for nonnegative quadratic programming in support vector machines. In *Advances in neural information processing systems* 1041-1048 (2002).
- [20] Wang, F., Li, T., & Zhang, C. Semi-Supervised Clustering via Matrix Factorization. In *SDM* 1-12 (2008).
- [21] Zarei, M., Izadi, D., & Samani, K. A. Detecting overlapping community structure of networks based on vertex vertex correlations. *J Stat Mech-Theory E*, P11013 (2009).
- [22] Psorakis, I., Roberts, S., Ebden, M., & Sheldon, B. Overlapping community detection using bayesian non-negative matrix factorization. *Phys Rev E* 83, 066114 (2011).
- [23] Cao, X., Wang, X., Jin, D., Cao, Y., & He, D. Identifying overlapping communities as well as hubs and outliers via nonnegative matrix factorization. *Sci Rep*, 3 (2013).
- [24] Kleinberg, J.M. Authoritative Sources in a Hyperlinked Environment. *J ACM* 46, 604-632 (1999).
- [25] Nefedov, N. Multiple-membership communities detection and its applications for mobile networks. INTECH Open Access Publisher (2011).
- [26] Zhang, Z. Y., Wang, Y., & Ahn, Y. Y. Overlapping community detection in complex networks using symmetric binary matrix factorization. *Phys Rev E*, 87, 062803 (2013).
- [27] Boss, M., Elsinger, H., Summer, M., & Thurner, S. An empirical analysis of the network structure of the Austrian interbank market. *Financial Stability Report 7*, 77-87 (2004).
- [28] De Masi, G., Iori, G., & Caldarelli G. A fitness model for the Italian interbank money market. *Phys Rev E* 74, 66112 (2006).
- [29] Girvan, M., & Newman, M. E. Community structure in social and biological networks. *Proceedings of the national academy of sciences*, 99, 7821-7826 (2002).
- [30] Newman, M. E. Analysis of weighted networks. *Phys Rev E* 70, 056131 (2004).

- [31] Blondel, V. D., Guillaume, J. L., Lambiotte, R., & Lefebvre, E. Fast unfolding of communities in large networks. *J Stat Mech-Theory E*, P10008 (2008).
- [32] MacQueen, J. Some methods for classification and analysis of multivariate observations. In Proceedings of the fifth Berkeley symposium on mathematical statistics and probability, 281-297 (1967).
- [33] Dunn, J. C. *A fuzzy relative of the ISODATA process and its use in detecting compact well-separated clusters*. Taylor & Francis (1973).
- [34] Bezdek, J. C. *Pattern recognition with fuzzy objective function algorithms*. Springer Science & Business Media (2013).
- [35] Palla, G., Derényi, I., Farkas, I., & Vicsek, T. Uncovering the overlapping community structure of complex networks in nature and society. *Nature* 435, 814-818 (2005).
- [36] Mankad, S., & Michailidis, G. Structural and functional discovery in dynamic networks with non-negative matrix factorization. *Phys Rev E* 88, 042812 (2013).
- [37] Wang, D., Li, T., Zhu, S., & Ding, C. Multi-document summarization via sentence-level semantic analysis and symmetric matrix factorization. In Proceedings of the 31st annual international ACM SIGIR conference on Research and development in information retrieval, 307-314. (2008).

1 Supplementary Information

1.1 Relationship between NMF and HITS algorithm

The HITS algorithm ([?]) was originally intended to discover the most central pages for broad search topics in the context of the www. It uses appropriate eigenvectors (or singular vectors) decomposition to compute the authority (a_i) and the hubness (h_i) of node i . Authority measures prestige: nodes who many other nodes point to are called authorities. If a node has a high number of nodes pointing to it, it has a high authority value and this quantifies its role as a source of information. On the contrary, a hub is an actor referring to many authorities and its score measures acquaintance. Essentially, a good hub points to many good authorities and a good authority is pointed to by many good hubs

If we denote the weighed-adjacency transaction matrix as W , the HITS algorithm leads to hubs and authorities computed iteratively

$$\mathbf{h}^{(t+1)} = \mathbf{W}\mathbf{a}^{(t)} \tag{1}$$

$$\mathbf{a}^{(t+1)} = \mathbf{W}^T\mathbf{h}^{(t+1)} \tag{2}$$

where vectors $a = [a_1, \dots, a_n]^T$ and $h = [h_1, \dots, h_n]^T$ yield respectively the authority and the hub scores on all nodes. Performing power iteration method on WW^T and W^TW , $a^{(t)}$ and $h^{(t)}$ will converge respectively to the principal eigenvectors a and h of the symmetric semi-positive definite matrices WW^T and W^TW .

Given the square $n \times n$ non-negative matrix W (i.e. $w_{ij} \geq 0$) and a reduced rank $K = 1$ approximation, the NMF rank-one decompositions is equivalent to a SVD-decomposition problem of finding two non-negative vectors $a \in \mathbb{R}_+^{n \times 1}$ and $h \in \mathbb{R}_+^{n \times 1}$ together with a scalar $d \in \mathbb{R}_+$ that approximate W , i.e.

$$\mathbf{W} \approx \mathbf{a}d\mathbf{h}^T = \hat{\mathbf{W}}$$

We need to prove that a and h are local minimizers of the function $\frac{1}{2} \|W - \hat{W}\|_F^2$, i.e. we have to solve the following optimization problem

$$\min_{\mathbf{a} \in \mathbb{R}_+^{n \times 1}, \mathbf{h} \in \mathbb{R}_+^{n \times 1}, d \in \mathbb{R}} \frac{1}{2} \|W - ad\mathbf{h}^T\|_F^2 \tag{3}$$

where $\|\bullet\|_F^2$ represents the Frobenius norm. Equivalently \hat{W} can be written as

$$\begin{aligned} \hat{\mathbf{W}} &= \mathbf{a}d\mathbf{h}^T = \mathbf{a}d^{1/2}d^{1/2}\mathbf{h}^T \\ &= \left(\mathbf{a}d^{1/2}\right) \left(\mathbf{h}d^{1/2}\right)^T = \mathbf{b}\mathbf{l}^T \end{aligned} \tag{4}$$

where b and l are the borrowing and the lending score vectors respectively.

Thus, we can transform the optimization problem (3) to be an equivalent problem of NMF:

$$\min_{\mathbf{b} \in \mathbb{R}_+^{n \times 1}, \mathbf{l} \in \mathbb{R}_+^{n \times 1}}, \frac{1}{2} \|W - \mathbf{b}\mathbf{l}^T\|_F^2 \quad (5)$$

The following lemma holds

Lemma 1 *The pair of vectors \mathbf{b} and \mathbf{l} are local minimizers of the function $\frac{1}{2} \|W - \hat{W}\|_F^2$ being \mathbf{l} and \mathbf{b} the non-negative right eigenvectors of WW^T and W^TW respectively.*

Proof. We permute matrix W in a way such that vectors \mathbf{b} and \mathbf{l} are partitioned as $(\mathbf{b}_+, \mathbf{0})^T$ and $(\mathbf{l}_+, \mathbf{0})^T$ with $b_+, l_+ > 0$. As a consequence W becomes

$$\mathbf{W} = \begin{pmatrix} \mathbf{W}_{11} & \mathbf{W}_{12} \\ \mathbf{W}_{21} & \mathbf{W}_{22} \end{pmatrix}$$

We can rewrite equation (??) as

$$\begin{pmatrix} \mathbf{b}_+ \mathbf{l}_+^T & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{pmatrix} \begin{pmatrix} \mathbf{l}_+ \\ \mathbf{0} \end{pmatrix} + \\ - \begin{pmatrix} \mathbf{W}_{11} & \mathbf{W}_{12} \\ \mathbf{W}_{21} & \mathbf{W}_{22} \end{pmatrix} \begin{pmatrix} \mathbf{l}_+ \\ \mathbf{0} \end{pmatrix} \geq \mathbf{0}$$

and

$$\begin{pmatrix} \mathbf{l}_+ \mathbf{b}_+^T & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{pmatrix} \begin{pmatrix} \mathbf{b}_+ \\ \mathbf{0} \end{pmatrix} + \\ - \begin{pmatrix} \mathbf{W}_{11}^T & \mathbf{W}_{21}^T \\ \mathbf{W}_{12}^T & \mathbf{W}_{22}^T \end{pmatrix} \begin{pmatrix} \mathbf{b}_+ \\ \mathbf{0} \end{pmatrix} \geq \mathbf{0}$$

This implies that $W_{21}l_+ \leq 0$ and $W_{12}^T b_+ \leq 0$. Since $W_{21}, W_{12}^T \geq 0$ and $b_+, l_+ > 0$ we can conclude that $W_{21} = 0$ and $W_{12}^T = 0$. From equation (??) we have

$$\mathbf{b}_+ \otimes (\|\mathbf{l}_+\|_2^2 \mathbf{b}_+ - \mathbf{W}_{11} \mathbf{l}_+) = 0, \\ \mathbf{l}_+ \otimes (\|\mathbf{b}_+\|_2^2 \mathbf{l}_+ - \mathbf{W}_{11}^T \mathbf{b}_+) = 0$$

Since $b_+, l_+ > 0$ we have

$$\|\mathbf{l}_+\|_2^2 \mathbf{b}_+ = \mathbf{W}_{11} \mathbf{l}_+, \\ \|\mathbf{b}_+\|_2^2 \mathbf{l}_+ = \mathbf{W}_{11}^T \mathbf{b}_+$$

or, equivalently

$$\|\mathbf{b}_+\|_2^2 \|\mathbf{l}_+\|_2^2 \mathbf{b}_+ = \mathbf{W}_{11} \mathbf{W}_{11}^T \mathbf{b}_+, \\ \|\mathbf{b}_+\|_2^2 \|\mathbf{l}_+\|_2^2 \mathbf{l}_+ = \mathbf{W}_{11}^T \mathbf{W}_{11} \mathbf{l}_+$$

This means that the pair (b, l) is a local minimizer of (5) if and only if b, l are non-negative eigenvectors of WW^T and W^TW respectively of the singular value $d = \|\mathbf{b}_+\|_2 \|\mathbf{l}_+\|_2$. ■

Therefore we can retrieve the original vectors a and h from (4), using the fact that the singular value of W are the square root of the eigenvalues of WW^T or W^TW

$$\mathbf{a} = \frac{\mathbf{b}}{\sqrt{\|\mathbf{b}_+\|_2 \|\mathbf{l}_+\|_2}},$$

$$\mathbf{h} = \frac{\mathbf{l}}{\sqrt{\|\mathbf{b}_+\|_2 \|\mathbf{l}_+\|_2}}$$

In so doing we established a relationship between the HITS algorithm and the non-negative matrix factorization outcome for the rank-1 matrix case, namely the rank-one NMF solution is always a rescaled version of authority and hub scores obtained with the HITS algorithm.

1. L. Colombo, H. Dawid, *Strategic Location Choice under Dynamic Oligopolistic Competition and Spillovers*, Novembre 2013.
2. M. Bordignon, M. Gamalerio, G. Turati, *Decentralization, Vertical Fiscal Imbalance, and Political Selection*, Novembre 2013.
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