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Cultural Transmission with Incomplete Information: Parental Perceived Efficacy and Group Misrepresentation*

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Abstract

This paper introduces incomplete information in the standard model of cultural transmission (Bisin and Verdier, 2001). We allow parents to ignore own group size and the efficiency of their cultural transmission technology, while receiving a feedback from their children. Using the selfconfirming equilibrium concept, parents may end up to sustain, and be confirmed about, wrong conjectures. We show that in equilibrium optimal socialization efforts display cultural complementarity with respect to own population share, while the standard substitution result holds with respect their own conjectured population shares. Considering the population dynamics, if conjectures about population shares are shaped by cultural leaders who want to maximize the presence of own traits in the next period, then conjectures are characterized by negative biases. Our main finding is that, depending on the magnitude of the bias, the dynamics can display stable or unstable polymorphic equilibria, or just a stable homomorphic equilibrium, potentially reverting standard predictions.

Journal of Economic Literature Classification Numbers: C72, D10, D80, J10, Z10
Keywords: Cultural Transmission; Incomplete Information; Selfconfirming Equilibrium; Group Under-Representation; Parental Perceived Efficacy; Cultural leaders.

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1 Introduction

It is a well-established fact that members of cultural or ethnic groups find it hard to have an unbiased perception of own population shares in the society, even if the correct information is publicly available. In 2016 the magazine "The Economist" analyzed the perception that European citizens of different countries have about the share of muslim population in own country[1]. It turned out that this perception is extremely biased. For example, the less biased ones result to be the Germans who think muslims to be 19% of the population, while they are just about 6%. The most biased ones are Hungarians, who think muslims to be 7%, while they are 70 times less (0.1%). A recent paper by Alesina et al. (2018) finds evidence that citizens strongly overestimate the share of migrants in the population. In some cases, as for the US, while migrants accounts for a 10% of the population, citizens think they are almost 40%. This strong misperception, paired with the populist narrative of immigrants' invasion and with xenophobic ideas that are diffusing in many countries, may have consequences on the way agents of different groups decide to transmit own cultural traits to the next generation, and can affect the long run composition of the society.

Considering the transmission of cultural traits, it is well-known in the social psychology literature that the **parental efficacy** (PE), i.e. the technology by which parents try to transmit own traits to offspring, may not be fully efficient. Social psychologists agree on the fact that parental efficacy may depend on several factors, such as neighborhood composition, children characteristics, or other ecological variables (Belsky et al., 1984; Van Bakel and Riksen-Walraven, 2002; Belsky and Jaffee, 2006). Given these methodological difficulties, parents may not be able to objectively measure their own parental efficacy, and in social psychology literature there are no data nor measures about its actual value. Therefore scholars focus on the **perceived parental efficacy** (PPE), i.e. parents' conjectures about their own parental efficacy ("beliefs or judgments parents hold of their capabilities to organize and execute a set of tasks related to parenting a child", de Montigny and Lacharité, 2005).[2] Notably, higher levels of PPE are associated with higher quality of parent-child interactions that leads to better offsprings' development, higher parental socialization effort and lower stress (Bandura, 1993; Coleman and Karraker, 2000; de Montigny and Lacharité, 2005, among others).

We study the issues of own group size misperception and perceived parental efficacy introducing incomplete information in a standard model of cultural transmission (Bisin and Verdier, 2001) in which parents of two different cultural groups exert efforts to try to

[1] *The Economist*, "Islam in Europe: perception and reality", 03/23/2016

[2] In literature, what we call PPE is also know as *perceived parental self-efficacy* or simply *parental self-efficacy.*

induce own type to children.[3] However, differently from standard literature, we assume that parents may ignore the actual population shares and their parental efficacy. Parents just have conjectures, possibly wrong, about the unknowns (the conjecture about parental efficacy is by definition PPE) and then exert socialization efforts maximizing their subjective expected utility. This double uncertainty is new with respect to the literature, in which parents know population shares, and the transmission technology is fully efficient and known to every agent. Notice that conjectures about population shares impact on the *oblique socialization* process, while conjectures about parental efficacy impact on the *vertical socialization* one.[4]

To model the conjectures formation, we assume parents to receive feedbacks about how much children have been convinced by each trait during the (vertical and oblique) socialization process. Feedbacks enable parents to make inference about population shares and PE. As discussed above, it is impossible for parents to have a precise measure about PE, and available information about population shares may be biased. Therefore feedbacks may not allow agents to perfectly disentangle the relative importance of the two unknowns. It follows that **confirmed conjectures**, i.e. conjectures about PE and population shares that are compatible with feedbacks received, may be wrong. Parents can only rely on confirmed conjectures since, given our information structure, they are indistinguishable from correct ones. This translates in long-run consequences about composition of the society.

Parent-child interactions are characterized by an implicit process of learning. We model this with the solution concept of **selfconfirming equilibrium**, early proposed by Battigalli (1987); Battigalli and Guaitoli (1988) as *comjectural equilibrium*, and by Fudenberg and Levine (1993a).[5] Selfconfirming equilibrium requires that, under incomplete information, agents maximize their subjective expected utility and have conjectures that must be confirmed by the feedback they receive. Although selfconfirming equilibrium is a static concept, as we discuss in the paper, it also has a strong learning foundation that allows for repeated parent-child interactions, where parents revise their conjectures until the feedback confirms them. In details, any form of adaptive learning process, if converges, it does so to a selfconfirming equilibrium (Milgrom and Roberts, 1991; Gilli, 1999; Battigalli, 2018).[6] Thus selfconfirming equilibrium fits with the spirit of cultural transmission processes.

We first provide results about the static setting, that is the choice of a single cohort

[3]For an extensive review about cultural transmission in economics, we refer to Bisin and Verdier (2011).
[4]We refer to Cavalli-Sforza and Feldman (1981) and Bisin and Verdier (2001) for the terminology.
[5]For a deeper discussion and for generalizations refer Battigalli (2012, 2018); Battigalli et al. (2015, 2019).
[6]For further discussion about learning foundation of selfconfirming equilibrium we remand to Section 2.

of parents. At first, in Section 3, we characterize the set of selfconfirming equilibria, showing that a higher PPE induces a higher socialization effort, in line social psychology literature. Then we discuss how the key concepts of cultural substitution and cultural complementarity change in our setting.[7] Indeed, while with complete information there is no difference between conjectures and true parameters, with incomplete information this difference is essential. We propose the definition of **conjectured cultural substitution (complementarity)**, where optimal socialization efforts are decreasing (increasing) in the conjecture about own population share, as opposed to **actual cultural substitution (complementarity)**, where optimal socialization efforts are decreasing (increasing) in own population share. Our main finding is that it is possible to obtain actual cultural complementarity in the standard cultural transmission mechanism if incomplete information is considered, as opposed to the case of complete information in which just cultural substitution is possible. Moreover, whenever there is conjectured cultural substitution (complementarity) then actual cultural complementarity (substitution) is shown. In particular, if agents underestimate the share of their traits in the society there is conjectured cultural substitution and, surprisingly, actual cultural complementarity. On the other hand, whenever agents overestimate their own group in the society we obtain multiple equilibria in which both actual cultural complementarity and substitution exist. We also show that no agent can largely overestimate own group size and still have conjectures confirmed, while the underestimation of own group size by any magnitude can be confirmed. We then study the welfare loss due to incomplete information with respect to the standard case, and show that it is increasing in the difference between the conjecture and the true parameter.

Section 4 presents the analysis of population dynamics. To model the dynamics of conjectures about population shares, we introduce a minimal model of leadership. We postulate the existence of two cultural leaders, one for each group.[8] Leaders can choose to induce a positive or a negative bias in the conjectures about population shares of their community members. Thus, the population dynamics induces a dynamics on the population shares conjectures. Each cultural leader, in order to maximize the share of the trait of her own community in the next period, at each time, chooses to instill a negative bias in agents belonging to their cultural group. The implications are particularly rich. We identify thresholds for the magnitude of biases of the two communities. Depending on the biases' magnitude with respect the threshold values, all possible social compositions can arise in the long run. In particular, if the bias of one cultural group overcomes the corresponding threshold, the equilibrium in which the whole society converges to that

[7] Cultural substitution (complementarity) captures the idea that the vertical socialization level negatively (positively) depends on the presence of own cultural trait in the population (Bisin and Verdier, 2001).

[8] In recent years the study of cultural leaders has become of interest in the cultural transmission literature, for example Verdier and Zenou (2015), Prummer and Siedlarek (2017) and Verdier and Zenou (2018)

culture is always locally stable. The most important result is that if biases of both groups do not overcome threshold values then, despite having actual cultural complementarity, there exists a stable equilibrium with cultural heterogeneity. In the opposite case an unstable polymorphic equilibrium is shown and long run cultural homogeneity is the result. This is particularly relevant because in previous cultural transmission papers with complete information (Bisin and Verdier, 2001; Cheung and Wu, 2018) cultural complementarity leads always to cultural homogeneity. We further discuss the role of different magnitude of population conjectures' bias showing that a larger negative bias translates in a higher population share in the long-run.

2 The Model

We consider, as in standard Bisin and Verdier (2001), a society composed of a continuum of agents. Each agent belongs to one cultural group. Each group is characterized by a specific discrete cultural trait. Let the set of traits be $I := \{a, b\}$ and, at each time period $t \in \mathbb{N}$, the fraction of individuals with trait $i \in I$ be $q_t^i \in [0, 1]$. We assume that in each group all agents are equal. Then, with a little abuse of notation, we refer to i as the representative agent displaying trait $i \in I$. In each period, each agent reproduces asexually giving birth to just one child. Children are born without any specific trait, and traits are acquired during the cultural transmission process. Cultural transmission from one generation to the next one is a probabilistic process that is the result of a *vertical socialization* step, that transmits parental trait to child with probability d_t^i, and an *oblique socialization* step, that transmits with probability $1 - d_T^i$ a random trait of the population. Each parent $i \in I$ exerts an effort $\tau_t^i \in \mathcal{T} := [0, 1]$ to induce own type to child in the vertical socialization step. Let $d_t^i = \varphi(\tau^i)$ where $\varphi : \mathcal{T} \to [0, 1]$ and it is increasing in τ_t^i. At the end of the socialization process each child acquires a traits, becomes an adult, reproduces asexually, and the process starts again.
For notation simplicity we now drop the time index t until Section 4, in which we study the population dynamics.

Parental Efficacy In the standard cultural transmission literature, from Bisin and Verdier (2001) on, it is assumed that, during *vertical socialization*, parents are fully effective, namely for each $i \in I$ $\varphi(\tau^i) = \tau^i$. However, as discussed in the introduction, the efficacy of parental transmission may not be perfect (Belsky et al., 1984). For this reason, we consider a generalization of $\varphi(\tau^i)$ allowing for a parental efficacy parameter $\alpha^i \in \mathbb{R}_+$, by letting $d^i = \alpha^i \tau^i$.[9] The parental efficacy is group specific. Define $S := [0, 1]^2$ and,

[9]Notice that the parameter α^i can both reduce or magnifying the parental efficacy. Even if, with a generic $\alpha^i \in \mathbb{R}_+$, we may have that $\alpha^i \tau^i > 1$, we will show later that it will never be the case so that $\varphi : \mathcal{T} \to [0, 1]$ is always true.

for each $i \in I$, let $s^i \in S$ be a generic pair (α^i, q^i). We define the consequence function mapping from the triple of effort, parental efficacy, and population shares, to transition probability p^{ii}. p^{ii} is the probability that a given parent i has a child who acquires her own trait. For each $i \in I$,

$$\begin{array}{rcl} p^{ii}: \quad \mathcal{T} \times S & \rightarrow & [0,1] \\ (\tau^i, \alpha^i, q^i) & \mapsto & \alpha^i \tau^i + (1-\alpha^i \tau^i) q^i. \end{array} \tag{1}$$

We also define $p^{ij} := 1 - p^{ii}$ that is the probability that own child is socialized to the different trait. This is given by $p^{ij} = (1-\alpha^i\tau^i)(1-q^i)$.

Incomplete information We assume that parents have *incomplete information* about own parental efficacy, α^i, and about their population share, q^i.[10] Then, each parent $i \in I$ has conjectures about s^i. $\hat{s}^i := (\hat{\alpha}^i, \hat{q}^i) \in S$ defines the pair of perceived parental efficacy (PPE) and conjectured population shares.[11] Given $i \in I$, and given $\hat{s}^i$, conjectures induce a **conjectured transition probability** function, describing what parent i conjectures to be the probability that own child is socialized to the same trait as her own.

$$\begin{array}{rcl} \hat{p}^{ii}: \quad \mathcal{T} \times S & \rightarrow & [0,1] \\ (\tau^i, \hat{\alpha}^i, \hat{q}^i) & \mapsto & \hat{\alpha}^i \tau^i + (1-\hat{\alpha}^i \tau^i) \hat{q}^i. \end{array} \tag{2}$$

As we did for transition probabilities, we define $\hat{p}^{ij} := 1 - \hat{p}^{ii} = (1-\hat{\alpha}^i\tau^i)(1-\hat{q}^i)$.

Subjective expected utility maximization We assume that each parent $i \in I$ prefers having a child of own type than one of a different type. As standard, for every $i \in I$ and $j \in I \setminus \{i\}$, we model these preferences as a vector $(V^{ii}, V^{ij}) \in [0,1]^2$ where $V^{ii} > V^{ij}$. For each $i \in I$, let $\Delta V^i := V^{ii} - V^{ij}$. Parents choose the level of socialization

[10]Note that, oblique socialization may depends on composition of the social environment the child is embedded into. However, although children may have a neighborhood composition different from the overall population shares due to homophily, in this framework we choose to ignore this issue. Even if homophily may be somehow relevant, the main contribution of this paper regards how parental cultural transmission works when oblique socialization effects, however defined, are not known to the parents, together with parental parental efficacy. In the first part of the analysis, when no population dynamics occurs, it would be indifferent to allow for homophily or not, since what is relevant is the overall effect of oblique socialization on transition probabilities. When it comes to the population dynamic, the dynamics with and without homophily would be a bit different. However, we decided to ignore this issue since the presence of homophily does not change the quality of the results, but makes the framework more complicated without changing the quality of results. For the network effect sin cultural transmission we refer to Büchel et al. (2014); Panebianco (2014); Panebianco and Verdier (2017); Hellmann and Panebianco (2018)

[11]Note that we consider here just deterministic conjectures. We can potentially have probabilistic conjectures of the form $\hat{s}^i \in \Delta(S)$. Given the structure of the utility function we describe below, for every probabilistic conjecture there exists one deterministic conjecture inducing the same subjectively optimal action. However, studying population dynamics with probabilistic conjectures is intractable. Thus we decided to focus just on deterministic conjectures.

effort using **imperfect empathy**, namely they evaluate the types of children using their own preferences (Bisin and Verdier, 2001). Assuming quadratic socialization costs, parents maximize their subjective expected utility given own conjectures. Then, for every $i \in I$ and $j \in I \setminus \{i\}$, we get the following problem

$$\max_{\tau^i \in [0,1]} \mathbb{E}^i_{\hat{p}^{ii}}[u^i] = \hat{p}^{ii} V^{ii} + \hat{p}^{ij} V^{ij} - \frac{1}{2}(\tau^i)^2. \tag{3}$$

Feedbacks and confirmed conjectures We assume that each parent $i \in I$, during the socialization process, receives some message m^i from own child and wants her conjectures not to contrast with the message received. For simplicity we assume that the message received by parents is exactly the transition probability, that is $m^i = p^{ii}$. The assumption here is that parents are able to understand, through deep communication, how much the children are convinced by parental traits in a $[0,1]$ scale, by getting some messages about how children are prone to get one trait against the other one.[12] This is, we think, a particularly relevant aspect of socialization, somehow ignored in standard literature, since parents interact, by communication and feedbacks, with children during the socialization. Notice that the message parents receive regards the overall socialization process and not only the vertical part of it. The information obtained by a parent $i \in I$ can be described by a feedback function $f : \mathcal{T} \times S \to [0,1]$. If a parent $i \in I$ receives a particular message m^i, what i can infer, conditioned on her socialization effort τ^i, is that the pairs (α^i, q^i) consistent with the message is given by $f^{-1}_{i,\tau^i}(m^i) := \{(\alpha^i, q^i) : f_i(\tau^i, \alpha^i, q^i) = m^i\}$.[13] Notably, the true parameters pair is possibly a strict subset of $f^{-1}_{i,\tau^i}(m^i)$.

Selfconfirming equilibrium We now define the equilibrium concept used in the paper. We refer to the notion of selfconfirming equilibrium that, in our setting, requires that parents produce a subjectively optimal socialization effort, and the conjectures on which maximization is based are confirmed.Let $r : S \to \mathcal{T}$ be the best response operator, so that $r(\hat{\alpha}^i, \hat{q}^i)$ is the subjectively optimal effort agent $i \in I$ exerts if she displays conjectures $(\hat{\alpha}^i, \hat{q}^i)$.[14]

Definition 1 *A profile $(\tau^i, \hat{\alpha}^i, \hat{q}^i)_{i \in I}$ of socialization choice and conjectures is a selfconfirming equilibrium at $(\alpha_i, q_i, \Delta V^i)_{i \in I}$ if, for each $i \in I$*

1. *(subjective rationality) $\tau^i \in r(\hat{\alpha}^i, \hat{q}^i)$*

[12]Below in the paper we provide an interpretation of the proposed concept of equilibrium and about confirmed conjectures in which parents update time by time their effort depending on the message sent by children, and converge to the equilibrium effort. For the time being we assume that the effort is chosen by each parent once and for all at the beginning of each period.

[13]In our model, the mechanism through which PPE ($\hat{\alpha}$) develops is the actual parental experiences; In fact, the feedback provided from adult-child interactions is considered a key determinant of the formation of PPE (Coleman and Karraker, 2000).

[14]At this stage we have not proved that the correspondence r is single-valued. However, given the concavity of the problem this is ensured and we define directly from here r as a function and not as a correspondence.

2. *(confirmed conjectures)* $(\hat{\alpha}^i, \hat{q}^i) \in f^{-1}_{i,\tau^i}(m^i)$

The first condition simply states that agents exert an effort that is an optimal response given their conjectures. The second condition implies that these conjectures must be confirmed by the message received, namely $f_i(\tau^i, \alpha^i, q^i) = f_i(\tau^i, \hat{\alpha}^i, \hat{q}^i)$. However parents just keep conjectures that are compatible with the message received. Since in our framework the dimension of the uncertainty is larger than the dimension of the signal, then f is not invertible, and multiple conjectures are compatible with the same signal. Therefore some wrong conjectures belong to the set of confirmed ones.

Learning foundation of selfconfirming equilibrium The selfconfirming equilibrium has a learning foundation that makes it particularly fit to model parental socialization choices. A selfconfirming equilibrium can be seen as the steady state of an adaptive learning dynamics in which conjectures are updated given the feedback agents receive at each period, when agents maximize their instantaneous expected utility (Battigalli et al., 1992; Fudenberg and Levine, 1993b). Notice that, as shown by Milgrom and Roberts (1991), Gilli (1999), and Battigalli (2018), the convergence to selfconfirming equilibrium does not depend on specific learning rules but holds for any possible adaptive learning. A process is consistent with adaptive learning if the agents can find a way to justify choices in terms of the past realizations. *Best Reply Dynamics*, *Fictitious Play*, and *Bayesian Learning* are all example of adaptive learning.[15] Therefore, selfconfirming equilibrium, although static, is consistent with all possible parental learning processes of this kind, and it is fit to capture the implicit process of learning that occurs during parenting.

In terms of our socialization process it is as if the socialization time is composed of infinitely many periods, and the child adopts a type at the end of all the periods. At the beginning of each period, each parent chooses her socialization choice. Then the vertical and oblique socialization schemes proceed as previously described, with each child producing a message for her parent. Each message is how much the child has been convinced, in that period, by each trait. Parents observe the message and update their conjectures, with an arbitrary adaptive learning dynamics. Then the process restarts again with a new round of vertical and oblique socialization. This mimics the fact that parents during socialization actually change their efforts depending on the feedback they can get from children. A fixed point of this adaptive learning process is a selfconfirming equilibrium.

[15] It is important to underline how selfconfirming equilibrium is the fixed point of the learning process even if agents update their conjecture as a Bayesian. This is relevant because it shows that even fully rational Bayesian parents may have, in equilibrium, wrong conjecture about their parental efficacy and population shares. Intuitively, given parents' conjectures optimal effort levels can induce outcome that confirm the previous conjectures. In this case, parents do not revise their conjectures (for a deeper discussion see for example Battigalli, 2018).

3 Characterization of Equilibra

We can now characterize the set of selfconfirming equilibria of the socialization problem.

Proposition 1 *A selfconfirming equilibrium at* $(\alpha^i, q^i, \Delta V^i)_{i \in I}$ *is a profile* $(\tau^i, \hat{\alpha}^i, \hat{q}^i)_{i \in I}$ *in which, for each* $i \in I$

$$\tau^i = \hat{\alpha}^i(1 - \hat{q}^i)\Delta V^i, \tag{4}$$

$$\hat{\alpha}^i(1 - \hat{q}^i)[\alpha^i(1 - q^i) - \hat{\alpha}^i(1 - \hat{q}^i)]\Delta V^i = \hat{q}^i - q^i. \tag{5}$$

Proof in the Appendix. □

The loci of points described by (4) and (5) stem from *subjective rationality* and *confirmed conjectures* conditions in Definition 1, respectively. The socialization effort positively depends on the perceived parental efficacy (PPE) $\hat{\alpha}^i$. This result is in line with the social psychology literature where, as outlined in Coleman and Karraker (2000), high PPE has been found to predict high parental effort and performance.[16] Notice that, although parents with trait i correctly observe p^{ii} from the messages they receive, this may not be enough to make a correct inference about the underlying parameters (α^i, q^i). Indeed, since the feedback function f is surjective, agents are not able to derive the exact parameter values but only the locus of points (α^i, q^i) consistent with the feedback as described by condition (5). We can observe that if an agent has a correct conjecture about population shares, namely $\hat{q}^i - q^i = 0$, then a conjectures pair that satisfies confirmed conjectures condition in (5) is $(\hat{\alpha}^i, \hat{q}^i) = (\alpha^i, q^i)$. In such a case parents have correct conjectures and the socialization choice described in (3) boils down to the standard Bisin and Verdier (2001) socialization choice

$$\tau^i_{bv} = \alpha^i(1 - q^i)\Delta V^i. \tag{6}$$

From now on we will use τ^i_{bv} as a benchmark for our analysis.

If, differently from our assumption, parents are perfectly able to observe their own parental efficacy ($\hat{\alpha}^i = \alpha^i$), then $\hat{q}^i = q^i$ if and only if the signal about $\hat{p}^{ii}$ is correct. As a consequence, parents may have incorrect conjectures about population share if only if they receive a wrong message about transition probabilities.[17]

We now go further in the equilibrium characterization analyzing how conjectures are shaped and the derived selfconfirming efforts differ from the standard literature ones. Figure 1 provides a generic representation of the pairs $(\hat{\alpha}^i, \hat{q}^i)$ consistent with (5).

[16] For example, responsivity to children's needs (Donovan and Leavitt, 1985; Donovan et al., 1997; Unger and Wandersman, 1985), engagement in direct parenting interactions (Mash and Johnston, 1983), and active parental coping orientations (Wells-Parker et al., 1990).

[17] This may happen if the message about p^{ii} is a random variable.

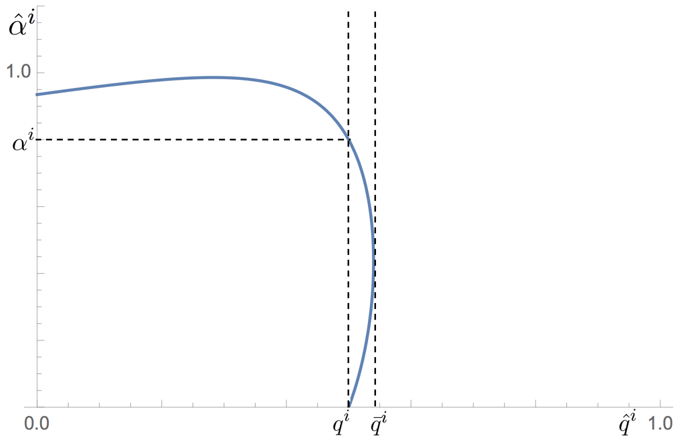


Figure 1: Loci of points $(\hat{\alpha}^i, \hat{q}^i)$ satisfying (5), with parametrization $\alpha^i = 0.8$, $\Delta V^i = 1$, and $q^i = 0.4$

Let us fix a conjecture $\hat{q}^i$. If $\hat{q}^i$ is too large with respect to q^i there is no $\hat{\alpha}^i$ compatible with the confirmed conjectures requirement. In details, defining $\bar{q}^i := q^i + \frac{(\tau^i_{bv})^2}{4\Delta V^i}$, then if $\hat{q}^i > \bar{q}^i$ no confirmed $\hat{\alpha}^i$ can be find, while if $\hat{q}^i < \bar{q}^i$ at least an $\hat{\alpha}^i$ consistent with (5) always exists. In other words, if agents strongly over-represent own group, they may find they are committing a mistake because they cannot be confirmed in their conjectures, while if they under-represent it they may not have a way to understand they are wrong. Moreover, as shown in Figure 1, if $\hat{q}^i \in [q^i, \bar{q}^i]$ there are two $\hat{\alpha}^i$ satisfying the confirmed conjectures requirement. In details, for each $i \in I$, define $\xi^i := \sqrt{4(q^i - \hat{q}^i)\Delta V^i + (\tau^i_{bv})^2}$, and solve (5) for $\hat{\alpha}^i$. Then, for each $i \in I$ there are two potential conjectures about parental efficacy, $\hat{\alpha}^i_h$ and $\hat{\alpha}^i_l$, that satisfy (5), given by the following[18]

$$\hat{\alpha}^i_h = \frac{\tau^i_{bv} + \xi^i}{2(1 - \hat{q}^i)\Delta V^i}, \; \hat{\alpha}^i_l = \frac{\tau^i_{bv} - \xi^i}{2(1 - \hat{q}^i)\Delta V^i}. \tag{7}$$

As a consequences, for each $i \in I$ and $\hat{q}^i$ there are two possible equilibrium efforts. Define $\tau^i_h := \frac{\tau^i_{bv} + \xi^i}{2}$, and $\tau^i_l := \frac{\tau^i_{bv} - \xi^i}{2}$. Next proposition characterizes the set of *selfconfirming* equilibria given any profile $(\hat{q}^i)_{i \in I}$. For each $i \in I$, define by $E_i \in \mathcal{T} \times S$ the set of selfconfirming equilibrium socialization choices and conjectures at $(\alpha^i, q^i, \Delta V^i)$.

Proposition 2 *For each $i \in I$:*

i) If $\hat{q}^i > \bar{q}^i$, it does not exist any $\hat{\alpha}^i$ satisfying the confirmed conjectures condition (5)*, and $E_i = \emptyset$;*

ii) If $q^i < \hat{q}^i < \bar{q}^i$, $E_i = \{(\tau^i_h, \hat{\alpha}^i_h, \hat{q}^i), (\tau^i_l, \hat{\alpha}^i_l, \hat{q}^i)\}$, with $\tau^i_l < \tau^i_h < \tau^i_{bv}$ and $\hat{\alpha}^i_l < \hat{\alpha}^i_h < \alpha^i$;

[18] There are some issues that we need to address. First of all if $\hat{q}^i < q^i$ $\hat{\alpha}^i_l$, while satisfying (5), is unfeasible because negative. Second, even if feasible, each of the conjectures $(\hat{\alpha}_h, \hat{q}^i)$ and $(\hat{\alpha}_l, \hat{q}^i)$ induces a subjectively optimal vertical socialization effort that needs to be feasible, $\tau^i \in [0, 1]$. In the next proposition we address both these issues.

iii) If $\hat{q}^i = q^i$, $E_i = \{(\tau^i_{bv}, \alpha^i, q^i), (0, 0, q^i)\}$*;*

iv) If $\hat{q}^i < q^i < \bar{q}^i$, $E_i = \{(\tau^i_h, \hat{\alpha}^i_h, \hat{q}^i)\}$*, with* $\tau^i_h > \tau^i_{bv}$ *and* $\hat{\alpha}^i_h > \alpha^i$*.*

Proof. in the Appendix. □.

This proposition analyzes how efforts differ with respect to the Bisin Verdier framework, and how conjectures relate to the true parameters values. As a direct consequence of the multiplicity of confirmed conjectures discussed above, we observe in some cases a multiplicity of equilibria. This is discussed in Figure 2 that summarizes the results of Proposition 2, and in which the diagonal represents the equilibrium with correct conjectures about population shares. If a group underestimates its own presence in the society, $\hat{q}^i < q^i$, the socialization choice of its members is unique and always higher than in the baseline model with complete information; on the other hand, if a group overestimates the presence of its own trait in the society, $\hat{q}^i > q^i$, there is room to two different conjectures and two different selfconfirming socialization choices, both below to the Bisin-Verdier benchmark. This multiplicity derives from the multiplicity of confirmed conjectures discussed above. Consider now the case of $\hat{q}^i < q^i$ where the equilibrium is unique. In this case, since $\hat{\alpha}^i_l < 0$ and thus unfeasible, only one equilibrium is possible. When $q^i = \hat{q}^i$ there are two equilibria. In the first one correct conjectures about population shares are paired with correct conjectures about parental efficacy, and thus τ^i_{bv} is displayed. In the second one, correct conjectures about population shares are paired with parents thinking to be not effective at all in vertical transmission ($\hat{\alpha}^i = 0$). This would induce a null socialization effort. Then the message parents receive is $p^{ii} = q^i$ and correct conjectures about population shares are displayed. Notice that $\hat{\alpha}^i > \alpha^i$ if and only if $\hat{q}^i < q^i$. Then in equilibrium parents can underestimate their parental efficacy only overestimating own group share in the society, and viceversa.

3.1 Cultural Complementarity and Substitution

One of the main results of the standard models of cultural transmission with complete information is the **cultural substitution** property. This property requires that optimal socialization efforts are decreasing in own population shares. There is still debate about how to get cases in which the opposite property, **cultural complementarity**, holds. The two properties are important since they have different consequences in terms of the derived population dynamics and the stability of polymorphic equilibria. In the case of complete information, cultural complementarity and substitution are unambiguously defined since parents, having correct conjectures, react to actual population shares. In the case of incomplete information, effort decisions directly depend on conjecture $\hat{q}^i$. On the other hand, q^i indirectly affects the socialization effort τ^t since, given $\hat{q}^i$, the confirmed conjecture $\hat{\alpha}^i$ depends on the true q^i. Therefore it is worth defining the concept of cultural

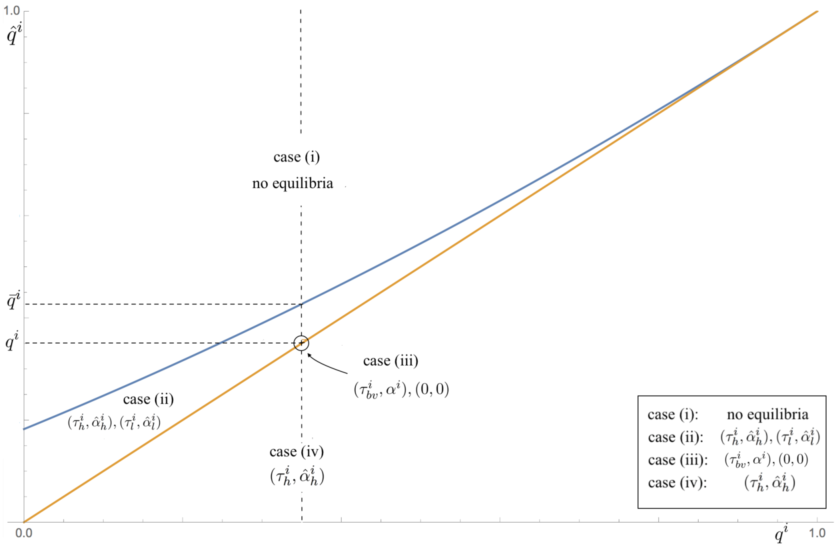


Figure 2: Set of selfconfirming equilibria for different $(q^i, \hat{q}^i) \in [0,1]^2$, at $\alpha^i = \Delta V^i = 0.9$

substitution or complementarity properties with respect to both the true parameter q^i and the conjecture $\hat{q}^i$.

Definition 2 *For each $i \in I$ and for each $\tau^i \in \{\tau_l^i, \tau_h^i\}$,*

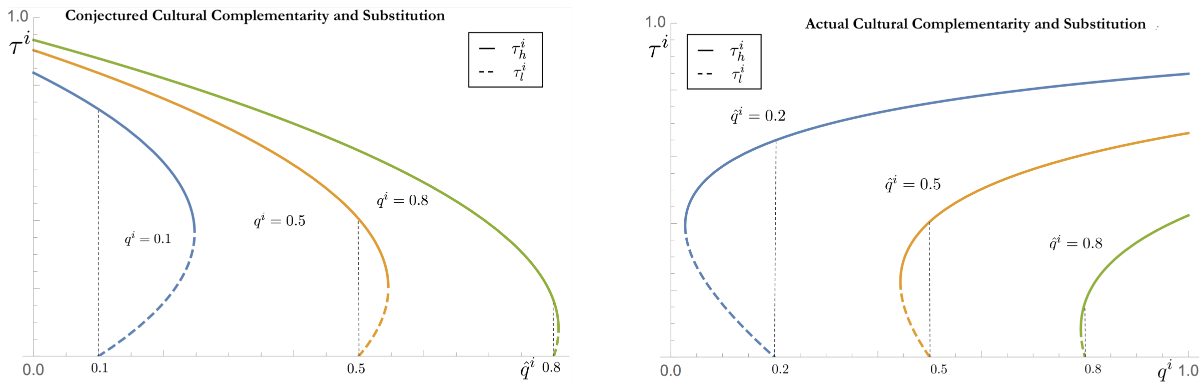
- *τ^i displays* **actual *cultural substitution (complementarity)*** *if it is decreasing (increasing) in q^i.*
- *τ^i displays* **conjectured *cultural substitution (complementarity)*** *if it is decreasing (increasing) in $\hat{q}^i$.*

This distinction is relevant because actions are motivated by conjectures, but a potential policy maker is likely to observe actual population shares. Next proposition describes when equilibrium socialization choices display actual and conjectured cultural complementarity or substitution.

Proposition 3 *For each $i \in I$, τ_h^i displays actual cultural complementarity and conjectured cultural substitution. τ_l^i displays actual cultural substitution and conjectured cultural complementarity.*

Proof. in the Appendix $\square$.

It is possible to see results of Proposition 3 in Figure 3. In Figure 3a we consider the *conjectured* cultural complementarity and substitution. If $\hat{q}^i < q^i$ the unique equilibrium socialization choice is negatively related to $\hat{q}^i$, so that there is conjectured cultural substitution. This result is not surprising since, in all models of cultural transmission, we observe socialization effort to be decreasing in what people consider to be own population share. However, if $\hat{q}^i > q^i$ we have already seen that there are two possible equilibria



(a) (b)

Figure 3: Socialization choice with respect to: (a) $\hat{q}^i$, with $q^i \in \{0.1, 0.5, 0.9\}$, and (b) q^i, with $\hat{q}^i \in \{0.2, 0.5, 0.8\}$. For both panels, $\alpha^i = \Delta V^i = 0.9$

and we observe both cultural substitution and complementarity associated respectively to the high and low socialization choices. In fact, if parents have the the low efficiency conjecture $\hat{\alpha}_l^i$ then, when $\hat{q}^i > q^i$, they choose τ_l^i and an increase in their conjecture leads to an increase in the socialization choice, so that cultural complementarity is shown. The opposite occurs with the higher socialization effort.
In Figure 3b, we consider *actual* cultural complementarity and substitution. When $\hat{q}^i < q^i$ the unique equilibrium τ_h^i exhibits actual cultural complementarity. This result should be interpreted keeping in mind that, fixing the conjecture $\hat{q}^i$, the effect of q^i on the socialization choice is indirect and passes through the conjecture $\hat{\alpha}_h^i$. Using the learning interpretation of the equilibrium, if q^i increases parents can misinterpret the feedback they receive and attribute the increase of p^{ii} to an increase on $\hat{\alpha}_h^i$. Keeping fixed their conjecture $\hat{q}^i$ they exert higher socialization effort.

3.2 Transition Probabilities

We have seen how incomplete information about the true parameters may produce multiple selfconfirming equilibria. In this section we are interested in studying the transition probabilities stemming from selfconfirming equilibria, and compare them with the benchmark Bisin-Verdier transition probability.

From the evaluation of the consequence function p^{ii} in (1) at the two possible socialization efforts τ_h^i and τ_l^i, we get

$$p_h^{ii*} = q^i + \alpha^i(1-q^i)\frac{\tau_{bv}^i + \xi^i}{2}, \quad p_l^{ii*} = q^i + \alpha^i(1-q^i)\frac{\tau_{bv}^i - \xi^i}{2}. \tag{8}$$

For each $i \in I$, let p^{ii*} the equilibrium transition probability.

Proposition 4 *For each $i \in I$:*

- *If $\hat{q}^i < q^i$, $p^{ii*} = p_h^{ii}$ with $p^{ii*} > p_{bv}^{ii}$;*
- *If $\hat{q}^i > q^i$, $p^{ii*} \in \{p_h^{ii}, p_l^{ii}\}$ with $p_l^{ii} < p_h^{ii} < p_{bv}^{ii}$.*

Proof. in the Appendix $\square$.

The presence of multiple equilibria when $\hat{q}^i > q^i$, induces a bifurcation in the population dynamics through the multiplicity of the induced transition probabilities. To avoid this problem of the dynamics, we define a way to select one of the two transition probabilities at each period. Recall that the multiplicity of transition probability derives from the presence of two feasible conjectures $\hat{\alpha}_h^i$ and $\hat{\alpha}_l^i$. Notice that $\hat{\alpha}_h^i$ and $\hat{\alpha}_l^i$ provide different levels of subjective expected utility, since they induce different effort levels and different transition probabilities. In particular the following result holds.

Proposition 5 *For each $i \in I$, and for each triple $(\alpha^i, q^i, \hat{q}^i)$ for which the set of selfconfirming equilibria is $E_i = \{(\tau_h^i, \hat{\alpha}_h^i, \hat{q}^i), (\tau_l^i, \hat{\alpha}_l^i, \hat{q}^i)\}$, then $\mathbb{E}^i_{p_h^{ii}}[u^i(q^i)] > \mathbb{E}^i_{p_l^{ii}}[u^i(q^i)]$.*

Proof. in the Appendix $\square$.

Since parents know their subjective expected utility functional form, and since they want to maximize it, they may be conscious that choosing $\hat{\alpha}_h^i$, as opposed to $\hat{\alpha}_l^i$, provides a higher level of subjective expected utility, while both satisfying the confirmed conjectures condition. Then, we assume that each parent chooses $\hat{\alpha}_h^i$.[19] Notice that $\hat{\alpha}_h^i$ also induces higher effort level and higher transition probability than $\hat{\alpha}_l^i$.

Assumption 1 *For each $i \in I$, if $q^i < \hat{q}^i$, $E_i = \{\tau_h^i, \hat{\alpha}_h^i, \hat{q}^i\}$.*

This equilibrium selection rule induces continuity in the effort with respect to conjectured population shares. Indeed, fix an $\epsilon \in \mathbb{R}_+$ arbitrarily small, and consider $\hat{q}'^i := q^i - \epsilon$. Then $\tau_h^i(\hat{q}'^i)$ is chosen. If parents experience a little increase in their conjectures of magnitude 2ϵ, the new conjecture is $\hat{q}''^i := q^i + \epsilon$. Thus the effort choice is likely to be not far from the previous one. Then $\tau_h^i(\hat{q}''^i)$ is more likely to be chosen than $\tau_l^i(\hat{q}''^i)$.
Notice also that given the equilibrium selection process, since τ_h^i is chosen, conjectured cultural substitution is always displayed. This equilibrium selection also induces continuity of socialization effort on actual population shares, for the same reasoning explained above but applied to actual rather than to conjectured population share. At last, by Proposition 3 we always observe actual cultural complementarity. This is a main difference with respect to literature about cultural transmission with complete information

[19]Notice that this equilibrium selection is corroborated by social psychology literature. Indeed, higher PPE is empirically associated with lower parental depression, anxiety, and stress, therefore higher parent's utility, and has positive effect on the development of the offspring (Jackson and Huang, 2000; Kuhn and Carter, 2006).

where parents' choices always display cultural substitution. Relatedly, with incomplete information parents' choices display conjectured cultural substitution and actual cultural complementarity. Figure 4 shows how the transition probability changes for different values of population shares.

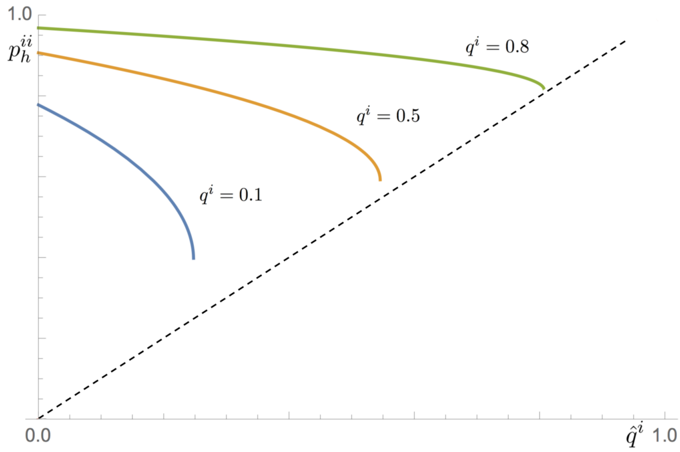


Figure 4: Transition probabilities with respect $\hat{q}^i$ with $q^i \in \{0.1, 0.5, 0.8\}$

3.3 Welfare with Wrong Conjectures

Before moving to the analysis of the population dynamics, we study how much having wrong but confirmed conjectures may cost in terms of utility loss. Whenever parents have correct conjectures they produce an effort that is optimal to the environment. Define $u^i_{bv} := p^{ii*}_{\tau^i_{bv}} V^{ii} + (1 - p^{ii*}_{\tau^i_{bv}}) V^{ij} - \frac{1}{2}(\tau^i_{bv})^2$ as the average utility realized by i agents at parameters (α^i, q^i) when they have correct conjectures and perform the Bisin-Verdier effort, τ^i_{bv}. This is our benchmark, and in this case ex-ante expected utility coincides with the average realized one. However, when parents misperceive the population shares, the equilibrium effort is ex-ante subjectively optimal but ex-post suboptimal. Define $u^i_{\hat{q}^i} := p^{ii*}_{\tau^i_h} V^{ii} + (1 - p^{ii*}_{\tau^i_h}) V^{ij} - \frac{1}{2}(\tau^i_h)^2$ as the average utility realized by i agents at parameters (α^i, q^i) when they have conjectures $(\hat{\alpha}^i_h, \hat{q}^i)$ and exert effort τ^i_h. Notice however that parents do not know the correct parameter q^i, otherwise they would have used it in the decision process. Then they cannot compute how much is their welfare loss associated with wrong conjectures. A policymaker, however, knows the correct parameters and compute the loss in terms of ex post utility when agents use wrong conjectures.[20] Let $\Delta u^i_{\hat{q}^i} := u^i_{bv} - u^i_{\hat{q}^i}$ be the average loss i agents experience from wrong conjectures. Then

Proposition 6 *For each $i \in I$, $\Delta u^i_{\hat{q}^i} = \frac{1}{8}(\tau^i_{bv} - \xi^i)^2 > 0$.*

[20]Notice that, while for an individual parent the realized utility depends on the realization of the socialization process, when talking about representative agents of a group, as we do in this paper, the utility function is exactly the average utility parents of type i gets, and the policy maker just looks at this.

Proof. in the Appendix □

To inspect the properties of the loss it is worth looking at Figure 5 to see how it changes with respect to conjectures. The loss function is convex in q^i, with a minimum in $q^i = \hat{q}^i$.

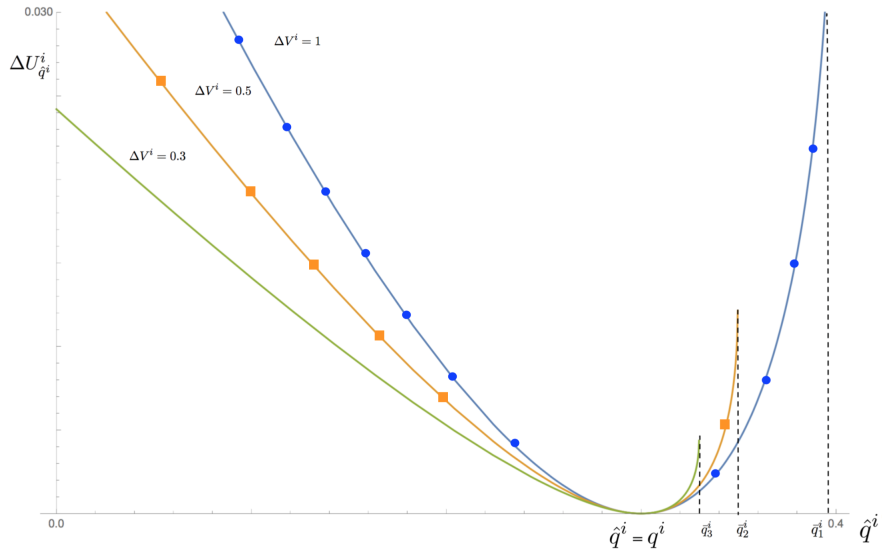


Figure 5: Average loss $\Delta u^i_{\hat{q}^i}$ at $\Delta V^i \in \{0.3, 0.5, 1\}$, with parametrization $q^i = 0.3$ and $\alpha^i = 0.9$.

In details, in $q^i = \hat{q}^i$ it reaches, by definition, a null value. Interestingly, it is steeper for positive biases about population shares' conjectures than for negative ones. This is due to the fact that a negative bias induces a higher effort than a positive bias, and the induced transmission probability partially counterbalances the loss. Another source of asymmetry is given by different levels of ΔV^i. In details, given a downward biased $\hat{q}^i$, more intolerant agents experience a higher loss, while this relation is reverted for positive biases. The welfare loss of the whole society, given $\hat{q}^i$ and $\hat{q}^j$, is $\frac{1}{8}[q^i(\tau^i_{bv} - \xi^i)^2 + (1 - q^i)(\tau^j_{bv} - \xi^j)^2]$. The previous results have two main consequence. First of all, since ξ^i positively depends on $(q^i - \hat{q}^i)$, fixing a bias $|q^i - \hat{q}^i|$, agents have a lower loss by under-perceiving than over-perceiving own population share. Intuitively this depends on the fact that, when parents under-perceive the presence of their own cultural traits in the society, by conjectured cultural substitution, they exert higher effort and this has a positive effect on $p^{ii*}_{\tau_h}$. Second, whenever agents of a group under-perceive their own presence in the society, the more intolerant agents are the ones who suffer a higher welfare loss, that is the loss is increasing in ΔV^i. This is due to the convexity of the cost function; indeed fixing conjecture $\hat{q}^i$, the most intolerant parents exert higher effort. The associated higher cost dominates the gains on the transition probability side.

4 Long-run Dynamics

In this section we study the consequences of incomplete information for the long run dynamics of population traits. We find that incomplete information may totally revert standard predictions of cultural transmission literature. Incomplete information about population shares is a particularly relevant issue since these conjectures can be shaped by social media, (fake) news, and exploited by cultural leaders. We introduce the presence of a cultural leader for each group who instill (possibly biased) conjectures about population shares in own group. Conversely, conjectures about parental efficacy are strictly related to the parent-child relationship, and then we assume that each parent has some (possibly wrong) conjecture about own PE, that cannot be shaped by the policy maker. We discuss how, when PPE differs from PE, the bias in the population shares conjectures induced by the cultural leaders affects the long run dynamics of the population.

We introduce time indexes for all the quantities. In details, for each time $t \in \mathbb{N}$, the equilibrium transition probabilities are given by p_t^{ii*} and p_t^{jj*}. Notice that equilibrium transition probabilities are defined given conjectures $\hat{q}_t^i$ and $\hat{q}_t^j$ and the derived $\hat{\alpha}_{h,t}^i$ and $\hat{\alpha}_{h,t}^j$. We first set the dynamic system. Recalling that, for each $i \in I$ and $t \in \mathbb{N}$, $d_t^{i*} := \alpha_t^i \tau_t^{i*}$ the population share of type i in $t+1$ is described by the following law

$$\begin{aligned} q_{t+1}^i &= p_t^{ii*} q_t^i + p_t^{ji*}(1 - q_t^i) && (9) \\ &= q_t^i[1 + (d_t^{i*} - d_t^{j*})(1 - q_t^i)]. && (10) \end{aligned}$$

Using a continuous time approximation we get

$$\dot{q}_t^i = q_t^i(1 - q_t^i)(d_t^{i*} - d_t^{j*}). \tag{11}$$

At this stage, for group $i \in I$, we introduce a cultural leader ℓ^i.[21] Leaders can instill a positive or a negative bias in the perception of their community members, but cannot control the intensity of biases, which we consider exogenous and constant along all the dynamics. For each $i \in I$, intensity of the bias if given by $\beta^i \in (0, 1)$.[22] Thus leader ℓ^i chooses an action $a_t^i \in \{p, n\}$, where p stands for positive and n stands for negative bias. Define an indicator function $\mathbb{I}_{a^i}$ which takes value 0 if the leader i chooses $a_t^i = n$ and 1 if she chooses $a_t^i = p$. Then, given β^i, we define biased conjectures for community i as

$$\hat{q}_{\beta,t}^i := \beta^i \mathbb{I}_{a_t^i} + (1 - \beta^i) q_t^i. \tag{12}$$

[21] Refer to these previous works about cultural leaders: Nteta and Wallsten (2012), Acemoglu and Jackson (2014),Verdier and Zenou (2015), Prummer and Siedlarek (2017), Verdier and Zenou (2018)

[22] Endogenizing the intensity of the bias β^i does not bring any meaningful insight.

Thus, if a leader chooses a positive bias the derived conjecture about population share will be $\hat{q}^i_{\beta,t} = \beta + (1-\beta)q^i_t > q^i_t$; if the leader choses a negative bias, then $\hat{q}^i_{\beta,t} = (1-\beta)q^i_t < q^i_t$. It follows that according to (12), the conjectures about population shares coevolve with q^i_t.[23] At each time t, both leaders are not forward looking and are interested only in maximizing the presence of their trait in the society one period ahead.[24] Therefore for each group $i \in I$, its leader maximizes the following

$$\max_{a^i_t \in \{p,n\}} \quad u^{\ell^i}_t(a^i_t, a^j_t) = q^i_{t+1}(a^i_t, a^j_t). \tag{13}$$

Proposition 7 *A leader who faces the maximization problem* (13) *always chooses* $a^i_t = n$. *Then* $\hat{q}^i_{\beta,t} = (1-\beta^i)q^i_t$.

Proof. in the Appendix □

Following Proposition 7, cultural leaders, in order to maximize the share of the trait of own community in the next period, always choose to instill a negative bias. This is a simple result that derives from the fact that choosing to instill a negative bias is a dominant strategy. Indeed, a negative bias induces a higher socialization effort (due to the conjectured cultural substitution property) and a higher share of own type in next generation than choosing a positive bias.[25] To make it clear that the agents' choices, and thus the dynamics, depend on the biases instilled by leaders on population shares conjectures, we write the dynamics in (11) as

$$\dot{q}^i_t = q^i_t(1-q^i_t)(d^{i*}_{\beta,t} - d^{j*}_{\beta,t}), \tag{14}$$

in which we index equilibrium quantities with the bias β. Define $Q^{ss} := \{q \in [0,1] : \dot{q}_t = 0\}$ and the following thresholds:

$$\bar{\beta}^i := \frac{\alpha^{j4}}{\alpha^{i2}}\frac{\Delta V^{j2}}{\Delta V^i}, \; \bar{\beta}^j := \frac{\alpha^{i4}}{\alpha^{j2}}\frac{\Delta V^{i2}}{\Delta V^j}. \tag{15}$$

Proposition 8 *Given that, for each* $i \in I$ *and* $t \in \mathbb{N}$, $a^i_t = n$, *then for all* $q^i \in [0,1]$ *the dynamics* (14) *is well defined. Moreover,* $\{0,1\} \subseteq Q^i_{ss}$ *and there exists at most one polymorhic* $q^* \in Q_{ss} \cap (0,1)$. *For each* $i \in I$,

[23] In Appendix B we show what happens if conjectures about population shares do not evolve along time and prove that this may not be sustainable in the long run with these conjectures being confirmed.

[24] We are assuming that leader are agents themselves why die at the end of each period.

[25] We have assumed that cultural leaders are myopic in their decisions to choose their group's bias, thus we implicitly assume that leaders are individuals with a finite life. We can also think about leaders as centralized cultural institutions with a more forward looking perspective. In that case, the game between cultural leaders would be a differential game. However, results do not change. The space of action is dichotomous, positive or negative bias. Moreover, as shown in Section 3.1, with negative biases there is always cultural complementarity in socialization effort, thus a negative bias leads to a higher parental effort. For these reasons, maximizing myopically the presence of leader's trait in the society is equivalent to maximize the long-run one.

- $q^i = 1$ ($q^i = 0$) *is stable if and only if* $\beta^i > \bar{\beta}^i$ ($\beta^j > \bar{\beta}^j$)*;*
- q^* *is stable if and only if* $\beta^i < \bar{\beta}^i$ *and* $\beta^j < \bar{\beta}^j$*.*

Proof. in the Appendix □

Proposition 8 states that, depending on primitive parameters of the model $(\alpha^i, \Delta V^i)_{i \in I}$, and bias profile $(\beta^i)_{i \in I}$, the cultural dynamics can show both stable and unstable polymorphic equilibria, and also no polymorphic equilibria at all.
Consider Figure 6 that describes all the possibile cases that can happen in the dynamics. If the negative biases are both large (part I of the graph), then an unstable polymorphic equilibrium is shown. This derives from the fact that for large negative biases, each group thinks to be extremely smaller than what it is in reality. Then, as we have already seen in previous sections, while we have conjectured cultural substitution, our model predicts actual cultural complementarity, and then unstable polymorphic equilibrium is observed. The opposite occurs for small negative biases (part III of the graph). In this case agents have almost correct conjectures, and then results in the standard model with complete information are good proxies of the dynamics. If, on the contrary, the biases are unbalanced, then one group takes over the other and invades the society.
If there is parameters homogeneity, i.e. $\alpha^i = \alpha^j$, $\Delta V^i = \Delta V^j$, and $\beta^i = \beta^j$, then the dynamics have always a polymorphic equilibria, either stable or unstable (I and III in Figure 6). This is due to the fact that in this case $\bar{\beta}^i = \bar{\beta}^j$. Thus, all pairs (β^i, β^j) lay on the main diagonal and thus only intersect areas I and III of Figure 6. Notice that when there is heterogeneity in parameters then the biases β^i and β^j determine which of the four possible outcomes is selected (Figure 6).

Focus on the difference between the population dynamic with incomplete information as compared to the case in which agents have complete information. In the standard case (as in Bisin and Verdier, 2001; Cheung and Wu, 2018), cultural substitution always leads to cultural heterogeneity while cultural complementarity leads to cultural homogeneity. In an incomplete information setting, however, in spite of having actual cultural complementarity in socialization choices, a stable equilibrium with cultural heterogeneity con exist. Notice that (15) shows us that the more a cultural group is intolerant or efficient in parenting, the more the threshold of the other group is higher. This implies that the set of possibile biases such that the dominance of the other group in the long run is a globally stable equilibrium shrinks. Moreover, starting from the case of a stable polymorphic equilibrium ($\beta^i < \bar{\beta}^i$ and $\beta^j < \bar{\beta}^j$) and keeping β^j fixed, the more β^i grows and approaches $\bar{\beta}^i$, the more the polymorphic equilibrium moves to the right increasing the share of agents belonging to i in the society.

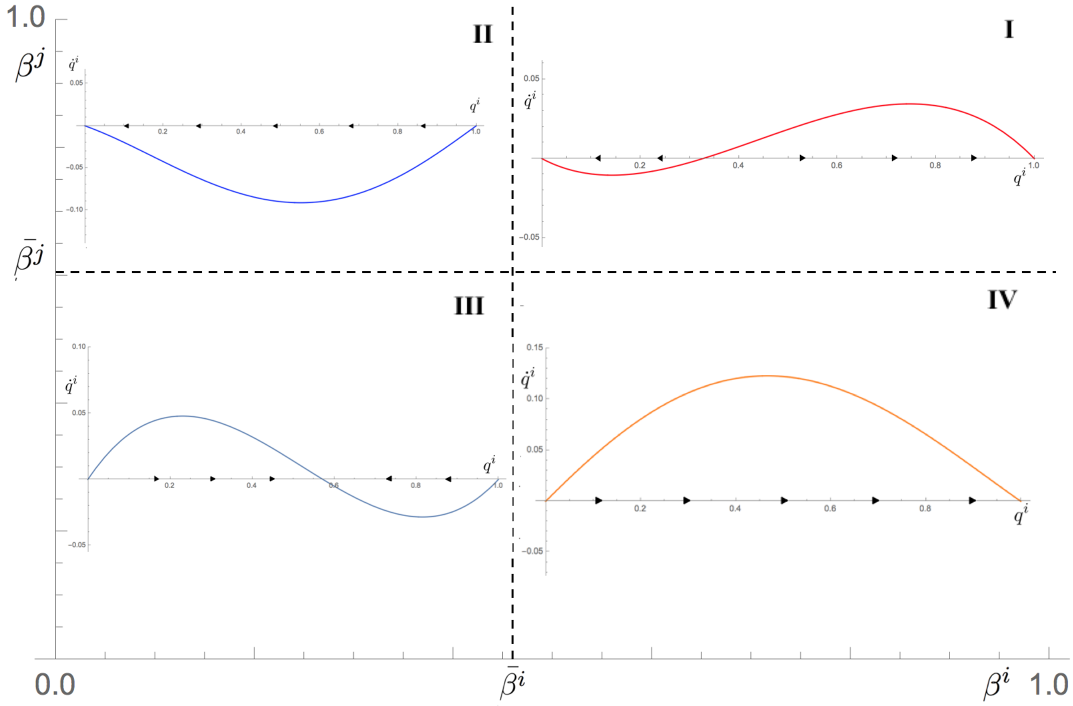


Figure 6: Cultural dynamics $\dot{q}^i$ for different β^i, β^j with $\alpha^i = \alpha^j = 1$, $\Delta V^i = 0.55$, $\Delta V^j = 0.5$.

Let us now consider a society in which one of the two leaders (ℓ_i) is identitarian and maximizes (13), while the other leader (ℓ_j) is non-identitarian, or group utilitarian, and maximize the welfare (as defined in Section 3.3). The identitarian leader decides to instill a negative bias, while the non-identitarian one truthfully reports the true population shares. In such a situation, looking at Figure 6, with ℓ_j being the non-identitarian leader, then $\beta^j = 0$. Thus the only two possible social outcomes are in areas III and IV. Depending on β^i a stable polymorphic equilibrium or a globally stable homomorphic equilibrium where group i dominates can exist. In particular, we can see that, as in the previous case, the more the agents belonging to the group with identitarian leader are intolerant (ΔV^i) or effective as parents (α^i), the larger is the set of possibile biases β^i such that $q^i = 1$ is a globally stable equilibrium.

5 Conclusion

This paper generalizes the classical cultural transmission model proposed by Bisin and Verdier (2001) to an incomplete information setting. Using selfconfirming equilibrium as solution concept, we show that if parents are not fully aware of own group size and about the parental efficacy, they may end up to sustain wrong conjectures about both quantities. While in the standard setting there is no difference between conjectures and true parameters, with incomplete information this difference is crucial. Thus, we propose the definition of “conjectured cultural substitution (complementarity)” as opposed to the “actual cultural substitution (complementarity)”. The main finding is that, with incomplete information, it is possible to obtain “actual cultural complementarity” in the standard cultural transmission mechanism. In the long run we are able to reproduce all the possible social outcome depending on the magnitude of bias about the population share. The most interesting result is that, in spite of having actual cultural complementarity in socialization choice, it can exist, in the long run, a stable equilibrium with cultural heterogeneity. This result is particularly relevant because in previous cultural transmission papers, cultural complementarity leads always to cultural homogeneity.

We are convinced that the approach we propose can be fruitfully used to analyze different theoretical and applied cultural transmission issues. From a theoretical perspective, one of the first source of information incompleteness is the ignorance of parents about the specific friendships network children are exposed to. Our approach can thus inform the growing literature analyzing network effects in cultural transmission. From an applied perspective, the framework we propose is strictly related to the present debate about the spread of fake news agents are not able to detect as such. Finally, a specific focus on leaders exploiting incomplete information to induce specific cultural dynamics, can shed some light on the debate about populism.

A Proofs of Propositions

A.1 Proof of Proposition 1

Substituting (2) in (3) and solving the maximization problem, we can see that *subjective rationality* condition is satisfied by (4):

$$\tau^i = \hat{\alpha}^i(1-\hat{q}^i)\Delta V^i,$$

where $\Delta V^i = V^{ii} - V^{ij}$ represents the *cultural intolerance* of a parent of trait i.
Consider now the *confirmed conjectures* condition, and since $m^i = p^{ii}$ (i.e. $f = g$) then the conjectures are confirmed if and only if

$$\alpha^i\tau^i + (1-\alpha^i\tau^i)q^i = \hat{\alpha}^i\tau^i + (1-\hat{\alpha}^i\tau^i)\hat{q}^i. \tag{16}$$

Therefore substituting (4) into (16) we can see how conjectures that support *selfconfirming* equilibria are described by (5):

$$(\hat{\alpha}^i, \hat{q}^i): \quad \hat{\alpha}^i(1-\hat{q}^i)\Delta V^i[\alpha^i(1-q^i) - \hat{\alpha}^i(1-\hat{q}^i)] = \hat{q}^i - q^i.$$

□

A.2 Proof of Proposition 2

From equation (5) we get the following condition on $\hat{\alpha}^i$ and $\hat{q}^i$

$$\hat{\alpha}^i\alpha^i(1-q^i)(1-\hat{q}^i)\Delta V^i - \hat{\alpha}^{i2}(1-\hat{q}^i)^2\Delta V^i + (q^i - \hat{q}^i) = 0.$$

Solving for $\hat{\alpha}^i$ we get

$$\hat{\alpha}^i = \frac{\alpha^i(1-q^i)\Delta V^i \pm \sqrt{4(q^i-\hat{q}^i)\Delta V^i + (\alpha^i(1-q^i)\Delta V^i)^2}}{2(1-\hat{q}^i)\Delta V^i} \tag{17}$$

then an $\hat{\alpha}^i \in \mathbb{R}_+$ exists if and only if $4(q^i-\hat{q}^i)\Delta V^i + (\tau^i_{bv})^2 \geq 0$. Let $\bar{q}^i := q^i + \frac{\alpha^{i2}(1-q^i)^2\Delta V^i}{4}$. Then, the previous inequality is satisfied if and only if $\hat{q}^i \leq \bar{q}^i$. This proves point (i).

Substituting (17) in (4) we get the optimal socialization efforts:

$$\tau^i = \frac{\alpha^i(1-q^i)\Delta V^i \pm \sqrt{4(q^i-\hat{q}^i)\Delta V^i + (\tau^i_{bv})^2}}{2}. \tag{18}$$

Define $\xi^i := \sqrt{4(q^i - \hat{q}^i)\Delta V^i + (\tau^i_{bv})^2}$. We can write (18) as

$$\tau^i = \frac{\tau^i_{bv} \ \pm \ \xi^i}{2} \tag{19}$$

Notice that $\xi^i \lesseqgtr \tau^i_{bv}$ if and only if $q^i \lesseqgtr \hat{q}^i$.

We now need to verify that $\tau^i \in [0,1]$. Define

$$\tau^i_h := \frac{\tau^i_{bv} \ + \ \xi^i}{2}, \quad \tau^i_l := \frac{\tau^i_{bv} \ - \ \xi^i}{2} \tag{20}$$

Consider first τ^i_h. It is trivial to see that $\tau^i_h > 0$. We now prove that $\tau^i_h \leq 1$. Indeed, $\frac{\tau^i_{bv} \ + \ \xi^i}{2} < 1$ can be written as

$$\frac{\sqrt{4(q^i - \hat{q}^i)\Delta V^i + (\tau^i_{bv})^2}}{2} \leq 1 - \frac{\tau^i_{bv}}{2}.$$

We can perform some simple algebra to get the following steps:

$$\begin{aligned}
\sqrt{4(q^i - \hat{q}^i)\Delta V^i + (\tau^i_{bv})^2} &\leq 2 - \tau^i_{bv} \\
4(q^i - \hat{q}^i)\Delta V^i + (\tau^i_{bv})^2 &\leq 4 + (\tau^i_{bv})^2 - 4\tau^i_{bv} \\
4(q^i - \hat{q}^i)\Delta V^i &\leq 4 - 4\tau^i_{bv} \\
(q^i - \hat{q}^i)\Delta V^i &\leq 1 - \tau^i_{bv} \\
q^i - \hat{q}^i &\leq \underbrace{\frac{1}{\Delta V^i}}_{\geq 1} - \underbrace{\alpha(1 - q^i)}_{\leq 1}.
\end{aligned}$$

Since the right hand side is always positive, the last inequality is always satisfied if $\hat{q}^i > q^i$. We now have to verify if the inequality holds when $\hat{q}^i < q^i$. Rewriting the inequality as $q^i\,(1-\alpha) - \hat{q}^i \leq \frac{1}{\Delta V^i} - \alpha$. The left-hand side is maximized when $q^i = 1$ and $\hat{q}^i = 0$, computing the inequality in that point we get $1 - \frac{1}{2}\alpha \leq \frac{1}{\Delta V^i} - \frac{1}{2}\alpha$, which is always true, implying that $\tau_h \in [0,1]$. Thus conditions (ii) and (iv), in their part regarding τ^i_h, are always satisfied.

We now verify under which condition $\tau^i_l \in [0,1]$.
Since $\tau^i_l \leq \tau^i_h \leq 1$ it is enough to verify conditions for $\tau^i_l > 0$. Given the definition, τ^i_l is

positive if and only if

$$\alpha(1-q^i)\sqrt{\Delta V^i} \geq \sqrt{4(q^i-\hat{q}^i)+\alpha^2(1-q^i)^2\Delta V^i}$$
$$\alpha^2(1-q^i)^2\Delta V^i \geq 4(q^i-\hat{q}^i)+\alpha^2(1-q^i)^2\Delta V^i$$

$$\Rightarrow \tau_l^i \in [0,1] \iff \hat{q}^i \geq q^i$$

This, jointly with previous argument about τ_h^i, proves points (ii) and (iv). In the end, it is trivial to see by (18) that whenever $\hat{q}^i = q^i$ then $\tau^i = \tau_h^i = \tau_{bv}^i$, which proves point (iii).

$\square$

A.3 Proof of Proposition 3

Recall from Proposition 2 that, for each $i \in I$, if $\hat{q}^i < q^i$ then there exists $\tau_h^i \in [0,1]$ but $\tau_l^i \notin [0,1]$, while if $\bar{q}^i > \hat{q}^i > q^i$ then $(\tau_h^i, \tau_l^i) \in [0,1]^2$.
Consider first the case of Conjectured cultural substitution. This holds if and only if

$$\frac{\partial \tau_h^i}{\partial \hat{q}^i} = -\frac{\Delta V^i}{\sqrt{4(q^i-\hat{q}^i)+\alpha^{i2}(1-q^i)^2\Delta V^i}} < 0$$

that is always true. We show that in this case there is Actual cultural complementarity. Indeed, actual cultural complementarity is shown if

$$\frac{\partial \tau_h^i}{\partial q^i} = \frac{1}{2}\Delta V^i\left(\frac{2-\alpha^{i2}(1-q^i)\Delta V^i}{\sqrt{\Delta V^i(4(q^i-\hat{q}^i)+\alpha^{i2}(1-q^i)^2\Delta V^i)}} - \alpha\right) > 0$$

that is

$$2-\alpha^{i2}(1-q^i)\Delta V^i - \alpha\sqrt{\Delta V^i(4(q^i-\hat{q}^i)+\alpha^{i2}(1-q^i)^2\Delta V^i)} \geq 0$$

Since this function is always decreasing in both α^i and ΔV^i and increasing in $\hat{q}^i$ then it reaches its minimum at $\alpha^i = \Delta V^i = 1$ and $\hat{q}^i = 0$. We then check that

$$2-(1-q^i)-\sqrt{4q^i+(1-q^i)^2} \geq 0$$

that is

$$2 \geq (1-q^i) \pm (1+q^i) = 2$$

which is always true. Thus $\frac{\partial \tau_h^i}{\partial q^i} \geq 0$ which proves actual cultural complementarity.

Consider now the case of conjectured cultural complementarity.

$$\frac{\partial \tau_l^i}{\partial \hat{q}^i} = \frac{\Delta V^i}{\sqrt{4(q^i-\hat{q}^i)+\alpha^{i2}(1-q^i)^2\Delta V^i}} > 0$$

that is always satisfied, Moreover, in this case, actual cultural substitution is satisfied if

$$\frac{\partial \tau_h^i}{\partial q^i} = \frac{1}{2}\Delta V^i \left(-\alpha - \frac{2 - \alpha^{i2}(1-q^i)\Delta V^i}{\sqrt{\Delta V^i(4(q^i - \hat{q}^i) + \alpha^{i2}(1-q^i)^2 \Delta V^i)}} \right) < 0$$

which is always true.

A.4 Proof of Proposition 4

Recall that

$$p^{ii*} = \alpha^i \tau^i + (1 - \alpha^i \tau^i) q^i$$

plugging the definition of τ_h^i and after a little algebra we get

$$p^{ii*} = q^i + \frac{\tau_{bv}^i \pm \xi^i}{2} \alpha^i (1 - q^i).$$

By Proposition 2 we know that if $\hat{q}^i < q^i$ then $\tau_l^i = \frac{\tau_{bv}^i - \xi^i}{2}$ does not exist, thus p^{ii*} is unique and, if $\hat{q}^i > q^i$, there are two p^{ii*}: (i) p_h^{ii*} associated to τ_h^i and (ii) p_l^{ii*} associated to τ_l^i. Moreover, p^{ii*} is increasing in τ^i and we know that $\tau_h^i > \tau_{bv}^i > \tau_l^i$, therefore

- If $\hat{q}^i < q^i$ then $\tau^i = \tau_h^i > \tau_{bv}^i$, therefore $p^{ii*} = p_h^{ii*} > p_{bv}^{ii*}$ and
- If $\hat{q}^i > q^i$ then $\tau^i = \{\tau_l^i, \tau_h^i\} < \tau_{bv}^i$, therefore $p^{ii*} \in \{p_l^{ii*}, p_h^{ii*}\} < p_{bv}^{ii*}$

□

A.5 Proof of Proposition 5

Substituting τ_h^i and τ_l^i in the expected utility function in (3) we get

$$\mathbb{E}[u^i(\hat{q}^i, \tau_h^i)] = V^{ij} + \Delta V^i \left(\hat{q}^i + (1 - \hat{q}^i)\hat{\alpha}^i \frac{\tau_{bv}^i + \xi^i}{2} \right) - \frac{1}{2} \left(\frac{\tau_{bv}^i + \xi^i}{2} \right)^2$$

$$\mathbb{E}[u^i(\hat{q}^i, \tau_l^i)] = V^{ij} + \Delta V^i \left(\hat{q}^i + (1 - \hat{q}^i)\hat{\alpha}^i \frac{\tau_{bv}^i - \xi^i}{2} \right) - \frac{1}{2} \left(\frac{\tau_{bv}^i - \xi^i}{2} \right)^2$$

. Considering their difference:

$$\mathbb{E}[u^i(\hat{q}^i, \tau_h^i)] - \mathbb{E}[u^i(\hat{q}^i, \tau_l^i)] = \hat{\alpha}^i \Delta V^i \xi^i \left(1 - \hat{q} - \frac{1}{2}(1 - q) \right)$$

This is always positive. Indeed $\frac{1}{2} - \hat{q}^i + \frac{1}{2} q^i \geq 0$ if $\hat{q}^i < \frac{1}{2}(1 + q^i)$ that is always satisfied, since $\frac{1}{2}(1 + q^i) \geq \bar{q}^i$ and $\hat{q}^i \leq \bar{q}^i$.

□

A.6 Proof of Proposition 6

The utility gained under complete information is

$$u^i_{bv}(q^i, \tau^i_{bv}) = V^{ij} + \Delta V^i \left(q^i + (1-q^i)\alpha^i \tau^i_{bv}\right) - \frac{1}{2}\left(\tau^i_{bv}\right)^2$$

Let us call

$$\Delta u^i_{\hat{q}^i} := u^i_{bv} - u^i_{\hat{q}^i}.$$

The following steps prove the proposition

$$\begin{aligned}
\Delta U^i_{\hat{q}^i} &= V^{ij} + \Delta V^i \left(q^i + (1-q^i)\alpha^i \tau^i_{bv}\right) - \frac{1}{2}\left(\tau^i_{bv}\right)^2 - V^{ij} - \Delta V^i \left(q^i + (1-q^i)\alpha^i \frac{\tau^i_{bv} + \xi^i}{2}\right) + \frac{1}{2}\left(\frac{\tau^i_{bv} + \xi^i}{2}\right)^2 \\
&= \Delta V^i (1-q^i)\alpha^i \tau^i_{bv} - \frac{1}{2}\left(\tau^i_{bv}\right)^2 - \Delta V^i (1-q^i)\alpha^i \frac{\tau^i_{bv} + \xi^i}{2} + \frac{1}{2}\left(\frac{\tau^i_{bv} + \xi^i}{2}\right)^2 \\
&= \tau^{2i}_{bv} - \frac{1}{2}\left(\tau^i_{bv}\right)^2 - \tau^i_{bv}\frac{\tau^i_{bv} + \xi^i}{2} + \frac{1}{2}\left(\frac{\tau^i_{bv} + \xi^i}{2}\right)^2 \\
&= \frac{1}{2}\left(-\tau^i_{bv}\xi^i + \left(\frac{\tau^i_{bv} + \xi^i}{2}\right)^2\right) \\
&= \frac{1}{2}\left(-\tau^i_{bv}\xi^i + \frac{\tau^{i2}_{bv} + \xi^{i2} + 2\tau^i_{bv}\xi^i}{4}\right) \\
&= \frac{1}{8}(\tau_{bv} - \xi^i)^2 > 0 \qquad \forall \alpha^i, \Delta V^i, q^i, \hat{q}^i
\end{aligned}$$

□

A.7 Proof of Proposition 7

Given the shape of socialization probability functions, for each $i \in I$, p^{ii}_t only depends on a^i_t and not on a^j_t. Then, the leader's utility is given by

$$q^i_{t+1} = p^{ii*}_t(a^i_t)q^i_t + (p^{jj*}_t(a^j_t))(1 - q^i_t)$$

Since $p^{ii*}_t(a^i_t)$ is the only element of q^i_{t+1} that depends on a^i_t and $\frac{q^i_{t+1}}{p^{ii*}_t(a^i_t)} > 0$ then

$$\max_{a^i_t}\ q^i_{t+1}(a^i_t, a^j_t) = \max_{a^i_t}\ p^{ii}_t(a^i_t)$$

Given assumption 1, the transition probability of each period is

$$p^{ii}_t = p^{ii}_{h,t} = q^i_t + \frac{\tau^i_{bv,t} + \xi^i_t(\hat{q}^i_{\beta,t})}{2}\alpha^i_t(1 - q^i_t)$$

Defining p^{ii}_- the transition probability associated with a negative biased $\hat{q}^i$ and p^{ii}_+ the transition probability associated with a positive biased $\hat{q}^i$ and since $\xi^i \lesseqqgtr \tau^i_{bv}$ if and only if $q^i \lesseqgtr \hat{q}^i$, then we can state that $p^{ii}_{-} > p^{ii}_{+}$. Therefore $a^i_t = n$ (and $a^j_t = n$) is a dominant strategy for leader ℓ^i (and ℓ^j).

$\square$

A.8 Proof of Proposition 8

To show that the dynamics (14) is well defined at each $t \in \mathbb{N}$, we need to show that, for each period $t \in \mathbb{N}$ and each $i \in I$, $\hat{q}^i_t < \bar{q}^i_t = q^i_t + \frac{\tau^{i2}_{bv,t}}{4\Delta V^i}$. Indeed if this condition is not satisfied there exists some $q^i_t \in [0,1]$ for which τ^i does not exists. We can express this condition with respect to β. Since by Proposition 7 biases induced by leaders are always negative, then this condition becomes

$$(1-\beta^i)q^i_t < q^i_t + \frac{\tau^{i2}_{bv,t}}{4\Delta V^i} \implies \beta^i > -\frac{\tau^{i2}_{bv,t}}{4\Delta V^i q^i_t}$$

Which is always satisfied. This proves the first part of the proposition.

We can rewrite (14) as

$$\dot{q}^i_t = \frac{1}{2} q^i_t(1-q^i_t)\phi(q^i_t, \hat{q}^i_{t,\beta}, \hat{q}^j_{t,\beta}) \tag{21}$$

In particualr $\phi(q^i_t, \hat{q}^i_{t,\beta}, \hat{q}^j_{t,\beta}) := (\alpha^i)^2(1-q^i_t)\Delta V^i + \alpha^i \xi^i_t(\hat{q}^i_{t,\beta}) - (\alpha^j)^2 q^i_t \Delta V^j - \alpha^j_t \xi^j_t(\hat{q}^j_{t,\beta})$, where

$$\begin{cases} \xi_i(\hat{q}^i_{t,\beta}) := \sqrt{4\beta^i q^i_t \Delta V^i + \alpha^{i2}(1-q^i_t)^2 \Delta V^{i2}} \\ \xi_j(\hat{q}^j_{t,\beta}) := \sqrt{4(\beta^j(1-q^i_t))\Delta V^j + \alpha^{j2} q^{i2}_t \Delta V^{j2}} \end{cases}$$

Solving $\phi(q^i_t, \hat{q}^i_{t,\beta}, \hat{q}^j_{t,\beta}) = 0$ it admits only one real solution, thus there exists al most one polymorphic steady state.

Let's now drop time indices for notation convenience. Notice that

$$\frac{\partial \dot{q}}{\partial q^i} = \frac{1}{2}\left(q^i(1-q^i)\frac{\partial \phi}{\partial q^i} + (1-2q^i)\phi\right),$$

therefore

$$\frac{\partial \dot{q}}{\partial q^i}|_{q^i=0} = \phi \;\text{ and }\; \frac{\partial \dot{q}}{\partial q^i}|_{q^i=1} = \frac{1}{2}\phi.$$

Thus, to study the stability of dynamics at the extreme points, we should look at the sign of $\frac{\partial \dot{q}}{\partial q^i}$. It is thus enough to look at ϕ, in fact

$$\text{if }\; \phi|_{q^i=0} > 0 \;\Rightarrow\; \frac{\partial \dot{q}}{\partial q^i}|_{q^i=0} > 0 \;\Rightarrow\; q^i = 0 \;\text{ is locally unstable}$$

$$\text{if } \phi|_{q^i=1} > 0 \quad \Rightarrow \quad \frac{\partial \dot{q}}{\partial q^i}|_{q^i=1} > 0 \quad \Rightarrow \quad q^i = 1 \text{ is locally unstable}$$

Thus

$$\begin{aligned}\phi(q^i, \hat{q}^i_\beta, \hat{q}^j_\beta)|_{q^i=0} &= \alpha^{i2}\Delta V^i + \alpha^i \xi^i(\hat{q}^i_\beta)|_{q^i=0} - \alpha^j \xi^j(\hat{q}^j_\beta)|_{q^i=0} \\ &= 2\alpha^{i2}\Delta V^i - 2\alpha^j\sqrt{\beta^j \Delta V^j}\end{aligned}$$

Then $\phi(q^i, \hat{q}^i_\beta, \hat{q}^j_\beta)|_{q^i=0} \geq 0$ is satisfied if and only if

$$\beta^j \leq \frac{\alpha^{i4}}{\alpha^{j2}} \frac{\Delta V^{i2}}{\Delta V^j}.$$

Similarly, $\phi(q^i, \hat{q}^i_\beta, \hat{q}^j_\beta)|_{q^i=1} \geq 0$ if and only if

$$\beta^i \leq \frac{\alpha^{j4}}{\alpha^{i2}} \frac{\Delta V^{j2}}{\Delta V^i}$$

□

B Discontinuity of $\dot{q}$

B.1 The case for time invariant conjectures

Consider the case in which, for each $i \in I$, conjectures $\hat{q}^i_t$ are time invariant.[26] After a little algebra, substituting equilibrium quantities into (11), we get

$$\dot{q}^i_t = \frac{1}{2} q^i_t (1 - q^i_t) \phi(q^i_t, \hat{q}^i, \hat{q}^j) \tag{22}$$

where $\phi(q^i_t, \hat{q}^i, \hat{q}^j) := (\alpha^i)^2(1 - q^i_t)\Delta V^i + \alpha^i \xi^i_t - (\alpha^j)^2 q^i_t \Delta V^j - \alpha^j_t \xi^j_t$. Recall that, even if we consider $\hat{q}^i_t$ as fixed, the selfconfirming equilibrium efforts and conjectures are defined as long as, for each $i \in I$, and for each time $t \in \mathbb{N}$, $\hat{q}^i < \bar{q}^i_t$. Indeed, the threshold $\bar{q}^i_t$ depends on q_t and thus varies with time. It is then possibile that, for a given conjecture $\hat{q}^i_t$ the population dynamics evolve such that at some point the existence condition is not satisfied anymore. Recall that $\bar{q}^i_t := q^i_t + \frac{\alpha^{i^2}\Delta V^i}{4}$. Then a sufficient condition for the equilibrium to exists for any possible population dynamics is that, for each $i \in I$, $\hat{q}^i_t < \frac{\alpha^{i^2}\Delta V^i}{4} \leq 0.25$. Of course, even though this is a sufficient condition, it is quite restrictive since, for example, a group with q^i_t close to 1 must think to be less that a fourth of the overall population. For this reason, in the paper we have endogenized the conjectures. Next proposition, however, uses this sufficient condition to characterize the

[26]Notice that albeit $\hat{q}^i_t$ are time invariant $\hat{\alpha}^i_t$ are not . This because they stem from (17), which depends on the true parameter q^i_t at the specific period t.

dynamics for the cases in which conjectures do not change, thus ensuring that the dynamic is well defined at every $q_t^i \in [0,1]$.

Proposition 9 *For each $i \in I$, let $\hat{q}^i \leq \frac{{\alpha^i}^2 \Delta V^i}{4}$. $\{0,1\} \subseteq Q_{ss}^i$ and $q^i = 1$ or $q^i = 0$, or both, are stable. Moreover, if there exists a polymorphic steady state $q^* \in (0,1)$, $Q_{ss} = \{0,1,q^*\}$, $q^i = 0$ and $q^i = 1$ are stable and $q^i = q^{*i}$ is unstable.*

Proof. in the Appendix B.2 □.

Figure 7 helps explaining the results of Proposition 9. If conjectures are fixed and satisfy the sufficient condition for existence of confirmed conjectures at each point of the dynamics, there are three possibile cases. The first two cases, represented in panels (a) and (b)

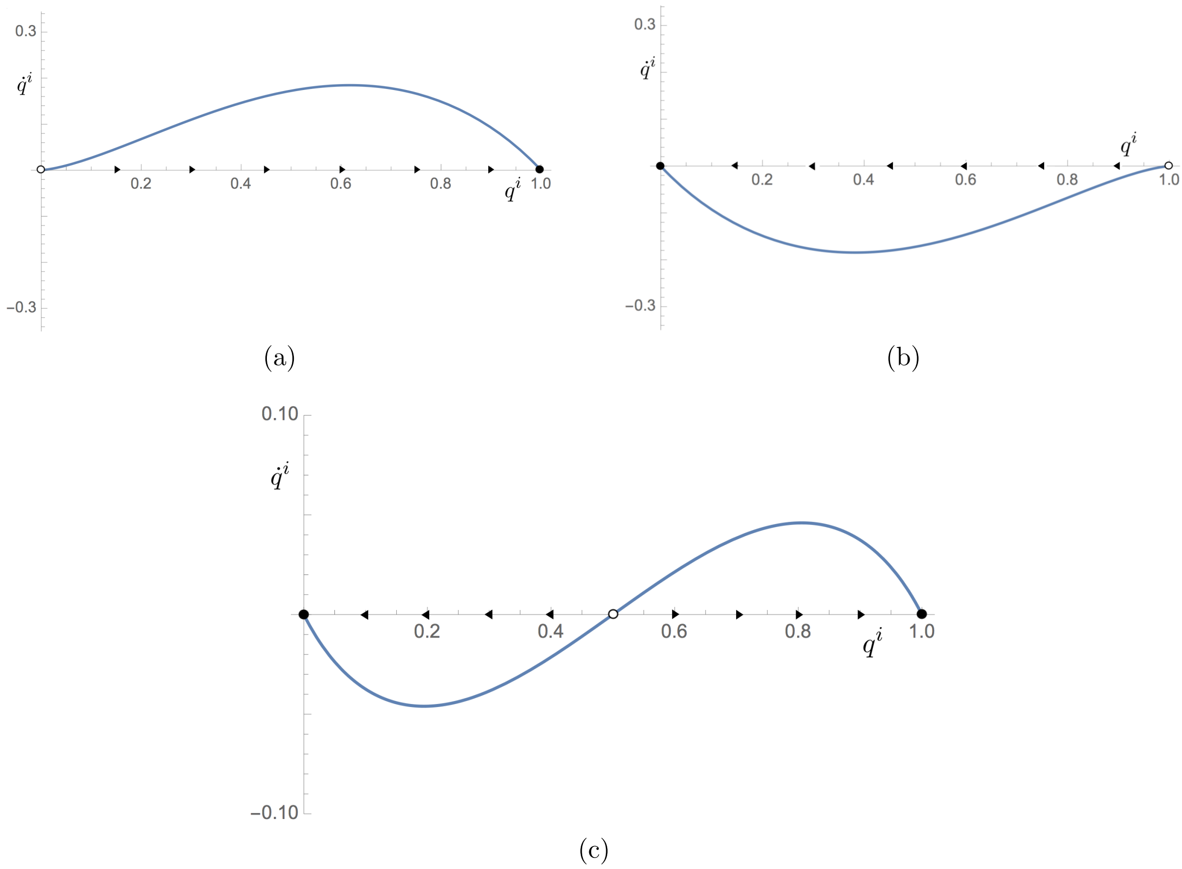


Figure 7: Cultural dynamics $\dot{q}^i$ with $\alpha^i = \alpha^j = 1$: (a) $\hat{q}^i = 0.1$, $\hat{q}^j = 0.025$, $\Delta V^i = 0.5$, $\Delta V^j = 0.1$ (b) $\hat{q}^i = 0.025$ $\hat{q}^j = 0.1$, $\Delta V^i = 0.1$, $\Delta V^j = 0.5$ and (c) $\hat{q}^i = 0.1$ $\hat{q}^j = 0.1$, $\Delta V^i = 0.5$, $\Delta V^j = 0.5$.

describe the situation in which there exists only one homomorphic stable steady state. These two cases occur when the two groups have quite unbalanced conjectures. Consider for example panel (a). In this case group i has a conjecture about q^i that is four times the conjectures j have about q^j. Even if, through the feedback, parents update differently the conjectures about α^i and α^j, the large imbalance of the conjectures about population shares make i agents produce a much lower effort than j agents, at every point t in the dynamics. Then j agents always increase in size and, at the end, win in the cultural dynamics. The same happens, with inverted roles, in panel j. Consider now panel (c) representing the case of balanced conjectures. In this case the difference in the exerted efforts is then given by the difference in the confirmed $\hat{\alpha}^i$ and $\hat{\alpha}^j$. Now, as we have seen in Figure 3, fixing $\hat{q}^i$, socialization effort increases with population shares. Indeed, one of the main results on our model is that it produces cultural complementarity with respect to actual population shares. In the case of panel (c), given that groups have same conjecture about own share, then the group with the larger actual population share produces the highest effort, and this determines the shape of the dynamics with one unstable polymorphic equilibrium.

If the sufficient condition for the existence of confirmed conjectures at any q_t^i is not satisfied, then we may have points in the dynamics in which the equilibrium does not exists and, thus, the dynamics cannot be determined. This is shown in Figure 7 in which we show that, if conjectures of group i do not satisfy the condition, then the dynamics for small q^i cannot be determined. Notice that the condition implies that conjectures must be quite small. If this is not the case, then when q^i gets close to 0, conjectures are very far away from the real value, and then no $\hat{\alpha}^i$ can counterbalance the bias. The fact that the dynamics cannot be computed simply underlines that in the long run conjectures must somehow be related, at least to a certain degree, to actual shares.

B.2 Proof of Proposition 9

It is trivial to see that $q^i = 0$ and $q^i = 1$ are steady states of $\dot{q}$.

In order to verify that it may exist only another possible steady state we prove that the function ϕ has at most one solution. Moreover we have to show that if a solution exists the steady state associated with that solution is unstable. Therefore, we prove that the function is monotone (uniqueness of solution) and increasing (instability of steady state, with cultural heterogeneity). Thus, we study the first derivative of ϕ with respsct q^i:

$$\frac{\partial \phi}{\partial q^i} = -\alpha^{i2}\Delta V^i - \alpha^{j2}\Delta V^j + \alpha^i \frac{\partial \xi^i}{\partial q^i} - \alpha^j \frac{\partial \xi^j}{\partial q^i}$$

where

$$\xi_i := \sqrt{4(q^i - \hat{q}^i)\Delta V^i + \alpha^{i2}(1-q^i)^2 \Delta V^{i2}}$$

$$\xi_j := \sqrt{4(1 - q^i - \hat{q}^j)\Delta V^j + \alpha^{j2} q^{i2} \Delta V^{j2}}$$

and

$$\frac{\partial \xi^i}{\partial q^i} = \frac{\Delta V^i (2 - \alpha^i \tau^i_{bv})}{\xi^i}$$

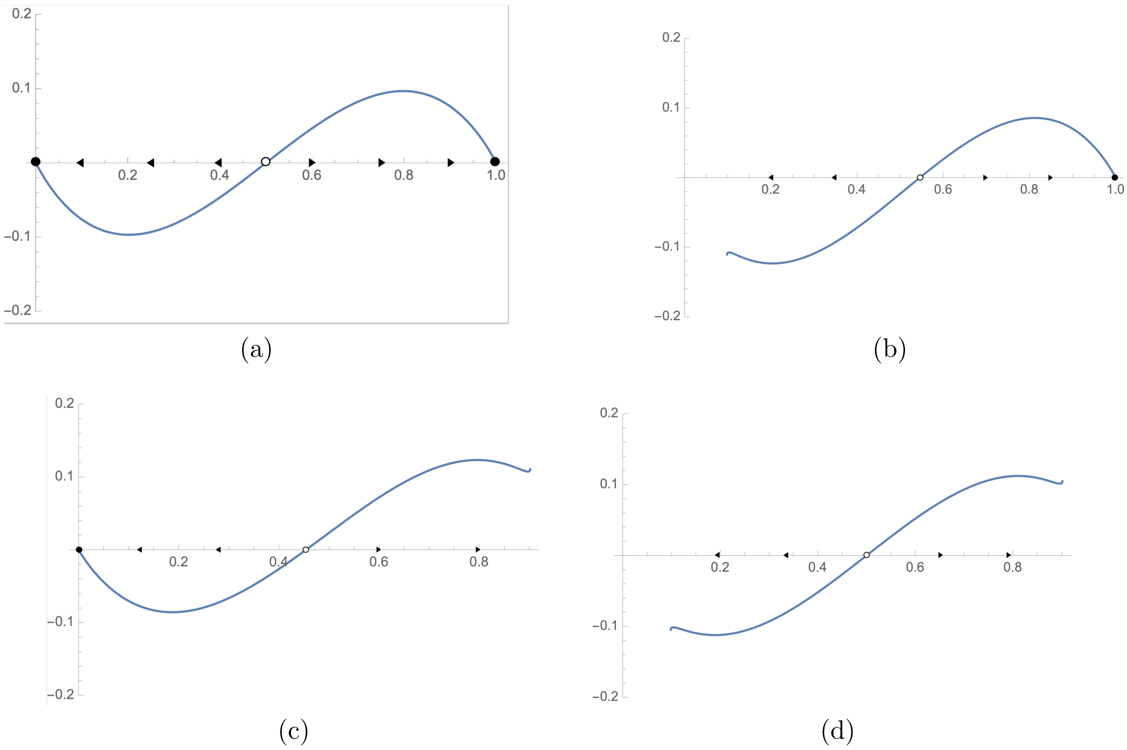


Figure 8: $\hat{q}^i = 0.12 < \bar{q}^i$ and $\hat{q}^j = 0.12 < \bar{q}^j$ (a) $\hat{q}^i = \hat{q}^j = 0.12$, (b) $\hat{q}^i = 0.12$ and $\hat{q}^j = 0.2$, (c) $\hat{q}^i = 0.2$ and $\hat{q}^j = 0.12$ and (d) $\hat{q}^i = \hat{q}^j = 0.2$

$$\frac{\partial \xi^j}{\partial q^i} = \frac{\Delta V^j(\alpha^j \tau^j_{bv} - 2)}{\xi^j}$$

$$\Rightarrow \frac{\partial \phi}{\partial q^i} = -\alpha^{i2}\Delta V^i - \alpha^{j2}\Delta V^j + \alpha^i \frac{\Delta V^i(2 - \alpha^i \tau^i_{bv})}{\xi^i} + \alpha^j \frac{\Delta V^j(2 - \alpha^j \tau^j_{bv})}{\xi^j}.$$

This quantity is decreasing both in $\Delta V^i, \Delta V^j, \alpha^i, \alpha^j$ and increasing in $\hat{q}^i$, $\hat{q}^j$ thus, to see if ϕ is increasing we can study the case $\Delta V^i = \Delta V^j = \alpha^i = \alpha^j = 1$ and $\hat{q}^i = \hat{q}^j = 0$

$$\begin{aligned}
\frac{\partial \phi}{\partial q^i} &= -2 + \frac{2 - (1 - q^i)}{\sqrt{4q^i + (1 - q^i)^2}} + \frac{(2 - q^i)}{\sqrt{4(1 - q^i) + q^{i2}}} \geq 0 \\
&= -2 + \frac{2 - (1 - q^i)}{\sqrt{2q^i + 1 + q^{i2}}} + \frac{(2 - q^i)}{\sqrt{4 - 4q^i + q^{i2}}} \geq 0 \\
&= -2 + \frac{2 - (1 - q^i)}{1 + q^i} + \frac{(2 - q^i)}{2 - q^i} \geq 0 \\
&= -2 + \frac{1 + q^i}{1 + q^i} + \frac{(2 - q^i)}{2 - q^i} = 0
\end{aligned}$$

Since the first derivative computed at his minimum is equal to zero then $\frac{\partial \phi}{\partial q^i} \geq 0$ for all parameters and ϕ is always increasing, therefore ϕ has no more than 1 solution.

□

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